NCERT Solutions Class 12 Maths Chapter 9: Differential Equations Exercise 9.4

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Quick Summary: In NCERT Solutions Class 12 Maths Chapter 9 Exercise 9.4, students learn systematic methods for solving first-order differential equations, particularly focusing on homogeneous differential equations. This exercise covers the identification and solution techniques for homogeneous DEs using substitution methods, which are crucial for CBSE board exams and competitive entrance tests.

Key Takeaways:

- Master the identification of homogeneous differential equations where (dy)/(dx) = f\left((y)/(x) \right)
- Learn the substitution method using y = vx where v is a function of x
- Understand how to separate variables after substitution to get (dv)/(dx) = (f(v) - v)/(x)
- Practice finding both general and particular solutions by applying initial conditions correctly

Complete Solutions

Question 1

QUESTION

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve it:

$(x^2+xy)\,dy=(x^2+y^2)\,dx$

SOLUTION

We are asked to show that the given differential equation is homogeneous and then solve it.

Step 1: Check for Homogeneity

The given differential equation is . We can rewrite this as:

To check for homogeneity, we replace with and with :

Since the expression remains unchanged, the differential equation is homogeneous.

Step 2: Solve the Homogeneous Differential Equation

Let , so . Substituting into the differential equation:

Now, isolate :

Separate the variables:

Integrate both sides. We can rewrite the left side as:

So,

And . Therefore,

Substitute :

Multiply by -1:

Final Answer:

ANSWER

$(x-y)^2=Cx\,e^{(-y)/(x)}$

Question 2

QUESTION

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve it:

y'=(x+y)/(x)

SOLUTION

We are asked to show that the given differential equation is homogeneous and then solve it.

Step 1: Check for homogeneity

The given differential equation is , which can be written as .

Let . To check for homogeneity, we replace with and with :

Since , the differential equation is homogeneous of degree 0.

Step 2: Solve the homogeneous differential equation

To solve, we substitute , so .

Substituting into the differential equation:

Separating variables:

Step 3: Integrate both sides

Step 4: Substitute back for

Since , we have:

Final Answer:

ANSWER

y=x\log|x|+Cx

Question 3

QUESTION

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve it:

(x-y)\,dy-(x+y)\,dx=0

SOLUTION

We are given the differential equation and asked to show that it is homogeneous and then solve it.

Step 1: Show that the differential equation is homogeneous

First, rewrite the equation as:

Now, let's check for homogeneity. A function is homogeneous of degree if . In our case, . Let's substitute with and with :

Since , the function is homogeneous of degree 0. Therefore, the given differential equation is homogeneous.

Step 2: Solve the homogeneous differential equation

Let . Then, .

Substitute into the differential equation:

Now, isolate :

Separate the variables:

Integrate both sides:

Substitute :

, where

Final Answer:

ANSWER

\tan^-1\left((y)/(x)\right)=-(1)/(2)\log(x^2+y^2)+C

Question 4

QUESTION

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve it:

$(x^2-y^2)\,dx+2xy\,dy=0$

SOLUTION

We are asked to show that the given differential equation is homogeneous and then solve it.

Step 1: Check for Homogeneity

The given differential equation is: . We can rewrite this as:

Now, let's check if the equation is homogeneous. Replace with and with :

Since the equation remains unchanged, the given differential equation is homogeneous.

Step 2: Solve the Homogeneous Differential Equation

Let . Then, . Substituting into the differential equation:

Now, separate the variables:

Step 3: Integrate both sides

Integrating both sides, we get:

Step 4: Substitute back

Final Answer:

ANSWER

$x^2+y^2=Cx$

Question 5

QUESTION

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve it:

x^2(dy)/(dx)=x^2-2y^2+xy

SOLUTION

We are asked to show that the given differential equation is homogeneous and then solve it.

Step 1: Rewrite the differential equation

The given differential equation is:

Divide both sides by :

Step 2: Check for homogeneity

Let . If we replace with and with , we get:

Since , the differential equation is homogeneous (of degree 0).

Step 3: Solve the homogeneous differential equation

Let . Then .

Substitute into the differential equation:

Integrate both sides:

The left integral can be evaluated using partial fractions or a standard integral formula:

The right integral is:

So, we have:

Substitute :

Final Answer:

ANSWER

(1)/(2√2)\log\left|x+√2yx-√2y\right|=\log|x|+C

Question 6

QUESTION

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve it:

x\,dy-y\,dx=√x^2+y^2\,dx

SOLUTION

We are asked to show that the given differential equation is homogeneous and then solve it. The given differential equation is .

