NCERT Solutions Class 12 Maths
Chapter 6: Application of Derivatives
Exercise 6.3

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Quick Summary: In NCERT Solutions Class 12 Maths Chapter 6 Exercise 6.3, students learn to find maximum and minimum values of functions using derivative applications. This exercise covers critical concepts of increasing/decreasing functions, tangents and normals, and optimization problems which are essential for CBSE board exams and competitive tests like JEE.

Key Takeaways:

- Use first derivative test: $f'(x) = 0$ to find critical points for maxima and minima
- Apply second derivative test: $f''(x) > 0$ for local minimum, $f''(x) < 0$ for local maximum
- Master finding absolute maximum and minimum values within closed intervals using boundary analysis
- Learn to prove functions have no extrema by analyzing sign changes in derivatives

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Question 1

QUESTION

Find the maximum and minimum values, if any, of the following functions given by

(i) f(x) = (2x - 1)^2 + 3

(ii) f(x) = 9x^2 + 12x + 2

(iii) f(x) = -(x - 1)^2 + 10

(iv) g(x) = x^3 + 1

SOLUTION

We are asked to find the maximum and minimum values of the given functions using the application of derivatives.

(i)

Step 1: Analyze the function

Notice that is always non-negative, i.e., for all real values of .

Step 2: Find the minimum value

The minimum value of is 0, which occurs when , or .

Therefore, the minimum value of is .

Step 3: Check for maximum value

As increases or decreases, increases without bound. Hence, has no maximum value.

Answer: Minimum Value = 3

(ii)

Step 1: Complete the square

We can rewrite the function by completing the square:

Step 2: Analyze the function

Since for all real , the minimum value of is 0.

Step 3: Find the minimum value

The minimum value occurs when , or .

The minimum value of is .

Step 4: Check for maximum value

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As increases or decreases, increases without bound. Hence, has no maximum value.

Answer: Minimum Value = -2

(iii)

Step 1: Analyze the function

Since for all real , then .

Step 2: Find the maximum value

The maximum value of is 0, which occurs when .

Therefore, the maximum value of is .

Step 3: Check for minimum value

As increases or decreases, increases without bound, so decreases without bound. Hence, has no minimum value.

Answer: Maximum Value = 10

(iv)

Step 1: Analyze the function

Consider the behavior of as varies.

Step 2: Check for maximum and minimum values

As increases, increases without bound. As decreases (becomes more negative), decreases without bound.

Therefore, has neither a maximum nor a minimum value.

Answer: Neither minimum nor maximum value

ANSWER

(i) Minimum Value = 3

(ii) Minimum Value = -2

(iii) Maximum Value = 10

(iv) Neither minimum nor maximum value

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Question 2

QUESTION

Find the maximum and minimum values, if any, of the following functions given by

(i) f(x) = |x + 2| - 1

(ii) g(x) = -|x + 1| + 3

(iii) h(x) = \sin(2x) + 5

(iv) f(x) = |\sin 4x + 3|

(v) h(x) = x + 1,\ x \in (-1, 1)

SOLUTION

This question asks us to find the maximum and minimum values of various functions. We will analyze each function separately, considering their properties and ranges.

(i)

Step 1: Analyze the absolute value function

We know that for all real numbers . The minimum value of is 0, which occurs when .

Step 2: Find the minimum value of f(x)

Therefore, the minimum value of is . This occurs at .

Step 3: Determine if there is a maximum value

As moves away from -2 in either direction, increases without bound. Therefore, also increases without bound, and there is no maximum value.

Answer: Minimum Value = -1; No maximum value

(ii)

Step 1: Analyze the absolute value function

We know that for all real numbers . Therefore, .

Step 2: Find the maximum value of g(x)

The maximum value of is 0, which occurs when . Therefore, the maximum value of is . This occurs at .

Step 3: Determine if there is a minimum value

As moves away from -1 in either direction, increases without bound. Therefore, decreases without bound, and also decreases without bound. Thus, there is no minimum value.

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Answer: Maximum Value = 3; No minimum value

(iii)

Step 1: Analyze the sine function

We know that for all real numbers .

Step 2: Find the minimum value of h(x)

The minimum value of is -1. Therefore, the minimum value of is .

Step 3: Find the maximum value of h(x)

The maximum value of is 1. Therefore, the maximum value of is .

Answer: Minimum Value = 4; Maximum Value = 6

(iv)

Step 1: Analyze the sine function

We know that for all real numbers .

Step 2: Find the range of sin(4x) + 3

Adding 3 to all parts of the inequality, we get .

Step 3: Analyze the absolute value

Since is always positive, .

Step 4: Find the minimum and maximum values

Therefore, the minimum value of is 2, and the maximum value is 4.

Answer: Minimum Value = 2; Maximum Value = 4

(v)

Step 1: Analyze the function and the interval

The function is a linear function. The interval is , which means $-1 < x < 1$.

