

NCERT Solutions Class 12 Maths

Chapter 5: Continuity and Differentiability

Exercise 5.5

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Quick Summary: In NCERT Solutions Class 12 Maths Chapter 5 Exercise 5.5, students learn advanced differentiation techniques for complex functions including logarithmic differentiation, derivatives of exponential functions, and product rule applications. This exercise covers essential calculus concepts like Mean Value Theorem and Rolle's Theorem, which are crucial for CBSE Class 12 board exams and competitive entrance tests.

Key Takeaways:

- Logarithmic differentiation for functions like $y = (\log x)^{\cos x}$ using $(dy)/(dx) = y \cdot (d)/(dx)[\ln y]$
- Product rule application for trigonometric functions such as $y = \cos x \cdot \cos 2x \cdot \cos 4x$
- Differentiation of exponential functions like x^x using the formula $(d)/(dx)[x^x] = x^x(1 + \ln x)$
- Application of Mean Value Theorem and Rolle's Theorem in solving derivative problems for CBSE exam preparation

Complete Solutions

Question 1

QUESTION

Differentiate: $\cos x \cdot \cos 2x \cdot \cos 3x$

SOLUTION

We are asked to differentiate the function with respect to x .

Step 1: Apply logarithms to simplify the product

Since we have a product of functions, it's easier to use logarithmic differentiation. Take the natural logarithm of both sides:

Using the property of logarithms, $\ln(ab) = \ln a + \ln b$, we get:

Step 2: Differentiate both sides with respect to x

Differentiating both sides with respect to x , we have:

Applying the chain rule:

Step 3: Simplify the expression

We can rewrite the expression using the definition of the tangent function, $\tan x = \frac{\sin x}{\cos x}$:

Step 4: Solve for $\frac{dy}{dx}$

Multiply both sides by $\cos x \cdot \cos 2x \cdot \cos 3x$ to isolate $\frac{dy}{dx}$:

Substitute back into the equation:

Final Answer:

ANSWER

$$-\cos x \cdot \cos 2x \cdot \cos 3x \cdot [\tan x + 2 \tan 2x + 3 \tan 3x]$$

Question 2

QUESTION

Differentiate: $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

SOLUTION

We are asked to differentiate the given function, which involves a square root and a fraction of polynomial terms. We will use logarithmic differentiation to simplify the process.

Step 1: Define the function

Let

Step 2: Take the natural logarithm of both sides

Taking the natural logarithm (ln) of both sides, we get:

Using the properties of logarithms, we can simplify this to:

Further simplifying using logarithm properties:

Step 3: Differentiate both sides with respect to x

Differentiating both sides with respect to x , we get:

Step 4: Solve for $\frac{dy}{dx}$

Multiply both sides by y to isolate $\frac{dy}{dx}$:

Step 5: Substitute the original expression for y

Substitute back into the equation:

Final Answer:

The derivative is:

ANSWER

$$\frac{1}{2} \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}} \left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5} \right]$$

Question 3

QUESTION

Differentiate: $(\log x)^{\cos x}$

SOLUTION

We need to find the derivative of the function . This is a variable raised to another variable, so we'll use logarithmic differentiation.

Step 1: Take the natural logarithm of both sides

Taking the natural logarithm (\ln) of both sides of the equation , we get:

Using the power rule of logarithms, we can simplify the right side:

Step 2: Differentiate both sides with respect to x

Now, we differentiate both sides of the equation with respect to . We'll need to use the chain rule on the left side and the product rule on the right side.

Differentiating the left side:

Differentiating the right side using the product rule:

Now we need to find the derivatives of and :

So, the derivative of the right side becomes:

Therefore, we have:

Step 3: Solve for dy/dx

Multiply both sides by to isolate :

Substitute :

Final Answer:

ANSWER

$$(\log x)^{\cos x} \left[(\cos x) / (x \log x) - \sin x, \log(\log x) \right]$$

Question 4

QUESTION

Differentiate: $x^x \cdot 2^{\sin x}$

SOLUTION

We are asked to differentiate the function . This problem involves differentiating functions of the form , which requires logarithmic differentiation.

