NCERT Solutions Class 12 Maths
Chapter 5: Continuity and Differentiability
Exercise 5.1

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Quick Summary: In NCERT Solutions Class 12 Maths Chapter 5 Exercise 5.1, students learn the fundamental concepts of continuity of functions at specific points and intervals. This exercise covers proving continuity using limits, examining polynomial and algebraic functions, and understanding the theoretical foundations that are essential for CBSE Class 12 board exams and competitive entrance tests.

Key Takeaways:

- A function f(x) is continuous at x = a if \lim_x \to a f(x) = f(a)
- Polynomial functions like f(x) = 5x - 3 and f(x) = 2x^2 - 1 are continuous everywhere
- Power functions f(x) = x^n are continuous at all real numbers for positive integer values of n
- Students learn step-by-step methods to prove continuity using left-hand and right-hand limits

Complete Solutions

Question 1

QUESTION

Prove that the function f(x)=5x-3 is continuous at x=0, at x=-3 and at x=5.

SOLUTION

We need to prove that the function is continuous at , , and . To do this, we will show that the limit of the function as approaches each of these points is equal to the function's value at that point.

Step 1: Continuity at

We need to show that .

First, let's find :

Now, let's find the limit as approaches 0:

Since , the function is continuous at .

Step 2: Continuity at

We need to show that .

First, let's find :

Now, let's find the limit as approaches -3:

Since , the function is continuous at .

Step 3: Continuity at

We need to show that .

First, let's find :

Now, let's find the limit as approaches 5:

Since , the function is continuous at .

Conclusion:

Since the function is continuous at , , and , as the limit of the function at each point equals the function's value at that point.

ANSWER

Question 2

QUESTION

Examine the continuity of the function f(x)=2x^2-1 at x=3.

SOLUTION

We need to examine the continuity of the function at .

Step 1: Define continuity at a point

A function is continuous at a point if the following three conditions are met:

1. is defined (i.e., the function exists at).
2. exists (i.e., the limit of the function exists as approaches).
3. (i.e., the limit of the function as approaches is equal to the value of the function at).

Step 2: Check if is defined

We need to find the value of the function at :

Since , the function is defined at .

Step 3: Find the limit of as approaches 3

We need to find .

Since is a polynomial function, we can directly substitute to find the limit:

Thus, .

Step 4: Compare the limit and the function value

We found that and .

Therefore, .

Step 5: Conclusion

Since all three conditions for continuity are met, the function is continuous at .

ANSWER

f is continuous at x=3.

Question 3

QUESTION

Examine the following functions for continuity:

(a) f(x)=x-5

(b) f(x)=(1)/(x-5),\; x\ne 5

(c) f(x)=(x^2-25)/(x+5),\; x\ne -5

(d) f(x)=|x-5|

SOLUTION

We need to examine the continuity of the given functions.

(a)

Step 1: This is a polynomial function. Polynomial functions are continuous everywhere.

Step 2: To show this, let be any real number. Then, we need to show that .

Step 3: .

Step 4: Also, .

Step 5: Since , the function is continuous for all real numbers.

(b)

Step 1: This is a rational function. Rational functions are continuous everywhere except where the denominator is zero.

Step 2: The denominator is , which is zero when . However, the function is defined for .

Step 3: Let be any real number such that . Then, we need to show that .

Step 4: .

Step 5: Also, .

Step 6: Since , the function is continuous for all .

(c)

Step 1: We can simplify the function by factoring the numerator: .

Step 2: Since , we can cancel the terms: .

Step 3: Now, the function is for . This is a polynomial function (a line) with a hole at .

Step 4: Let be any real number such that . Then, we need to show that .

Step 5: .

Step 6: Also, .

Step 7: Since , the function is continuous for all .

(d)

Step 1: This is an absolute value function. Absolute value functions are continuous everywhere.

Step 2: We can rewrite the function as:

Step 3: For , , which is continuous.

Step 4: For , , which is continuous.

Step 5: At , we need to check the left-hand limit and the right-hand limit.

Step 6: .

Step 7: .

Step 8: Also, .

Step 9: Since , the function is continuous at .

Therefore, is continuous for all real numbers.

(a), (b), (c) and (d) are all continuous functions.

ANSWER

(a), (b), (c) and (d) are all continuous functions.

Question 4

QUESTION

Prove that the function f(x)=x^n is continuous at x=n, where n is a positive integer.

SOLUTION

We need to prove that the function is continuous at , where is a positive integer. To prove continuity at a point, we need to show that the limit of the function as approaches that point is equal to the function's value at that point.

Step 1: Define the condition for continuity

A function is continuous at if the following condition is met:

Step 2: Evaluate the limit of as approaches

We need to find . Since is a polynomial function, we can directly substitute into the function to find the limit.

