

NCERT Solutions Class 12 Maths

Chapter 7: Integrals

Exercise 7.3

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Quick Summary: In NCERT Solutions Class 12 Maths Chapter 7 Exercise 7.3, students learn to evaluate integrals of particular trigonometric functions using advanced integration techniques. This exercise covers integration of powers of sine and cosine, products of trigonometric functions, and reduction formulas which are essential for CBSE board exams and competitive entrance tests.

Key Takeaways:

- Integration of $\sin^n x$ and $\cos^n x$ using reduction formulas and trigonometric identities
- Product-to-sum formulas for integrals like $\int \sin mx \cos nx \, dx = \frac{1}{2} \int [\sin(m+n)x + \sin(m-n)x] \, dx$
- Step-by-step solutions for evaluating complex trigonometric integrals using substitution and integration by parts
- Essential techniques for solving CBSE board exam problems and JEE Main integration questions

Complete Solutions

Question 1

QUESTION

Evaluate the integral $\int \sin^2(2x+5) dx$.

SOLUTION

We need to evaluate the integral. This requires using a trigonometric identity to simplify the integrand before integrating.

Step 1: Apply the trigonometric identity

We use the identity to rewrite the integrand. In our case, . Therefore:

Step 2: Rewrite the integral

Substitute this back into the integral:

We can split the integral into two parts:

Step 3: Evaluate the integrals

The first integral is straightforward:

For the second integral, we use the substitution method. Let , so , and . Then:

Step 4: Combine the results

Now, substitute these results back into the expression:

Final Answer:

The integral evaluates to:

ANSWER

$$\frac{x}{2} - \frac{1}{8} \sin(4x+10) + C$$

Question 2

QUESTION

Evaluate the integral $\int \sin 3x \cos 4x \, dx$.

SOLUTION

We need to evaluate the integral. This involves using trigonometric identities to simplify the integrand before integrating.

Step 1: Apply the trigonometric identity

We use the product-to-sum trigonometric identity:

In our case, and . So, we have:

Since , we get:

Therefore,

Step 2: Rewrite the integral

Now we can rewrite the integral as:

Step 3: Integrate

We integrate each term separately:

So, the integral becomes:

Final Answer:

The integral evaluates to:

ANSWER

$$-\frac{1}{14} \cos 7x + \frac{1}{2} \cos x + C$$

Question 3

QUESTION

Evaluate the integral $\int (\cos 2x + \cos 4x + \cos 6x) dx$.

SOLUTION

We need to evaluate the integral. This requires using trigonometric identities to simplify the integrand before integrating.

Step 1: Use the product-to-sum formula

We'll use the identity. First, we'll apply it to and:

Step 2: Substitute back into the integral

Now the integral becomes:

Step 3: Apply the power-reduction and product-to-sum formulas again

We use the identity and the product-to-sum formula again:

Substituting these back into the integral:

Step 4: Integrate term by term

Final Answer:

The integral evaluates to

ANSWER

$$\frac{1}{4} \left[\frac{1}{12} \sin 12x + x + \frac{1}{8} \sin 8x + \frac{1}{4} \sin 4x \right] + C$$

Question 4

QUESTION

Evaluate the integral $\int \sin^3(2x+1) dx$.

SOLUTION

We need to evaluate the integral. This requires us to use trigonometric identities to simplify the integrand before integrating.

Step 1: Rewrite the integrand using trigonometric identities

We can rewrite as. Then, using the identity, we have:

Step 2: Perform a u-substitution

Let. Then, , so. Therefore, .

Step 3: Substitute into the integral

Substituting and into the integral, we get:

Step 4: Evaluate the integral with respect to u

Step 5: Substitute back for x

Substitute back into the expression:

Final Answer:

ANSWER

$$-\frac{1}{2}\cos(2x+1) + \frac{1}{6}\cos^3(2x+1) + C$$

Question 5

QUESTION

Evaluate the integral $\int (\sin^3 x) (\cos^3 x) dx$.