Step 1: Define the functions

Let . We can write , where and .

Step 2: Differentiate $u = x^x$

Taking the natural logarithm of both sides of , we get:

Differentiating both sides with respect to :

Therefore,

Step 3: Differentiate $v = 2^{(\sin x)}$

Taking the natural logarithm of both sides of , we get:

Differentiating both sides with respect to :

Therefore,

Step 4: Find dy/dx

Since , we have

Final Answer:

ANSWER

$$x^x(1+\log x) \cdot 2^{\sin x} \cdot \cos x \cdot \log 2$$

Question 5

QUESTION

Differentiate: $(x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$

SOLUTION

We need to find the derivative of the function .

Since the function is a product of several terms, it's easier to use logarithmic differentiation.

Step 1: Take the natural logarithm of both sides

Taking the natural logarithm (ln) of both sides of the equation, we get:

Using the properties of logarithms, we can expand the right side:

Step 2: Differentiate both sides with respect to x

Differentiating both sides with respect to , we get:

Step 3: Isolate

Multiply both sides by to isolate :

Step 4: Substitute the original expression for y

Substitute back into the equation:

Step 5: Simplify the expression

Factor out from the expression:

Now, expand and simplify the expression inside the brackets:

Therefore,

Final Answer:

ANSWER

$$(x+3)(x+4)^2(x+5)^3(9x^2+70x+133)$$

Question 6

QUESTION

Differentiate: $\left(x+1\right)^x+x^{\left(1+x\right)}$

SOLUTION

We are asked to differentiate the function . This requires using logarithmic differentiation.

Step 1: Define the function and split it into two parts

Let , where and .

Then .

Step 2: Differentiate

Taking logarithm on both sides, we get .

Differentiating with respect to , we have

Step 3: Differentiate

Taking logarithm on both sides, we get .

Differentiating with respect to , we have

Step 4: Combine the results

Final Answer:

ANSWER

$$\left(x+1\right)^x\left[\frac{\left(x^2-1\right)}{\left(x^2+1\right)}+\log\left(x+1\right)\right]+x^{\left(1+x\right)}\left[\frac{\left(x+1-\log x\right)}{\left(x^2\right)}\right]$$

Question 7

QUESTION

Differentiate: $(\log x)^x + x^{\log x}$

SOLUTION

We need to find the derivative of the function. Let's denote $y = (\log x)^x + x^{\log x}$. We will differentiate each term separately using logarithms.

Step 1: Separate the terms

Let $u = (\log x)^x$ and $v = x^{\log x}$. Then $y = u + v$.

Step 2: Differentiate

Take the natural logarithm of both sides: $\log u = x \log(\log x)$.

Differentiate both sides with respect to x :

Using the product rule:

Therefore, $\frac{1}{u} \frac{du}{dx} = \log(\log x) + \frac{1}{\log x}$.

Step 3: Differentiate

Take the natural logarithm of both sides: $\log v = \log x \cdot \log x$.

Differentiate both sides with respect to x :

Therefore, $\frac{1}{v} \frac{dv}{dx} = \log x + 1$.

Step 4: Combine the derivatives

.

Final Answer:

ANSWER

$$(\log x)^{x-1} \left[1 + \log x \cdot \log(\log x) \right] + 2x^{\log x - 1} \cdot \log x$$

Question 8

QUESTION

Differentiate: $(\sin x)^x + \sin^{-1}\sqrt{x}$

SOLUTION

We need to find the derivative of the function . This requires using logarithmic differentiation for the first term and the chain rule for the second term.