Step 1: Rewrite the equation

First, let's rewrite the equation to isolate :

Step 2: Check for homogeneity

A differential equation is homogeneous if it can be written in the form . Let's check if our equation satisfies this condition:

Since the equation can be expressed as a function of , it is a homogeneous differential equation.

Step 3: Solve the homogeneous equation

Let , so . Then, .

Substitute these into the equation:

Step 4: Separate variables and integrate

Integrate both sides:

Alternatively, using a standard integral:

Exponentiate both sides:

where .

Step 5: Substitute back

Multiply both sides by :

Final Answer:

ANSWER

y+√x^2+y^2=Cx^2

Question 7

QUESTION

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve it:

\(x\cos(y)/(x))+y\sin((y)/(x))\\,y\,dx=\(y\sin(y)/(x))-x\cos((y)/(x))\\,x\,dy

SOLUTION

This question requires us to first prove that the given differential equation is homogeneous and then solve it.

Step 1: Rewrite the equation

We are given the differential equation:

Rearrange the terms to get on one side:

Step 2: Check for homogeneity

A differential equation is homogeneous if it can be written in the form . In our case, we can see that the right-hand side is a function of only. Therefore, the given differential equation is homogeneous.

Step 3: Substitute

Let . Then, differentiating with respect to , we get:

Substitute and into the differential equation:

Step 4: Separate variables and integrate

This step seems to have an error. Let's correct the algebra:

The correct separation is:

Let's go back to:

Final Answer:

ANSWER

xy\cos\left|(y)/(x)\right|=C

Question 8

QUESTION

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve it:

x(dy)/(dx)-y+x\sin((y)/(x))=0

SOLUTION

We are given the differential equation and asked to show it is homogeneous and then solve it.

Step 1: Show that the equation is homogeneous

A differential equation is homogeneous if it can be written in the form . Let's rewrite the given equation:

Since the right-hand side is a function of , the given differential equation is homogeneous.

Step 2: Solve the homogeneous differential equation

Let , so . Substituting these into the homogeneous equation:

Separate the variables:

Integrate both sides:

Recall that . So, . Also, .

Let . Then:

Substitute :

Rearrange to match the given answer:

Final Answer:

ANSWER

x\left[1-\cos\left((y)/(x)\right)\right]=C\sin\left((y)/(x)\right)

Question 9

QUESTION

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve it:

y\,dx+x\log\left((y)/(x)\right)\,dy-2x\,dy=0

SOLUTION

We are given a differential equation and asked to show that it is homogeneous and then solve it.

Step 1: Check for homogeneity

The given differential equation is:

Rearrange the equation to get on one side:

Since the equation can be expressed in the form , it is a homogeneous differential equation.

Step 2: Solve the homogeneous differential equation

Let , so . Then, .

Substitute into the differential equation:

Separate the variables:

Let , then , and . So .

Integrate both sides:

, where

, where

Final Answer:

ANSWER

cy=\log\left|(y)/(x)\right|-1

Question 10

QUESTION

In each of the Exercises 1 to 10, show that the given differential equation is homogeneous and solve it:

$(1+e^{\frac{x}{y}})\,dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)\,dy=0$

SOLUTION

We are given a differential equation and asked to show that it is homogeneous and then solve it.

Step 1: Check for Homogeneity

A differential equation of the form is homogeneous if and are homogeneous functions of the same degree. A function is homogeneous of degree if for any non-zero constant .

Let's rewrite the given equation:

We can express this as:

Now, let's check for homogeneity:

Both and are homogeneous functions of degree 0. Therefore, the given differential equation is homogeneous.

Step 2: Solve the Homogeneous Differential Equation

Since the equation is homogeneous, we can use the substitution , so .

Substituting into the given equation:

Separate the variables:

Integrate both sides:

Multiply and divide the first term by :

Let , then . This substitution doesn't seem to simplify the integral.

Let's try another approach. Divide the original equation by :

Instead, let's go back to and rearrange:

This doesn't seem right. Let's try dividing by instead:

Let's go back to and try a different substitution. Let , so . Then:

Consider . Then

From the original equation,

Instead, consider . Then .