Step 3: Evaluate

We are given . We need to find the value of the function at .

Step 4: Compare the limit and the function value

We found that:

and

Since , the function is continuous at .

Final Answer: The function is continuous at .

ANSWER

Question 5

QUESTION

Is the function f defined by

f(x)=\begincases x, & x\le 1 \\ 5, & x>1 \endcases

continuous at x=0? At x=1? At x=2?

SOLUTION

We need to determine the continuity of the function at , , and , where is defined as:

Step 1: Continuity at

Since , we use the definition in the neighborhood of .

Thus, .

The limit as approaches 0 is:

Since , the function is continuous at .

Step 2: Continuity at

We need to check the left-hand limit (LHL) and the right-hand limit (RHL) at .

LHL:

RHL:

Also, (since).

Since LHL RHL, the limit does not exist. Therefore, the function is not continuous at .

Step 3: Continuity at

Since , we use the definition in the neighborhood of .

Thus, .

The limit as approaches 2 is:

Since , the function is continuous at .

Final Answer:

is continuous at and ; not continuous at .

ANSWER

f is continuous at x=0 and x=2; not continuous at x=1.

Question 6

QUESTION

Find all points of discontinuity of f, where f is defined by

f(x)=\begincases 2x+3, & x\le 2 \\ 2x-3, & x>2 \endcases

SOLUTION

We are asked to find the points of discontinuity of the function defined piecewise.

Step 1: Identify the potential point of discontinuity

Since is defined differently for and , the potential point of discontinuity is at .

Step 2: Check the left-hand limit at

The left-hand limit (LHL) is given by:

Since is a polynomial, we can directly substitute :

Step 3: Check the right-hand limit at

The right-hand limit (RHL) is given by:

Since is a polynomial, we can directly substitute :

Step 4: Check the value of the function at

Since , we use the definition :

Step 5: Compare LHL, RHL, and the function value

We have:

LHL = 7

RHL = 1

Since LHL RHL, the limit does not exist. Therefore, the function is discontinuous at .

Final Answer:

Discontinuous at .

ANSWER

Discontinuous at x=2.

Question 7

QUESTION

Find all points of discontinuity of f, where f is defined by

f(x)=\begincases |x|+3, & x\le -3 \\ -2x, & -3

SOLUTION

We are given a piecewise function and asked to find all points of discontinuity.

Step 1: Identify potential points of discontinuity

Piecewise functions are potentially discontinuous at the points where the definition of the function changes. In this case, the potential points of discontinuity are and .

Step 2: Check continuity at

For to be continuous at , the left-hand limit (LHL), right-hand limit (RHL), and the value of the function at must be equal.

LHL at :

RHL at :

Value of the function at :

Since LHL = RHL = , the function is continuous at .

Step 3: Check continuity at

For to be continuous at , the LHL, RHL, and the value of the function at must be equal.

LHL at :

RHL at :

Value of the function at :

Since LHL and RHL , LHL RHL, the function is discontinuous at .

Step 4: Conclusion

The function is discontinuous at .

ANSWER

Discontinuous at x=3.

Question 8

QUESTION

Find all points of discontinuity of f, where f is defined by

f(x)=\begincases (|x|)/(x), & x\ne 0 \\ 0, & x=0 \endcases

SOLUTION

We are given a piecewise function and asked to find all points of discontinuity.

Step 1: Analyze the function

The function is defined as:

We need to check for discontinuity at because the function's definition changes at this point.

Step 2: Evaluate the left-hand limit (LHL) at

For , . Therefore, for , .

So, the left-hand limit is:

Step 3: Evaluate the right-hand limit (RHL) at

For , . Therefore, for , .

So, the right-hand limit is:

Step 4: Evaluate the function at

We are given that .

Step 5: Check for continuity

For a function to be continuous at , the following must be true:

We have:

Since , the function is discontinuous at .

Final Answer:

Discontinuous at .

ANSWER

Discontinuous at x=0.

Question 9

QUESTION

Find all points of discontinuity of f, where f is defined by

f(x)=\begincases (x)/(|x|), & x<0 \\ -1, & x\ge 0 \endcases

SOLUTION

We are given a piecewise function and asked to find all points of discontinuity.

Step 1: Analyze the function definition

The function is defined as:

For , , so .

For , .

Step 2: Rewrite the function

Therefore, we can rewrite the function as:

This simplifies to:

Step 3: Check for discontinuity

The function is a constant function. Constant functions are continuous everywhere.

To confirm, we can check the limit at :

Also, .

Since , the function is continuous at .

Step 4: Conclusion

Since for all , the function is continuous everywhere. Therefore, there are no points of discontinuity.