Step 2: Find the values at the endpoints

As approaches -1, approaches -1 + 1 = 0. As approaches 1, approaches 1 + 1 = 2.

Step 3: Determine if there are minimum and maximum values

Since the interval is open, never actually reaches -1 or 1. Therefore, never actually reaches 0 or 2. There is no minimum or maximum value.

Answer: Neither minimum nor Maximum Value

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ANSWER

(i) Minimum Value = -1; No maximum value

(ii) Maximum Value = 3; No minimum value

(iii) Minimum Value = 4; Maximum Value = 6

(iv) Minimum Value = 2; Maximum Value = 4

(v) Neither minimum nor Maximum Value

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Question 3

QUESTION

Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be:

(i) $f(x) = x^2$

(ii) $g(x) = x^3 - 3x$

(iii) $h(x) = \sin x + \cos x,\ 0 < x < \frac{\pi}{2}$

(iv) $f(x) = \sin x - \cos x,\ 0 < x < 2\pi$

(v) $f(x) = x^3 - 6x^2 + 9x + 15$

(vi) $g(x) = \frac{x}{2} + \frac{2}{x},\ x > 0$

(vii) $g(x) = \frac{1}{x^2 + 2}$

(viii) $f(x) = x\sqrt{1 - x},\ 0 < x < 1$

SOLUTION

This question tests our understanding of finding local maxima and minima of various functions using the first and second derivative tests.

(i)

Step 1: Find the first derivative

Step 2: Find critical points by setting the first derivative to zero

Step 3: Find the second derivative

Step 4: Apply the second derivative test

Since , there is a local minimum at .

Step 5: Find the local minimum value

Answer: Local minimum at , local minimum value = 0

(ii)

Step 1: Find the first derivative

Step 2: Find critical points by setting the first derivative to zero

Step 3: Find the second derivative

Step 4: Apply the second derivative test

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At , , so there is a local minimum at .

At , , so there is a local maximum at .

Step 5: Find the local minimum and maximum values

Answer: Local minimum at , local minimum value = -2; local maximum at , local maximum value = 2

(iii)

Step 1: Find the first derivative

Step 2: Find critical points by setting the first derivative to zero

Step 3: Find the second derivative

Step 4: Apply the second derivative test

At , , so there is a local maximum at .

Step 5: Find the local maximum value

Answer: Local maximum at , local maximum value =

(iv)

Step 1: Find the first derivative

Step 2: Find critical points by setting the first derivative to zero

In the interval ,

Step 3: Find the second derivative

Step 4: Apply the second derivative test

At , , so there is a local maximum at .

At , , so there is a local minimum at .

Step 5: Find the local maximum and minimum values

Answer: Local maximum at , local maximum value = ; local minimum at , local minimum value =

(v)

Step 1: Find the first derivative

Step 2: Find critical points by setting the first derivative to zero

Step 3: Find the second derivative

Step 4: Apply the second derivative test

At , , so there is a local maximum at .

At , , so there is a local minimum at .

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Step 5: Find the local maximum and minimum values

Answer: Local maximum at , local maximum value = 19; local minimum at , local minimum value = 15

(vi)

Step 1: Find the first derivative

Step 2: Find critical points by setting the first derivative to zero

Since , we only consider .

Step 3: Find the second derivative

Step 4: Apply the second derivative test

At , , so there is a local minimum at .

Step 5: Find the local minimum value

Answer: Local minimum at , local minimum value = 2

(vii)

Step 1: Find the first derivative

Step 2: Find critical points by setting the first derivative to zero

Step 3: Find the second derivative

Step 4: Apply the second derivative test

At , , so there is a local maximum at .

Step 5: Find the local maximum value

Answer: Local maximum at , local maximum value =

(viii)

Step 1: Find the first derivative

Step 2: Find critical points by setting the first derivative to zero

Step 3: Find the second derivative

Step 4: Apply the second derivative test

At , , so there is a local maximum at .

Step 5: Find the local maximum value

Answer: Local maximum at , local maximum value =

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ANSWER

(i) local minimum at x = 0, local minimum value = 0

(ii) local minimum at x = 1, local minimum value = -2; local maximum at x = -1, local maximum value = 2

(iii) local maximum at $x = \frac{\pi}{4}$, local maximum value = $\sqrt{2}$

(iv) local maximum at $x = \frac{3\pi}{4}$, local maximum value = $\sqrt{2}$; local minimum at $x = \frac{7\pi}{4}$, local minimum value = $-\sqrt{2}$

(v) local maximum at x = 1, local maximum value = 19; local minimum at x = 3, local minimum value = 15

(vi) local minimum at x = 2, local minimum value = 2

(vii) local maximum at x = 0, local maximum value = $\frac{1}{2}$

(viii) local maximum at $x = \frac{2}{3}$, local maximum value = $2\sqrt{39}$

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Question 4

QUESTION

Prove that the following functions do not have maxima or minima:

(i) f(x) = e^x

(ii) g(x) = \log x

(iii) h(x) = x^3 + x^2 + x + 1

SOLUTION

This question asks us to prove that certain functions do not have any points of maxima or minima. To do this, we will analyze their first and second derivatives.

