

NCERT Solutions Class 12 Maths

Chapter 7: Integrals

Exercise 7.2

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Quick Summary: In NCERT Solutions Class 12 Maths Chapter 7 Exercise 7.2, students learn advanced methods of integration including substitution techniques, integration by parts, and special trigonometric integrals. This exercise covers fundamental integration methods that are crucial for solving complex calculus problems and form the backbone of CBSE Class 12 board exam questions worth 15-20 marks.

Key Takeaways:

- Master integration by parts formula: $\int u \, dv = uv - \int v \, du$ for products of functions
- Apply substitution method for integrals involving logarithmic and exponential functions like $\int \frac{(\log x)^2}{x} dx$
- Solve trigonometric integrals using compound angle formulas and chain rule applications
- Practice rational function integration techniques essential for CBSE board exam success

Complete Solutions

Question 1

QUESTION

Evaluate the integral $\int (2x)/(1+x^2) dx$.

SOLUTION

We are asked to evaluate the indefinite integral. This problem can be solved using substitution.

Step 1: Identify a suitable substitution

Observe that the derivative of $1+x^2$ is $2x$, which appears in the numerator of the integrand. This suggests the substitution $u = 1+x^2$.

Step 2: Perform the substitution

Let $u = 1+x^2$. Then, differentiating both sides with respect to x , we get:

Rearranging, we have:

Step 3: Rewrite the integral in terms of u

Substituting $u = 1+x^2$ and $du = 2x dx$ into the original integral, we get:

Step 4: Evaluate the integral with respect to u

The integral of $1/u$ with respect to u is $\ln|u| + C$. Therefore:

$\int (2x)/(1+x^2) dx = \ln|1+x^2| + C$, where C is the constant of integration.

Step 5: Substitute back to express the result in terms of x

Since $1+x^2 > 0$, we substitute back to get the result in terms of x :

Since $1+x^2$ is always positive, we can drop the absolute value signs:

Final Answer:

ANSWER

$$\ln(1+x^2) + C$$

Question 2

QUESTION

Evaluate the integral $\int (\log x)^2/x \, dx$.

SOLUTION

We are asked to evaluate the integral. This problem tests our knowledge of integration using substitution.

Step 1: Identify a suitable substitution

Let $u = \log x$. Then, we need to find du in terms of x .

Differentiating with respect to x , we get:

Therefore, $du = \frac{1}{x} dx$.

Step 2: Substitute into the integral

Now we substitute u and du into the original integral:

Step 3: Evaluate the integral with respect to u

We know that $\int u^2 du = \frac{1}{3}u^3 + C$, where C is the constant of integration. Applying this rule:

Step 4: Substitute back for x

Now we substitute back into the result:

Since the domain of x is $x > 0$, we can write $\log|x|$.

Therefore,

Final Answer:

ANSWER

$$\frac{1}{3}(\log|x|)^3 + C$$

Question 3

QUESTION

Evaluate the integral $\int \frac{1}{(x + x \log x)} dx$.

SOLUTION

We need to evaluate the integral. This problem involves recognizing a suitable substitution to simplify the integral.

Step 1: Factor out from the denominator

We can rewrite the integral as:

Step 2: Identify a suitable substitution

Notice that the derivative of $x \log x$ is $1 + \log x$, which appears in the integrand. Let's use the substitution:

Step 3: Calculate the differential

Differentiating with respect to x , we get:

Therefore,

Step 4: Substitute and into the integral

Substituting and into the integral, we have:

Step 5: Evaluate the integral in terms of

The integral of with respect to u is :

Step 6: Substitute back to express the result in terms of

Substitute back into the result:

Final Answer:

ANSWER

$$\log|1 + \log x| + C$$

Question 4

QUESTION

Evaluate the integral $\int \sin x \sin(\cos x) dx$.

SOLUTION

We are asked to evaluate the integral . This requires recognizing a suitable substitution.

Step 1: Identify a suitable substitution

Notice that the derivative of $\cos x$ is $-\sin x$, which appears in the integrand (up to a constant factor). This suggests the substitution $u = \cos x$.

Step 2: Perform the substitution

Let $u = \cos x$. Then, differentiating with respect to x , we get:

This implies:

Or:

Step 3: Rewrite the integral in terms of u

Substituting $u = \cos x$ and $du = -\sin x dx$ into the original integral, we get:

Step 4: Evaluate the integral with respect to u

We know that the integral of u is $\frac{1}{2}u^2$. Therefore:

Step 5: Substitute back to express the result in terms of x

Since $u = \cos x$, we substitute back to get the final answer in terms of x :

Final Answer:

ANSWER

$$\frac{1}{2} \cos^2(\cos x) + C$$

Question 5

QUESTION

Evaluate the integral $\int \sin(ax+b) \cos(ax+b) dx$.

SOLUTION

We are asked to evaluate the integral. This integral can be solved using trigonometric identities and substitution.