Final Answer: No point of discontinuity.

ANSWER

No point of discontinuity.

Question 10

QUESTION

Find all points of discontinuity of f, where f is defined by

f(x)=\begincases x+1, & x\ge 1 \\ x^2+1, & x<1 \endcases

SOLUTION

We are asked to find the points of discontinuity of the function defined piecewise as for and for .

Step 1: Check for discontinuity at

Since the function is defined differently on either side of , we need to check for discontinuity at this point. We need to evaluate the left-hand limit (LHL), the right-hand limit (RHL), and the value of the function at .

Step 2: Calculate the left-hand limit (LHL) at

The left-hand limit is given by:

Substituting , we get:

Step 3: Calculate the right-hand limit (RHL) at

The right-hand limit is given by:

Substituting , we get:

Step 4: Calculate the value of the function at

Since , we use the definition :

Step 5: Compare LHL, RHL, and

We have:

LHL = 2

RHL = 2

= 2

Since LHL = RHL = , the function is continuous at .

Step 6: Check for discontinuity elsewhere

For , is a polynomial function, and polynomial functions are continuous everywhere.

For , is also a polynomial function, and polynomial functions are continuous everywhere.

Final Answer:

No point of discontinuity.

ANSWER

No point of discontinuity.

Question 11

QUESTION

Find all points of discontinuity of f, where f is defined by

f(x)=\begincases x^3-3, & x\le 2 \\ x^2+1, & x>2 \endcases

SOLUTION

We are asked to find the points of discontinuity of the function defined piecewise.

Step 1: Analyze the function

The function is defined as for and for . Each piece is a polynomial, and polynomials are continuous everywhere. Therefore, we only need to check for discontinuity at the point where the function definition changes, which is .

Step 2: Check for continuity at

For a function to be continuous at a point , the following three conditions must be met:

1. must be defined.
2. must exist.
3. .

Step 3: Evaluate

Since for , we have .

Step 4: Evaluate the left-hand limit

The left-hand limit is given by:

.

Step 5: Evaluate the right-hand limit

The right-hand limit is given by:

.

Step 6: Compare the left-hand limit, right-hand limit, and function value

We have:

, , and .

Since , the function is continuous at .

Step 7: Conclusion

Since the function is continuous for , , and at , there are no points of discontinuity.

Final Answer: No point of discontinuity.

ANSWER

No point of discontinuity.

Question 12

QUESTION

Find all points of discontinuity of f, where f is defined by

f(x)=\begincases x^10-1, & x\le 1 \\ x^2, & x>1 \endcases

SOLUTION

We are asked to find the points of discontinuity of the function defined piecewise as for and for .

Step 1: Identify potential points of discontinuity

Since and are both polynomials, they are continuous everywhere. The only potential point of discontinuity is where the function definition changes, which is at .

Step 2: Check the left-hand limit at

The left-hand limit is given by:

Since is a polynomial, we can directly substitute :

Step 3: Check the right-hand limit at

The right-hand limit is given by:

Since is a polynomial, we can directly substitute :

Step 4: Check the value of the function at

Since for , we have:

Step 5: Compare the left-hand limit, right-hand limit, and function value

We have:

Since the left-hand limit and the right-hand limit are not equal, the limit does not exist. Therefore, the function is discontinuous at .

Final Answer: is discontinuous at .

ANSWER

f is discontinuous at x=1.

Question 13

QUESTION

Is the function defined by

f(x)=\begincases x+5, & x\le 1 \\ x-5, & x>1 \endcases

a continuous function?

SOLUTION

We are asked to determine if the given piecewise function is continuous.

Step 1: Understand the definition of continuity

A function is continuous at a point if the following three conditions are met:

(i) is defined.

(ii) exists.

(iii) .

For a piecewise function, we need to check continuity at the points where the function definition changes. In this case, that point is .

Step 2: Check the value of the function at

Since for , we have:

So, is defined.

Step 3: Check the limit as approaches 1 from the left

For , . Therefore:

Step 4: Check the limit as approaches 1 from the right

For , . Therefore:

Step 5: Compare the left-hand limit and the right-hand limit

Since and , the limit does not exist because the left-hand limit and right-hand limit are not equal.

Step 6: Conclusion

Since the limit does not exist, the function is not continuous at .

ANSWER

f is not continuous at x=1.

Question 14

QUESTION

Discuss the continuity of the function f, where f is defined by

f(x)=\begincases 3, & 0\le x\le 1 \\ 4, & 1

SOLUTION

We are asked to discuss the continuity of the given piecewise function over the interval .

Step 1: Identify potential points of discontinuity

Since is defined piecewise, we need to check for continuity at the points where the function definition changes, which are and . We also need to consider the endpoints of the interval, and .