(i)

Step 1: Find the first derivative

The first derivative of is:

Step 2: Find critical points

To find critical points, we set :

However, is always positive and never equals 0 for any real value of . Therefore, there are no critical points.

Step 3: Conclusion

Since there are no critical points, has no maxima or minima.

(ii)

Step 1: Find the first derivative

The first derivative of is:

Step 2: Find critical points

To find critical points, we set :

This equation has no solution for any real value of . Also, the domain of is , so we only consider positive values of .

Step 3: Conclusion

Since there are no critical points in the domain of , has no maxima or minima.

(iii)

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Step 1: Find the first derivative

The first derivative of is:

Step 2: Find critical points

To find critical points, we set :

We can find the discriminant of this quadratic equation:

Since the discriminant is negative, the quadratic equation has no real roots. Therefore, there are no real critical points.

Step 3: Conclusion

Since there are no real critical points, has no maxima or minima.

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Question 5

QUESTION

Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:

(i) $f(x) = x^3,\ x \in [-2, 2]$

(ii) $f(x) = \sin x + \cos x,\ x \in [0, \pi]$

(iii) $f(x) = 4x - \frac{1}{2}x^2,\ x \in \left[-2, \frac{9}{2}\right]$

(iv) $f(x) = (x - 1)^2 + 3,\ x \in [-3, 1]$

SOLUTION

This question tests our understanding of how to find absolute maximum and minimum values of a function within a given interval using derivatives.

(i)

Step 1: Find the derivative

We first find the derivative of with respect to :

Step 2: Find critical points

To find the critical points, we set :

Step 3: Evaluate the function at critical points and endpoints

We evaluate at the critical point and the endpoints and :

Step 4: Determine absolute maximum and minimum values

Comparing the values, we find:

Absolute minimum value = -8

Absolute maximum value = 8

(ii)

Step 1: Find the derivative

Step 2: Find critical points

Set :

In the interval ,

Step 3: Evaluate the function at critical points and endpoints

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Step 4: Determine absolute maximum and minimum values

Absolute minimum value = -1

Absolute maximum value =

(iii)

Step 1: Find the derivative

Step 2: Find critical points

Set :

Step 3: Evaluate the function at critical points and endpoints

Step 4: Determine absolute maximum and minimum values

Absolute minimum value = -10

Absolute maximum value = 8

(iv)

Step 1: Find the derivative

Step 2: Find critical points

Set :

Step 3: Evaluate the function at critical points and endpoints

Step 4: Determine absolute maximum and minimum values

Absolute minimum value = 3

Absolute maximum value = 19

ANSWER

(i) Absolute minimum value = -8, absolute maximum value = 8

(ii) Absolute minimum value = -1, absolute maximum value = $\sqrt{2}$

(iii) Absolute minimum value = -10, absolute maximum value = 8

(iv) Absolute minimum value = 19, absolute maximum value = 3

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Question 6

QUESTION

Find the maximum profit that a company can make, if the profit function is given by

p(x) = 41 - 72x - 18x^2.

SOLUTION

We are asked to find the maximum profit for a company, given the profit function .

Step 1: Find the first derivative of the profit function

To find the maximum profit, we first need to find the critical points of the profit function. This involves finding the first derivative, , and setting it equal to zero.

Step 2: Find the critical points

Set the first derivative equal to zero and solve for :

So, the critical point is .

Step 3: Find the second derivative of the profit function

To determine whether the critical point corresponds to a maximum or minimum, we need to find the second derivative, :

Step 4: Check the sign of the second derivative at the critical point

Since is negative for all , including , the profit function has a maximum at .

Step 5: Calculate the maximum profit

Substitute into the original profit function to find the maximum profit:

Final Answer: Maximum profit = 113 unit.

ANSWER

Maximum profit = 113 unit.

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Question 7

QUESTION

Find both the maximum value and the minimum value of 3x^4 - 8x^3 + 12x^2 - 48x + 25 on the interval [0, 3].

SOLUTION

We are asked to find the maximum and minimum values of the function on the interval . This involves finding critical points and evaluating the function at the endpoints of the interval.

Step 1: Find the derivative of the function

First, we need to find the derivative of with respect to :

Step 2: Find the critical points

To find the critical points, we set and solve for :

Divide by 12:

Factor by grouping:

The solutions are and , which gives imaginary roots. Since we are only interested in real values, the only critical point is .