Step 1: Differentiate

Let . Taking the natural logarithm of both sides:

Differentiating both sides with respect to :

Using the product rule:

Step 2: Differentiate

Let . Using the chain rule:

Step 3: Combine the derivatives

Let . Then

Final Answer:

The derivative is

ANSWER

$$(\sin x)^x(x \cot x + \log \sin x) + \frac{1}{2\sqrt{x-x^2}}$$

Question 9

QUESTION

Differentiate: $x^{\sin x} + (\sin x)^{\cos x}$

SOLUTION

We need to find the derivative of the function. This involves differentiating functions raised to variable powers, which requires logarithmic differentiation.

Step 1: Define the functions

Let $y = x^{\sin x} + (\sin x)^{\cos x}$. Then the given function is y , and we want to find $\frac{dy}{dx}$.

Step 2: Differentiate

Take the natural logarithm of both sides:

Differentiate both sides with respect to x :

Solve for $\frac{dy}{dx}$:

Step 3: Differentiate

Take the natural logarithm of both sides:

Differentiate both sides with respect to x :

Solve for $\frac{dy}{dx}$:

Step 4: Combine the derivatives

Now, $\frac{dy}{dx}$, so

Final Answer:

ANSWER

$$x^{\sin x} \left[\frac{\sin x}{x} + \cos x \log x \right] + (\sin x)^{\cos x} \left[\cos x \cot x - \sin x \log(\sin x) \right]$$

Question 10

QUESTION

Differentiate: $x^x \cos x + (x^2+1)/(x^2-1)$

SOLUTION

We need to find the derivative of the function . Let's break this down into two parts and differentiate them separately.

Step 1: Differentiate

Let . To differentiate this, we'll use logarithmic differentiation. Take the natural logarithm of both sides:

Now differentiate both sides with respect to :

Use the product rule:

Substitute :

Step 2: Differentiate

Let . We'll use the quotient rule:

Step 3: Combine the results

The derivative of the entire function is the sum of the derivatives of the two parts:

Final Answer:

ANSWER

$$x^x \cos x \left[\cos x (1 + \log x) - x \sin x, \log x \right] - (4x) / ((x^2 - 1)^2)$$

Question 11

QUESTION

Differentiate: $(x \cos x)^x + (x \sin x)^{1/x}$

SOLUTION

We are asked to differentiate the function . This problem involves differentiating functions of the form , which requires the use of logarithms.

Step 1: Define the functions

Let and . Then the given function is , and we need to find .

Step 2: Differentiate

Take the natural logarithm of both sides:

Differentiate both sides with respect to :

Step 3: Differentiate

Take the natural logarithm of both sides:

Differentiate both sides with respect to :

Step 4: Combine the results

Final Answer:

ANSWER

$$(x \cos x)^x \left[1 - x \tan x + \log(x \cos x) \right] + (x \sin x)^{1/x} \left[(x \cot x + 1 - \log(x \sin x)) / (x^2) \right]$$

Question 12

QUESTION

Find $(dy)/(dx)$ if $x^y + y^x = 1$.

SOLUTION

We need to find for the implicit function .

Step 1: Differentiate both sides with respect to

Differentiating with respect to , we get:

Since the derivative of a constant is 0:

Step 2: Differentiate with respect to

Let . Taking the natural logarithm of both sides:

Differentiating both sides with respect to :

Step 3: Differentiate with respect to

Let . Taking the natural logarithm of both sides:

Differentiating both sides with respect to :

Step 4: Substitute the derivatives back into the equation

Step 5: Solve for

Final Answer:

ANSWER

$$-y \cdot x^y - 1 + y^x \log yx^y \log x + x \cdot y^x - 1$$

Question 13

QUESTION

Find $(dy)/(dx)$ if $y^x = x^y$.

SOLUTION

We are asked to find the derivative for the implicit function .