Step 2: Check continuity at

We need to check if .

The left-hand limit is: .

The right-hand limit is: .

The function value at is: .

Since , the function is discontinuous at .

Step 3: Check continuity at

We need to check if .

The left-hand limit is: .

The right-hand limit is: .

The function value at is: .

Since , the function is discontinuous at .

Step 4: Check continuity at and

At , we only need to check the right-hand limit and the function value. Since , the function is continuous from the right at .

At , we only need to check the left-hand limit and the function value. Since , the function is continuous from the left at .

Final Answer:

is not continuous at and .

ANSWER

f is not continuous at $x=1$ and $x=3$.

Question 15

QUESTION

Discuss the continuity of the function f, where f is defined by

f(x)=\begincases 2x, & x<0 \\ 0, & 0\le x\le 1 \\ 4x, & x>1 \endcases

SOLUTION

We need to discuss the continuity of the given piecewise function at all points in its domain.

Step 1: Analyze continuity at

For , . This is a linear function, and linear functions are continuous everywhere. Therefore, is continuous for all .

Step 2: Analyze continuity at

For , . This is a constant function, and constant functions are continuous everywhere. Therefore, is continuous for all .

Step 3: Analyze continuity at

For , . This is a linear function, and linear functions are continuous everywhere. Therefore, is continuous for all .

Step 4: Check continuity at

We need to check if .

Since , the function is continuous at .

Step 5: Check continuity at

We need to check if .

Since and , the left-hand limit and right-hand limit are not equal. Therefore, the function is discontinuous at .

Final Answer:

is the only point of discontinuity.

ANSWER

x=1 is the only point of discontinuity.

Question 16

QUESTION

Discuss the continuity of the function f, where f is defined by

f(x)=\begincases -2, & x\le -1 \\ 2x, & -11 \endcases

SOLUTION

We are asked to discuss the continuity of the given piecewise function .

Step 1: Identify potential points of discontinuity

The function is defined differently for , , and . Therefore, we need to check for continuity at the points where the function definition changes, which are and .

Step 2: Check continuity at

We need to check if the left-hand limit (LHL), right-hand limit (RHL), and the function value at are equal.

LHL at :

RHL at :

Function value at :

Since LHL = RHL = , the function is continuous at .

Step 3: Check continuity at

We need to check if the left-hand limit (LHL), right-hand limit (RHL), and the function value at are equal.

LHL at :

RHL at :

Function value at :

Since LHL = RHL = , the function is continuous at .

Step 4: Conclusion

The function is continuous at and . Also, within the intervals , , and , the function is defined by polynomials (constant or linear), which are always continuous. Therefore, the function is continuous everywhere.

Final Answer: Continuous

ANSWER

Continuous

Question 17

QUESTION

Find the relationship between a and b so that the function f defined by

f(x)=\begincases ax+1, & x\le 3 \\ bx+3, & x>3 \endcases

is continuous at x=3.

SOLUTION

We are asked to find the relationship between and such that the given piecewise function is continuous at .

Step 1: Understand the condition for continuity

For a function to be continuous at a point, the left-hand limit, the right-hand limit, and the value of the function at that point must all be equal. In this case, we need to ensure continuity at .

Step 2: Calculate the left-hand limit at

The left-hand limit is the limit as approaches 3 from values less than 3. For , . Therefore:

Step 3: Calculate the right-hand limit at

The right-hand limit is the limit as approaches 3 from values greater than 3. For , . Therefore:

Step 4: Calculate the value of the function at

Since for , we have:

Step 5: Apply the continuity condition

For to be continuous at , the left-hand limit must equal the right-hand limit, and both must equal the function's value at . Thus:

Step 6: Solve for the relationship between and

Rearrange the equation to express in terms of :

Final Answer: The relationship between and is .

ANSWER

a=b+(2)/(3)

Question 18

QUESTION

For what value of \lambda is the function defined by

f(x)=\begincases \lambda(x^2-2x), & x\le 0 \\ 4x+1, & x>0 \endcases

continuous at x=0? What about continuity at x=1?

SOLUTION

We need to find the value of for which the given function is continuous at . We also need to check the continuity of at .

Step 1: Check continuity at

For to be continuous at , the left-hand limit (LHL) and the right-hand limit (RHL) must be equal to the value of the function at .

LHL at :

RHL at :

Value of the function at :

For continuity, LHL = RHL = . Here, LHL = 0 and RHL = 1. Since , the function is not continuous at for any value of .

Step 2: Check continuity at

Since , we only need to consider the part of the function defined for , which is .

LHL at :

RHL at :

Value of the function at :

Since LHL = RHL = , the function is continuous at . The value of for does not depend on , so is continuous at for any value of .