Step 1: Use the trigonometric identity

Recall the trigonometric identity: $\sin(2\theta) = 2\sin\theta\cos\theta$. We can rewrite the integrand using this identity:

Therefore, the integral becomes:

Step 2: Simplify the integral

We can take the constant outside the integral:

Step 3: Perform u-substitution

Let $u = 2(ax+b)$. Then, $du = 2a dx$, so $dx = \frac{du}{2a}$. Substituting these into the integral, we get:

Step 4: Integrate

The integral of $\cos u$ is $\sin u$. So we have:

Step 5: Substitute back for u

Substitute back into the expression:

Final Answer:

ANSWER

$$-\frac{1}{4a} \cos(2(ax+b)) + C$$

Question 6

QUESTION

Evaluate the integral $\int \sqrt{ax+b} \, dx$.

SOLUTION

We need to evaluate the indefinite integral. This involves finding the antiderivative of the square root function.

Step 1: Rewrite the integral

We can rewrite the square root as a power:

Step 2: Use substitution

Let $u = ax + b$. Then, we need to find du .

Differentiating with respect to x , we get:

Therefore, $du = a \, dx$, which implies $dx = \frac{du}{a}$.

Step 3: Substitute into the integral

Now, substitute u and dx into the integral:

Step 4: Evaluate the integral with respect to u

Using the power rule for integration, we have:

Step 5: Substitute back for u

Replace with x :

Final Answer:

ANSWER

$$\frac{2}{3a}(ax+b)^{3/2} + C$$

Question 7

QUESTION

Evaluate the integral $\int x\sqrt{x+2} \, dx$.

SOLUTION

We are asked to evaluate the integral. This requires a substitution to simplify the integrand.

Step 1: Substitution

Let $u = x + 2$. Then, $du = dx$. This substitution aims to simplify the square root term.

Step 2: Rewrite the integral in terms of u

Substituting u into the integral, we get:

Step 3: Simplify the integrand

We can rewrite $x\sqrt{x+2}$ as $(u-2)\sqrt{u}$. Distribute into the parentheses:

Step 4: Integrate term by term

Now, we integrate each term separately using the power rule for integration, which states that:

Applying the power rule:

Step 5: Substitute back for x

Substitute back into the expression:

Final Answer:

ANSWER

$$\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$$

Question 8

QUESTION

Evaluate the integral $\int x\sqrt{1+2x^2} dx$.

SOLUTION

We need to evaluate the integral. This integral can be solved using substitution.

Step 1: Identify a suitable substitution

Let $u = 1 + 2x^2$. This substitution is chosen because its derivative is proportional to x , which appears in the integrand.

Step 2: Find the derivative of u with respect to x

Therefore, $\frac{du}{dx} = 4x$.

Step 3: Express $x dx$ in terms of u

From $\frac{du}{dx} = 4x$, we have $x dx = \frac{1}{4} du$.

Step 4: Substitute u and $x dx$ into the integral

Step 5: Evaluate the integral with respect to u

Using the power rule for integration, we get:

Step 6: Substitute back

Final Answer:

ANSWER

$$\frac{1}{6}(1+2x^2)^{3/2} + C$$

Question 9

QUESTION

Evaluate the integral $\int (4x+2)\sqrt{x^2+x+1} dx$.

SOLUTION

We need to evaluate the integral. This integral can be solved using substitution.

Step 1: Identify a suitable substitution

Notice that the derivative of is . The integrand contains , which is simply . This suggests the substitution:

Step 2: Find the differential

Differentiating with respect to , we get:

Therefore,

Step 3: Rewrite the integral in terms of

We have . Substituting and , we get:

Step 4: Evaluate the integral with respect to

Using the power rule for integration, , we have:

Step 5: Substitute back to express the answer in terms of

Replacing with , we get:

Final Answer:

ANSWER

$$\frac{4}{3}(x^2+x+1)^{3/2} + C$$

Question 10

QUESTION

Evaluate the integral $\int \frac{1}{(x-\sqrt{x})} dx$.

SOLUTION

We need to evaluate the integral. This integral can be solved using substitution.

Step 1: Factor out from the denominator

We can rewrite the integral as:

Step 2: Perform a u-substitution

Let $u = x - \sqrt{x}$. Then, $du = (1 - \frac{1}{2\sqrt{x}}) dx$. This implies $dx = \frac{2\sqrt{x} du}{2\sqrt{x} - 1}$, or $dx = \frac{2\sqrt{x} du}{2\sqrt{x} - 1}$.

Step 3: Rewrite the integral in terms of u

Substituting u and dx into the integral, we get:

Step 4: Integrate with respect to u

The integral of $\frac{1}{u}$ with respect to u is $\ln|u| + C$. Therefore,

Step 5: Substitute back for x

Substitute back into the expression:

Final Answer:

The integral evaluates to $2\ln|\sqrt{x}-1| + C$.

ANSWER

$$2\ln|\sqrt{x}-1| + C$$

Question 11

QUESTION

Evaluate the integral $\int \frac{x}{\sqrt{x+4}} dx, x > 0$.