Final Answer:

ANSWER

ye^(x)/(y)+x=C

Question 11

QUESTION

For each of the differential equations in Exercises from 11 to 15, find the particular solution satisfying the given condition:

(x+y)\,dy+(x-y)\,dx=0;\ y=1\ \textwhen\ x=1

SOLUTION

We are asked to find the particular solution of the given differential equation with the condition when .

Step 1: Rewrite the differential equation

Rearrange the equation to isolate :

Step 2: Recognize the homogeneity

The equation is homogeneous because each term has a degree of 1. We can solve this by substituting , so .

Step 3: Substitute and simplify

Substituting into the differential equation, we get:

Now, isolate the derivative term:

Step 4: Separate variables and integrate

Separate the variables and :

Integrate both sides:

Step 5: Substitute back

Step 6: Apply the initial condition when

Step 7: Write the particular solution

Final Answer:

ANSWER

\log(x^2+y^2)+2\tan^-1\left((y)/(x)\right)=(π)/(2)+\log 2

Question 12

QUESTION

For each of the differential equations in Exercises from 11 to 15, find the particular solution satisfying the given condition:

$x^2\,dy+(xy+y^2)\,dx=0;\ y=1\ \text{when}\ x=1$

SOLUTION

We are given a homogeneous differential equation and an initial condition, and we need to find the particular solution.

Step 1: Rewrite the differential equation

The given differential equation is:

Rearrange the terms to isolate :

Step 2: Check for homogeneity and substitute

The equation is homogeneous because each term has a degree of 2. Let , then .

Substitute and into the differential equation:

Step 3: Separate variables and integrate

Integrate both sides:

We can write . Solving for A and B, we get and .

, where

, where

Step 4: Substitute back

Step 5: Apply the initial condition when

Step 6: Write the particular solution

Therefore, the particular solution is .

ANSWER

$y+2x=3x^2y$

Question 13

QUESTION

For each of the differential equations in Exercises from 11 to 15, find the particular solution satisfying the given condition:

[x\sin^2((y)/(x))-y]dx+x\,dy=0;\ y=(π)/(4)\ \textwhen\ x=1

SOLUTION

We are asked to find the particular solution to the given differential equation with the specified initial condition.

Step 1: Check if the equation is homogeneous

The given differential equation is:

Rearranging, we have:

Let . Then, . Thus, the equation is homogeneous.

Step 2: Solve the homogeneous differential equation

Substitute , so . The equation becomes:

Separating variables:

Integrating both sides:

Step 3: Substitute back

Step 4: Apply the initial condition when

, so

Step 5: Write the particular solution

Final Answer:

ANSWER

\cot\left((y)/(x)\right)=\log|ex|

Question 14

QUESTION

For each of the differential equations in Exercises from 11 to 15, find the particular solution satisfying the given condition:

(dy)/(dx)-(y)/(x)+\cosec((y)/(x))=0;\ y=0\ \textwhen\ x=1

SOLUTION

We are asked to find the particular solution to the given differential equation with the initial condition when .

Step 1: Check if the equation is homogeneous

The given differential equation is: . This is a homogeneous differential equation because we can write it in the form .

Step 2: Substitute

Let . Then, differentiating with respect to , we get .

Step 3: Substitute in the differential equation

Substituting and into the given equation, we have:

Simplifying, we get:

Step 4: Separate variables

Separating the variables, we have:

Step 5: Integrate both sides

Integrating both sides, we get:

, where is the constant of integration.

Step 6: Substitute back

Substituting , we get:

Step 7: Apply the initial condition when

Substituting and , we have:

, so

Step 8: Write the particular solution

Substituting into the general solution, we get:

Final Answer:

ANSWER

\cos\left((y)/(x)\right)=\log|ex|

Question 15

QUESTION

For each of the differential equations in Exercises from 11 to 15, find the particular solution satisfying the given condition:

2xy+y^2-2x^2(dy)/(dx)=0;\ y=2\ \textwhen\ x=1

SOLUTION

We are given a homogeneous differential equation and an initial condition, and we need to find the particular solution.

Step 1: Rewrite the differential equation

The given differential equation is:

Rearrange to isolate :

Step 2: Check for homogeneity and substitute

The equation is homogeneous because each term has a degree of 2. Let , so .

Substitute into the differential equation:

Step 3: Separate variables and integrate

Integrate both sides:

Step 4: Substitute back

Step 5: Apply the initial condition when

Step 6: Write the particular solution

Final Answer:

ANSWER

y=(2x)/(1-\log|x|)\ \ (x\ne 0,\ x\ne e)

Question 16

QUESTION

A homogeneous differential equation of the form (dx)/(dy)=h\left((x)/(y)\right) can be solved by making the substitution.