Step 1: Take the natural logarithm of both sides

Taking the natural logarithm (ln) of both sides allows us to use properties of logarithms to simplify the equation:

Using the power rule of logarithms, we get:

Step 2: Differentiate both sides with respect to x

We will use the product rule for differentiation, which states that . Remember that is a function of , so we'll need to use the chain rule when differentiating terms involving .

So we have:

Step 3: Isolate terms

Rearrange the equation to group all terms containing on one side:

Factor out :

Step 4: Solve for

Divide both sides by :

Step 5: Simplify the expression

Multiply the numerator and denominator by to clear the fractions within the fraction:

Factor out a from the numerator:

Final Answer:

ANSWER

$$(y)/(x)\left((y-x)\log y)/(x-y)\log x\right)$$

Question 14

QUESTION

Find $(dy)/(dx)$ if $(\cos x)^y = (\cos y)^x$.

SOLUTION

This question tests our understanding of implicit differentiation and logarithmic differentiation. We need to find $(dy)/(dx)$ for the given equation .

Step 1: Take the natural logarithm of both sides

Taking the natural logarithm (\ln) of both sides simplifies the equation by allowing us to bring down the exponents:

Using the property $\ln(a^b) = b \ln a$, we get:

Step 2: Differentiate both sides with respect to x

Now, differentiate both sides of the equation with respect to x , using the product rule and chain rule:

Applying the product rule, $\frac{d}{dx}(x \ln y) = x \frac{dy}{dx} + \ln y$, we have:

Step 3: Apply the chain rule

Using the chain rule, we get:

Step 4: Rearrange to solve for $(dy)/(dx)$

Collect terms with $(dy)/(dx)$ on one side:

Now, isolate $(dy)/(dx)$:

Final Answer:

ANSWER

$$\frac{(y \tan x + \log \cos y)}{(x \tan y + \log \cos x)}$$

Question 15

QUESTION

Find $(dy)/(dx)$ if $xy=e^{(x-y)}$.

SOLUTION

We are asked to find the derivative for the implicit function .

Step 1: Take the natural logarithm of both sides

Taking the natural logarithm (ln) of both sides of the equation simplifies the exponential term:

Using logarithm properties, we get:

Step 2: Differentiate both sides with respect to x

Differentiate both sides of the equation with respect to :

This gives us:

Step 3: Isolate terms

Rearrange the equation to group all terms containing on one side:

Factor out :

Step 4: Solve for

Divide both sides by to isolate :

Step 5: Simplify the expression

Simplify the numerator and denominator by finding a common denominator:

Final Answer:

ANSWER

$$(y(x-1))/(x(y+1))$$

Question 16

QUESTION

Find the derivative of the function $f(x)=(1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find $f'(1)$.

SOLUTION

We are asked to find the derivative of the function and then evaluate the derivative at .

Step 1: Simplify the function

Notice that we can simplify the function by multiplying by and dividing by :

Using the identity , we can simplify the numerator:

Step 2: Differentiate the simplified function

Now we differentiate using the quotient rule:

Step 3: Evaluate

We cannot directly substitute into the expression for because it results in an indeterminate form . However, we can use L'Hôpital's rule. Differentiating the numerator and denominator with respect to :

Numerator derivative:

Denominator derivative:

Applying L'Hôpital's rule:

Applying L'Hôpital's rule again:

Numerator derivative:

Denominator derivative:

Now, substitute :

Step 4: Alternative approach using logarithmic differentiation

Taking logarithm on both sides of :

Differentiating both sides with respect to :

Now, substitute :

Final Answer:

ANSWER

$$(1+x)(1+x^2)(1+x^4)(1+x^8)\left[11+x+(2x)/(1+x^2)+(4x^3)/(1+x^4)+(8x^7)/(1+x^8)\right]; f'(1)=120$$

Question 17

QUESTION

Differentiate $(x^2-5x+8)(x^3+7x+9)$ in three ways: (i) by using product rule (ii) by expanding the product (iii) by logarithmic differentiation. Do they all give the same answer?