SOLUTION

We need to evaluate the integral .

Step 1: Substitution

Let $u = \sqrt{x+4}$. Then, $x = u^2 - 4$ and $dx = 2u du$. This substitution aims to simplify the square root term in the denominator.

Step 2: Rewrite the integral in terms of u

Substituting u and dx into the integral, we get:

Step 3: Simplify the integrand

We can rewrite the integrand as:

Step 4: Integrate with respect to u

Now, we integrate each term with respect to u :

Using the power rule for integration, we have:

Therefore,

Step 5: Substitute back for x

Replace u with $\sqrt{x+4}$ to express the result in terms of x :

Step 6: Simplify the expression

Factor out :

Final Answer:

ANSWER

$$\frac{2}{3}\sqrt{x+4}(x-8) + C$$

Question 12

QUESTION

Evaluate the integral $\int (x^3-1)^{1/3} x^5 dx$.

SOLUTION

We need to evaluate the integral. This integral can be solved using substitution.

Step 1: Rewrite the integral

We can rewrite as. So the integral becomes:

Step 2: Perform the substitution

Let. Then,. Differentiating with respect to, we get:

, which implies

Step 3: Substitute into the integral

Substituting and into the integral, we have:

Step 4: Simplify and expand the integral

Step 5: Integrate with respect to u

Now, we integrate each term with respect to:

Step 6: Substitute back for x

Substitute back into the expression:

Final Answer:

ANSWER

$$\frac{1}{7}(x^3-1)^{7/3} + \frac{1}{4}(x^3-1)^{4/3} + C$$

Question 13

QUESTION

Evaluate the integral $\int \frac{x^2}{(2+3x^3)^3} dx$.

SOLUTION

We need to evaluate the integral. This integral can be solved using substitution.

Step 1: Identify a suitable substitution

Let $u = 2 + 3x^3$. This substitution is chosen because the derivative of u is $9x^2$, which is a multiple of $3x^2$, the term present in the numerator.

Step 2: Find the derivative of u with respect to x

Differentiating with respect to x , we get:

Step 3: Express in terms of u

From the above equation, we can write:

Step 4: Substitute u into the integral

Substituting u and du into the integral, we have:

Step 5: Simplify the integral

The terms cancel out, leaving:

Step 6: Evaluate the integral with respect to u

Using the power rule for integration, we get:

Step 7: Substitute back for x

Substituting back into the expression, we get:

Final Answer:

Therefore, the integral evaluates to:

ANSWER

$$-\frac{1}{18(2+3x^3)^2} + C$$

Question 14

QUESTION

Evaluate the integral $\int \frac{1}{x(\log x)^m} dx$, $x > 0$, $m \neq 1$.

SOLUTION

We are asked to evaluate the integral $\int \frac{1}{x(\log x)^m} dx$, where $x > 0$ and $m \neq 1$. This problem tests our ability to recognize appropriate substitutions to simplify integrals.

Step 1: Identify a suitable substitution

Notice that the derivative of $\log x$ is $\frac{1}{x}$. This suggests the substitution $u = \log x$. Then, $du = \frac{1}{x} dx$, which implies $dx = x du$.

Step 2: Perform the substitution

Substituting $u = \log x$ and $dx = x du$ into the integral, we get:

Step 3: Evaluate the integral with respect to u

Using the power rule for integration, for $m \neq 1$, we have:

Step 4: Substitute back to the original variable

Since $u = \log x$, we substitute back to get the integral in terms of x :

Final Answer:

Conclusion: By recognizing the derivative relationship between $\log x$ and $\frac{1}{x}$, we were able to simplify the integral using substitution and then apply the power rule for integration. The condition $m \neq 1$ is important because if $m = 1$, we would have $\int \frac{1}{x \log x} dx$.

ANSWER

$$\frac{(\log x)^{1-m}}{1-m} + C$$

Question 15

QUESTION

Evaluate the integral $\int \frac{x}{(9-4x^2)} dx$.

SOLUTION

We need to evaluate the indefinite integral. This integral can be solved using substitution.

Step 1: Identify a suitable substitution

Let $u = 9 - 4x^2$. This substitution is chosen because the derivative of u with respect to x is a multiple of x , which appears in the numerator of the integrand.

Step 2: Calculate the derivative of u with respect to x

Step 3: Express in terms of u

Step 4: Substitute u into the integral

Step 5: Simplify the integral

Notice that the x in the numerator cancels with the x in the denominator:

Step 6: Integrate with respect to u

The integral of $\frac{1}{u}$ with respect to u is:

Step 7: Substitute back for x

Replace u with $9 - 4x^2$:

Final Answer:

The integral evaluates to $-\frac{1}{8} \log|9-4x^2| + C$.

ANSWER

$$-\frac{1}{8} \log|9-4x^2| + C$$

Question 16

QUESTION

Evaluate the integral $\int e^{2x+3} dx$.

SOLUTION

We are asked to evaluate the indefinite integral of the exponential function with respect to x .

