NCERT Solutions Class 12 Maths Chapter 9: Differential Equations Exercise 9.3

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Quick Summary: In NCERT Solutions Class 12 Maths Chapter 9 Exercise 9.3, students learn the formation of differential equations and methods to find general solutions of first-order differential equations. This exercise covers separable equations, homogeneous equations, and linear differential equations which are essential for CBSE board exams and competitive entrance tests.

Key Takeaways:

- General solution contains arbitrary constants while particular solutions are obtained by applying initial conditions
- Separable equations of the form (dy)/(dx) = f(x)g(y) can be solved by separating variables
- Linear first-order equations (dy)/(dx) + Py = Q use integrating factor method with I.F. = e^\int P dx
- Homogeneous equations where (dy)/(dx) = F((y)/(x)) are solved using substitution y = vx

Complete Solutions

Question 1

QUESTION

For the differential equation, find the general solution:

(dy)/(dx) = (1-\cos x)/(1+\cos x)

SOLUTION

We are asked to find the general solution of the given differential equation.

Step 1: Rewrite the differential equation

The given differential equation is:

Step 2: Simplify the trigonometric expression

We can simplify the right-hand side using trigonometric identities. Recall the half-angle formulas:

Substituting these into the differential equation, we get:

Step 3: Integrate both sides with respect to x

We have:

Integrate both sides with respect to :

We know that , so:

Step 4: Evaluate the integral

Now, we integrate term by term:

The integral of is (because the derivative of is). The integral of 1 is . Thus:

where is the constant of integration.

Final Answer:

ANSWER

y = 2\tan(x)/(2) - x + C

Question 2

QUESTION

For the differential equation, find the general solution:

$\frac{dy}{dx} = \sqrt{4-y^2}\ (-2<y<2)$

SOLUTION

We are asked to find the general solution of the given differential equation: where .

Step 1: Separate the variables

We rewrite the equation so that terms involving are on one side and terms involving are on the other side:

Step 2: Integrate both sides

Integrate both sides of the equation with respect to their respective variables:

Step 3: Evaluate the integrals

The integral on the left side is a standard integral. Recall that . In our case, , so we have:

The integral on the right side is straightforward:

Therefore, we have:

where is an arbitrary constant.

Step 4: Solve for y

To isolate , take the sine of both sides:

Multiply both sides by 2:

Final Answer:

The general solution of the differential equation is .

ANSWER

$y = 2\sin(x + C)$

Question 3

QUESTION

For the differential equation, find the general solution:

(dy)/(dx) + y = 1 (y \ne 1)

SOLUTION

We are asked to find the general solution of the given first-order differential equation.

Step 1: Recognize the type of differential equation

The given differential equation is:

This is a first-order linear differential equation of the form:

where and .

Step 2: Find the integrating factor

The integrating factor (IF) is given by:

In our case, , so:

Step 3: Multiply the differential equation by the integrating factor

Multiplying the given differential equation by , we get:

Step 4: Recognize the left-hand side as a derivative

The left-hand side is the derivative of with respect to :

So, we can rewrite the equation as:

Step 5: Integrate both sides with respect to x

Integrating both sides with respect to , we get:

, where A is the constant of integration.

Step 6: Solve for y

Divide both sides by :

Final Answer:

ANSWER

y = 1 + Ae^-x

Question 4

QUESTION

For the differential equation, find the general solution:

\sec^2 x\,\tan y\,dx + \sec^2 y\,\tan x\,dy = 0

SOLUTION

We are asked to find the general solution of the given differential equation: .

Step 1: Separate the variables

Rearrange the equation to separate the and terms:

Divide both sides by :

Step 2: Integrate both sides

Integrate both sides of the separated equation:

Step 3: Evaluate the integrals

Let , then . Similarly, let , then . The integrals become:

Substituting back and :

Step 4: Simplify the solution

Rearrange the equation:

Using the property of logarithms, :

Exponentiate both sides:

Let , where is a positive constant. We can also absorb the absolute value into the constant, allowing to be any non-zero constant.

Final Answer:

ANSWER

\tan x\,\tan y = C

Question 5

QUESTION

For the differential equation, find the general solution:

(e^x + e^-x)\,dy - (e^x - e^-x)\,dx = 0

SOLUTION

We are asked to find the general solution to the given differential equation: .