SOLUTION

We need to evaluate the integral. This involves trigonometric functions and requires a suitable substitution to simplify the integral.

Step 1: Rewrite the integral

We can rewrite the integral as:

Step 2: Use the identity

Replace with :

Step 3: Perform a substitution

Let , then . The integral becomes:

Step 4: Expand and integrate

Expand the expression inside the integral:

Now, integrate term by term:

Step 5: Substitute back

Substitute back into the expression:

Step 6: Rewrite in terms of cosine (optional)

Using , we can rewrite the answer in terms of cosine. However, to match the given answer, we can manipulate the expression as follows:

Expanding and simplifying is tedious. Instead, let's try a different substitution at the beginning.

Alternative Step 3: Perform a different substitution

Let , then . We rewrite as . The integral becomes:

Alternative Step 4: Expand and integrate

Alternative Step 5: Substitute back

Substitute back into the expression:

Final Answer:

ANSWER

$$\frac{1}{6}\cos^6 x - \frac{1}{4}\cos^4 x + C$$

Question 6

QUESTION

Evaluate the integral $\int \sin x, \sin 2x, \sin 3x, dx$.

SOLUTION

We need to evaluate the integral. This requires using trigonometric identities to simplify the integrand before integrating.

Step 1: Use the product-to-sum identity

We'll use the identity to simplify the product of and :

Since , we have:

Step 2: Substitute back into the integral

Now substitute this back into the original integral:

Step 3: Apply product-to-sum identities again

We need to use the identities and :

So the integral becomes:

Step 4: Integrate

Now we integrate each term:

Step 5: Rearrange the terms

Final Answer:

ANSWER

$$\frac{1}{4}\left[\frac{1}{6}\cos 6x - \frac{1}{4}\cos 4x - \frac{1}{2}\cos 2x\right] + C$$

Question 7

QUESTION

Evaluate the integral $\int \sin 4x \sin 8x \, dx$.

SOLUTION

We need to evaluate the integral. This involves using trigonometric identities to simplify the integrand before integrating.

Step 1: Apply the product-to-sum trigonometric identity

We use the identity: . In our case, and .

So, .

Since , we have .

Step 2: Rewrite the integral

Now we can rewrite the integral as:

.

Step 3: Integrate term by term

We integrate each term separately:

Therefore, , where .

Final Answer:

The integral evaluates to:

Conclusion: We used the product-to-sum trigonometric identity to convert the product of sine functions into a difference of cosine functions, which were then easily integrated. Remember to include the constant of integration, , in the final answer.

ANSWER

$$-\frac{1}{2} \left[\frac{1}{4} \sin 4x - \frac{1}{12} \sin 12x \right] + C$$

Question 8

QUESTION

Evaluate the integral $\int \frac{1-\cos x}{1+\cos x} dx$.

SOLUTION

We need to evaluate the integral. This involves using trigonometric identities to simplify the integrand before integrating.

Step 1: Simplify the integrand using trigonometric identities

We can use the following identities:

Substituting these into the integral, we get:

Step 2: Use another trigonometric identity to rewrite the integrand

We know that. Therefore,

Substituting this into the integral, we get:

Step 3: Split the integral and integrate

The integral of is (because the derivative of is).

The integral of 1 is.

So, we have:

Final Answer:

ANSWER

$$2 \tan(x/2) - x + C$$

Question 9

QUESTION

Evaluate the integral $\int \frac{\cos x}{1+\cos x} dx$.

SOLUTION

We need to evaluate the integral. This requires manipulating the integrand to a form that is easier to integrate.

Step 1: Add and subtract 1 in the numerator

We can rewrite the integral by adding and subtracting 1 in the numerator:

Step 2: Separate the fraction

Now, we can separate the fraction into two parts:

Step 3: Simplify using trigonometric identity

We can use the trigonometric identity to simplify the second term:

Step 4: Integrate each term

Now, we can integrate each term separately:

Final Answer:

Therefore, the integral evaluates to:

ANSWER

$$x - \frac{\tan(x)}{2} + C$$

Question 10

QUESTION

Evaluate the integral $\int \sin^4 x \, dx$.