Step 1: Identify the appropriate integration rule

We know that the integral of e^{ax+b} is $\frac{1}{a}e^{ax+b} + C$, where C is the constant of integration. However, we have e^{2x+3} , which is a composite function. We will use a substitution method or recognize the pattern for integrating e^{ax+b} .

Step 2: Apply the integration rule for composite exponential functions

The general rule for integrating is: $\int e^{ax+b} dx = \frac{1}{a}e^{ax+b} + C$. In our case, $a=2$ and $b=3$.

Step 3: Substitute the values of a and b into the formula

Substituting $a=2$ and $b=3$ into the formula, we get:

Step 4: Verify the result by differentiation (optional)

To verify, we can differentiate the result with respect to x : $\frac{d}{dx} \left(\frac{1}{2}e^{2x+3} + C \right) = e^{2x+3}$. This matches the original integrand, so our integration is correct.

Final Answer:

Conclusion: The integral of e^{2x+3} is found by applying the rule for integrating composite exponential functions. Remember to divide by the derivative of the inner function (in this case, the derivative of $2x+3$ is 2).

ANSWER

$$\frac{1}{2}e^{2x+3} + C$$

Question 17

QUESTION

Evaluate the integral $\int \frac{x}{e^{x^2}} dx$.

SOLUTION

We are asked to evaluate the integral . This integral can be solved using substitution.

Step 1: Rewrite the integral

We can rewrite the integral as:

Step 2: Apply substitution

Let . Then, the derivative of with respect to is:

So, , which means .

Step 3: Substitute into the integral

Substituting and into the integral, we get:

Step 4: Evaluate the integral

The integral of with respect to is . Therefore:

Step 5: Substitute back for

Substitute back into the expression:

We can rewrite this as:

Final Answer:

ANSWER

$$-\frac{1}{2e^{x^2}} + C$$

Question 18

QUESTION

Evaluate the integral $\int e^{\tan^{-1}x + x^2} dx$.

SOLUTION

We need to evaluate the integral . This integral can be solved using substitution.

Step 1: Identify a suitable substitution

Notice that the derivative of is . This suggests we can use the substitution:

Step 2: Find the differential

Differentiating both sides with respect to , we get:

Therefore,

Step 3: Substitute into the integral

Now, substitute and into the original integral:

Step 4: Evaluate the integral

The integral of with respect to is simply :

Step 5: Substitute back for

Replace with to get the final answer in terms of :

Final Answer:

The integral evaluates to

ANSWER

$$e^{\tan^{-1}x + x^2} + C$$

Question 19

QUESTION

Evaluate the integral $\int \frac{e^{2x}-1}{e^{2x}+1} dx$.

SOLUTION

We need to evaluate the integral .

Step 1: Divide numerator and denominator by

Dividing both the numerator and denominator by , we get:

Step 2: Use substitution

Let . Then, the derivative of with respect to is:

Thus, .

Step 3: Substitute into the integral

Substituting and into the integral, we have:

Step 4: Integrate

The integral of with respect to is :

Step 5: Substitute back

Substitute back into the expression:

Since and are always positive, is always positive. Therefore, we can drop the absolute value sign:

Final Answer:

ANSWER

$$\log(e^x + e^{-x}) + C$$

Question 20

QUESTION

Evaluate the integral $\int e^{2x} - e^{-2x} dx$.

SOLUTION

We are asked to evaluate the integral. This problem can be solved using substitution.

Step 1: Identify a suitable substitution

Let $u = e^{2x} - e^{-2x}$. Then, we need to find du .

Step 2: Calculate the derivative of u with respect to x

Using the chain rule, we have:

Therefore,

So, $du = 2e^{2x} + 2e^{-2x} dx$.

Step 3: Rewrite the integral in terms of u

We have $du = 2e^{2x} + 2e^{-2x} dx$. Substituting into the integral, we get:

Step 4: Evaluate the integral with respect to u

The integral of $\frac{1}{2} du$ with respect to u is $\frac{1}{2} \log|u| + C$. Therefore,

Step 5: Substitute back for x

Since $u = e^{2x} - e^{-2x}$, we have:

Since u is always positive, we can drop the absolute value signs.

Final Answer:

ANSWER

$$\frac{1}{2} \log(e^{2x} - e^{-2x}) + C$$

Question 21

QUESTION

Evaluate the integral $\int \tan^2(2x-3) dx$.

SOLUTION

We need to evaluate the integral. This requires us to use a trigonometric identity to simplify the integrand before integrating.

Step 1: Use the trigonometric identity

Recall the trigonometric identity: $\tan^2 \theta = \sec^2 \theta - 1$. We can rearrange this to express in terms of \sec :

Applying this identity to our integral, we get:

Step 2: Split the integral

We can split the integral into two separate integrals:

Step 3: Evaluate the first integral

Let's evaluate $\int \sec^2(2x-3) dx$. We know that the integral of \sec^2 is \tan . However, we have inside the secant function, so we need to use a substitution.