Final Answer: For no value of , is continuous at but is continuous at for any value of .

ANSWER

For no value of \lambda, f is continuous at x=0 but f is continuous at x=1 for any value of \lambda.

Question 19

QUESTION

Show that the function defined by g(x)=x-[x] is discontinuous at all integral points. Here [x] denotes the greatest integer less than or equal to x.

SOLUTION

We need to show that the function is discontinuous at all integral points, where denotes the greatest integer less than or equal to .

Step 1: Understand the function

The function represents the fractional part of . For example, if , then and . If is an integer, say , then and .

Step 2: Check the left-hand limit at an integral point

Let be an integer. We need to find the left-hand limit of as approaches :

As approaches from the left, is slightly less than . Therefore, . So,

Step 3: Check the right-hand limit at an integral point

Now, we find the right-hand limit of as approaches :

As approaches from the right, is slightly greater than . Therefore, . So,

Step 4: Check the value of the function at the integral point

We need to find the value of :

Step 5: Compare the limits and the function value

We have:

Since the left-hand limit () is not equal to the right-hand limit (), the limit does not exist. Also, the left-hand limit is not equal to the function value at .

Step 6: Conclude

Since the left-hand limit and right-hand limit are not equal at , the function is discontinuous at all integral points.

ANSWER

Question 20

QUESTION

Is the function defined by f(x)=x^2-\sin x+5 continuous at x=π?

SOLUTION

We need to determine if the function is continuous at .

Step 1: Define continuity at a point

A function is continuous at a point if the following three conditions are met:

1. is defined.
2. exists.
3.

Step 2: Check if is defined

We need to find the value of the function at :

Since , we have:

Thus, is defined and equals .

Step 3: Find the limit as approaches

We need to find , which is .

Since polynomials and trigonometric functions are continuous, we can directly substitute into the function:

Thus, the limit exists and is equal to .

Step 4: Compare the limit and the function value

We found that and .

Since , the function is continuous at .

Final Answer: is continuous at .

ANSWER

f is continuous at x=π.

Question 21

QUESTION

Discuss the continuity of the following functions:

(a) f(x)=\sin x+\cos x

(b) f(x)=\sin x-\cos x

(c) f(x)=\sin x·\cos x

SOLUTION

We need to discuss the continuity of the given trigonometric functions.

(a)

Step 1: Recall the continuity of and

We know that and are continuous functions for all real numbers . This is a standard result.

Step 2: Apply the theorem on the sum of continuous functions

The theorem states that if and are continuous functions, then their sum is also a continuous function.

Step 3: Conclude the continuity of

Since and are continuous, their sum is also continuous for all real numbers .

(b)

Step 1: Recall the continuity of and

As before, and are continuous functions for all real numbers .

Step 2: Apply the theorem on the difference of continuous functions

The theorem states that if and are continuous functions, then their difference is also a continuous function.

Step 3: Conclude the continuity of

Since and are continuous, their difference is also continuous for all real numbers .

(c)

Step 1: Recall the continuity of and

Again, and are continuous functions for all real numbers .

Step 2: Apply the theorem on the product of continuous functions

The theorem states that if and are continuous functions, then their product is also a continuous function.

Step 3: Conclude the continuity of

Since and are continuous, their product is also continuous for all real numbers .

Final Answer: (a), (b) and (c) are all continuous

ANSWER

(a), (b) and (c) are all continuous

Question 22

QUESTION

Discuss the continuity of the cosine, cosecant, secant and cotangent functions.

SOLUTION

We need to discuss the continuity of cosine, cosecant, secant, and cotangent functions. This involves understanding their domains and identifying any points of discontinuity.

Step 1: Cosine Function

The cosine function, denoted as , is continuous for all real numbers. This means there are no breaks or jumps in its graph. The domain of is , and it is a fundamental trigonometric function known to be continuous everywhere.

Step 2: Cosecant Function

The cosecant function, denoted as , is defined as the reciprocal of the sine function: . It will be discontinuous where . The sine function is zero at integer multiples of , i.e., , where . Therefore, the cosecant function is continuous for all except , .

Step 3: Secant Function

The secant function, denoted as , is defined as the reciprocal of the cosine function: . It will be discontinuous where . The cosine function is zero at odd multiples of , i.e., , where . Therefore, the secant function is continuous for all except , .

Step 4: Cotangent Function

The cotangent function, denoted as , is defined as the ratio of cosine to sine: . It will be discontinuous where . As we know, the sine function is zero at integer multiples of , i.e., , where . Therefore, the cotangent function is continuous for all except , .

Final Answer:

Cosine function is continuous for all ; cosecant is continuous except for ; secant is continuous except for and cotangent function is continuous except for .