Step 1: Separate the variables

Rearrange the equation to separate the and terms:

Divide both sides by to isolate :

Step 2: Integrate both sides

Integrate both sides of the equation with respect to their respective variables:

Step 3: Evaluate the integrals

The left side integral is straightforward:

For the right side integral, use substitution. Let . Then, the derivative of with respect to is:

So, . Substitute these into the integral:

The integral of with respect to is :

Step 4: Substitute back for u

Substitute back into the equation:

Since and are always positive, is always positive, so we can drop the absolute value signs.

Step 5: Write the general solution

Combine the results from both sides:

Final Answer:

ANSWER

y = \log(e^x + e^-x) + C

Question 6

QUESTION

For the differential equation, find the general solution:

(dy)/(dx) = (1+x^2)(1+y^2)

SOLUTION

We are asked to find the general solution of the given differential equation: .

Step 1: Separate the variables

The given differential equation is:

We can separate the variables and to get:

Step 2: Integrate both sides

Now, integrate both sides of the equation with respect to their respective variables:

Step 3: Evaluate the integrals

The integral on the left side is a standard integral:

The integral on the right side is:

where is the constant of integration.

Step 4: Write the general solution

Combining the results from Step 3, we have:

Final Answer:

ANSWER

\tan^-1y = x + (x^3)/(3) + C

Question 7

QUESTION

For the differential equation, find the general solution:

y\log y\,dx - x\,dy = 0

SOLUTION

We are asked to find the general solution of the given differential equation: .

Step 1: Separate the variables

Rearrange the equation to separate the variables and on opposite sides:

Divide both sides by :

Step 2: Integrate both sides

Integrate both sides of the equation with respect to their respective variables:

Step 3: Evaluate the integrals

The integral on the left side is straightforward:

For the integral on the right side, use the substitution method. Let , then , so . The integral becomes:

Step 4: Combine the results and simplify

Now we have:

, where

Rewrite the equation to isolate the constant:

Exponentiate both sides:

Let , where is another constant. Then:

Exponentiate again:

Final Answer:

ANSWER

y = e^cx

Question 8

QUESTION

For the differential equation, find the general solution:

x^5(dy)/(dx) = -y^5

SOLUTION

We are asked to find the general solution of the given differential equation .

Step 1: Separate the variables

We want to get all the terms on one side and all the terms on the other side. Divide both sides by to get:

This can be rewritten as:

Step 2: Integrate both sides

Now, we integrate both sides of the equation with respect to their respective variables:

Using the power rule for integration, , we have:

where is the constant of integration.

Step 3: Simplify the equation

Multiply both sides by -4:

Let , which is another arbitrary constant. Then:

Rearranging the terms, we get:

We can also write this as:

Final Answer: The general solution of the differential equation is .

ANSWER

x^-4 + y^-4 = C

Question 9

QUESTION

For the differential equation, find the general solution:

(dy)/(dx) = \sin^-1 x

SOLUTION

We are asked to find the general solution to the differential equation .

Step 1: Separate the variables

The given differential equation is already in a separated form. We have:

Step 2: Integrate both sides

Integrate both sides with respect to their respective variables:

The left side is straightforward:

For the right side, we need to integrate with respect to . We will use integration by parts.

Step 3: Integrate by parts

Let and . Then, and .

Using the integration by parts formula, , we get:

Step 4: Solve the remaining integral

Now we need to solve . Let , so , and .

Then,

Step 5: Substitute back into the integration by parts result

Step 6: Write the general solution

So, we have:

, where is the constant of integration.

Final Answer:

ANSWER

y = x\sin^-1x + √1-x^2 + C

Question 10

QUESTION

For the differential equation, find the general solution:

e^x\tan y\,dx + (1-e^x)\sec^2 y\,dy = 0

SOLUTION

We are asked to find the general solution to the given differential equation: .

Step 1: Rearrange the equation

First, we rearrange the given equation to separate the variables:

Step 2: Separate the variables

Divide both sides by and to separate the variables and :

Step 3: Integrate both sides

Integrate both sides of the equation with respect to their respective variables:

Step 4: Evaluate the integrals

For the left side, let , then . So, .

For the right side, let , then . So, .

Therefore, we have:

, where is the constant of integration.

Step 5: Simplify the equation

Multiply both sides by -1:

Let , then

Exponentiate both sides:

Let , then

Rearranging, we get:

Let , then

Replacing with , we get:

Final Answer:

ANSWER

$\tan y = C(1 - e^x)$

Question 11

QUESTION

Find a particular solution satisfying the given condition:

$(x^3 + x^2 + x + 1)\frac{dy}{dx} = 2x^2 + x$, $y = 1$ when $x = 0$

SOLUTION

We are asked to find a particular solution to the given differential equation with the initial condition when .

