

NCERT Solutions Class 12 Maths

Chapter 7: Integrals

Exercise 7.7

Document Information:

Class: 12 | Subject: Mathematics | Chapter: 7 | Exercise: 7.7

Total Questions: 11 | Academic Year: 2025-26

Source: www.ncertbooks.net | Generated: February 21, 2026

Quick Summary: In NCERT Solutions Class 12 Maths Chapter 7 Exercise 7.7, students learn to integrate complex expressions involving square roots and quadratic forms under radicals. This exercise covers advanced integration techniques including trigonometric substitutions and completing the square method, which are essential for CBSE Class 12 board exams and competitive entrance tests.

Key Takeaways:

- Master integration of $\sqrt{a^2 - x^2}$ using trigonometric substitution $x = a\sin\theta$
- Apply completing the square technique to simplify quadratic expressions under square roots
- Learn standard integrals like $\int \sqrt{x^2 + a^2} \, dx$ and $\int \sqrt{a^2 - x^2} \, dx$
- Practice step-by-step solutions for definite integrals with radical expressions commonly asked in CBSE exams

Complete Solutions

Question 1

QUESTION

Integrate $\sqrt{4-x^2}$.

SOLUTION

We need to evaluate the integral . This integral can be solved using trigonometric substitution.

Step 1: Trigonometric Substitution

Let . Then, . This substitution is chosen because it simplifies the expression inside the square root.

Step 2: Substitute into the Integral

Substituting and into the integral, we get:

Step 3: Simplify the Integral

Factor out the 4 from the square root:

Since , we have:

Step 4: Use the Double-Angle Identity

Recall the identity . Substitute this into the integral:

Step 5: Integrate

Step 6: Convert back to

Since , we have , so .

Also, . We know , and we can find using the Pythagorean identity: .

Therefore, .

Step 7: Final Answer

Substituting back into the result, we get:

ANSWER

$$\frac{1}{2}x\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right) + C$$

Question 2

QUESTION

Integrate $\sqrt{1-4x^2}$.

SOLUTION

We need to evaluate the integral . This integral can be solved using a trigonometric substitution.

Step 1: Rewrite the integral

First, rewrite the integral to make it easier to apply a trigonometric substitution:

Step 2: Apply trigonometric substitution

Let . Then, and .

Substitute these into the integral:

Step 3: Simplify the integral

Since , we have:

Step 4: Use the identity for

Recall the identity: . Substitute this into the integral:

Step 5: Integrate

Step 6: Substitute back for

Since , we have . Also, .

Substitute these back into the expression:

Final Answer:

ANSWER

$$\frac{1}{4} \sin^{-1}(2x) + \frac{1}{2} x \sqrt{1-4x^2} + C$$

Question 3

QUESTION

Integrate $\sqrt{x^2+4x+6}$.

SOLUTION

We need to integrate the given expression: .

Step 1: Complete the square

First, we complete the square for the quadratic expression inside the square root:

So, the integral becomes:

Step 2: Apply the standard integral formula

We use the standard integral formula:

In our case, we have . Let , so . Then the integral becomes:

Step 3: Apply the formula with substitution

Using the formula with , we get:

Step 4: Substitute back

Substitute back into the expression:

Step 5: Simplify

Since , the integral becomes:

Final Answer:

ANSWER

$$(x+2)/(2)\sqrt{x^2+4x+6}+\log\left|x+2+\sqrt{x^2+4x+6}\right|+C$$

Question 4

QUESTION

Integrate $\sqrt{x^2+4x+1}$.

SOLUTION

We need to integrate the expression with respect to x .

Step 1: Complete the square

First, we complete the square for the quadratic expression inside the square root:

So, the integral becomes:

Step 2: Apply trigonometric substitution

Let $x+2 = 3\cos\theta$. Then, $dx = -3\sin\theta d\theta$.

Substituting these into the integral gives:

Step 3: Integrate

We know that $\int \cos\theta d\theta = \sin\theta + C$ and $\int \frac{1}{\cos\theta} d\theta = \log|\sec\theta + \tan\theta| + C$.

So,

Step 4: Substitute back

Since $x+2 = 3\cos\theta$, we have $\cos\theta = \frac{x+2}{3}$. Then, $\sin\theta = \frac{\sqrt{9-(x+2)^2}}{3}$.

Substituting back, we get:

Final Answer:

ANSWER

$$\frac{(x+2)\sqrt{x^2+4x+1}}{2} - \frac{3}{2} \log\left|x+2+\sqrt{x^2+4x+1}\right| + C$$

Question 5

QUESTION

Integrate $\sqrt{1-4x-x^2}$.

