

**MATHEMATICS – Code No. 041**  
**MARKING SCHEME**  
**CLASS – XII (2025-26)**

SECTION-A (MCQs of 1 mark each)		
Sol. N.	Hint / Solution	Marks
1	Clearly from the graph Domain is $\left[-\frac{1}{2}, \frac{1}{2}\right]$ So graph is of the function $\sin^{-1}(2x)$ <b>Answer is (B) <math>\sin^{-1}(2x)</math></b>	1
1 (V.I.)	Domain is $\left[-\frac{1}{3}, \frac{1}{3}\right]$ So the function is $\cos^{-1}(3x)$ <b>Answer is (C) <math>\cos^{-1}(3x)</math></b>	1
2	AB is defined so $n=4$ AC is defined so $p=4$ AB and AC are square matrices of same order so $m \times 3 = m \times q \Rightarrow q = 3 = m$ <b>Answer is (A) <math>m = q = 3</math> and <math>n = p = 4</math></b>	1
3	As A is skew symmetric So $p = 0, q = 2, r = -3, t = 4$ So $\frac{q+t}{p+r} = \frac{6}{-3} = -2$ <b>Answer is (A) -2</b>	1
4	$ adj A  = 27 \Rightarrow  A ^3 = 27 = 3^3 \Rightarrow  A  = 3$ $A (adj A) =  A  I = 3 I$ <b>Answer is (C) <math>3 I</math></b>	1
5	Inverse of the matrix $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ <b>Answer is (B)</b>	1
6	$\begin{vmatrix} \cos 67^\circ & \sin 67^\circ \\ \sin 23^\circ & \cos 23^\circ \end{vmatrix} = \cos 67^\circ \cos 23^\circ - \sin 67^\circ \sin 23^\circ = \cos(67^\circ + 23^\circ) = \cos 90^\circ = 0$ <b>Answer is (A) 0</b>	1
7	$f(x)$ is continuous at $x = \pi$ $\Rightarrow \lim_{x \rightarrow \pi^-} (kx + 1) = \lim_{x \rightarrow \pi^+} \cos x = f(\pi)$ $\Rightarrow \lim_{h \rightarrow 0^+} [k(\pi - h) + 1] = \lim_{h \rightarrow 0} \cos(\pi + h) = k\pi + 1$ $\Rightarrow k\pi + 1 = -1 \quad \Rightarrow k = \frac{-2}{\pi}$ <b>Answer is (D) <math>\frac{-2}{\pi}</math></b>	1

8	$f(x) = x \tan^{-1} x$ $f'(x) = 1 \cdot \tan^{-1} x + x \cdot \frac{1}{1+x^2}$ $f'(1) = 1 \cdot \tan^{-1} 1 + \frac{1}{1+1} = \frac{\pi}{4} + \frac{1}{2}$ <b>Answer is (B) <math>\frac{\pi}{4} + \frac{1}{2}</math></b>	1
9	$f(x) = 10 - x - 2x^2$ $\Rightarrow f'(x) = -1 - 4x$ For increasing function $f'(x) \geq 0$ $\Rightarrow -(1 + 4x) \geq 0$ $\Rightarrow (1 + 4x) \leq 0$ $\Rightarrow x \leq -\frac{1}{4}$ $\Rightarrow x \in \left(-\infty, -\frac{1}{4}\right]$ <b>Answer is (A) <math>\left(-\infty, -\frac{1}{4}\right]</math></b>	1
10	$xdx + ydy = 0$ $\Rightarrow \int xdx = -\int ydy$ $\Rightarrow \frac{x^2}{2} = -\frac{y^2}{2} + k$ $\Rightarrow x^2 + y^2 = 2k$ Solution is $x^2 + y^2 = 2k$ , $k$ being an arbitrary constant. <b>Answer is (C) Circles</b>	1
11	$I = \int_a^b x f(x) dx = \int_a^b (a + b - x) f(a + b - x) dx$ $\Rightarrow I = \int_a^b (a + b - x) f(x) dx$ (given $f(a + b - x) = f(x)$ ) $\Rightarrow I = \int_a^b (a + b) f(x) dx - \int_a^b x f(x) dx$ $\Rightarrow 2I = (a + b) \int_a^b f(x) dx$ $\Rightarrow I = \frac{1}{2} (a + b) \int_a^b f(x) dx$ <b>Answer is (D) <math>\frac{a+b}{2} \int_a^b f(x) dx</math></b>	1
12	Let $I = \int x^3 \sin^4(x^4) \cos(x^4) dx$ Let $\sin(x^4) = t \Rightarrow 4x^3 \cos(x^4) dx = dt \Rightarrow x^3 \cos(x^4) = \frac{1}{4} dt$ Thus $I = \int t^4 \left(\frac{1}{4} dt\right) = \frac{1}{20} t^5 + C = \frac{1}{20} \sin^5(x^4) + C$ $\Rightarrow I = \frac{1}{20} \sin^5(x^4) + C = a \sin^5(x^4) + C$ So, $a = \frac{1}{20}$ <b>Answer is (B) <math>\frac{1}{20}</math></b>	1
13	The projection of the vector $\hat{i} + 2\hat{j} + \hat{k}$ on the line $\vec{r} = (3\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ is $\frac{1 \times 1 + 2 \times 2 + 1 \times 3}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{8}{\sqrt{14}}$ units <b>Answer is (C) <math>\frac{8}{\sqrt{14}}</math> units</b>	1