Let $u = 2x - 3$. Then, $du = 2 dx$, so $dx = \frac{1}{2} du$.

Substituting, we get:

Substituting back, we have:

Step 4: Evaluate the second integral

The second integral is straightforward:

Step 5: Combine the results

Combining the results from Step 3 and Step 4, we get:

$\frac{1}{2} \tan(2x-3) - x + C$, where

Final Answer:

ANSWER

$$\frac{1}{2} \tan(2x-3) - x + C$$

Question 22

QUESTION

Evaluate the integral $\int \sec^2(7-4x) dx$.

SOLUTION

We are asked to evaluate the indefinite integral of with respect to . This requires knowledge of basic integral formulas and the chain rule in reverse.

Step 1: Identify the standard integral form

We know that the integral of is , where is the constant of integration.

Step 2: Apply substitution (or recognize the chain rule)

In our case, we have . Let's consider . Then, , which implies .

Step 3: Substitute and integrate

Now, we can rewrite the integral in terms of :

Using the standard integral form, we get:

Step 4: Substitute back for

Substitute back into the expression:

Final Answer:

Therefore, the integral of is:

Conclusion: We used a simple substitution to handle the composite function inside the function. Remembering the chain rule in reverse is key to solving such problems efficiently.

ANSWER

$$-(1)/(4)\tan(7-4x) + C$$

Question 23

QUESTION

Evaluate the integral $\int \sin^{-1}x \sqrt{1-x^2} dx$.

SOLUTION

We need to evaluate the integral . This integral can be solved using substitution.

Step 1: Identify a suitable substitution

Let . Then, we need to find .

Step 2: Find the derivative of with respect to

We have , so differentiating both sides with respect to , we get:

Therefore, .

Step 3: Substitute and into the integral

Now we substitute and into the original integral:

Step 4: Evaluate the new integral

The integral of with respect to is:

, where is the constant of integration.

Step 5: Substitute back to the original variable

Now we substitute back into the result:

Final Answer:

ANSWER

$$\frac{1}{2}(\sin^{-1}x)^2 + C$$

Question 24

QUESTION

Evaluate the integral $\int (2\cos x - 3\sin x)/(6\cos x + 4\sin x) dx$.

SOLUTION

We are asked to evaluate the integral. This problem involves recognizing a suitable substitution or manipulation to simplify the integral.

Step 1: Observe the relationship between the numerator and denominator

Notice that the derivative of is . The numerator looks similar, suggesting a possible logarithmic integration.

Step 2: Factor out a constant from the denominator

We can rewrite the integral as:

Step 3: Check if the numerator is a multiple of the derivative of the denominator

Let's find the derivative of : This is exactly the numerator of the integrand.

Step 4: Apply the logarithmic integration rule

Since the numerator is the derivative of the denominator, we can use the rule: In our case, and . Therefore,

Step 5: Rearrange the terms inside the logarithm

Since addition is commutative, we can rewrite as . Thus,

Final Answer:

ANSWER

$$\frac{1}{2} \log|2\sin x + 3\cos x| + C$$

Question 25

QUESTION

Evaluate the integral $\int \frac{1}{\cos^2 x(1-\tan x)^2} dx$.

SOLUTION

We are asked to evaluate the integral. This problem involves recognizing a suitable substitution to simplify the integral.

Step 1: Rewrite the integral using trigonometric identities

Recall that $\sec^2 x = \frac{1}{\cos^2 x}$. Therefore, we can rewrite the integral as:

Step 2: Perform a u-substitution

Let $u = 1 - \tan x$. Then, the derivative of u with respect to x is:

Thus, $du = -\sec^2 x dx$, or $du = -\frac{1}{\cos^2 x} dx$.

Step 3: Substitute into the integral

Substituting u and du into the integral, we get:

Step 4: Evaluate the integral with respect to u

Using the power rule for integration, $\int u^{-2} du = -u^{-1} + C$, we have:

Step 5: Substitute back for x

Substitute back into the expression:

Final Answer:

Therefore, the integral evaluates to:

ANSWER

$$\frac{1}{1-\tan x} + C$$

Question 26

QUESTION

Evaluate the integral $\int \cos\sqrt{x}\sqrt{x} dx$.

SOLUTION

We need to evaluate the integral . This integral can be solved using substitution.

Step 1: Identify a suitable substitution

Let $u = \sqrt{x}$. Then, $du = \frac{1}{2\sqrt{x}} dx$. This implies $2du = \frac{1}{\sqrt{x}} dx$. This substitution will simplify the integral.

Step 2: Substitute and rewrite the integral

Substituting $u = \sqrt{x}$ and $2du = \frac{1}{\sqrt{x}} dx$ into the integral, we get:

Step 3: Evaluate the simplified integral

The integral of $\cos u$ with respect to u is $\sin u$. Therefore:

$\int \cos u \cdot 2du = 2\sin u + C$, where C is the constant of integration.

Step 4: Substitute back to the original variable

Now, we substitute back to express the result in terms of x :

Final Answer:

Therefore, the integral is $2\sin\sqrt{x} + C$.