SOLUTION

This question requires us to differentiate the given function using three different methods: the product rule, expansion, and logarithmic differentiation. We then need to verify if all methods yield the same result.

(i) Using the Product Rule

The product rule states that if u and v are functions of x , then $(uv)' = u'v + uv'$. Let $u = x^2 - 5x + 8$ and $v = x^3 + 7x + 9$.

Step 1: Find the derivatives of u and v .

$u' = 2x - 5$

Step 2: Apply the product rule.

Step 3: Expand and simplify.

(ii) Expanding the Product First

Step 1: Expand the original function.

Step 2: Differentiate the expanded polynomial.

(iii) Logarithmic Differentiation

Step 1: Take the natural logarithm of both sides of $y = (x^2 - 5x + 8)(x^3 + 7x + 9)$.

Step 2: Differentiate both sides with respect to x .

Step 3: Multiply both sides by y .

Step 4: Simplify.

Final Answer: In all three methods, we obtain the same derivative: $5x^4 - 20x^3 + 45x^2 - 52x + 11$.

ANSWER

$$5x^4 - 20x^3 + 45x^2 - 52x + 11$$

Question 18

QUESTION

If u , v and w are functions of x , show that $\frac{d}{dx}(u \cdot v \cdot w) = \frac{du}{dx}vw + u\frac{dv}{dx}w + uv\frac{dw}{dx}$.

SOLUTION

This question asks us to prove the derivative of a product of three functions. It tests our understanding of the product rule of differentiation and how to apply it iteratively.

Step 1: Apply the product rule to the first two functions

Let's consider $u \cdot v$ as a single function. Then we have:

Applying the product rule, which states $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$, we get:

Step 2: Apply the product rule again

Now we need to find $\frac{d}{dx}(u \cdot v \cdot w)$. Applying the product rule again:

Step 3: Substitute back into the original equation

Substitute this result back into the equation from Step 1:

Step 4: Simplify

Distribute the $\frac{d}{dx}$ in the first term:

Final Answer:

This result shows that the derivative of a product of three functions is the sum of the derivatives of each function multiplied by the other two functions. This is a direct extension of the product rule for two functions.

ANSWER

$$\frac{d}{dx}(uvw) = \frac{du}{dx}vw + u\frac{dv}{dx}w + uv\frac{dw}{dx}$$

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Key Formulas

Important Formulas for Exercise 5.5

Formula / Concept	Description
Logarithmic Differentiation	A method used to differentiate functions by first taking the natural logarithm of both sides and then differentiating implicitly. It is particularly useful for functions of the form $y = [f(x)]^{g(x)}$ or functions involving products, quotients, or powers.
Logarithm Product Rule	$\log(ab) = \log(a) + \log(b)$ This rule converts a product inside a logarithm into a sum of logarithms, simplifying differentiation.
Logarithm Quotient Rule	$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$ This rule converts a division inside a logarithm into a difference of logarithms.
Logarithm Power Rule	$\log(a^b) = b \log(a)$ This rule brings the exponent down as a multiplier, which is key for differentiating functions of the form $f(x)^{g(x)}$.
Derivative of Natural Logarithm	$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$ The basic derivative of the natural logarithm function.
Chain Rule with Natural Logarithm	$\frac{d}{dx}(\ln(u)) = \frac{1}{u} \cdot \frac{du}{dx}$ Used when the argument of the logarithm is a function of x . This is the foundation of logarithmic differentiation.
Implicit Differentiation	A technique used to find the derivative of a function that is not explicitly solved for one variable. When differentiating $\ln(y)$ with respect to x , we get $\frac{1}{y} \frac{dy}{dx}$.
Product Rule for Differentiation	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$ Often used after applying logarithmic properties to differentiate the resulting expression.
Derivative of x^x	$\frac{d}{dx}(x^x) = x^x(1 + \ln(x))$ A common example in this exercise, solved using logarithmic differentiation.

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