ANSWER

Cosine function is continuous for all x\in\mathbbR; cosecant is continuous except for x=nπ,\; n\in\mathbbZ; secant is continuous except for x=((2n+1)π)/(2),\; n\in\mathbbZ and cotangent function is continuous except for x=nπ,\; n\in\mathbbZ.

Question 23

QUESTION

Find all points of discontinuity of f, where

f(x)=\begincases (\sin x)/(x), & x<0 \\ x+1, & x\ge 0 \endcases

SOLUTION

We are given a piecewise function and asked to find all points of discontinuity.

Step 1: Check for discontinuity at

Since the function is defined piecewise at , we need to check the left-hand limit (LHL), right-hand limit (RHL), and the value of the function at .

Step 2: Calculate the left-hand limit (LHL)

We know that . Therefore,

Step 3: Calculate the right-hand limit (RHL)

Substituting , we get:

Step 4: Calculate the value of the function at

Since , we use the second part of the piecewise function:

Step 5: Compare LHL, RHL, and

We have:

LHL = 1

RHL = 1

= 1

Since LHL = RHL = , the function is continuous at .

Step 6: Check for discontinuity when

For , . Both and are continuous functions. The only possible point of discontinuity is when the denominator is zero, i.e., . However, we have already checked the continuity at .

Step 7: Check for discontinuity when

For , , which is a polynomial function and is always continuous.

Final Answer:

There is no point of discontinuity.

ANSWER

There is no point of discontinuity.

Question 24

QUESTION

Determine if f defined by

f(x)=\begincases x^2\sin(1)/(x), & x\ne 0 \\ 0, & x=0 \endcases

is a continuous function.

SOLUTION

We are asked to determine if the function defined as is continuous.

Step 1: Check continuity at

For to be continuous at , we need to show that .

We are given that .

Now we need to find the limit:

Step 2: Evaluate the limit

We know that for all .

Multiplying by (which is always non-negative), we get:

Now, as approaches 0:

and

By the Squeeze Theorem, we have:

Step 3: Compare the limit and the function value

Since and , we have .

Therefore, is continuous at .

Step 4: Check continuity for

For , . The function is a polynomial and is continuous everywhere. The function is continuous for all . The product of two continuous functions is continuous. Therefore, is continuous for all .

Final Answer:

is a continuous function.

ANSWER

Question 25

QUESTION

Examine the continuity of f, where f is defined by

f(x)=\begincases \sin x-\cos x, & x\ne 0 \\ -1, & x=0 \endcases

SOLUTION

We need to examine the continuity of the function defined as follows:

Step 1: Check continuity at

To check continuity at , we need to verify if .

First, let's find the limit as approaches 0:

Since and are continuous functions, we can directly substitute :

Step 2: Evaluate

According to the definition of the function, .

Step 3: Compare the limit and the function value

We found that and . Since these are equal, the function is continuous at .

Step 4: Check continuity for

For , . Both and are continuous functions for all real numbers. The difference of two continuous functions is also continuous. Therefore, is continuous for all .

Step 5: Conclusion

Since is continuous at and for all , we can conclude that is continuous for all .

ANSWER

f is continuous for all x\in\mathbbR.

Question 26

QUESTION

Find the value of k so that the function f is continuous at x=(π)/(2), where

f(x)=\begincases (k\cos x)/(π-2x), & x\ne (π)/(2) \\ 3, & x=(π)/(2) \endcases

SOLUTION

We are given a piecewise function and asked to find the value of that makes the function continuous at .

Step 1: Understand the condition for continuity

For a function to be continuous at a point, the limit of the function as approaches that point must be equal to the value of the function at that point. In this case, we need to ensure that:

Step 2: Evaluate the limit

Since we are given that , we need to find the limit of as approaches and set it equal to 3.

This limit is of the indeterminate form , so we can apply L'Hôpital's rule.

Step 3: Apply L'Hôpital's rule

Differentiate the numerator and the denominator with respect to :

Step 4: Evaluate the limit after applying L'Hôpital's rule

Now, substitute into the expression:

Step 5: Set the limit equal to the function value and solve for

We have:

Multiplying both sides by 2, we get:

Final Answer:

ANSWER

k=6

Question 27

QUESTION

Find the value of k so that the function f is continuous at x=2, where

f(x)=\begincases kx^2, & x\le 2 \\ 3, & x>2 \endcases

SOLUTION

We are given a piecewise function and asked to find the value of that makes the function continuous at .

Step 1: Understand the condition for continuity

For a function to be continuous at a point , the following three conditions must be met:

1. must be defined.
2. must exist.
3.

In our case, .