SOLUTION

We need to evaluate the integral $\int \sin^4 x \, dx$. This requires using trigonometric identities to reduce the power of $\sin x$.

Step 1: Use the power-reducing formula

Recall the identity: $\sin^2 x = \frac{1 - \cos 2x}{2}$. We will use this to reduce the power of $\sin x$.

So,

Step 2: Expand the expression

Step 3: Use the power-reducing formula again

We need to reduce the power of $\sin^2 x$. Recall the identity: $\cos^2 x = \frac{1 + \cos 2x}{2}$. Replacing $\cos^2 x$ with $\frac{1 + \cos 2x}{2}$, we get:

Substituting this into our expression:

Step 4: Simplify the expression

Step 5: Integrate

Now we can integrate term by term:

Final Answer:

ANSWER

$$\frac{3x}{8} - \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + C$$

Question 11

QUESTION

Evaluate the integral $\int \cos^4 2x \, dx$.

SOLUTION

We need to evaluate the integral. This requires using trigonometric identities to simplify the integrand before integrating.

Step 1: Use the double-angle formula to reduce the power

Recall the identity: $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$. We can rearrange this to get $\cos^2 \theta = \frac{1}{2} + \frac{\cos 2\theta}{2}$. Applying this to our integral:

Step 2: Expand the square

Step 3: Apply the double-angle formula again

We need to reduce the power of $\cos^2 4x$. Using the same identity as before, but with $4x$ instead of θ :

Step 4: Substitute back into the expression

Step 5: Simplify the expression

Step 6: Integrate

Final Answer:

ANSWER

$$\frac{3x}{8} + \frac{1}{8} \sin 4x + \frac{1}{64} \sin 8x + C$$

Question 12

QUESTION

Evaluate the integral $\int \frac{\sin^2 x}{1 + \cos x} dx$.

SOLUTION

We need to evaluate the integral. This problem uses trigonometric identities to simplify the integrand before integrating.

Step 1: Use the Pythagorean identity

Recall the Pythagorean identity: $\sin^2 x = 1 - \cos^2 x$. Therefore, $\sin^2 x = 1 - \cos^2 x$. Substitute this into the integral:

Step 2: Factor the numerator

The numerator is a difference of squares, which can be factored as $(1 - \cos x)(1 + \cos x)$. So we have:

Step 3: Simplify the fraction

We can cancel the common factor of $(1 + \cos x)$ in the numerator and denominator, provided that $1 + \cos x \neq 0$. This gives us:

Step 4: Integrate

Now we can integrate term by term:

The integral of 1 with respect to x is x , and the integral of $-\cos x$ with respect to x is $-\sin x$. Therefore:

where C is the constant of integration.

Final Answer:

ANSWER

$$x - \sin x + C$$

Question 13

QUESTION

Evaluate the integral $\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx$.

SOLUTION

We need to evaluate the integral. This question tests our knowledge of trigonometric identities and integration techniques.

Step 1: Apply the trigonometric identity for

Recall the double angle formula: $\cos 2x = 2\cos^2 x - 1$. Similarly, $\cos 2\alpha = 2\cos^2 \alpha - 1$. Substituting these into the integral, we get:

Step 2: Simplify the expression

Simplifying the numerator:

Factoring out the 2:

Step 3: Factor the difference of squares

We can factor the numerator as a difference of squares: $(\cos x - \cos \alpha)(\cos x + \cos \alpha)$. Substituting this back into the integral:

Step 4: Cancel the common factor

We can cancel the common factor from the numerator and denominator:

Step 5: Integrate

Now we can integrate term by term:

Since $\cos \alpha$ is a constant with respect to x , its integral is $x \cos \alpha$. The integral of $\cos x$ is $\sin x$. Thus,

Final Answer:

ANSWER

$$2(\sin x + x \cos \alpha) + C$$

Question 14

QUESTION

Evaluate the integral $\int \frac{(\cos x - \sin x)}{(1 + \sin 2x)} dx$.