SOLUTION

We need to integrate the expression with respect to x .

Step 1: Complete the square

First, we rewrite the expression inside the square root by completing the square:

To complete the square for x^2 , we need to add and subtract 4 :

So,

Now the integral becomes:

Step 2: Apply the standard integral formula

We use the standard integral formula:

In our case, $a = 1$, $b = 2$, and we have $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) + C$.

Let $x = 2 - 2t$, then $dx = -2dt$. The integral becomes:

Applying the formula:

Step 3: Substitute back for x

Substituting back into the expression, we get:

Since $x = 2 - 2t$, we can rewrite the expression as:

Final Answer:

The integral of $\sqrt{1-4x-x^2}$ is $\frac{x}{2} \sqrt{1-4x-x^2} + \frac{5}{2} \sin^{-1} \left(\frac{x+2}{\sqrt{5}} \right) + \frac{(x+2)}{2} \sqrt{1-4x-x^2} + C$.

ANSWER

$\frac{x}{2} \sqrt{1-4x-x^2} + \frac{5}{2} \sin^{-1} \left(\frac{x+2}{\sqrt{5}} \right) + \frac{(x+2)}{2} \sqrt{1-4x-x^2} + C$

Question 6

QUESTION

Integrate $\sqrt{x^2+4x-5}$.

SOLUTION

We are asked to integrate the expression with respect to .

Step 1: Complete the square

We need to rewrite the quadratic expression inside the square root by completing the square:

So, the integral becomes:

Step 2: Apply the standard integral formula

We use the standard integral formula:

In our case, we have in place of and .

So, we replace with in the formula:

Step 3: Simplify the expression

We know that , so we substitute it back into the expression:

Final Answer:

Therefore, the integral is:

ANSWER

$$\frac{(x+2)}{(2)}\sqrt{x^2+4x-5}-\frac{(9)}{(2)}\log\left|x+2+\sqrt{x^2+4x-5}\right|+C$$

Question 7

QUESTION

Integrate $\sqrt{1+3x-x^2}$.

SOLUTION

We need to evaluate the integral .

Step 1: Complete the square

First, rewrite the expression inside the square root by completing the square:

To complete the square, we need to add and subtract inside the parenthesis:

So, the integral becomes:

Step 2: Apply the standard integral formula

We use the formula:

Here, and is replaced by . So we have:

Step 3: Simplify the expression

Final Answer:

ANSWER

$$\frac{(2x-3)\sqrt{1+3x-x^2} + \frac{13}{8} \sin^{-1} \left(\frac{2x-3}{\sqrt{13}} \right) + C}{4}$$

Question 8

QUESTION

Integrate $\sqrt{x^2+3x}$.

SOLUTION

We need to integrate the function with respect to x .

Step 1: Complete the square

First, we complete the square for the expression inside the square root:

So, the integral becomes:

Step 2: Use a trigonometric substitution

Let $x = \frac{3}{2} - \frac{1}{2}t^2$. Then, $dx = -t dt$.

Substituting these into the integral, we get:

Step 3: Integrate and

We know that $\int \frac{1}{t} dt = \ln|t| + C$ and $\int \frac{1}{t^2} dt = -\frac{1}{t} + C$.

So, the integral becomes:

Step 4: Substitute back for x

Since $x = \frac{3}{2} - \frac{1}{2}t^2$, we have $t = \sqrt{2(3-x)}$. Also, $dx = -t dt$.

Substituting back, we get:

Final Answer:

ANSWER

$$\frac{(2x+3)\sqrt{x^2+3x}-9}{8} \log\left|\frac{x+(3/2)+\sqrt{x^2+3x}}{2}\right| + C$$

Question 9

QUESTION

Integrate $\sqrt{(1+x^2)/9}$.

SOLUTION

We need to integrate the given expression: .

Step 1: Simplify the expression inside the square root

First, we simplify the expression inside the square root by finding a common denominator:

Step 2: Rewrite the integral

Now we can rewrite the integral as:

Step 3: Use the standard integral formula

We use the standard integral formula: , where .

So,

Step 4: Substitute back into the original integral

Now we substitute this back into our original integral:

Step 5: Simplify the result

Finally, we simplify the expression:

Final Answer:

ANSWER

$$(x)/(6)\sqrt{x^2+9}+(3)/(2)\log\left|x+\sqrt{x^2+9}\right|+C$$

Question 10

QUESTION

$\int \sqrt{1+x^2} dx$ is equal to

SOLUTION

We are asked to evaluate the indefinite integral . This requires using integration by parts or recognizing a standard integral form.

