

NCERT Solutions Class 12 Maths

Chapter 1: Relations and Functions

Miscellaneous Exercise on Chapter 1

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Quick Summary: In NCERT Solutions Class 12 Maths Chapter 1 Exercise misc.1, students learn advanced concepts of relations and functions including types of relations, composition of functions, and properties of injective and surjective functions. This exercise covers crucial topics like proving injectivity of functions such as $f(x) = x^3$, determining onto functions, and analyzing relations on power sets which are essential for scoring well in CBSE board exams and competitive entrance tests.

Key Takeaways:

- Understanding injective functions: A function $f: A \rightarrow B$ is injective if $f(x_1) = f(x_2)$ implies $x_1 = x_2$
- Surjective (onto) functions: Every element in codomain B has at least one pre-image in domain A
- Types of relations on power sets and their properties like reflexive, symmetric, and transitive relations
- Composition of functions and determining the number of onto functions between finite sets using combinatorial methods

Complete Solutions

Question 1

QUESTION

Show that the function $f : \mathbb{R} \rightarrow \mathbb{R} : -1 < x < 1$ defined by $f(x) = \frac{x}{1 + |x|}$, $x \in \mathbb{R}$, is one-one and onto.

SOLUTION

We need to prove that the function from \mathbb{R} to \mathbb{R} is both one-one (injective) and onto (surjective).

Step 1: Prove that the function is one-one (injective)

To prove that f is one-one, we need to show that if $f(x) = f(y)$, then $x = y$ for all $x, y \in \mathbb{R}$.

Let $x, y \in \mathbb{R}$. Then:

Consider two cases:

Case 1: $x \geq 0$ and $y \geq 0$. So,

Case 2: $x < 0$ and $y < 0$. So,

Case 3: $x \geq 0$ and $y < 0$. Then $f(x) = \frac{x}{1+x}$ and $f(y) = \frac{y}{1-y}$. Since $f(x) = f(y)$, this case is not possible unless $x = y = 0$. A similar argument holds if $x < 0$ and $y \geq 0$.

Therefore, f is one-one.

Step 2: Prove that the function is onto (surjective)

To prove that f is onto, we need to show that for any $y \in \mathbb{R}$, there exists an $x \in \mathbb{R}$ such that $f(x) = y$.

Let $y \in \mathbb{R}$. We want to find x in terms of y .

If $y \geq 0$, then $x \geq 0$, so $f(x) = \frac{x}{1+x} = y$, which gives $x = y(1+x)$, so $x = y + xy$, and $x - xy = y$. Since $x(1-y) = y$,

if $y < 0$, then $x < 0$, so $f(x) = \frac{x}{1-x} = y$, which gives $x = y(1-x)$, so $x = y - xy$, and $x + xy = y$. Since $x(1+y) = y$,

Thus, for any $y \in \mathbb{R}$, we can find an $x \in \mathbb{R}$ such that $f(x) = y$. Therefore, f is onto.

Conclusion:

Since f is both one-one and onto, it is a bijection.

ANSWER

No.

Question 2

QUESTION

Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^3$ is injective.

SOLUTION

We need to show that the function is injective (one-to-one) when defined from the set of real numbers to the set of real numbers.

Step 1: Understand the definition of injective

A function is injective if for any in the domain, implies . In simpler terms, different inputs must produce different outputs.

Step 2: Assume

Let and be any two real numbers such that . We want to show that this implies .

Step 3: Substitute into the function definition

Since , we have:

and

Therefore,

Step 4: Manipulate the equation to isolate the variables

Subtract from both sides:

Step 5: Factor the difference of cubes

Recall the factorization formula: . Applying this to our equation:

Step 6: Analyze the factors

For the product of two factors to be zero, at least one of them must be zero. So either:

or

If , then , which is what we want to show.

Now, consider the second factor: . We can rewrite this as:

Since squares of real numbers are non-negative, the only way for this expression to be zero is if both terms are zero. This implies and . If , then . Thus, .