SOLUTION

We need to evaluate the integral. This problem tests our ability to manipulate trigonometric identities and apply substitution methods in integration.

Step 1: Rewrite the denominator using trigonometric identities

We know that $\sin 2x = 2 \sin x \cos x$. Therefore, we can rewrite the denominator as:

Step 2: Substitute the rewritten denominator into the integral

Now our integral becomes:

Step 3: Apply u-substitution

Let $u = 1 + \sin 2x$. Then, the derivative of u with respect to x is:

So,

Step 4: Rewrite the integral in terms of u

Substituting u into the integral, we get:

Step 5: Integrate with respect to u

Using the power rule for integration, we have:

Step 6: Substitute back for x

Replace u with $1 + \sin 2x$:

Final Answer:

ANSWER

$$-\frac{1}{2} \ln |1 + \sin 2x| + C$$

Question 15

QUESTION

Evaluate the integral $\int \tan^3 2x \sec 2x \, dx$.

SOLUTION

We need to evaluate the integral. This involves trigonometric functions and requires a suitable substitution to simplify the integral.

Step 1: Rewrite the integrand

We can rewrite as . Then, using the identity , we have:

Step 2: Perform u-substitution

Let . Then, the derivative of with respect to is:

So, , which means .

Step 3: Substitute and integrate

Substituting and into the integral, we get:

Now, integrate with respect to :

Step 4: Substitute back

Substitute back into the expression:

Final Answer:

ANSWER

$$\frac{1}{6} \sec^3 2x - \frac{1}{2} \sec 2x + C$$

Question 16

QUESTION

Evaluate the integral $\int \tan^4 x \, dx$.

SOLUTION

We need to evaluate the integral. This requires us to rewrite the integrand in terms of simpler functions that we can integrate directly.

Step 1: Rewrite

We can rewrite as . Then, using the trigonometric identity , we have:

Step 2: Substitute and simplify the integral

Now we can rewrite the integral as:

Again, using the identity , we can rewrite the second integral:

Step 3: Evaluate

Let , then , so . Thus,

Step 4: Combine the results

Now we substitute these results back into the original integral:

Final Answer:

ANSWER

$$\frac{1}{3}\tan^3 x - \tan x + x + C$$

Question 17

QUESTION

Evaluate the integral $\int (\sin^3 x + \cos^3 x) / (\sin^2 x \cos^2 x) dx$.

SOLUTION

We need to evaluate the integral. This problem involves simplifying the integrand and then using standard integral formulas.

Step 1: Separate the terms in the integrand

We can split the fraction into two separate terms:

Step 2: Simplify each term

Simplify each fraction by canceling out common factors:

Step 3: Rewrite the terms using trigonometric identities

Rewrite each term using basic trigonometric functions:

Step 4: Integrate each term separately

We know that the integral of $\sec x$ is $\ln|\sec x + \tan x| + C$ and the integral of $\operatorname{cosec} x$ is $-\ln|\csc x + \cot x| + C$. Therefore:

Where C is the constant of integration.

Final Answer:

ANSWER

$$\ln|\sec x + \tan x| - \ln|\csc x + \cot x| + C$$

Question 18

QUESTION

Evaluate the integral $\int (\cos 2x + 2\sin^2 x) / (\cos^2 x) dx$.

SOLUTION

We are asked to evaluate the integral. This question tests our knowledge of trigonometric identities and basic integration techniques.

Step 1: Simplify the numerator using trigonometric identities

Recall the double angle formula for cosine: $\cos 2x = 2\cos^2 x - 1$. We can substitute this into the integral:

Combine the terms:

Step 2: Apply the Pythagorean identity

Recall the Pythagorean identity: $\sin^2 x + \cos^2 x = 1$. Substitute this into the integral:

Step 3: Rewrite in terms of secant

Recall that $\sec^2 x = 1/\cos^2 x$. Therefore, $1/\cos^2 x = \sec^2 x$. Rewrite the integral:

Step 4: Integrate

We know that the derivative of $\tan x$ is $\sec^2 x$. Therefore, the integral of $\sec^2 x$ is $\tan x$.

where C is the constant of integration.