Step 1: Recall the integration by parts formula

The integration by parts formula is: .

Step 2: Choose and

Let and . Then we need to find and .

To find , we differentiate with respect to :

Step 3: Apply integration by parts

Step 4: Manipulate the integral

We can rewrite the integral as follows:

Step 5: Substitute back into the original equation

Step 6: Evaluate the remaining integral

The integral

Step 7: Solve for the original integral

Final Answer:

Option 0 is correct.

Option 1 is incorrect because it represents the integral of a different function.

Option 2 is incorrect as it also represents the integral of a different function.

Option 3 is incorrect as it also represents the integral of a different function.

ANSWER

0

Question 11

QUESTION

$\int \sqrt{x^2 - 8x + 7} dx$ is equal to

SOLUTION

We need to evaluate the integral . This involves completing the square and using a standard integral formula.

Step 1: Complete the square

We rewrite the expression inside the square root by completing the square:

So, the integral becomes:

Step 2: Apply the standard integral formula

We use the formula:

In our case, we have in place of and , so . Therefore:

Step 3: Simplify the expression

We simplify the expression inside the square root:

So, the integral becomes:

Final Answer:

The correct answer is

Why other options are incorrect:

Option 1 is incorrect because the sign of the logarithmic term is wrong and the constant is incorrect.

Option 2 is incorrect because it has instead of and an incorrect logarithmic term.

Option 3 is incorrect because it has instead of as the coefficient of the logarithmic term.

ANSWER

3

Relevant Resources

Explore more NCERT solutions (click links to visit):

| Resource | Visit Link |
|-----------------------------------|---------------------------------|
| NCERT Class 12 Sociology Textbook | Download Book → |

| Resource | Visit Link |
|-----------------------------------|----------------------------------|
| NCERT Class 10 Maths (Foundation) | View Solutions → |

Key Formulas

Important Formulas for Exercise 7.7

| Formula / Concept | Description |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\int \sqrt{x^2 - a^2} \, dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log x + \sqrt{x^2 - a^2} + C$ | This formula is used to integrate functions that are in the form of the square root of a variable squared minus a constant squared. |
| $\int \sqrt{x^2 + a^2} \, dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log x + \sqrt{x^2 + a^2} + C$ | This formula is used for integrating functions that involve the square root of the sum of a variable squared and a constant squared. |
| $\int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + C$ | This formula is applied to integrate functions in the form of the square root of a constant squared minus a variable squared. |
| Integration by Parts | A technique of integration based on the product rule of differentiation. |
| $\int u \, dv = uv - \int v \, du$ | The formula for integration by parts, where the integral of the product of two functions is found. The choice of 'u' and 'dv' is often determined by the ILATE (Inverse, Logarithmic, Algebraic, Trigonometric, Exponential) rule. |
| Fundamental Theorem of Calculus | A theorem that links the concepts of differentiating a function and integrating a function. It has two parts. |
| If f is continuous on [a, b], then the function g defined by $g(x) = \int_a^x f(t) \, dt$ for $x \in [a, b]$ is continuous on [a, b] and differentiable on (a, b), and $g'(x) = f(x)$. | The first fundamental theorem of calculus states that the derivative of the integral of a function is the original function itself. |
| If f is a continuous function on the closed interval [a, b] and F is an antiderivative of f, then $\int_a^b f(x) \, dx = F(b) - F(a)$. | The second fundamental theorem of calculus provides a way to evaluate definite integrals by using an antiderivative of the function. |

More Exercises

Visit all exercises from Chapter 7:

[Exercise 7.1 →](#)

[Exercise 7.2 →](#)

[Exercise 7.3 →](#)

[Exercise 7.4 →](#)

[Exercise 7.5 →](#)

[Exercise 7.6 →](#)

[Exercise 7.7 ✓ →](#)

[Exercise 7.8 →](#)

[Exercise 7.9 →](#)

[Exercise 7.10 →](#)

[Exercise 7. Miscellaneous →](#)

 **Complete Chapter:** [Class 12 Maths Ch 7: Integrals →](#)

© NCERT Solutions - www.ncertbooks.net

All solutions verified by subject experts for CBSE 2025-26 | **Share this PDF to help other students!**

www.ncertbooks.net