Step 1: Separate the variables

Rewrite the given differential equation:

Separate the variables and :

Step 2: Factor the denominator

Factor the denominator :

So, the equation becomes:

Step 3: Integrate both sides

Integrate both sides of the equation:

Let's use partial fraction decomposition on the right-hand side:

Comparing coefficients:

, ,

Solving this system of equations, we get , ,

So, the integral becomes:

Step 4: Apply the initial condition

Given when :

Step 5: Write the particular solution

Substitute into the general solution:

Final Answer:

ANSWER

$y = \frac{1}{4}\log\left[(x+1)^2(x^2+1)^3\right] - \frac{1}{2}\tan^{-1}x + 1$

Question 12

QUESTION

Find a particular solution satisfying the given condition:

x(x^2 - 1)(dy)/(dx) = 1, y = 0 when x = 2

SOLUTION

We are asked to find a particular solution to the given differential equation with the condition when .

Step 1: Separate the variables

We can rewrite the given differential equation as:

Separating the variables, we get:

Step 2: Integrate both sides

Integrating both sides, we have:

The left side integrates to . For the right side, we use partial fractions.

Step 3: Partial fraction decomposition

We can write

Multiplying both sides by , we get:

Let :

Let :

Let :

So,

Step 4: Integrate the right side

Therefore,

Step 5: Apply the initial condition

Given when , we have:

Step 6: Write the particular solution

Substituting the value of , we get:

Final Answer:

ANSWER

$y = \frac{1}{2}\log\left(\frac{x^2-1}{x^2}\right) - \frac{1}{2}\log\left(\frac{3}{4}\right)$

Question 13

QUESTION

Find a particular solution satisfying the given condition:

\cos\left((dy)/(dx)\right) = a (a \in \mathbbR), y = 1 when x = 0

SOLUTION

We are given a differential equation and an initial condition when . We need to find the particular solution that satisfies this condition.

Step 1: Solve for

We have . Taking the inverse cosine of both sides, we get:

Let , where is a constant since is a constant.

So,

Step 2: Integrate both sides with respect to

Integrating both sides with respect to , we have:

, where is the constant of integration.

Step 3: Apply the initial condition when

Substituting and into the general solution , we get:

Step 4: Write the particular solution

Substituting back into the general solution, we have:

Since , we can write:

Rearranging the terms, we get:

Taking cosine of both sides:

Final Answer: The particular solution is

ANSWER

\cos\left((y-2)/(x)\right) = a

Question 14

QUESTION

Find a particular solution satisfying the given condition:

(dy)/(dx) = y\tan x, y = 1 when x = 0

SOLUTION

We are asked to find a particular solution to the differential equation given the initial condition when .

Step 1: Separate the variables

We rewrite the equation so that terms involving are on one side and terms involving are on the other side:

Step 2: Integrate both sides

Integrate both sides of the separated equation:

We know that and . Therefore,

where is an arbitrary constant.

Step 3: Solve for y

To solve for , we exponentiate both sides:

Let , where is another arbitrary constant. Then,

Step 4: Apply the initial condition

We are given that when . Substitute these values into the general solution:

Since , we have:

Step 5: Write the particular solution

Substitute back into the general solution:

Final Answer:

ANSWER

y = \sec x

Question 15

QUESTION

Find the equation of a curve passing through the point (0,0) and whose differential equation is y' = e^x\sin x.

SOLUTION

We are asked to find the equation of a curve that passes through the point and satisfies the differential equation . This involves solving the differential equation and then using the given point to find the particular solution.

Step 1: Separate variables and integrate

The given differential equation is:

Since the variables are already separated, we can integrate both sides with respect to :

The left side integrates to :

Step 2: Evaluate the integral on the right side

To evaluate , we'll use integration by parts twice. Let .

First, let and . Then and . Using integration by parts:

Now, integrate by parts. Let and . Then and :

Substitute this back into the expression for :

Therefore,

Step 3: Apply the initial condition

The curve passes through , so when , :

Step 4: Write the final equation

Substitute back into the equation for :

Multiply by 2 to eliminate fractions:

Final Answer:

ANSWER

2y - 1 = e^x(\sin x - \cos x)

Question 16

QUESTION

For the differential equation xy(dy)/(dx) = (x+2)(y+2), find the solution curve passing through the point (1,-1).

SOLUTION

We are given the differential equation and asked to find the solution curve passing through the point .