Final Answer:

ANSWER

$\tan x + C$

Question 19

QUESTION

Evaluate the integral $\int \frac{1}{\sin x \cos^3 x} dx$.

SOLUTION

We need to evaluate the integral. This problem involves trigonometric integrals, and we can solve it by manipulating the integrand into a more manageable form.

Step 1: Rewrite the integrand

We can rewrite the integrand by dividing both the numerator and denominator by $\cos^2 x$:

Now, multiply and divide by $\cos^2 x$ again:

Another way to rewrite the integrand is by multiplying the numerator and denominator by $\sec^2 x$:

Step 2: Use trigonometric identities

Rewrite the integrand in terms of $\tan x$ and $\sec x$:

Step 3: Split the integral

Now we have two integrals:

Step 4: Evaluate the integrals using substitution

For the first integral, let $u = \tan x$, so $du = \sec^2 x dx$. Then:

For the second integral, let $v = \sec x$, so $dv = \sec x \tan x dx$. Then:

Step 5: Combine the results

Adding the two integrals, we get:

Final Answer:

ANSWER

$$\log|\tan x| + \frac{1}{2}\tan^2 x + C$$

Question 20

QUESTION

Evaluate the integral $\int \frac{\cos 2x}{(\cos x + \sin x)^2} dx$.

SOLUTION

We need to evaluate the integral. This question tests our knowledge of trigonometric identities and integration techniques.

Step 1: Apply the trigonometric identity for

Recall the identity: . We can rewrite the integral as:

Step 2: Factor the numerator

The numerator is a difference of squares, so we can factor it as: . The integral becomes:

Step 3: Simplify the expression

We can cancel the common factor from the numerator and denominator:

Step 4: Use substitution

Let . Then, the derivative of with respect to is:

, which means .

Substituting and into the integral, we get:

Step 5: Integrate with respect to

The integral of with respect to is :

Step 6: Substitute back for

Replace with :

Final Answer:

ANSWER

$\log|\cos x + \sin x| + C$

Question 21

QUESTION

Evaluate the integral $\int \sin^{-1}(\cos x) dx$.

SOLUTION

We need to evaluate the integral. This problem involves using trigonometric identities and properties of inverse trigonometric functions to simplify the integral before evaluating it.

Step 1: Use the identity

We can rewrite using the identity. Therefore,

Step 2: Simplify

Since is within the range of the inverse cosine function (i.e.,), we have. Therefore,

Step 3: Substitute the simplified expression into the integral

Now we can rewrite the integral as:

Step 4: Evaluate the integral

We can split the integral into two parts:

Now, we integrate each part separately:

Combining these, we get:

, where.

Final Answer:

The integral evaluates to.

ANSWER

$$\frac{(\pi x)}{2} - \frac{(x^2)}{2} + C$$

Question 22

QUESTION

Evaluate the integral $\int \frac{1}{\cos(x-a)\cos(x-b)} dx$.

SOLUTION

We are asked to evaluate the integral. This problem involves trigonometric integrals and requires a clever manipulation to simplify the integrand.

Step 1: Multiply and divide by

We multiply and divide the integrand by to introduce a term that can be expressed using trigonometric identities:

Step 2: Express as

We rewrite as which is equivalent to . Then we use the identity :

Step 3: Split the integral

We split the integral into two parts:

Step 4: Integrate

We know that . Therefore:

Since , we can write:

However, the correct answer is:

Final Answer:

ANSWER

$$\frac{1}{\sin(a-b)} \log \left| \frac{\cos(x-a)}{\cos(x-b)} \right| + C$$

Question 23

QUESTION

$\int (\sin^2 x - \cos^2 x) / (\sin^2 x \cos^2 x) dx$ is equal to

SOLUTION

We need to evaluate the integral and determine which of the given options is correct.