Step 3: Evaluate the function at the critical point and endpoints

We need to evaluate at , , and :

Step 4: Determine the maximum and minimum values

Comparing the values, we have:

The maximum value is 25 at , and the minimum value is -39 at .

Final Answer: Minima at , minimum value = -39; Maxima at , maximum value = 25.

ANSWER

Minima at x = 2, minimum value = -39; Maxima at x = 0, maximum value = 25.

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Question 8

QUESTION

At what points in the interval [0, 2π], does the function \sin 2x attain its maximum value?

SOLUTION

We need to find the points in the interval where the function attains its maximum value.

Step 1: Find the maximum value of the sine function

The maximum value of the sine function is 1. Therefore, we need to find the values of for which .

Step 2: Solve the equation

We know that when , where is an integer.

Therefore, we have:

Dividing both sides by 2, we get:

Step 3: Find the values of in the interval

We need to find the integer values of such that .

For :

. This value lies in the interval .

For :

. This value lies in the interval .

For :

. This value is greater than , so it's outside the interval.

For :

. This value is less than 0, so it's outside the interval.

Step 4: State the points where the function attains its maximum value

The function attains its maximum value at and in the interval .

Final Answer: At and

ANSWER

At x = (π)/(4) and (5π)/(4)

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Question 9

QUESTION

What is the maximum value of the function \sin x + \cos x?

SOLUTION

We need to find the maximum value of the function .

Step 1: Rewrite the function

We can rewrite the function in the form , where is a constant and is a phase angle. To do this, we can express as:

Comparing coefficients, we have:

and

Squaring and adding these equations, we get:

Since , we have , so .

Thus, , where is such that and , which means .

So, .

Step 2: Find the maximum value

The maximum value of the sine function is 1. Therefore, the maximum value of is 1.

The maximum value of is .

Final Answer:

Maximum value =

ANSWER

Maximum value = √2

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Question 10

QUESTION

Find the maximum value of 2x^3 - 24x + 107 in the interval [1, 3]. Find the maximum value of the same function in [-3, -1].

SOLUTION

This question requires us to find the maximum value of a cubic function within given intervals using the concepts of derivatives.

Step 1: Find the derivative of the function

Let . We first find the derivative to locate critical points:

Step 2: Find the critical points

To find the critical points, we set :

So, the critical points are and .

Step 3: Analyze the interval [1, 3]

In the interval , the critical point lies within the interval. We need to evaluate the function at the endpoints and the critical point:

The maximum value in the interval is 89, which occurs at .

Step 4: Analyze the interval [-3, -1]

In the interval , the critical point lies within the interval. We need to evaluate the function at the endpoints and the critical point:

The maximum value in the interval is 139, which occurs at .

Final Answer:

Maximum at , maximum value 89; maximum at , maximum value = 139

ANSWER

Maximum at x = 3, maximum value 89; maximum at x = -2, maximum value = 139

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Question 11

QUESTION

It is given that at x = 1, the function x^4 - 62x^2 + ax + 9 attains its maximum value on the interval [0, 2]. Find the value of a.

SOLUTION

We are given a function and told that it attains its maximum value at on the interval . We need to find the value of .

Step 1: Find the derivative of the function

To find the maximum value, we first need to find the derivative of the function with respect to :

Step 2: Apply the condition for maximum at x = 1

Since the function attains its maximum value at , the derivative at must be equal to 0 (necessary condition for maxima or minima):

Step 3: Solve for a

Now we solve for :

Step 4: Check the second derivative (optional, but good practice)

To confirm that is indeed a point of maxima, we can check the second derivative:

Evaluate the second derivative at :

Since , the function has a local maximum at .

Final Answer:

The value of is 120.

ANSWER

a = 120

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Question 12

QUESTION

Find the maximum and minimum values of x + \sin 2x on [0, 2π].

SOLUTION

We need to find the maximum and minimum values of the function on the interval .

Step 1: Find the derivative of the function

First, we find the derivative of with respect to :

Step 2: Find the critical points

To find the critical points, we set :

The general solution for is:

Dividing by 2, we get the critical points in the interval :

Step 3: Evaluate the function at the critical points and endpoints

We evaluate at the critical points and the endpoints and :

Step 4: Determine the maximum and minimum values

Comparing the values, we see that the minimum value is 0 at and the maximum value is at .

Final Answer: Maximum at , maximum value = ; Minimum at , minimum value = 0

ANSWER

Maximum at $x = 2\pi$, maximum value = 2π; Minimum at $x = 0$, minimum value = 0

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Question 13

QUESTION

Find two numbers whose sum is 24 and whose product is as large as possible.

SOLUTION

We need to find two numbers that add up to 24 and have the maximum possible product. This is an optimization problem that can be solved using derivatives.

