

NCERT Solutions Class 12 Maths

Chapter 7: Integrals

Exercise 7.9

Document Information:

Class: 12 | Subject: Mathematics | Chapter: 7 | Exercise: 7.9

Total Questions: 10 | Academic Year: 2025-26

Source: www.ncertbooks.net | Generated: February 21, 2026

Quick Summary: In NCERT Solutions Class 12 Maths Chapter 7 Exercise 7.9, students learn to evaluate definite integrals using properties and advanced techniques. This exercise covers the Fundamental Theorem of Calculus, integration by parts for definite integrals, and symmetry properties which are essential for solving complex integration problems in CBSE board exams and competitive tests.

Key Takeaways:

- Fundamental Theorem of Calculus: $\int_a^b f(x)dx = F(b) - F(a)$ where $F'(x) = f(x)$
- Integration by parts for definite integrals: $\int_a^b u \, dv = [uv]_a^b - \int_a^b v \, du$
- Symmetry property: $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$ when $f(x)$ is even
- Property of definite integrals: $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ for substitution techniques

Complete Solutions

Question 1

QUESTION

Evaluate the integral $\int_0^1 \frac{x}{x^2+1} dx$.

SOLUTION

We need to evaluate the definite integral. This problem involves using substitution to solve the integral.

Step 1: Identify a suitable substitution

Let $u = x^2 + 1$. Then, the derivative of u with respect to x is $2x$. We can rewrite this as $\frac{1}{2} du = x dx$, or $x dx = \frac{1}{2} du$.

Step 2: Change the limits of integration

Since we are changing the variable of integration from x to u , we must also change the limits of integration.

When $x = 0$, $u = 1$.

When $x = 1$, $u = 2$.

So, the new limits of integration are from 1 to 2.

Step 3: Substitute and rewrite the integral

Using the substitution and $x dx = \frac{1}{2} du$, the integral becomes:

Step 4: Evaluate the integral

The integral of $\frac{1}{u}$ with respect to u is $\ln|u|$. Therefore:

Step 5: Apply the limits of integration

Since $x = 0$ corresponds to $u = 1$ and $x = 1$ corresponds to $u = 2$, we have:

Final Answer:

ANSWER

$$\frac{1}{2} \ln 2$$

Question 2

QUESTION

Evaluate the integral $\int_0^{\pi/2} \sqrt{\sin \phi} \cos^5 \phi \, d\phi$.

SOLUTION

We need to evaluate the definite integral .

Step 1: Rewrite the integral

We can rewrite the integral as:

Step 2: Use the Gamma function representation of the Beta function

Recall the integral representation of the Beta function:

Comparing this with our integral, we have:

and

Solving for and :

Thus, our integral becomes:

Step 3: Express the Beta function in terms of Gamma functions

Recall that . Therefore:

Step 4: Simplify using Gamma function properties

We know that . Thus, and

Substituting these values:

Using the reflection formula: , with , we have

Also, using the duplication formula: , with , we have

After further simplification, we get:

Final Answer:

ANSWER

$(64)/(231)$

Question 3

QUESTION

Evaluate the integral $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$.

SOLUTION

We need to evaluate the definite integral .

Step 1: Simplify the integrand using trigonometric substitution

Let $x = \tan \theta$. Then, $dx = \sec^2 \theta d\theta$. Also, when $x = 0$, $\theta = 0$, and when $x = 1$, $\theta = \frac{\pi}{4}$. We can rewrite the integral as:

We know that $\sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right) = 2\theta$, so the integral becomes:

Step 2: Evaluate the integral using integration by parts

Let $u = 2\theta$ and $v = \sec^2 \theta$. Then, $du = 2 d\theta$ and $dv = 2 \sec^2 \theta d\theta$. Using integration by parts, we have:

Step 3: Evaluate the remaining integral and the limits

The integral of $2\theta \sec^2 \theta$ is $2\theta \tan \theta - \log |\sec \theta|$. So, we have:

Now, we evaluate the limits:

Final Answer:

ANSWER

$(\pi/2) - \log 2$

Question 4

QUESTION

Evaluate the integral $\int_0^2 x\sqrt{x+2} dx$ (Put $x+2=t^2$).

SOLUTION

We need to evaluate the definite integral using the substitution .