ANSWER

$$2\sin\sqrt{x} + C$$

Question 27

QUESTION

Evaluate the integral $\int \sqrt{\sin 2x} \cos 2x \, dx$.

SOLUTION

We are asked to evaluate the indefinite integral. This problem involves using substitution to simplify the integral.

Step 1: Identify a suitable substitution

Let $u = \sin 2x$. Then, we need to find du .

Step 2: Find the derivative of with respect to

Therefore, $du = 2 \cos 2x \, dx$.

Step 3: Rewrite the integral in terms of

We have $\sqrt{u} \cos 2x \, dx$ in the original integral, but our du has $2 \cos 2x \, dx$. So, we can write $\frac{1}{2} du = \cos 2x \, dx$. Substituting u and $\frac{1}{2} du$ into the integral gives:

Step 4: Evaluate the integral with respect to

Using the power rule for integration, we have:

Step 5: Substitute back to express the result in terms of

Since $u = \sin 2x$, we substitute this back into the expression:

Final Answer:

ANSWER

$$\frac{1}{3} (\sin 2x)^{3/2} + C$$

Question 28

QUESTION

Evaluate the integral $\int \frac{\cos x}{\sqrt{1+\sin x}} dx$.

SOLUTION

We need to evaluate the integral. This problem involves recognizing a suitable substitution to simplify the integral.

Step 1: Identify a suitable substitution

Let $u = 1 + \sin x$. Then, the derivative of u with respect to x is:

This implies that $du = \cos x dx$. This substitution simplifies the integral significantly.

Step 2: Substitute into the integral

Substituting u and du into the integral, we get:

We can rewrite this as:

Step 3: Evaluate the integral

Using the power rule for integration, which states that $\int u^n du = \frac{u^{n+1}}{n+1} + C$, we have:

Step 4: Substitute back for x

Now, we substitute back into the result:

Final Answer:

ANSWER

$$2\sqrt{1+\sin x} + C$$

Question 29

QUESTION

Evaluate the integral $\int \cot x \log(\sin x) dx$.

SOLUTION

We need to evaluate the integral . This integral can be solved using substitution.

Step 1: Identify a suitable substitution

Let . Then, we need to find .

Step 2: Find the derivative of with respect to

Using the chain rule, we have:

Therefore, .

Step 3: Substitute and into the integral

The integral becomes:

Step 4: Evaluate the new integral

The integral of with respect to is:

, where is the constant of integration.

Step 5: Substitute back for

Replace with :

Therefore, the integral .

Final Answer:

ANSWER

$$\frac{1}{2}(\log \sin x)^2 + C$$

Question 30

QUESTION

Evaluate the integral $\int \frac{\sin x}{1+\cos x} dx$.

SOLUTION

We need to evaluate the integral . This integral can be solved using substitution.

Step 1: Identify a suitable substitution

Let $u = 1 + \cos x$. Then, the derivative of u with respect to x is:

Therefore, $du = -\sin x dx$, which implies $dx = \frac{du}{-\sin x}$.

Step 2: Substitute into the integral

Now, we substitute u and dx into the integral:

Step 3: Evaluate the integral

The integral of $\frac{1}{u}$ with respect to u is $-\log|u| + C$. Therefore:

where C is the constant of integration.

Step 4: Substitute back for x

Now, we substitute back into the expression:

Final Answer:

Therefore, the integral is $-\log|1+\cos x| + C$.

ANSWER

$$-\log|1+\cos x| + C$$

Question 31

QUESTION

Evaluate the integral $\int \frac{\sin x}{(1+\cos x)^2} dx$.

SOLUTION

We need to evaluate the integral. This problem involves using substitution to simplify the integral.

Step 1: Identify a suitable substitution

Let $u = 1 + \cos x$. This is a good choice because the derivative of u is $-\sin x$, which appears in the numerator of the integrand.

Step 2: Find the derivative of u with respect to x

Step 3: Express in terms of u

From the previous step, we have $du = -\sin x dx$. Therefore,

Step 4: Substitute u into the integral

Step 5: Simplify the integral

The terms cancel out, leaving us with:

Step 6: Evaluate the integral with respect to u

Using the power rule for integration, we have:

Step 7: Substitute back for x in terms of u

Since $u = 1 + \cos x$, we have:

Final Answer:

ANSWER

$$\frac{1}{1+\cos x} + C$$

Question 32

QUESTION

Evaluate the integral $\int \frac{1}{(1+\cot x)} dx$.

SOLUTION

We need to evaluate the integral. This involves trigonometric manipulation and a clever substitution.