14	<p>The distance of the point (a, b, c) from the y-axis is <math>\sqrt{a^2 + c^2}</math></p> <p>So, the distance is <math>\sqrt{3^2 + 5^2} = \sqrt{34}</math> units.</p> <p><b>Answer is (B) <math>\sqrt{34}</math> units</b></p>	1
15	$(2\vec{a} \cdot \hat{i})\hat{i} - (\vec{b} \cdot \hat{j})\hat{j} + (\vec{c} \cdot \hat{k})\hat{k} = (2 \times 3)\hat{i} - (1)\hat{j} + (2)\hat{k}$ $= 6\hat{i} - \hat{j} + 2\hat{k} = \vec{c}$ <p><b>Answer is (D) <math>\vec{c}</math></b></p>	1
16	<p>The points (1,0) and (0,2) satisfy the equation <math>2x + y = 2</math></p> <p>And shaded region shows that (0,0) doesn't lie in the feasible solution region</p> <p>So, the inequality is <math>2x + y \geq 2</math></p> <p><b>Answer is (B) <math>2x + y \geq 2</math></b></p>	1
16 (V.I.)	<p>(4,0) and (0,3) gives maximum value so</p> $Z_{(4,0)} = Z_{(0,3)} \Rightarrow 4a + c = 3b + c \Rightarrow 4a = 3b$ <p><b>Answer is (A) <math>4a = 3b</math></b></p>	1
17	<p>The student may read the point (2,9) from the line on the graph.</p> <p>The student may find the equation <math>3x + y = 15</math> joining (5,0) and (0,15) and then verify the point (2,9) satisfies it.</p> <p><b>Answer is (A) (2,9)</b></p>	1
17 (V.I.)	<p><b>Answer is (C)</b> Open Half plane that contains origin, but not the points of the line <math>3x + 5y = 10</math></p>	1
18	<p><b>Answer is (B) <math>\frac{1}{100}</math></b></p> <p>The person knows the first 4 digits. So the person has to guess the remaining two digits.</p> $P(\text{guessing the PIN}) = 1 \times 1 \times 1 \times 1 \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$	1
19	$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \tan^{-1} 1 - \sec^{-1}(\sqrt{2}) = \frac{\pi}{3} + \frac{\pi}{4} - \frac{\pi}{4} = \frac{\pi}{3} \neq \frac{\pi}{4}$ <p>So, A is false.</p> <p>Principal Value branch of <math>\sin^{-1} x</math> is <math>\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]</math> and that of <math>\sec^{-1} x</math> is <math>[0, \pi] - \left\{\frac{\pi}{2}\right\}</math>.</p> <p>So, R is true</p> <p><b>Answer is (D) Assertion is false, but Reason is true</b></p>	1
20	<p><math>C. \vec{r} \times (\vec{a} + \vec{b}) = \vec{0} \Rightarrow \vec{r}</math> is parallel to <math>(\vec{a} + \vec{b})</math> and <math>(\vec{a} + \vec{b})</math> lies on the plane of <math>\vec{a}</math> and <math>\vec{b}</math>.</p> <p>So, <math>\vec{r}</math> is parallel to the plane of <math>\vec{a}</math> and <math>\vec{b} \Rightarrow \vec{r}</math> is perpendicular to <math>(\vec{a} \times \vec{b})</math>.</p> <p>So, Assertion is true</p> <p>But <math>(\vec{a} + \vec{b})</math> lies on the plane of <math>\vec{a}</math> and <math>\vec{b}</math>, so <math>(\vec{a} + \vec{b})</math> is not perpendicular to the plane of <math>\vec{a}</math> and <math>\vec{b}</math></p> <p>Therefore, Reason is false.</p> <p><b>Answer is (C) Assertion is true, but Reason is false</b></p>	1