Step 1: Perform the substitution

Let . Then, . Differentiating both sides with respect to , we get .

Step 2: Change the limits of integration

When , , so .

When , , so .

Step 3: Rewrite the integral in terms of

Substituting , , and , the integral becomes:

Step 4: Evaluate the integral

Final Answer:

ANSWER

$$16\sqrt{2}15(\sqrt{2}+1)$$

Question 5

QUESTION

Evaluate the integral $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx$.

SOLUTION

We need to evaluate the definite integral .

Step 1: Substitution

Let . Then, the derivative of with respect to is:

Therefore, , or .

Step 2: Change the limits of integration

When , .

When , .

So, the new limits of integration are from 1 to 0.

Step 3: Rewrite the integral with the substitution

The integral becomes:

Step 4: Reverse the limits of integration

We can change the sign of the integral by reversing the limits of integration:

Step 5: Evaluate the integral

The integral of is . Therefore:

Step 6: Apply the limits of integration

Since and , we have:

Final Answer:

Therefore, .

ANSWER

$(\pi)/4$

Question 6

QUESTION

Evaluate the integral $\int_0^2 \frac{dx}{x+4-x^2}$.

SOLUTION

We need to evaluate the definite integral. This problem involves integrating a rational function, which often requires completing the square and using partial fractions or a standard integral formula.

Step 1: Rewrite the denominator by completing the square

First, rewrite the denominator as . Now, complete the square:

So,

Step 2: Rewrite the integral

Now the integral becomes:

Step 3: Apply the standard integral formula

We use the formula: . Here, and is replaced by .

So, the integral becomes:

Step 4: Evaluate the definite integral

Now, we plug in the limits:

Final Answer:

ANSWER

$$\frac{1}{\sqrt{17}} \log \left(21 + 5\sqrt{174} \right)$$

Question 7

QUESTION

Evaluate the integral $\int_{-1}^1 \frac{dx}{x^2+2x+5}$.

SOLUTION

We need to evaluate the definite integral .

Step 1: Complete the square in the denominator

We rewrite the quadratic expression in the denominator by completing the square:

So, the integral becomes:

Step 2: Use the standard integral formula

Recall the standard integral formula:

In our case, we have . Let , so . The integral becomes:

Step 3: Evaluate the definite integral

Now, we evaluate the definite integral between the limits -1 and 1:

Step 4: Simplify using known values of arctangent

We know that and .

Therefore:

Final Answer: The value of the integral is .

ANSWER

$(\pi)/8$

Question 8

QUESTION

Evaluate the integral $\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2} \right) e^{2x} dx$.

SOLUTION

We are asked to evaluate the definite integral. This problem tests our ability to recognize and apply integration techniques, particularly integration by parts and recognizing special forms.

Step 1: Recognize the form for integration by parts

Notice that the integral has the form $\int u dv$, which often simplifies nicely. In our case, let's consider $u = \frac{1}{x} - \frac{1}{2x^2}$. Then, $dv = e^{2x}$. We can rewrite the integral to match this form.

Step 2: Rewrite the integral

We can rewrite the integral as: This is close to the form $\int u dv$, but not quite. We need to adjust the term and compensate.

Step 3: Apply integration by parts

Consider the integral $\int u dv$. We can use integration by parts, but it will not simplify easily. Instead, let's try to manipulate the integral to fit the form $\int u dv$. In our case, $u = \frac{1}{x} - \frac{1}{2x^2}$, so we want $dv = e^{2x}$. With $u = \frac{1}{x} - \frac{1}{2x^2}$, we have $dv = e^{2x}$, so $v = \frac{1}{2}e^{2x}$. Thus, the integral becomes:

Step 4: Evaluate the definite integral

Now we evaluate the expression at the limits of integration:

Step 5: Simplify the result

We can factor out to get:

Final Answer: The final answer is

ANSWER

$$\frac{e^2(e^2-2)}{4}$$

Question 9

QUESTION

The value of the integral $\int_{-1}^1 (x-x^3)^{1/3} x^4 dx$ is

SOLUTION

We need to evaluate the definite integral .

Step 1: Simplify the integrand

Let . We can rewrite the integrand as follows:

Step 2: Substitute

Let . Then, , so .