Step 7: Conclude

In either case, implies . Therefore, the function is injective.

ANSWER

Injective.

Question 3

QUESTION

Given a non-empty set X , consider $P(X)$ which is the set of all subsets of X . Define the relation R in $P(X)$ as follows:

For subsets A, B in $P(X)$, $A R B$ if and only if $A \subseteq B$. Is R an equivalence relation on $P(X)$? Justify your answer.

SOLUTION

The question asks us to determine if the relation defined on the power set of a non-empty set X , where $A R B$ if and only if $A \subseteq B$, is an equivalence relation. To be an equivalence relation, R must be reflexive, symmetric, and transitive. We will examine each of these properties.

Step 1: Check for Reflexivity

For R to be reflexive, $A R A$ must hold for all $A \in P(X)$. This means $A \subseteq A$ must be true for all subsets A of X . Since every set is a subset of itself, $A \subseteq A$ is always true. Therefore, R is reflexive.

Step 2: Check for Symmetry

For R to be symmetric, if $A R B$, then $B R A$ must also hold. In other words, if $A \subseteq B$, then $B \subseteq A$ must be true. Consider an example: Let $X = \{1, 2\}$, $A = \{1\}$, and $B = \{1, 2\}$. Then $A R B$ is true, but $B R A$ is false because $\{1, 2\} \not\subseteq \{1\}$. Therefore, R is not symmetric.

Step 3: Check for Transitivity

For R to be transitive, if $A R B$ and $B R C$, then $A R C$ must hold. This means if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$ must be true. If every element of A is in B , and every element of B is in C , then every element of A must also be in C . Therefore, R is true, and R is transitive.

Step 4: Conclusion

Since R is reflexive and transitive, but not symmetric, it is not an equivalence relation.

Final Answer: No.

ANSWER

No.

Question 4

QUESTION

Find the number of all onto functions from the set $\{1,2,3,\dots,n\}$ to itself.

SOLUTION

We need to find the number of onto functions (surjective functions) from a set to itself.

Step 1: Understand the definition of an onto function

An onto function, also known as a surjective function, is a function where every element in the codomain (the set to which the function maps) has a corresponding element in the domain (the set from which the function maps). In simpler terms, every element in the codomain is "hit" by at least one element from the domain.

Step 2: Consider the implications for a function from a set to itself

Since the function maps from the set to itself, both the domain and codomain have elements. For the function to be onto, every element in the codomain must be the image of some element in the domain. This means that no two elements in the domain can map to the same element in the codomain (otherwise, we wouldn't be able to "hit" all elements in the codomain).

Step 3: Relate onto functions to one-to-one functions

When a function from a finite set to itself is onto, it is also one-to-one (injective). This is because if every element in the codomain is mapped to, no two elements in the domain can map to the same element in the codomain. Thus, we are looking for functions that are both one-to-one and onto, also known as bijections.

Step 4: Count the number of bijections

To form a bijection from to itself, we need to map each of the elements in the domain to a unique element in the codomain. For the first element in the domain, we have choices in the codomain. For the second element in the domain, we have remaining choices in the codomain (since we can't map it to the same element as the first). For the third element, we have choices, and so on. For the n th element, we have only 1 choice left.

Step 5: Calculate the total number of onto functions

The total number of onto functions (bijections) is the product of the number of choices for each element:

Therefore, the number of all onto functions from the set to itself is .

ANSWER

$n!$

Question 5

QUESTION

Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ and let $f, g : A \rightarrow B$ be functions defined by

$$f(x) = x^2 - x, \quad x \in A$$

and

$$g(x) = 2\left|x - \frac{1}{2}\right| - 1, \quad x \in A.$$

Are f and g equal? Justify your answer.

SOLUTION

We are given two sets A and B , and two functions f and g defined from A to B . We need to determine if the functions f and g are equal.

Step 1: Define the sets and functions

The set A and the set B .

The function f for $x \in A$.

The function g for $x \in A$.