Step 1: Define the variables

Let the two numbers be and . We are given that their sum is 24:

Step 2: Express one variable in terms of the other

We can express in terms of as:

Step 3: Define the product function

Let be the product of the two numbers. Then:

Substitute into the product equation:

Step 4: Find the critical points by taking the derivative and setting it to zero

To maximize the product, we need to find the critical points of . Take the derivative of with respect to :

Set to find the critical points:

Step 5: Verify that the critical point is a maximum

To ensure that corresponds to a maximum, we can use the second derivative test. Find the second derivative of :

Since , the function has a maximum at .

Step 6: Find the other number

Now, find the value of when :

Final Answer: The two numbers are 12 and 12.

ANSWER

12, 12

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Question 14

QUESTION

Find two positive numbers x and y such that x + y = 60 and xy^3 is maximum.

SOLUTION

We need to find two positive numbers and such that their sum is 60, and the product is maximized. This is an optimization problem that can be solved using derivatives.

Step 1: Define the objective function

We want to maximize the function . This is our objective function.

Step 2: Express the objective function in terms of a single variable

We are given the constraint . We can express in terms of as . Substituting this into the objective function:

Step 3: Find the critical points

To find the critical points, we need to find the derivative of with respect to and set it to zero:

Setting :

This gives us two possible values for : or . Since we are looking for positive numbers, is not a valid solution. Therefore, .

Step 4: Check for maximum using the second derivative test

Find the second derivative of with respect to :

Evaluate the second derivative at :

Since the second derivative is negative at , this indicates a maximum.

Step 5: Find the value of x

Using the constraint , we can find the value of :

Final Answer: The two positive numbers are and .

ANSWER

45, 15

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Question 15

QUESTION

Find two positive numbers x and y such that their sum is 35 and the product x^2 y^5 is a maximum.

SOLUTION

We need to find two positive numbers and such that their sum is 35 and the product is maximized. This is an optimization problem using derivatives.

Step 1: Define the objective function and constraint

We want to maximize , subject to the constraint .

Step 2: Express the objective function in terms of one variable

From the constraint, we have . Substitute this into the objective function:

Step 3: Find the derivative of the objective function

Differentiate with respect to :

Step 4: Simplify the derivative and find critical points

Set and solve for :

So, , , or .

Since and are positive, and are not valid solutions. Thus, .

Step 5: Find the corresponding value of x

If , then .

Step 6: Verify that this is a maximum

We can use the second derivative test, but it's simpler to observe that when is slightly less than 25, is positive, and when is slightly greater than 25, is negative. This indicates a maximum.

Final Answer: The two numbers are and .

ANSWER

25, 10

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Question 16

QUESTION

Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

SOLUTION

This problem requires us to find two positive numbers that satisfy a given condition (their sum is 16) and minimize another expression (the sum of their cubes). This is a classic application of derivatives.

Step 1: Define the variables and the objective function

Let the two positive numbers be and . We are given that . We want to minimize the sum of their cubes, which we can represent as .

Step 2: Express the objective function in terms of a single variable

Since , we can write . Substitute this into the expression for :

Step 3: Find the derivative of the objective function

Now, we need to find the derivative of with respect to , denoted as :

Step 4: Find the critical points

To find the critical points, set :

or

The second equation gives which implies , which is not possible. So we consider only the first equation:

Step 5: Verify that the critical point corresponds to a minimum

Find the second derivative of with respect to :

Since , the critical point corresponds to a minimum.

Step 6: Find the value of the other variable

If , then .

Final Answer: The two positive numbers are 8 and 8.

ANSWER

8, 8

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Question 17

QUESTION

A square piece of tin of side 18\text cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible.

SOLUTION

We need to find the size of the square to cut from each corner of a tin sheet to maximize the volume of the box formed.

Step 1: Define the variables

Let be the side length of the square cut from each corner.

The side of the original square piece of tin is 18 cm.

Step 2: Express the dimensions of the box in terms of

After cutting the squares and folding up the flaps:

Length of the box,

Breadth of the box,

Height of the box,

Step 3: Write the volume of the box as a function of

The volume of the box is given by:

Step 4: Find the critical points by taking the derivative and setting it to zero

Differentiate with respect to :

Set :

Divide by 12:

Factor the quadratic equation:

So, or

Step 5: Determine the maximum volume using the second derivative test

Find the second derivative:

Evaluate at :

Since the second derivative is negative at , this indicates a maximum.

Evaluate at :

Since the second derivative is positive at , this indicates a minimum.

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Also, if , then the length and breadth of the box would be , which is not possible.

Step 6: State the final answer

Therefore, the side of the square to be cut off should be 3 cm to maximize the volume of the box.

ANSWER

3 cm

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Question 18

QUESTION

A rectangular sheet of tin 45\text cm by 24\text cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum?

SOLUTION

We need to find the side length of the square cut from each corner of a rectangular tin sheet to maximize the volume of the box formed.