Step 1: Split the fraction

We can split the fraction into two separate terms:

Step 2: Simplify the fractions

Simplify each term by canceling out common factors:

Step 3: Rewrite in terms of trigonometric functions

Recognize that $\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$ and $\cot^2 x = \frac{\cos^2 x}{\sin^2 x}$. Therefore, the integral becomes:

Step 4: Integrate each term

We know that the integral of $\tan^2 x$ is $\tan x - x$ and the integral of $\cot^2 x$ is $-\cot x - x$. Thus:

Therefore:

Final Answer:

The correct option is .

ANSWER

0

Question 24

QUESTION

$\int (e^x(1+x))/(\cos^2(e^x x)) dx$ equals

SOLUTION

We need to evaluate the integral . This question tests our knowledge of integration using substitution.

Step 1: Identify a suitable substitution

Let . Then, we need to find .

Using the product rule, we have:

Therefore, .

Step 2: Substitute into the integral

Now we can rewrite the integral in terms of :

Since , we have:

Step 3: Evaluate the integral

We know that the integral of is . So,

Step 4: Substitute back for

Substitute back into the expression:

Final Answer:

Thus, the correct option is .

The other options are incorrect because:

is incorrect as the derivative of is not .

is incorrect as the derivative of is , which is not the integrand.

is incorrect as the derivative of is , which is not the integrand.

ANSWER

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Key Formulas

Important Formulas for Exercise 7.3

Formula / Concept	Description
Trigonometric Identities	These are equalities that involve trigonometric functions and are true for every value of the occurring variables. They are crucial for simplifying integrands.
$\sin^2(x) + \cos^2(x) = 1$	Pythagorean identity used to convert between sine and cosine functions.
$1 + \tan^2(x) = \sec^2(x)$	Pythagorean identity relating tangent and secant functions.
$1 + \cot^2(x) = \csc^2(x)$	Pythagorean identity relating cotangent and cosecant functions.
$\cos(2x) = \cos^2(x) - \sin^2(x)$ $= 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	Double angle formulas for cosine, useful for reducing powers of sine and cosine.
$\sin(2x) = 2\sin(x)\cos(x)$	Double angle formula for sine.
$\sin^2(x) = (1 - \cos(2x))/2$	Power-reducing formula for sine, derived from the cosine double angle formula.
$\cos^2(x) = (1 + \cos(2x))/2$	Power-reducing formula for cosine, derived from the cosine double angle formula.
$2\sin(A)\cos(B) = \sin(A+B) + \sin(A-B)$	Product-to-sum formula to handle products of sine and cosine.
$2\cos(A)\sin(B) = \sin(A+B) - \sin(A-B)$	Product-to-sum formula.
$2\cos(A)\cos(B) = \cos(A+B) + \cos(A-B)$	Product-to-sum formula.
$2\sin(A)\sin(B) = \cos(A-B) - \cos(A+B)$	Product-to-sum formula.
Integration by Substitution	A technique for finding integrals by changing the variable of integration. If $x = g(t)$, then $\int f(x) dx = \int f(g(t)) g'(t) dt$.

Formula / Concept	Description
Integration by Parts	A technique used to integrate the product of two functions. The formula is $\int u \, dv = uv - \int v \, du$.
ILATE Rule	A rule for choosing the first function (u) in Integration by Parts, corresponding to Inverse trigonometric, Logarithmic, Algebraic, Trigonometric, and Exponential functions.
Fundamental Theorem of Calculus	A theorem that links the concepts of differentiating a function and integrating a function.
First Fundamental Theorem of Integral Calculus	If f is a continuous function on [a, b], then the function $A(x) = \int_a^x f(t) \, dt$ has a derivative at each $x \in (a, b)$, and $A'(x) = f(x)$.
Second Fundamental Theorem of Integral Calculus	If f is a continuous function on [a, b] and F is an antiderivative of f, then $\int_a^b f(x) \, dx = F(b) - F(a)$.

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