Step 1: Separate the variables

We rewrite the equation to separate the variables and on opposite sides:

Step 2: Integrate both sides

Integrate both sides of the equation with respect to their respective variables:

Step 3: Simplify the integrals

We can rewrite the integrals as follows:

Step 4: Evaluate the integrals

Now, we integrate each term:

Step 5: Apply the initial condition

We are given that the solution curve passes through the point . Substitute and into the equation:

Since , we have:

Step 6: Write the final solution

Substitute back into the equation:

Rearrange the terms:

Final Answer:

ANSWER

y - x + 2 = \log\left(x^2(y+2)^2\right)

Question 17

QUESTION

Find the equation of a curve passing through the point (0,-2) given that at any point (x,y) on the curve, the product of the slope of its tangent and y coordinate of the point is equal to the x coordinate of the point.

SOLUTION

This question requires us to find the equation of a curve given a condition involving the slope of the tangent at any point on the curve. This involves setting up and solving a differential equation.

Step 1: Translate the given information into a differential equation.

The slope of the tangent at any point on the curve is given by . The problem states that the product of the slope and the y-coordinate is equal to the x-coordinate. Therefore, we can write the differential equation as:

Step 2: Separate the variables.

To solve this differential equation, we separate the variables and on opposite sides of the equation:

Step 3: Integrate both sides.

Now, we integrate both sides of the equation with respect to their respective variables:

This gives us:

, where is the constant of integration.

Step 4: Simplify the equation.

Multiplying both sides by 2, we get:

We can replace with another constant, say , so the equation becomes:

Step 5: Use the given point to find the value of the constant.

The curve passes through the point . Substituting and into the equation, we get:

Step 6: Write the final equation of the curve.

Substituting back into the equation , we get:

Rearranging the terms, we get:

Final Answer:

The equation of the curve is .

ANSWER

$y^2 - x^2 = 4$

Question 18

QUESTION

At any point (x,y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (-4,-3). Find the equation of the curve given that it passes through (-2,1).

SOLUTION

This question involves forming and solving a differential equation based on the given geometric condition. We need to find the equation of a curve where the slope of the tangent at any point is twice the slope of the line segment joining that point to . Finally, we'll use the given point to find the particular solution.

Step 1: Define the slopes

Let the slope of the tangent at point be . The slope of the line segment joining to is .

Step 2: Form the differential equation

According to the problem, the slope of the tangent is twice the slope of the line segment. Therefore:

Step 3: Solve the differential equation

Separate the variables:

Integrate both sides:

Rewrite the equation:

Exponentiate both sides:

Let , then:

Step 4: Apply the given point

Substitute and into the equation:

Step 5: Write the final equation

Substitute back into the general solution:

Therefore, the equation of the curve is .

Answer:

ANSWER

(x+4)^2 = y + 3

Question 19

QUESTION

The volume of a spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units, find the radius of the balloon after t seconds.

SOLUTION

This problem involves finding the radius of a spherical balloon as a function of time, given that its volume changes at a constant rate. We'll use differential equations to model the situation.

Step 1: Define variables and rates

Let be the volume of the balloon and be its radius at time . We are given that , where is a constant.

Step 2: Volume of a sphere

The volume of a sphere is given by .

Step 3: Differentiate the volume with respect to time

Differentiating both sides of with respect to , we get:

Step 4: Substitute the constant rate

Since , we have . Rearranging, we get:

Step 5: Separate variables and integrate

Separating variables, we have . Integrating both sides:

Step 6: Apply initial conditions

At , . Substituting these values:

Step 7: Apply the second condition

At , . Substituting these values and :

Step 8: Find the radius as a function of time

Substituting and into :

Final Answer: The radius of the balloon after seconds is .

ANSWER

(63t + 27)^(1)/(3)

Question 20

QUESTION

In a bank, principal increases continuously at the rate of r\% per year. Find the value of r if Rs 100 doubles itself in 10 years (\log_e 2 = 0.6931).

SOLUTION

This question involves solving a differential equation to model continuous compounding interest and then using the given information to find the interest rate .

Step 1: Set up the differential equation

Let be the principal at time (in years). The rate of increase of the principal is given by , and it's stated that the principal increases at a rate of per year. Therefore, we can write the differential equation as:

Step 2: Solve the differential equation

This is a separable differential equation. We can separate the variables and integrate:

Integrating both sides:

Where is the constant of integration. Exponentiating both sides:

Let , then:

Step 3: Apply initial conditions

We are given that the initial principal is Rs 100. So, when , . Substituting these values into the equation:

So, . Therefore, the equation becomes:

Step 4: Use the doubling condition

We are given that the principal doubles in 10 years. So, when , . Substituting these values into the equation:

Step 5: Solve for

Take the natural logarithm of both sides:

We are given that , so:

Step 6: State the final answer

The value of is approximately .