Step 1: Define variables and express the volume

Let be the side length of the square cut off from each corner. The dimensions of the resulting box will be:

Length:

Width:

Height:

The volume of the box is given by:

Expanding this, we get:

Step 2: Find the first derivative of the volume with respect to x

To maximize the volume, we need to find the critical points by taking the derivative of with respect to and setting it to zero:

Step 3: Set the first derivative to zero and solve for x

Now, set :

Divide by 12:

Factor the quadratic equation:

So, or

Step 4: Find the second derivative of the volume with respect to x

To determine whether these points are maxima or minima, we find the second derivative:

Step 5: Evaluate the second derivative at the critical points

For :

Since , is a local maximum.

For :

Since , is a local minimum.

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Step 6: Check for feasibility

Since the width of the sheet is 24 cm, cutting a square of side 18 cm from each corner is not feasible because .

Therefore, the only feasible solution is .

Final Answer:

The side of the square to be cut off should be to maximize the volume of the box.

ANSWER

x = 5\text cm

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Question 19

QUESTION

Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

SOLUTION

This question requires us to prove that among all rectangles inscribed in a circle, the square has the maximum area. This involves using derivatives to find the maximum area.

Step 1: Draw a diagram and define variables

Imagine a rectangle inscribed in a circle of radius . Let the length and breadth of the rectangle be and respectively. The diagonal of the rectangle is the diameter of the circle, which is .

Step 2: Relate the variables using the circle's equation

Using the Pythagorean theorem, we have:

Thus,

Step 3: Express the area of the rectangle in terms of one variable

The area of the rectangle is given by:

Substituting , we get:

Step 4: Find the critical points by differentiating the area

To maximize the area, we need to find and set it to zero. It's easier to maximize instead of , since is always positive. Let .

Differentiating with respect to :

Setting :

Since , we have , which gives , so

Step 5: Determine the nature of the critical point

Differentiating again:

At , we have:

Since , the area is maximum at .

Step 6: Find the dimensions and conclude

If , then

Thus, , which means . The rectangle is a square.

Therefore, of all rectangles inscribed in a given circle, the square has the maximum area.

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Question 20

QUESTION

Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

SOLUTION

This question requires us to prove that for a right circular cylinder with a given surface area, the volume is maximized when the height equals the diameter of the base. This involves using derivatives to find the maximum volume.

Step 1: Define the variables and given condition

Let be the radius of the base and be the height of the cylinder. The surface area is given, and the volume is to be maximized.

The surface area of a closed cylinder is given by:

The volume of the cylinder is given by:

Step 2: Express in terms of and

From the surface area equation, we can express as:

Step 3: Express in terms of and

Substitute the expression for into the volume equation:

Step 4: Differentiate with respect to

To maximize , we need to find and set it to zero:

Step 5: Find the critical points

Set :

Step 6: Find the value of

Substitute back into the equation for :

Step 7: Interpret the result

We found that , which means the height of the cylinder is equal to twice the radius, or the diameter of the base.

Final Answer: The height of the cylinder is equal to the diameter of the base.

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Question 21

QUESTION

Of all the closed cylindrical cans (right circular), of a given volume of 100 cubic centimetres, find the dimensions of the can which has the minimum surface area?

SOLUTION

We need to find the dimensions (radius and height) of a closed cylindrical can with a fixed volume of 100 cubic centimeters such that its surface area is minimized. This is an optimization problem using derivatives.

Step 1: Define the variables and given information

Let be the radius and be the height of the cylinder. The volume is given as 100 cm. The formulas for volume and surface area are:

Step 2: Express surface area in terms of a single variable

From the volume equation, we can express in terms of :

Substitute this into the surface area equation:

Step 3: Find the critical points by taking the derivative and setting it to zero

Differentiate with respect to :

Set to find the critical points:

Step 4: Verify that this critical point gives a minimum

Take the second derivative of with respect to :

Since , , which means the surface area is minimized at this value of .

Step 5: Find the corresponding height

Substitute the value of back into the equation for :

Final Answer: The dimensions of the can with minimum surface area are:

Radius = cm and height = cm

ANSWER

Radius = \left((50)/(π)\right)^1/3 cm and height = 2\left((50)/(π)\right)^1/3 cm

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Question 22

QUESTION

A wire of length 28\text m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

SOLUTION

This question involves finding the minimum combined area of a square and a circle formed from a wire of fixed length. It tests the application of derivatives to optimization problems.

Step 1: Define variables

Let the length of the wire used for the square be meters. Then, the length of the wire used for the circle is meters.

Step 2: Express side of square and radius of circle in terms of x

The side of the square is . The radius of the circle, , can be found from the circumference: , so .

Step 3: Write the expression for the combined area

The area of the square is . The area of the circle is . The combined area, , is:

Step 4: Differentiate the area with respect to x

Step 5: Find the critical points by setting the derivative to zero

Step 6: Verify that this critical point corresponds to a minimum

Since the second derivative is positive, the critical point corresponds to a minimum.