**SECTION B**  
**(VSA type questions of 2 marks each)**

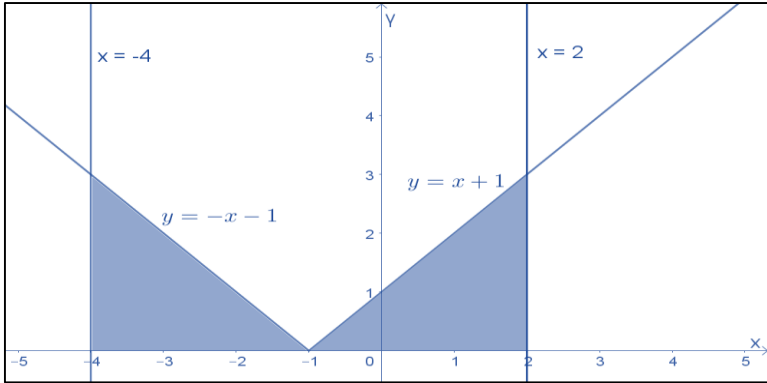
<p>21A</p>	$\tan\left(\tan^{-1}(-1) + \frac{\pi}{3}\right) = \tan\left(-\frac{\pi}{4} + \frac{\pi}{3}\right)$ $= \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$ $= \frac{\sqrt{3}-1}{1+\sqrt{3}} \text{ or } 2 - \sqrt{3}$ <p style="text-align: center;"><b>OR</b></p>	<p>1/2</p> <p>1</p> <p>1/2</p> <p>OR</p>
<p>21B</p>	<p>For domain, <math>-1 \leq 3x - 2 \leq 1</math>  <math>\Rightarrow 1 \leq 3x \leq 3</math>  <math>\Rightarrow \frac{1}{3} \leq x \leq 1</math></p> <p>So, domain of <math>\cos^{-1}(3x - 2)</math> is <math>\left[\frac{1}{3}, 1\right]</math></p>	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>
<p>22</p>	$y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ <p>Differentiating with respect to <math>x</math></p> $\frac{dy}{dx} = \frac{1}{\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)} \cdot \sec^2\left(\frac{\pi}{4} + \frac{x}{2}\right) \cdot \frac{1}{2}$ $= \frac{\cos\left(\frac{\pi}{4} + \frac{x}{2}\right)}{\sin\left(\frac{\pi}{4} + \frac{x}{2}\right)} \cdot \frac{1}{\cos^2\left(\frac{\pi}{4} + \frac{x}{2}\right)} \cdot \frac{1}{2}$ $= \frac{1}{2 \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) \cos\left(\frac{\pi}{4} + \frac{x}{2}\right)} = \frac{1}{\sin\left(\frac{\pi}{2} + x\right)} = \frac{1}{\cos x}$ $\Rightarrow \frac{dy}{dx} - \sec x = 0$	<p>1/2</p> <p>1</p> <p>1/2</p>
<p>23A</p>	$\int \frac{(x-3)e^x}{(x-1)^3} dx = \int \frac{(x-1-2)e^x}{(x-1)^3} dx$ $= \int \left( \frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right) e^x dx = \int \left( \frac{1}{(x-1)^2} + \frac{d}{dx} \left( \frac{1}{(x-1)^2} \right) \right) e^x dx$ $= \frac{e^x}{(x-1)^2} + c \quad (\text{as } \int (f(x) + f'(x))e^x dx = e^x f(x) + c)$ <p style="text-align: center;"><b>OR</b></p>	<p>1</p> <p>1</p> <p>OR</p>
<p>23B</p>	$A = \int_0^4 x dy = \int_0^4 \sqrt{y} dy$ $= \frac{2}{3} \times y^{3/2} \Big _{y=0}^{y=4} = \frac{16}{3} \text{ sq. units}$	<p>1</p> <p>1</p>
<p>23B</p>	<p><b>For Visually Impaired:</b></p> $A = \int_0^3 y dx = \int_0^3 \sqrt{x} dx$ $= \frac{2}{3} \times x^{3/2} \Big _{x=0}^{x=3} = 2\sqrt{3} \text{ sq. units}$	<p>1</p> <p>1</p>

24	Given $f(x+y) = f(x)f(y)$	
	$f(0+5) = f(0)f(5)$	
	$\Rightarrow f(0) = 1$	
	$f'(5) = \lim_{h \rightarrow 0} \frac{f(5+h)-f(5)}{h} = \lim_{h \rightarrow 0} \frac{f(5)f(h)-f(5)}{h}$ [ $\because f(x+y) = f(x)f(y)$ ]	
	$= \lim_{h \rightarrow 0} \frac{2f(h)-2}{h}$ [ $\because f(5) = 2$ ]	1
	$= 2 \lim_{h \rightarrow 0} \frac{f(h)-1}{h} = 2 \lim_{h \rightarrow 0} \frac{f(h)-f(0)}{h} = 2 f'(0)$	
	$= 2(3)$ [ $\because f'(0) = 3$ ]	$\frac{1}{2}$
	$= 6$	