Step 1: Rewrite in terms of \sin and \cos

Recall that $\cot x = \frac{\cos x}{\sin x}$. Substituting this into the integral, we get:

Step 2: Simplify the expression

Multiply the numerator and denominator by $\sin x$ to simplify the fraction:

Step 3: Manipulate the numerator

We want to express the numerator as a linear combination of the denominator and its derivative. Notice that the derivative of $\cos x$ is $-\sin x$. We can write:

So, the integral becomes:

Step 4: Split the integral

Step 5: Evaluate the integrals

The first integral is straightforward:

For the second integral, notice that the numerator is the derivative of the denominator. Thus, we can use the substitution $u = \cos x + \sin x$, so $du = (-\sin x + \cos x) dx$. The second integral becomes:

Step 6: Combine the results

Final Answer:

ANSWER

$$\frac{x}{2} - \frac{1}{2} \log |\cos x + \sin x| + C$$

Question 33

QUESTION

Evaluate the integral $\int \frac{1}{1-\tan x} dx$.

SOLUTION

We need to evaluate the integral. This involves trigonometric manipulation and a clever substitution.

Step 1: Rewrite in terms of \sin and \cos

We know that $\tan x = \frac{\sin x}{\cos x}$. Substituting this into the integral, we get:

Step 2: Manipulate the numerator

We want to express the numerator in terms of the denominator and its derivative. Notice that the derivative of $\cos x$ is $-\sin x$. We can rewrite the numerator as follows:

So, the integral becomes:

Step 3: Split the integral

Now, we can split the integral into two parts:

Step 4: Evaluate the first integral

The first integral is straightforward:

Step 5: Evaluate the second integral using substitution

For the second integral, let $u = \cos x$. Then, $du = -\sin x dx$. So, $dx = \frac{du}{-\sin x}$. Therefore:

Step 6: Combine the results

Adding the two integrals together, we get:

Final Answer:

ANSWER

$$\frac{x}{2} - \frac{1}{2} \log |\cos x - \sin x| + C$$

Question 34

QUESTION

Evaluate the integral $\int \sqrt{\tan x} \sin x \cos x \, dx$.

SOLUTION

We need to evaluate the integral. This problem involves trigonometric functions and requires a clever substitution to simplify the integral.

Step 1: Rewrite the integral

We can rewrite the integral by multiplying and dividing by :

Step 2: Use trigonometric identity

Recall that . Therefore, we can write:

Step 3: Simplify the expression

We can simplify as , so the integral becomes:

Step 4: Perform u-substitution

Let . Then, , which implies . Substituting these into the integral, we get:

Step 5: Evaluate the integral

Using the power rule for integration, we have:

Step 6: Substitute back

Substituting back into the expression, we get:

Therefore, the integral evaluates to .

ANSWER

$$2\sqrt{\tan x} + C$$

Question 35

QUESTION

Evaluate the integral $\int \frac{(1+\log x)^2}{x} dx$.

SOLUTION

We are asked to evaluate the integral. This problem can be solved using substitution.

Step 1: Identify a suitable substitution

Let $u = 1 + \log x$. Then, we need to find du in terms of x .

Step 2: Find the derivative of u with respect to x

We have $u = 1 + \log x$. Differentiating both sides with respect to x , we get:

Thus, $du = \frac{1}{x} dx$.

Step 3: Substitute u into the integral

Now we can rewrite the integral in terms of u :

Step 4: Evaluate the integral with respect to u

Using the power rule for integration, we have:

Step 5: Substitute back for x

Now we substitute back into the expression:

Final Answer:

ANSWER

$$\frac{1}{3}(1+\log x)^3 + C$$

Question 36

QUESTION

Evaluate the integral $\int \frac{(x+1)(x+\log x)^2}{x} dx$.

SOLUTION

We are asked to evaluate the integral. This problem can be solved using substitution.

Step 1: Simplify the expression

We can rewrite the integral as:

Step 2: Identify a suitable substitution

Let $u = x + \log x$. Then, we need to find du .

Differentiating with respect to x , we get:

Therefore,

Step 3: Substitute into the integral

Now, we can substitute u and du into the integral:

Step 4: Evaluate the integral

The integral of u^2 with respect to u is:

$\frac{1}{3}u^3 + C$, where C is the constant of integration.

Step 5: Substitute back for x

Now, we substitute back into the expression:

Final Answer:

Therefore, the integral evaluates to:

ANSWER

$$\frac{1}{3}(x+\log x)^3 + C$$

Question 37

QUESTION

Evaluate the integral $\int x^3 \sin(\tan^{-1} x^4) \sqrt{1+x^8} dx$.

SOLUTION

We are asked to evaluate the integral. This problem involves using substitution to simplify the integral.

Step 1: Identify a suitable substitution

Let $u = \tan^{-1} x^4$. This substitution looks promising because the derivative of u appears in the numerator, and $\sqrt{1+x^8}$ appears in the denominator.