Step 7: Calculate the lengths of the two pieces

The length of the piece for the square is meters.

The length of the piece for the circle is meters.

Final Answer: The length of the wire for the square should be m, and the length of the wire for the circle should be m.

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ANSWER

$(112)/(\pi + 4)$ m, $(28\pi)/(\pi + 4)$ m

Question 23

QUESTION

Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is (8)/(27) of the volume of the sphere.

SOLUTION

This question requires us to maximize the volume of a cone inscribed within a sphere. We'll use calculus to find the maximum volume and then compare it to the sphere's volume.

Step 1: Define the variables and the objective function

Let be the radius of the sphere. Let and be the radius and height of the inscribed cone, respectively. The volume of the cone is given by:

Step 2: Relate the cone's dimensions to the sphere's radius

Consider a cross-section of the sphere and the cone. The height of the cone can be expressed as , where is the distance from the center of the sphere to the base of the cone. By the Pythagorean theorem, we have .

Step 3: Express the volume in terms of a single variable

Substitute and into the volume equation:

Now, is a function of only.

Step 4: Find the critical points by differentiating and setting to zero

Differentiate with respect to :

Simplify:

Set :

This gives us or . Since , would give , which is not a valid cone. So, we take .

Step 5: Verify that it's a maximum

We can use the second derivative test.

At :

Since , we have a maximum at .

Step 6: Calculate the maximum volume

When , and .

So, the maximum volume is:

Step 7: Compare with the volume of the sphere

The volume of the sphere is .

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The ratio is:

Therefore, the volume of the largest cone that can be inscribed in a sphere of radius is of the volume of the sphere.

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Question 24

QUESTION

Show that the right circular cone of least curved surface and given volume has an altitude equal to √2 time the radius of the base.

SOLUTION

This question requires us to minimize the curved surface area of a right circular cone, given its volume, and then to establish a relationship between its altitude and radius.

Step 1: Define the variables and given condition

Let be the radius of the base, be the altitude (height), and be the slant height of the cone. Let be the given volume. The volume of the cone is given by:

Since is given, we have .

Step 2: Express the curved surface area in terms of only

The curved surface area is given by , where . Therefore,

Substitute into the expression for :

To minimize , it is sufficient to minimize . Let . Then,

Step 3: Differentiate and find critical points

Differentiate with respect to :

Set to find critical points:

Step 4: Find the relation between and

Substitute into the equation :

Divide both sides by :

Step 5: Verify that it is a minimum

Calculate the second derivative:

Since and , , which means we have a minimum at .

Final Answer:

The altitude is equal to times the radius of the base.

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Question 25

QUESTION

Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is \tan^-1 √2.

SOLUTION

This question tests our understanding of maximizing volume using derivatives. We need to find the semi-vertical angle of a cone with maximum volume, given its slant height.

Step 1: Define the variables and the objective function

Let be the radius, be the height, be the slant height, and be the semi-vertical angle of the cone. The volume of the cone is given by:

We want to maximize , given that is constant.

Step 2: Relate the variables using the given constraint

From the geometry of the cone, we have:

Step 3: Express the volume in terms of a single variable

Substituting and in the volume equation, we get:

Since is constant, we can consider as a function of .

Step 4: Differentiate the volume with respect to the angle

Differentiating with respect to , we get:

Step 5: Find the critical points

For maximum volume, set :

Since , . Thus:

Therefore, .

Step 6: Verify that it is a maximum

We can verify that this value of gives a maximum by checking the second derivative. However, for brevity, we assume it's a maximum.

Final Answer: The semi-vertical angle of the cone of maximum volume is .

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Question 26

QUESTION

Show that semi-vertical angle of right circular cone of given surface area and maximum volume is \sin^-1\left((1)/(3)\right).

SOLUTION

This question involves maximizing the volume of a right circular cone given its surface area. We need to show that the semi-vertical angle for maximum volume is .

Step 1: Define variables and given conditions

Let be the radius, be the height, be the slant height, be the volume, and be the surface area of the cone. We are given that is constant.

Step 2: Write the formulas for surface area and volume

The surface area is given by (curved surface area + base area). The volume is given by .

Step 3: Express in terms of and

We know that , so . Substituting this into the volume formula, we get .

Step 4: Express in terms of and

From the surface area formula, , we can express as . Substituting this into the volume formula is complex, so we'll keep for now and substitute later.

Step 5: Express in terms of

To simplify differentiation, let's consider . Now substitute :

Step 6: Differentiate with respect to and set to zero

Let . Then . Differentiating with respect to , we get:

. Setting , we have . Since , we can divide by to get , so .

Step 7: Find and in terms of

Since , we have , so . Then .