25	The vector $\overrightarrow{OP} = \frac{1}{2}(4\hat{i} + 4\hat{k}) = 2\hat{i} + 2\hat{k}$	$\frac{1}{2}$
	Area of the parallelogram formed by the two adjacent sides as OA and OP	
	$=  (\overrightarrow{OA} \times \overrightarrow{OP})  = \left  \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 0 & 2 \end{vmatrix} \right $	$\frac{1}{2}$
	$=  2\hat{i} - 2\hat{k} $	$\frac{1}{2}$
	$= 2\sqrt{2}$ square units.	$\frac{1}{2}$

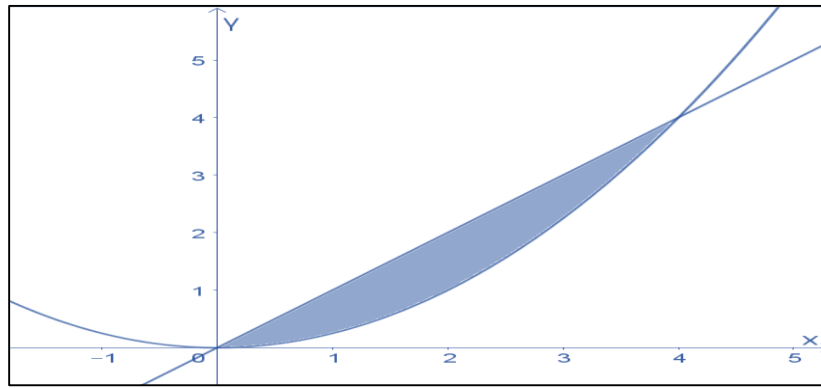
**SECTION C**  
(SA type questions of 3 marks each)

26A	$x^y = e^{x-y}$	
	Taking log of both sides	
	$y \log x = (x - y) \log e$	
	$y \log x + y = x$ ( since $\log e = 1$ )	
	$\Rightarrow y = \frac{x}{1+\log x}$	1
	Differentiating with respect to $x$	
	$\frac{dy}{dx} = \frac{(1+\log x) \cdot 1 - x \cdot \frac{1}{x}}{(1+\log x)^2}$	
	$= \frac{\log x}{(\log e + \log x)^2}$	
	$= \frac{\log x}{(\log(xe))^2}$	1
	Now $\left. \frac{dy}{dx} \right _{x=e} = \frac{\log e}{(\log e^2)^2} = \frac{1}{(2\log e)^2} = \frac{1}{2^2} = \frac{1}{4}$ (as $\log e = 1$ )	1
	<b>Alternative Solution:</b>	
	$x^y = e^{x-y}$	
	Taking log of both sides	
	$y \log x = (x - y) \log e$	
	$y \log x + y = x$ ( since $\log e = 1$ )	
	Differentiating both sides w.r.t. $x$	
	$\log x \frac{dy}{dx} + \frac{y}{x} + \frac{dy}{dx} = 1$	
	$\Rightarrow \frac{dy}{dx} (1 + \log x) = 1 - \frac{y}{x}$	
	$\Rightarrow \frac{dy}{dx} = \frac{x-y}{x(1+\log x)} = \frac{x - \frac{x}{1+\log x}}{x(1+\log x)} = \frac{x(1+\log x) - x}{x(1+\log x)^2} = \frac{x(1+\log x - 1)}{x(\log e + \log x)^2} = \frac{\log x}{(\log(xe))^2}$	
	Now $\left. \frac{dy}{dx} \right _{x=e} = \frac{\log e}{(\log e^2)^2} = \frac{1}{(2\log e)^2} = \frac{1}{2^2} = \frac{1}{4}$ (as $\log e = 1$ )	