Also, , so .

When , . When , .

Now, substitute these into the integral:

Since , . Thus,

Step 3: Evaluate the integral

There was a mistake in the calculation. Let's correct it.

Let's try the substitution . Then , so . Also, , so .

This substitution does not seem to simplify the integral.

Let . Then

Let's try . Then , so . When , . When , . Then .

Step 4: Final Answer

The correct answer is 6.

The correct option is .

Option is incorrect because the integral is not zero.

Option is incorrect because the calculation leads to 6.

Option is incorrect because the calculation leads to 6.

ANSWER

3

Question 10

QUESTION

If $f(x) = \int_0^x t \sin t \, dt$, then $f'(x)$ is

SOLUTION

We are given a function defined as an integral and asked to find its derivative. This question tests the Fundamental Theorem of Calculus.

Step 1: Recall the Fundamental Theorem of Calculus (Part 1)

The Fundamental Theorem of Calculus (Part 1) states that if $f(x) = \int_a^x g(t) \, dt$, then $f'(x) = g(x)$. In simpler terms, the derivative of an integral with a variable upper limit is just the integrand evaluated at that upper limit.

Step 2: Apply the Fundamental Theorem to the given function

We have $f(x) = \int_0^x t \sin t \, dt$. Comparing this with the theorem, we see that $g(t) = t \sin t$.

Step 3: Find the derivative

According to the Fundamental Theorem of Calculus, $f'(x) = g(x)$. Therefore, we substitute x for t in the expression for g :

Step 4: Check the options

The correct option is (A).

Final Answer:

Why other options are incorrect:

(B) is incorrect because it seems to involve integrating instead of differentiating the integral.

(C) is incorrect; it might arise from differentiating incorrectly.

(D) is also incorrect; it represents the derivative of $\int_0^x t \sin t \, dt$, obtained using the product rule: $\frac{d}{dx} \int_0^x t \sin t \, dt = \int_0^x t \cos t \, dt$, but this is not what the question asks.

ANSWER

1

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Key Formulas

Important Formulas for Exercise 7.9

Formula / Concept	Description
Fundamental Theorem of Calculus	If f is a continuous function on the closed interval $[a, b]$ and F is an antiderivative of f , then: $\int_a^b f(x) \, dx = F(b) - F(a)$
Integration by Parts (Definite Integral)	If u and v are functions of x , then: $\int_a^b u(x)v'(x) \, dx = [u(x)v(x)]_a^b - \int_a^b v(x)u'(x) \, dx$
Properties of Definite Integrals	Below are the key properties used for evaluating definite integrals.
$\int_a^b f(x) \, dx = \int_a^b f(t) \, dt$	The value of a definite integral is independent of the variable of integration.
$\int_a^b f(x) \, dx = - \int_b^a f(x) \, dx$	Interchanging the limits of integration changes the sign of the integral.
$\int_a^a f(x) \, dx = 0$	If the upper and lower limits of integration are equal, the value of the integral is zero.
$\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx$	The integral can be split at any point c within the interval $[a, b]$.
$\int_a^b f(x) \, dx = \int_a^b f(a+b-x) \, dx$	A property to change the integrand.
$\int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx$	A special case of the above property, very useful in simplifying integrals.
$\int_0^{2a} f(x) \, dx = \int_0^a f(x) \, dx + \int_0^a f(2a-x) \, dx$	Property to split an integral with an upper limit of $2a$.
$\int_0^{2a} f(x) \, dx = 2 \int_0^a f(x) \, dx$, $\text{if } f(2a-x) = f(x)$	A simplification of the above property when the function is symmetric about $x=a$.
$\int_0^{2a} f(x) \, dx = 0$, $\text{if } f(2a-x) = -f(x)$	A simplification when the function has a certain type of antisymmetry.

Formula / Concept	Description
$\int_{-a}^a f(x) \, dx = 2 \int_0^a f(x) \, dx$, \text{ if } f \text{ is an even function } (f(-x) = f(x))	Property for integrating an even function over a symmetric interval.
$\int_{-a}^a f(x) \, dx = 0$, \text{ if } f \text{ is an odd function } (f(-x) = -f(x))	Property for integrating an odd function over a symmetric interval.

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