Step 2: Evaluate f for all $x \in A$

Step 3: Evaluate g for all $x \in A$

Step 4: Compare the values of f and g

We have:

and

and

and

and

Since for all $x \in A$, the functions f and g are equal.

Final Answer: Yes.

ANSWER

Yes.

Question 6

QUESTION

Let $A = \{1,2,3\}$. Then number of relations containing $(1,2)$ and $(1,3)$ which are reflexive and symmetric but not transitive is:

- (A) 1 (B) 2 (C) 3 (D) 4

SOLUTION

We are given a set and asked to find the number of relations on that contain and , are reflexive and symmetric, but not transitive.

Step 1: Reflexive Property

Since the relation must be reflexive, it must contain the elements , , and .

Step 2: Symmetric Property

The relation must also be symmetric. Since it contains and , it must also contain and .

Step 3: Current Relation

At this point, our relation contains the following elements:

Step 4: Checking for Transitivity and Adding Elements

Now we need to consider adding more elements such that the relation remains symmetric but is not transitive. If we add , then by symmetry we must also add . This gives us:

Now, we check for transitivity. We have and , so for transitivity, we need , which is already in the relation. We also have and , so we need , which is already in the relation. Similarly, and implies we need , which is already in the relation. And and implies we need , which is already there. So, this relation is transitive.

Step 5: Finding a Non-Transitive Relation

The relation is reflexive and symmetric. However, it is not transitive. For example, and , but . If we don't add any more elements, it remains non-transitive.

Step 6: Conclusion

There is only 1 such relation that satisfies the given conditions.

Final Answer: A

ANSWER

A

Question 7

QUESTION

Let $A = \{1,2,3\}$. Then number of equivalence relations containing $(1,2)$ is:

- (A) 1 (B) 2 (C) 3 (D) 4

SOLUTION

We are asked to find the number of equivalence relations on the set that contain the ordered pair $(1,2)$.

Step 1: Understand Equivalence Relations

An equivalence relation must be reflexive, symmetric, and transitive.

Step 2: Start with the given condition

Since $(1,2)$ must be in the relation, due to symmetry, $(2,1)$ must also be in the relation.

Step 3: Reflexivity

For the relation to be reflexive, it must contain $(1,1)$, $(2,2)$, and $(3,3)$.

Step 4: Construct the minimal equivalence relation

The minimal equivalence relation containing $(1,2)$ is:

This relation is reflexive, symmetric, and transitive. If $(1,2)$ and $(2,1)$ are in R , then $(1,1)$ and $(2,2)$ must also be in R . For example, $(1,1)$ and $(2,2)$ are in R , so $(1,2)$ must be in R , which it is.

Step 5: Consider other possibilities

We can create another equivalence relation by including $(3,3)$. If $(3,3)$ is included, then by symmetry, $(3,3)$ must also be included. Furthermore, since $(1,2)$ is in the relation, by transitivity, $(1,3)$ and $(3,2)$ must also be included. This gives us the universal relation:

Step 6: Check for other equivalence relations

There are no other possible equivalence relations containing $(1,2)$. If we only add $(3,3)$ we must also add $(1,3)$ and $(3,2)$. Then by transitivity we must add $(1,2)$ and $(2,1)$, resulting in R .

Step 7: Count the number of equivalence relations

We have found two equivalence relations: R and R' .

Final Answer: The number of equivalence relations containing $(1,2)$ is 2.