	<b>OR</b>	<b>OR</b>
26B	$\frac{dx}{d\theta} = a(1 - \cos \theta), \frac{dy}{d\theta} = a(0 + \sin \theta),$ $\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a \sin \theta}{a(1 - \cos \theta)}$ $= \frac{2 \sin(\frac{\theta}{2}) \cos(\frac{\theta}{2})}{2 \sin^2(\frac{\theta}{2})} = \cot \frac{\theta}{2}$ $\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{2} \operatorname{cosec}^2\left(\frac{\theta}{2}\right) \frac{d\theta}{dx}$ $= -\frac{1}{2a} \operatorname{cosec}^2\left(\frac{\theta}{2}\right) \frac{1}{2 \sin^2(\frac{\theta}{2})}$ $= -\frac{1}{4a} \operatorname{cosec}^4\left(\frac{\theta}{2}\right)$	<p>1</p> <p>1</p> <p>1</p>
27	<p>Let <math>r</math> be the radius of ice ball at time <math>t</math>.</p> $V = \frac{4}{3} \pi r^3 \dots\dots\dots (1)$ $S = 4\pi r^2 \dots\dots\dots (2)$ <p>Given <math>\frac{dV}{dt} \propto S</math></p> $\Rightarrow \frac{dV}{dt} = -k S \text{ (where } k \text{ is some positive constant) } \dots\dots\dots (3)$ <p>Differentiating (1) w.r.t. <math>t</math>, we get</p> $\frac{dV}{dt} = \frac{4}{3} \pi \cdot (3 r^2) \frac{dr}{dt}$ $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \dots\dots\dots (4)$ $\Rightarrow -k S = 4\pi r^2 \frac{dr}{dt} \text{ (from (3) and (4))}$ $\Rightarrow -k S = S \frac{dr}{dt} \text{ (using (2))}$ $\Rightarrow \frac{dr}{dt} = -k$ <p><math>\Rightarrow</math> radius of the ice-ball decreases at a constant rate</p>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>
28A	 $\int_{-4}^2  x + 1  dx = \int_{-4}^{-1} (-x - 1) dx + \int_{-1}^2 (x + 1) dx$ $= -\left[\frac{(x+1)^2}{2}\right]_{-4}^{-1} + \left[\frac{(x+1)^2}{2}\right]_{-1}^2$ $= -\left(0 - \frac{9}{2}\right) + \left(\frac{9}{2} - 0\right) = 9$ <p>It represent the area of shaded region bounded by the curve <math>y =  x + 1 </math>, <math>x -</math> axis and the lines <math>x = -4</math> and <math>x = 2</math></p>	<p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>

OR

28B



$$\begin{aligned} \text{Required Area} &= \int_0^4 x \, dx - \int_0^4 \frac{x^2}{4} \, dx \\ &= \left. \frac{x^2}{2} \right|_0^4 - \frac{1}{12} [x^3]_0^4 \\ &= \frac{1}{2} (16 - 0) - \frac{1}{12} (64 - 0) = 8 - \frac{16}{3} = \frac{8}{3} \text{ sq. units} \end{aligned}$$

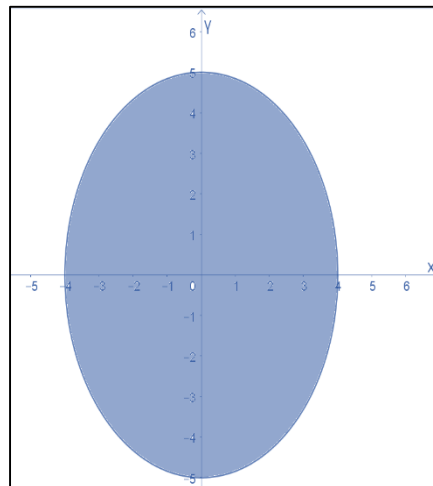
**For Visually Impaired:**

$$y = |x + 1| = f(x) = \begin{cases} -x - 1, & x < -1 \\ x + 1, & x \geq -1 \end{cases}$$

$$\begin{aligned} \int_{-4}^2 |x + 1| \, dx &= \int_{-4}^{-1} (-x - 1) \, dx + \int_{-1}^2 (x + 1) \, dx \\ &= -\left. \frac{(x+1)^2}{2} \right|_{-4}^{-1} + \left. \frac{(x+1)^2}{2} \right|_{-1}^2 \\ &= -\left(0 - \frac{9}{2}\right) + \left(\frac{9}{2} - 0\right) = 9 \end{aligned}$$

It represent the area of shaded region bounded by the curve  $y = |x + 1|$ ,  
 $x$  - axis and the lines  $x = -4$  and  $x = 2$