ANSWER

6.93\%

Question 21

QUESTION

In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it be worth after 10 years (e^0.5 = 1.648).

SOLUTION

This question involves solving a differential equation to model continuous compounding interest and then finding the value of an investment after a certain period.

Step 1: Define the differential equation

Let be the principal amount at time (in years). The rate of increase of the principal is given as 5% per year, which can be written as:

Step 2: Solve the differential equation

This is a separable differential equation. Separate the variables:

Integrate both sides:

Exponentiate both sides:

Let , then:

Step 3: Apply the initial condition

We are given that the initial amount is Rs 1000, so . Substitute into the equation:

Thus, , and the equation becomes:

Step 4: Find the amount after 10 years

We want to find . Substitute into the equation:

We are given that , so:

Final Answer:

The amount after 10 years will be Rs 1648.

ANSWER

Rs 1648

Question 22

QUESTION

In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present?

SOLUTION

This question involves solving a differential equation to model bacterial growth. We'll use the given information to find the growth constant and then determine the time it takes for the bacteria count to double.

Step 1: Set up the differential equation

Let be the number of bacteria at time . The rate of growth is proportional to the number present, so we have:

where is the constant of proportionality.

Step 2: Solve the differential equation

Separating variables and integrating, we get:

where is a constant.

Step 3: Use the initial condition

At , . So,

Thus, .

Step 4: Use the information about the 10% increase

At hours, . So,

Step 5: Find the time when the count reaches 200000

We want to find such that . So,

Final Answer: The number of hours required is .

ANSWER

(2\log 2)/(\log\left(11)10\right)

Question 23

QUESTION

The general solution of the differential equation (dy)/(dx) = e^x+y is

SOLUTION

We are asked to find the general solution of the differential equation . This is a problem involving separable differential equations.

Step 1: Separate the variables

We can rewrite the given differential equation as:

Now, separate the variables and by dividing both sides by and multiplying both sides by :

This can be written as:

Step 2: Integrate both sides

Integrate both sides of the equation with respect to their respective variables:

The integral of with respect to is , and the integral of with respect to is . So we have:

, where is the constant of integration.

Step 3: Rearrange the equation

Multiply both sides by -1:

Let , where is another arbitrary constant. Then:

Rearrange the terms to get:

Final Answer: The general solution of the differential equation is .

Therefore, the correct option is .

Why other options are incorrect:

: This is incorrect because the separation of variables and integration lead to , not .

: This is incorrect because the integration of results in , not .

: This is incorrect because the integration of results in , not .

ANSWER

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📐 Key Formulas

Important Formulas for Exercise 9.3

Formula / Concept	Description
Differential Equation	An equation that involves an independent variable, a dependent variable, and the derivatives of the dependent variable.
General Solution	A solution of a differential equation that contains a number of independent arbitrary constants equal to the order of the differential equation. It represents a family of curves.
Particular Solution	A solution obtained by assigning specific values to the arbitrary constants in the general solution. This solution is free from arbitrary constants.
Arbitrary Constants	Parameters in the general solution of a differential equation that are not fixed. The number of arbitrary constants in a general solution is equal to the order of the differential equation.
Formation of a Differential Equation	The process of eliminating the arbitrary constants from the equation of a family of curves to obtain a differential equation.
Step 1: Differentiate	Differentiate the given equation of the family of curves with respect to the independent variable (usually x).
Step 2: Repeat Differentiation	If the equation has 'n' arbitrary constants, differentiate it 'n' times to get a total of 'n+1' equations.
Step 3: Eliminate Constants	Eliminate the 'n' arbitrary constants using the 'n+1' equations. The resulting equation is the required differential equation.
Order of DE	The order of the highest derivative appearing in the differential equation. It is equal to the number of arbitrary constants in the general solution.
(d)/(dx)(x^n) = nx^n-1	Power rule for differentiation.
(d)/(dx)(e^ax) = ae^ax	Derivative of the exponential function.

Formula / Concept	Description
(d)/(dx)(\log x) = (1)/(x)	Derivative of the natural logarithm function.
(d)/(dx)(\sin(ax)) = a\cos(ax)	Derivative of the sine function.
(d)/(dx)(\cos(ax)) = -a\sin(ax)	Derivative of the cosine function.

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