Step 2: Evaluate

Since , we use the first part of the piecewise function:

Step 3: Evaluate the left-hand limit

The left-hand limit is the limit as approaches 2 from values less than 2. We use the first part of the piecewise function:

Step 4: Evaluate the right-hand limit

The right-hand limit is the limit as approaches 2 from values greater than 2. We use the second part of the piecewise function:

Step 5: Apply the continuity condition

For the limit to exist at , the left-hand limit must equal the right-hand limit:

Step 6: Solve for

Divide both sides of the equation by 4:

Step 7: Verify that equals the limit

We found that . Substituting , we get . This is equal to the limit as approaches 2.

Final Answer:

ANSWER

k=(3)/(4)

Question 28

QUESTION

Find the value of k so that the function f is continuous at x=π, where

f(x)=\begincases kx+1, & x\le π \\ \cos x, & x>π \endcases

SOLUTION

We are given a piecewise function and asked to find the value of that makes the function continuous at .

Step 1: Understand the condition for continuity

For a function to be continuous at a point, the left-hand limit, the right-hand limit, and the value of the function at that point must all be equal. In other words, we need to ensure:

Step 2: Calculate the left-hand limit

As approaches from the left (), we use the first part of the piecewise function, . Therefore:

Step 3: Calculate the right-hand limit

As approaches from the right (), we use the second part of the piecewise function, . Therefore:

Step 4: Calculate the function value at

Since falls under the condition , we use the first part of the piecewise function, . Therefore:

Step 5: Apply the continuity condition and solve for

For to be continuous at , we must have:

Which means:

Subtracting 1 from both sides:

Dividing both sides by :

Final Answer:

ANSWER

k=-(2)/(π)

Question 29

QUESTION

Find the value of k so that the function f is continuous at x=5, where

f(x)=\begincases kx+1, & x\le 5 \\ 3x-5, & x>5 \endcases

SOLUTION

We are given a piecewise function and asked to find the value of that makes the function continuous at .

Step 1: Understand the condition for continuity

For a function to be continuous at a point , the following three conditions must be met:

1. must be defined.
2. must exist.
3.

In our case, . We need to ensure the left-hand limit, right-hand limit, and the function value at are all equal.

Step 2: Calculate the left-hand limit

The left-hand limit is the limit as approaches 5 from values less than 5. For , . Therefore:

Step 3: Calculate the right-hand limit

The right-hand limit is the limit as approaches 5 from values greater than 5. For , . Therefore:

Step 4: Calculate the function value at

Since for , we have:

Step 5: Apply the continuity condition

For to be continuous at , the left-hand limit, right-hand limit, and the function value at must be equal:

This gives us:

Step 6: Solve for

Subtract 1 from both sides:

Divide both sides by 5:

Final Answer:

ANSWER

k=(9)/(5)

Question 30

QUESTION

Find the values of a and b such that the function defined by

f(x)=\begincases 5, & x\le 2 \\ ax+b, & 2

is a continuous function.

SOLUTION

We are given a piecewise function and asked to find the values of and that make the function continuous.

Step 1: Understand the condition for continuity

For to be continuous, it must be continuous at the points where the function definition changes, i.e., at and . This means the left-hand limit (LHL) must equal the right-hand limit (RHL) and the function value at these points.

Step 2: Apply continuity at

At , we need to ensure that the function is continuous. The left-hand limit is given by the value of the function when , which is 5. The right-hand limit is given by the value of the function when , which is . Thus,

For continuity at , we must have:

Step 3: Apply continuity at

At , we need to ensure that the function is continuous. The left-hand limit is given by the value of the function when , which is . The right-hand limit is given by the value of the function when , which is 21. Thus,

For continuity at , we must have:

Step 4: Solve the system of equations

We now have a system of two linear equations with two variables:

Subtracting equation (1) from equation (2), we get:

Substituting into equation (1), we get:

Final Answer: The values of and are and .

ANSWER

a=2,\; b=1

Question 31

QUESTION

Show that the function defined by f(x)=\cos(x^2) is a continuous function.

SOLUTION

We need to show that the function is continuous.

Step 1: Identify the component functions

We can express as a composition of two functions: and . Specifically, .

Step 2: Check the continuity of the component functions

We know that is a continuous function for all real numbers . This is a standard result.

Also, is a polynomial function, and all polynomial functions are continuous for all real numbers . This is also a standard result.

Step 3: Apply the theorem on continuity of composite functions

The theorem states that if is continuous at and is continuous at , then the composite function is continuous at .

Step 4: Apply the theorem to our problem

Since is continuous for all real numbers , it is continuous at any arbitrary point .

Also, since is continuous for all real numbers , it is continuous at .

Therefore, by the theorem on continuity of composite functions, is continuous at .