OR



	$25x^2 + 16y^2 = 400 \Rightarrow \frac{x^2}{16} + \frac{y^2}{25} = 1 \Rightarrow \frac{x^2}{4^2} + \frac{y^2}{5^2} = 1 \Rightarrow y = \frac{5}{4}\sqrt{4^2 - x^2}$ <p>Required Area = <math>4 \int_0^4 \frac{5}{4}\sqrt{4^2 - x^2} dx</math></p> $= 5 \left[ \frac{x\sqrt{4^2 - x^2}}{2} + \frac{4^2}{2} \sin^{-1} \left( \frac{x}{4} \right) \right]_0^4$ $= 5[0 + 8 \sin^{-1}(1) - 0]$ $= 40 \times \frac{\pi}{2} = 20\pi \text{ sq. units}$	1 1 1
29A	<p>The line through <math>(2, -1, 3)</math> parallel to the z-axis is given by</p> $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{k})$ <p>Any point on this line is <math>P(2, -1, 3 + \lambda)</math></p> <p>Any point on the given line <math>\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \mu(3\hat{i} + 6\hat{j} + 2\hat{k})</math> is</p> $Q(2 + 3\mu, -1 + 6\mu, 2 + 2\mu)$ <p>For the intersection point</p> $Q(2 + 3\mu, -1 + 6\mu, 2 + 2\mu) = P(2, -1, 3 + \lambda) \Rightarrow 2 = 2 + 3\mu \Rightarrow \mu = 0$ <p>The point of intersection is <math>(2, -1, 2)</math></p> <p>The distance from <math>(2, -1, 3)</math> to <math>(2, -1, 2)</math> is clearly 1 unit.</p> <p><b>Alternative Solution:</b></p> <p>Any point on the line through <math>(2, -1, 3)</math> parallel to the z-axis is <math>(2, -1, \lambda)</math></p> <p>Any point on the given line is <math>(2 + 3\mu, -1 + 6\mu, 2 + 2\mu)</math></p> <p>Therefore, <math>2 = 2 + 3\mu \Rightarrow \mu = 0</math></p> <p>The point of intersection is <math>(2, -1, 2)</math></p> <p>The distance from <math>(2, -1, 3)</math> to <math>(2, -1, 2)</math> is clearly 1 unit.</p>	1 1/2 1/2 1/2 1/2 1 1 1/2 1/2
29B	<p style="text-align: center;"><b>OR</b></p> <p>The line through <math>(2, -1, 1)</math> parallel to the z-axis is <math>\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(\hat{k})</math></p> <p>Any point on this line is <math>P(2, -1, 1 + \lambda)</math></p> <p>Any point on the given line is <math>A(3 + \mu, \mu, 1 + \mu)</math></p> $A(3 + \mu, \mu, 1 + \mu) = P(2, -1, 1 + \lambda) \Rightarrow \mu = -1$ <p>The point of intersection is <math>(2, -1, 0)</math></p> <p>The distance of <math>(2, -1, 0)</math> from the z-axis is <math>\sqrt{2^2 + (-1)^2} = \sqrt{5}</math> units.</p>	1 1 1/2 1/2
30	<p>Sketching the graph</p>	1 1/2



33A	<p>Let <math>x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta</math></p> $I = \int_0^{\frac{\pi}{4}} \frac{\log(1+\tan\theta)}{1+\tan^2\theta} \cdot \sec^2\theta d\theta$ $I = \int_0^{\frac{\pi}{4}} \log(1+\tan\theta) d\theta = \int_0^{\frac{\pi}{4}} \log\left[1 + \tan\left(\frac{\pi}{4} - \theta\right)\right] d\theta$ $= \int_0^{\frac{\pi}{4}} \log\left[1 + \frac{1-\tan\theta}{1+\tan\theta}\right] d\theta$ $= \int_0^{\frac{\pi}{4}} \log\left[\frac{1+\tan\theta+1-\tan\theta}{1+\tan\theta}\right] d\theta$ $= \int_0^{\frac{\pi}{4}} \log\left[\frac{2}{1+\tan\theta}\right] d\theta$ $= \int_0^{\frac{\pi}{4}} \log 2 d\theta - \int_0^{\frac{\pi}{4}} \log[1+\tan\theta] d\theta$ $= \log 2 \times x \Big _0^{\frac{\pi}{4}} - I$ $\Rightarrow 2I = \frac{\pi}{4} \log 2$ $\Rightarrow I = \frac{\pi}{8} \log 2$	<p>1/2</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1/2</p>
33B	<p style="text-align: center;"><b>OR</b></p> $I = \int \frac{(3 \sin \theta - 2) \cos \theta}{5 - \cos^2 \theta - 4 \sin \theta} d\theta = \int \frac{(3 \sin \theta - 2) \cos \theta}{5 - (1 - \sin^2 \theta) - 4 \sin \theta} d\theta$ <p>Let <math>\sin \theta = t \Rightarrow \cos \theta d\theta = dt</math></p> $I = \int \frac{(3t-2)}{5-(1-t^2)-4t} dt$ $= \int \frac{(3t-2)}{t^2-4t+4} dt = \int \frac{3t-2}{(t-2)^2} dt$ <p>Let <math>\frac{3t-2}{(t-2)^2} = \frac{A}{(t-2)} + \frac{B}{(t-2)^2}</math></p> $3t - 2 = A(t - 2) + B$ <p>Comparing the coefficients of t and constant terms on both sides</p> $A = 3, -2A + B = -2, B = 4$ $\int \frac{(3 \sin \theta - 2) \cos \theta}{5 - \cos^2 \theta - 4 \sin \theta} d\theta = \int \frac{3}{t-2} dt + \int \frac{4}{(t-2)^2} dt$ $= 3 \log t - 2  - \frac{4}{t-2} + C$ $= 3 \log \sin \theta - 2  - \frac{4}{\sin \theta - 2} + C$	<p>OR</p> <p>1/2</p> <p>1</p> <p>1/2</p> <p>+1/2</p> <p>1+1</p> <p>1/2</p>
34A	$y + \frac{d}{dx}(xy) = x(\sin x + x)$ $\Rightarrow y + \left(x \frac{dy}{dx} + y\right) = x(\sin x + x)$ $\Rightarrow 2y + x \frac{dy}{dx} = x(\sin x + x)$ $\Rightarrow \frac{dy}{dx} + \frac{2y}{x} = (\sin x + x)$ <p>This a linear differential equation of the form <math>\frac{dy}{dx} + Py = Q</math></p> <p><math>P = \frac{2}{x}, Q = (\sin x + x)</math></p> $I.F = e^{\int \frac{2}{x} dx} = e^{2 \log x} = e^{\log x^2} = x^2$ <p>Solution will be <math>y \cdot I.F = \int Q \cdot I.F dx</math></p> $yx^2 = \int (\sin x + x) x^2 dx$ $yx^2 = \int \sin x \cdot x^2 dx + \int x^3 dx$	<p>1</p> <p>1</p> <p>1</p>