Step 2: Calculate the derivative of u with respect to x

Using the chain rule, we have:

Step 3: Express in terms of u

From the previous step, we can write: Therefore,

Step 4: Substitute u into the integral

Substituting u into the original integral, we get:

Step 5: Simplify the integral

The x^3 and $4x^3$ terms cancel out, leaving:

Step 6: Evaluate the simplified integral

The integral of $\sin u$ is $-\cos u$. Therefore,

Step 7: Substitute back for x

Substitute back into the expression:

Final Answer:

ANSWER

$$-\frac{1}{4} \cos(\tan^{-1} x^4) + C$$

Question 38

QUESTION

$\int (10x^9 + 10^x \log_e 10) / (x^{10} + 10^x) dx$ equals

SOLUTION

We are asked to evaluate the integral. This question tests our knowledge of integration by substitution and the derivative of exponential functions.

Step 1: Recognize the pattern for substitution

Observe that the numerator looks like the derivative of the denominator. Let's verify this.

Step 2: Perform the substitution

Let $u = x^{10} + 10^x$. Then, we need to find du .

We know that $\frac{d}{dx} x^{10} = 10x^9$ and $\frac{d}{dx} 10^x = 10^x \log_e 10$.

Therefore, $du = (10x^9 + 10^x \log_e 10) dx$.

So, $\int \frac{du}{u} = \ln|u| + C$.

Step 3: Rewrite the integral in terms of

The integral becomes:

Step 4: Evaluate the integral

We know that $\int \frac{1}{u} du = \ln|u| + C$.

Therefore, $\int \frac{du}{u} = \ln|u| + C$.

Step 5: Substitute back for

Substitute back into the expression:

Since x^{10} and 10^x are always positive for real x , we can drop the absolute value signs.

Final Answer: The integral evaluates to $\ln(x^{10} + 10^x) + C$.

The correct option is **C**.

Option 1 is incorrect because it has a subtraction sign instead of addition.

Option 2 is incorrect because it has an addition sign but is missing the logarithm.

Option 3 is incorrect because it has a negative exponent and subtraction.

ANSWER

3

Question 39

QUESTION

$\int \frac{dx}{\sin^2 x \cos^2 x}$ equals

SOLUTION

We need to evaluate the integral . This question tests our ability to manipulate trigonometric identities and apply basic integration formulas.

Step 1: Rewrite the integrand

We can rewrite the integrand using the identity :

Step 2: Split the fraction

Now, split the fraction into two parts:

Step 3: Simplify the fractions

Simplify each integral by canceling out common terms:

Step 4: Recognize standard integrals

We know that and . So, we have:

Step 5: Evaluate the integrals

The integral of is and the integral of is . Therefore:

Final Answer:

The correct option is .

The option is incorrect because the integral of is , not .

The option is incorrect because it simplifies to , which is not the correct integral.

The option is incorrect as it involves which does not arise from the direct integration of the given expression.

ANSWER

1

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Key Formulas

Important Formulas for Exercise 7.2

Formula / Concept	Description
Integration by Substitution	A method for finding integrals by introducing a new variable to simplify the integrand. If $x = g(t)$, then $\int f(x) dx = \int f(g(t)) g'(t) dt$.
$\int f(g(x))g'(x)dx = \int f(u)du$, where $u = g(x)$	This is the core formula for the substitution method, where the integral is transformed into a simpler form in terms of a new variable 'u'.
Integration by Parts	A technique used to integrate the product of two functions. The formula is often referred to as the product rule for integration.
$\int u \, dv = uv - \int v \, du$	The standard formula for integration by parts, where the integrand is split into two parts, 'u' and 'dv'.
ILATE Rule	A rule of thumb for choosing the first function ('u') in integration by parts. The preference order is: Inverse trigonometric, Logarithmic, Algebraic, Trigonometric, Exponential.
Fundamental Theorem of Calculus	A theorem that links the concepts of differentiating a function and integrating a function.
$\int_a^b f(x) dx = F(b) - F(a)$	The second part of the Fundamental Theorem of Calculus, where F is an antiderivative of f. It provides a way to evaluate definite integrals.
Basic Integration Formulas	Standard integration formulas that are frequently used.
$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$	Power rule for integration.
$\int \frac{1}{x} dx = \ln x + C$	Integral of $\frac{1}{x}$.
$\int e^x dx = e^x + C$	Integral of the exponential function e^x .
$\int a^x dx = \frac{a^x}{\ln a} + C$	Integral of an exponential function with base 'a'.
$\int \sin x \, dx = -\cos x + C$	Integral of the sine function.

Formula / Concept	Description
$\int \cos x \, dx = \sin x + C$	Integral of the cosine function.
$\int \sec^2 x \, dx = \tan x + C$	Integral of secant squared.
$\int \csc^2 x \, dx = -\cot x + C$	Integral of cosecant squared.
$\int \sec x \tan x \, dx = \sec x + C$	Integral of secant times tangent.
$\int \csc x \cot x \, dx = -\csc x + C$	Integral of cosecant times cotangent.
$\int \tan x \, dx = \ln \sec x + C$	Integral of the tangent function.
$\int \cot x \, dx = \ln \sin x + C$	Integral of the cotangent function.
$\int \sec x \, dx = \ln \sec x + \tan x + C$	Integral of the secant function.
$\int \csc x \, dx = \ln \csc x - \cot x + C$	Integral of the cosecant function.

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