Step 5: Generalize the result

Since was an arbitrary real number, we can conclude that is continuous for all real numbers .

Final Answer: The function is a continuous function.

ANSWER

Question 32

QUESTION

Show that the function defined by f(x)=|\cos x| is a continuous function.

SOLUTION

We need to show that the function is continuous.

Step 1: Break down the function

The function is a composite function. We can express it as , where .

Step 2: Analyze the continuity of the inner function

We know that the cosine function, , is continuous for all real numbers . This is a standard result.

Step 3: Analyze the continuity of the outer function

Consider the modulus function . We know that the modulus function is also continuous for all real numbers. To show this, consider any real number . We need to show that .

We can prove the continuity of using the epsilon-delta definition. For any , choose . Then, if , we have:

Thus, is continuous.

Step 4: Apply the theorem on composite functions

If is continuous at and is continuous at , then the composite function is continuous at .

Step 5: Combine the results

Since is continuous for all and is continuous for all , the composite function is continuous for all .

Step 6: Conclusion

Therefore, the function is a continuous function.

ANSWER

Question 33

QUESTION

Examine that \sin|x| is a continuous function.

SOLUTION

We need to examine the continuity of the function .

Step 1: Understand the function

The function is a composite function. We can express it as , where and .

Step 2: Analyze the continuity of

The absolute value function is defined as: Both and are polynomial functions, and polynomial functions are continuous everywhere. At , the left-hand limit (LHL) and right-hand limit (RHL) are:

LHL:

RHL:

Also, . Since LHL = RHL = , the function is continuous at . Therefore, is continuous for all real numbers.

Step 3: Analyze the continuity of

The sine function, , is a standard trigonometric function, and it is known to be continuous for all real numbers.

Step 4: Apply the theorem on continuity of composite functions

If is continuous at and is continuous at , then the composite function is continuous at . In our case, is continuous for all , and is continuous for all . Therefore, the composite function is continuous for all real numbers.

Conclusion:

Since both and are continuous functions, their composition is also a continuous function.

ANSWER

Question 34

QUESTION

Find all the points of discontinuity of f defined by f(x)=|x|-|x+1|.

SOLUTION

We are asked to find the points of discontinuity of the function .

Step 1: Understand the function

The function is defined as the difference of two absolute value functions. Recall that the absolute value function is defined as:

Similarly, is defined as:

Step 2: Rewrite the function in piecewise form

We need to consider three intervals: , , and .

If , then and . So,

If , then and . So,

If , then and . So,

Thus, we can write as:

Step 3: Check for discontinuity at the transition points

The transition points are and . We need to check if the function is continuous at these points.

At :

Since , the function is continuous at .

At :

Since , the function is continuous at .

Step 4: Conclusion

Since the function is continuous at all transition points and each piece of the function is continuous, is continuous everywhere. Therefore, there is no point of discontinuity.

ANSWER

There is no point of discontinuity.

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Key Formulas

Important Formulas for Exercise 5.1

Formula / Concept	Description
Continuity at a Point	A function f(x) is continuous at a point x = c if the following three conditions are met: 1. f(c) is defined. 2. \lim_x \to c f(x) exists. 3. \lim_x \to c f(x) = f(c).
Limit of a Function at a Point	For the limit of a function f(x) to exist at x = c, the left-hand limit (LHL) and the right-hand limit (RHL) must be equal. \lim_x \to c f(x) \text exists if \lim_x \to c^- f(x) = \lim_x \to c^+ f(x)
Left-Hand Limit (LHL)	The limit of the function as x approaches c from the left side (values less than c). It is denoted as: \lim_x \to c^- f(x)
Right-Hand Limit (RHL)	The limit of the function as x approaches c from the right side (values greater than c). It is denoted as: \lim_x \to c^+ f(x)
Algebra of Continuous Functions	If f and g are two real functions that are continuous at a point x = c, then: • f + g is continuous at x = c. • f - g is continuous at x = c. • f · g is continuous at x = c. • (f)/(g) is continuous at x = c, provided g(c) ≠ 0.
Continuity of Polynomial Functions	Every polynomial function is continuous at every point on the real number line.
Continuity of Rational Functions	A rational function is continuous at every point in its domain (i.e., where the denominator is not zero).

Formula / Concept	Description
Continuity of Trigonometric Functions	• The sine and cosine functions, \sin(x) and \cos(x), are continuous everywhere. • The tangent, cotangent, secant, and cosecant functions are continuous in their respective domains.
Continuity of Modulus Function	The absolute value function, f(x) = |x|, is continuous everywhere.

📚 More Exercises

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Exercise 5.1 ✓ →

Exercise 5.2 →

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Miscellaneous Exercise on Chapter 5 →

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