34B	$\Rightarrow yx^2 = -x^2 \cos x + 2 \int x \cos x dx + \frac{x^4}{4} + C$ $\Rightarrow yx^2 = -x^2 \cos x + 2(x \sin x + \cos x) + \frac{x^4}{4} + C$ <p style="text-align: center;">Which is the required solution</p> <p style="text-align: center;"><b>OR</b></p> $2y e^{x/y} dx + (y - 2x e^{x/y}) dy = 0$ $\Rightarrow \frac{dx}{dy} = \frac{2x e^{x/y} - y}{2y e^{x/y}} = \frac{2 \frac{x}{y} e^{\frac{x}{y}} - 1}{2 e^{\frac{x}{y}}}$ <p>It is a homogeneous differential equation.</p> <p>Let <math>x = vy \Rightarrow \frac{dx}{dy} = v + y \frac{dv}{dy}</math></p> $v + y \frac{dv}{dy} = \frac{2ve^v - 1}{2e^v}$ $\Rightarrow y \frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v = \frac{2ve^v - 1 - 2ve^v}{2e^v}$ $\Rightarrow y \frac{dv}{dy} = \frac{-1}{2e^v}$ $\Rightarrow 2e^v dv = -\frac{dy}{y}$ $\int 2e^v dv = -\int \frac{dy}{y}$ $\Rightarrow 2e^v = -\log y  + C$ $\Rightarrow 2e^{\frac{x}{y}} + \log y  = C$ <p>When <math>x = 0, y = 1, C = 2</math></p> <p>Required solution <math>2 e^{\frac{x}{y}} + \log y  = 2</math></p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
35	<p>Let <math>\frac{x-1}{3} = \frac{y-0}{-1} = \frac{z+1}{0} = \lambda \Rightarrow</math> Any point on it is <math>(3\lambda + 1, -\lambda, -1)</math></p> <p>For the point where <math>y = 1 \Rightarrow \lambda = -1</math></p> <p><math>\Rightarrow</math> The point is <math>(-2, 1, -1)</math></p> <p>The directions of the two lines are <math>\vec{m} = 3\hat{i} - \hat{j}</math> and <math>\vec{n} = -2\hat{i} + 2\hat{j} + \hat{k}</math></p> $\vec{m} \times \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ -2 & 2 & 1 \end{vmatrix} = -\hat{i} - 3\hat{j} + 4\hat{k}$ <p>The equation of the required line is</p> $\vec{r} = (-2\hat{i} + \hat{j} - \hat{k}) + \mu(-\hat{i} - 3\hat{j} + 4\hat{k})$ <p><b>Alternative Solution:</b></p> <p>Let <math>\frac{x-1}{3} = \frac{y-0}{-1} = \frac{z+1}{0} = \lambda \Rightarrow</math> Any point on it is <math>(3\lambda + 1, -\lambda, -1)</math></p> <p>For the point where <math>y = 1 \Rightarrow \lambda = -1</math></p> <p><math>\Rightarrow</math> The point is <math>(-2, 1, -1)</math></p> <p>Let the direction ratios of the required line be a, b, c</p> <p>Then <math>3a - b = 0</math></p> <p>And <math>-2a + 2b + c = 0</math></p> <p>Solving we get <math>\frac{a}{-1} = \frac{-b}{3} = \frac{c}{4} \Rightarrow \frac{a}{-1} = \frac{b}{-3} = \frac{c}{4}</math></p> <p>The required line is <math>\frac{x+2}{-1} = \frac{y-1}{-3} = \frac{z+1}{4} = \mu</math></p> <p>In vector form <math>\vec{r} = (-2\hat{i} + \hat{j} - \hat{k}) + \mu(-\hat{i} - 3\hat{j} + 4\hat{k})</math></p>	<p>1/2</p> <p>1</p> <p>1/2</p> <p>1</p> <p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1</p> <p>1</p> <p>1/2</p> <p>1/2</p>