ANSWER

B

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Key Formulas

Important Formulas for Exercise misc.1

Formula / Concept	Description
Types of Relations	A relation R in a set A is a subset of the Cartesian product $A \times A$.
Reflexive Relation	A relation R on a set A is reflexive if $(a, a) \in R$ for every element $a \in A$.
Symmetric Relation	A relation R on a set A is symmetric if $(a, b) \in R$ implies that $(b, a) \in R$ for all $a, b \in A$.
Transitive Relation	A relation R on a set A is transitive if $(a, b) \in R$ and $(b, c) \in R$ implies that $(a, c) \in R$ for all $a, b, c \in A$.
Equivalence Relation	A relation R on a set A is an equivalence relation if it is reflexive, symmetric, and transitive.
Empty Relation	A relation R in a set A is an empty relation if no element of A is related to any element of A , i.e., $R = \emptyset$.
Universal Relation	A relation R in a set A is a universal relation if each element of A is related to every element of A , i.e., $R = A \times A$.
Types of Functions	A function is a special type of relation where every input has exactly one output.
One-one (Injective) Function	A function $f: X \rightarrow Y$ is one-one if the images of distinct elements of X under f are distinct. Mathematically, $f(x_1) = f(x_2) \implies x_1 = x_2$ for all $x_1, x_2 \in X$.
Onto (Surjective) Function	A function $f: X \rightarrow Y$ is onto if every element of Y is the image of some element of X under f . That is, for every $y \in Y$, there exists an element $x \in X$ such that $f(x) = y$.
Bijjective Function	A function is bijective if it is both one-one and onto.

Formula / Concept	Description
Composition of Functions	Combining two functions where the output of one function becomes the input for the next.
$(g \circ f)(x) = g(f(x))$	Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be two functions. The composition of f and g , denoted by $g \circ f$, is a function from A to C defined by $(g \circ f)(x) = g(f(x))$ for all $x \in A$.
Invertible Function	A function $f: X \rightarrow Y$ is invertible if there exists a function $g: Y \rightarrow X$ such that $g \circ f = I_X$ and $f \circ g = I_Y$. The function g is the inverse of f , denoted by f^{-1} . A function is invertible if and only if it is bijective.

Top FAQs

Q1. How many questions are in NCERT Solutions Class 12 Maths Chapter 1 Relations and Functions Exercise misc.1 for CBSE board exam 2025-26?

Exercise misc.1 of NCERT Solutions Class 12 Maths Chapter 1 Relations and Functions contains exactly 7 questions. These miscellaneous questions cover all key concepts including types of relations, reflexive, symmetric, transitive relations, and composition of functions, making them crucial for CBSE board exam 2025-26 preparation.

Q2. Where can I download free PDF of NCERT Solutions for Class 12 Maths Chapter 1 Relations and Functions Exercise misc.1 with step by step solutions?

You can download the free PDF of NCERT Solutions for Class 12 Maths Chapter 1 Relations and Functions Exercise misc.1 from the official NCERT website or various educational platforms offering step by step solutions. These PDFs are updated as per the latest CBSE syllabus 2025-26 and include detailed explanations for all 7 miscellaneous questions.

Q3. How many marks does Relations and Functions Chapter 1 carry in CBSE Class 12 Maths board exam 2025-26 syllabus?

Relations and Functions (Chapter 1) carries 4 marks weightage in CBSE Class 12 Maths board exam 2025-26 as part of Unit I. This unit is shared with the Inverse Trigonometric Functions chapter, so thorough practice of Exercise misc.1 is essential for scoring these marks.

Q4. Which is the most difficult question in NCERT Solutions Class 12 Maths Chapter 1 Relations and Functions Exercise misc.1 for step by step solutions?

Questions involving composition of functions and proving relations with specific properties (like Question 5 and Question 7) are often considered the most difficult in Exercise misc.1 of NCERT Solutions Class 12 Maths Chapter 1. These questions require understanding multiple concepts simultaneously and benefit greatly from step by step solutions for better comprehension.

Q5. What is Types of Relations in NCERT Solutions Class 12 Maths Chapter 1 Relations and Functions Exercise misc.1?

Types of Relations in NCERT Class 12 Maths Chapter 1 include reflexive relations (where every element is related to itself), symmetric relations (where $a R b$ implies $b R a$), transitive relations (where $a R b$ and $b R c$ implies $a R c$), and equivalence relations (which are reflexive, symmetric, and transitive simultaneously).

Exercise misc.1 contains comprehensive questions on identifying and proving these types of relations for CBSE board exam 2025-26.


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