SOLUTION

We are asked to identify the correct substitution to solve a homogeneous differential equation of the form .

Step 1: Understand Homogeneous Differential Equations

A homogeneous differential equation is one where the function depends only on the ratio or .

Step 2: Analyze the given form

The given equation is . This means we want to express in terms of and a new variable , such that can be replaced by .

Step 3: Test the substitution

If we let , then . This substitution directly addresses the form of the given differential equation.

Step 4: Differentiate the substitution

Differentiating with respect to , we get:

Now, we can substitute and into the original differential equation to solve it.

Step 5: Analyze why other options are incorrect

- : This substitution is used when the differential equation is in the form .
- : This substitution doesn't simplify the given homogeneous equation and makes it more complex.
- : This substitution doesn't account for the dependence on and is not suitable for solving homogeneous differential equations.

Final Answer: The correct substitution is .

ANSWER

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Question 17

QUESTION

Which of the following is a homogeneous differential equation?

SOLUTION

We need to identify which of the given differential equations is homogeneous. A differential equation of the form is homogeneous if and are homogeneous functions of the same degree.

Step 1: Define Homogeneous Function

A function is homogeneous of degree if for any constant .

Step 2: Analyze Option 1:

Rearranging, we have . Let and . Since and contain constant terms, they are not homogeneous functions.

Step 3: Analyze Option 2:

Rearranging, we have . Let and . The degree of is 2, and the degree of is 3. Since the degrees are different, this is not a homogeneous equation.

Step 4: Analyze Option 3:

Let and . . This is NOT a homogeneous function. Therefore, this equation is NOT homogeneous.

Step 5: Analyze Option 4:

Let and . . . Both and are homogeneous functions of degree 2. Therefore, this is a homogeneous differential equation.

Final Answer: Option 4 is the homogeneous differential equation.

ANSWER

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Key Formulas

Important Formulas for Exercise 9.4

Formula / Concept	Description
First Order, First Degree Differential Equation	An equation of the form $(dy)/(dx) = F(x, y)$ is a first-order, first-degree differential equation.
General Solution	A solution to a differential equation that contains arbitrary constants. It represents a family of curves.
Particular Solution	A solution obtained from the general solution by giving particular values to the arbitrary constants. It represents a specific curve.
Methods of Solving First Order, First Degree Differential Equations	
1. Variables Separable Method	A method used when the differential equation can be expressed in the form $(dy)/(dx) = g(x)h(y)$.
Separation of Variables	If $(dy)/(dx) = g(x)h(y)$, we can separate the variables to get $(1)/(h(y))dy = g(x)dx$.
General Solution (Variables Separable)	$\int (1)/(h(y))\, dy = \int g(x)\, dx + C$ where C is the constant of integration.
2. Homogeneous Differential Equations	A differential equation of the form $(dy)/(dx) = F(x, y)$ where $F(x, y)$ is a homogeneous function of degree zero. A function $F(x, y)$ is homogeneous of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$.
Substitution for Homogeneous Equations	To solve a homogeneous differential equation, substitute $y = vx$.
Resulting Differential	From $y = vx$, we get $(dy)/(dx) = v + x(dv)/(dx)$.
Resulting Separable Equation	After substitution, the equation becomes $v + x(dv)/(dx) = G(v)$, which can be solved by separating the variables: $(dv)/(G(v) - v) = (dx)/(x)$.
3. Linear Differential Equations	A differential equation of the form $(dy)/(dx) + P(x)y = Q(x)$, where P and Q are constants or functions of x only.
Integrating Factor (I.F.)	A function used to solve linear differential equations. The integrating factor is given by $I.F. = e^{\int P(x)dx}$.
General Solution (Linear DE)	The solution of the linear differential equation is given by: $y \cdot (I.F.) = \int (Q(x) \cdot I.F.)\, dx + C$
Alternative Form of Linear DE	An equation of the form $(dx)/(dy) + P_1(y)x = Q_1(y)$, where P_1 and Q_1 are constants or functions of y only.

Formula / Concept	Description
I.F. for Alternative Form	The integrating factor for the alternative form is I.F. = $e^{\int P_1(y)dy}$.
General Solution for Alternative Form	The solution for the alternative form is: $x \cdot (I.F.) = \int (Q_1(y) \cdot I.F.)\, dy + C$

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