<b>SECTION- E</b> <b>(3 case-study/passage-based questions of 4 marks each)</b>		
36	<p>I. Traffic flow is not reflexive as <math>(A, A) \notin R</math> (or no major spot is connected with itself)</p> <p>II. Traffic flow is not transitive as <math>(A, B) \in R</math> and <math>(B, E) \in R</math>, but <math>(A, E) \notin R</math></p> <p>III A. <math>R = \{(A, B), (A, C), (A, D), (B, C), (B, E), (C, E), (D, E), (D, C)\}</math>            Domain = <math>\{A, B, C, D\}</math>            Range = <math>\{B, C, D, E\}</math></p> <p style="text-align: center;"><b>OR</b></p> <p>III B. No, the traffic flow doesn't represent a function as A has three images.</p>	<p>1</p> <p>1</p> <p>1</p> <p><math>\frac{1}{2} +</math> <math>\frac{1}{2}</math></p> <p>1+1</p>
37	<p>I. <math>P(x) = R(x) - C(x) = -0.3x^2 + 20x - (0.5x^2 - 10x + 150)</math>  <math>= -0.8x^2 + 30x - 150</math></p> <p>II. For critical points <math>P'(x) = 0 \Rightarrow -1.6x + 30 = 0</math>  <math>\Rightarrow x = \frac{30}{1.6} = \frac{300}{16} = 18.75</math></p> <p>III A. Now <math>P''(x) = -1.6</math>            In particular <math>P''(18.75) = -1.6 &lt; 0</math>            So, critical value <math>x = 18.75</math> corresponds to a maximum profit.</p> <p style="text-align: center;"><b>OR</b></p> <p>III B. As <math>x</math> is the number of bulbs, so practically 18 bulbs correspond to a maximum profit.            Maximum profit is <math>P(18) = -0.8 \times 18^2 + 30 \times 18 - 150</math>  <math>= -259.2 + 540 - 150</math>  <math>= 540 - 409.2 = ₹130.80</math></p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
38	<p>Let the events be</p> <p><math>E_1</math>: the student is in the first group (time spent on screen is more than 4 hours)</p> <p><math>E_2</math>: the student is in the second group (time spent on screen is 2 to 4 hours)</p> <p><math>E_3</math>: the student is in third group (time spent on screen is less than 2 hours)</p> <p>A: the event of the student showing symptoms of anxiety and low retention</p> <p><math>P(E_1) = \frac{60}{100}</math>                      <math>P(E_2) = \frac{30}{100}</math>                      and <math>P(E_3) = \frac{10}{100}</math></p> <p><math>P(A E_1) = \frac{80}{100}</math>                      <math>P(A E_2) = \frac{70}{100}</math>                      and <math>P(A E_3) = \frac{30}{100}</math></p> <p>I. <math>P(A) = P(E_1) \times P(A E_1) + P(E_2) \times P(A E_2) + P(E_3) \times P(A E_3)</math>  <math>= \frac{60}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{70}{100} + \frac{10}{100} \times \frac{30}{100} = \frac{72}{100} = 72\%</math></p> <p>II. <math>P(E_1 A) = \frac{P(E_1 \cap A)}{P(A)}</math>  <math>= \frac{\left(\frac{60}{100} \times \frac{80}{100}\right)}{\left(\frac{72}{100}\right)} = \frac{48}{72} = \frac{2}{3}</math></p>	<p>2</p> <p>2</p>