Step 8: Find the semi-vertical angle

. Therefore, .

Final Answer: The semi-vertical angle of the cone is .

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Question 27

QUESTION

The point on the curve x^2 = 2y which is nearest to the point (0, 5) is

SOLUTION

We need to find the point on the curve that is nearest to the point .

Step 1: Define the distance function

Let be a point on the curve . The distance between and is given by the distance formula:

Step 2: Express the distance in terms of a single variable

Since , we can substitute in the distance formula:

To minimize , we can minimize , which simplifies the calculation. Let .

Step 3: Find the critical points

To find the minimum value of , we take the derivative with respect to and set it to zero:

Setting , we get:

Step 4: Verify that it is a minimum

Take the second derivative of with respect to :

Since the second derivative is positive, corresponds to a minimum.

Step 5: Find the corresponding x-coordinate

Since , and , we have:

Step 6: Determine the nearest point

The points on the curve nearest to are and . Since the question only provides one of these as an option, we select that one.

Final Answer: The point on the curve nearest to is .

Why other options are incorrect:

is incorrect because it does not lie on the curve .

is incorrect because it is farther from than .

is incorrect because it is farther from than .

ANSWER

0

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Question 28

QUESTION

For all real values of x, the minimum value of (1 - x + x^2)/(1 + x + x^2) is

SOLUTION

We are asked to find the minimum value of the given expression for all real values of .

Step 1: Let equal the given expression

Let . We want to find the minimum value of .

Step 2: Rearrange the equation

Multiply both sides by :

Rearrange to form a quadratic equation in terms of :

Step 3: Apply the discriminant condition for real roots

Since is real, the discriminant of the quadratic equation must be greater than or equal to zero.

The discriminant, , is given by , where , , and .

So,

Step 4: Solve the quadratic inequality

Factor the quadratic expression:

The critical points are and .

Since the quadratic expression is less than or equal to zero, must lie between the roots:

Step 5: Determine the minimum value

The minimum value of is .

Final Answer: The minimum value is .

The correct option is .

Option 0 is incorrect because the expression is always positive for real .

Option 1 is incorrect because the minimum value is less than 1.

Option 3 is incorrect because it's the maximum value, not the minimum.

ANSWER

3

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Question 29

QUESTION

The maximum value of [x(x - 1) + 1]^1/3, 0 \le x \le 1 is

SOLUTION

We need to find the maximum value of the function on the interval .

Step 1: Simplify the function

First, let's simplify the expression inside the cube root:

So, our function becomes:

Step 2: Find the critical points

To find the critical points, we need to find the derivative of and set it to zero.

Setting , we get:

This implies , so

Step 3: Evaluate the function at the critical point and endpoints

We need to evaluate at , , and .

Step 4: Determine the maximum value

Comparing the values, we have:

, , and

Since , the maximum value of the function is 1.

Final Answer: 1

The correct option is 1.

ANSWER

2

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Key Formulas

Important Formulas for Exercise 6.3

Formula / Concept	Description
Slope of Tangent	For a curve $y = f(x)$, the slope of the tangent at a point (x_1, y_1) is given by the value of the derivative at that point: $m = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = f'(x_1)$.
Equation of Tangent	The equation of the tangent to the curve $y = f(x)$ at the point (x_1, y_1) is given by the point-slope form: $y - y_1 = m(x - x_1)$, where m is the slope of the tangent at that point.
Slope of Normal	The normal to a curve at a point is the line perpendicular to the tangent at that point. Its slope is the negative reciprocal of the tangent's slope: $m_{normal} = -\frac{1}{m_{tangent}} = -\frac{1}{\left(dy/dx\right)_{(x_1, y_1)}}$.
Equation of Normal	The equation of the normal to the curve $y = f(x)$ at the point (x_1, y_1) is: $y - y_1 = m_{normal}(x - x_1)$, which is $y - y_1 = -\frac{1}{\left(dy/dx\right)_{(x_1, y_1)}}(x - x_1)$.
Horizontal Tangent	If the tangent line is parallel to the x-axis, its slope is 0. Therefore, $\frac{dy}{dx} = 0$. The equation of the tangent is $y = y_1$.
Vertical Tangent	If the tangent line is perpendicular to the x-axis (parallel to the y-axis), its slope is undefined. This occurs when $\frac{dx}{dy} = 0$. The equation of the tangent is $x = x_1$.
Condition for Parallel Lines	Two lines are parallel if and only if their slopes are equal. If m_1 and m_2 are the slopes of two lines, then the lines are parallel if $m_1 = m_2$.
Condition for Perpendicular Lines	Two lines are perpendicular if and only if the product of their slopes is -1. If m_1 and m_2 are the slopes of two lines, then the lines are perpendicular if $m_1 \cdot m_2 = -1$.

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Exercise 6.1 →

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Miscellaneous Exercise on Chapter 6 →

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