

NCERT Solutions Class 12 Maths

Chapter 1: Relations and Functions

EXERCISE 1.2

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Quick Summary: In NCERT Solutions Class 12 Maths Chapter 1 Exercise 1.2, students learn about different types of functions and their properties through comprehensive problem-solving. This exercise covers injective, surjective, and bijective functions along with special functions like modulus, signum, and greatest integer functions, which are essential topics frequently tested in CBSE board exams and competitive tests.

Key Takeaways:

- Understanding injective (one-to-one) and surjective (onto) functions to determine if a function is bijective
- Special functions: Greatest integer function $f(x) = [x]$, Modulus function $f(x) = |x|$, and Signum function
- Composition of functions and proving properties like $(f \circ g)(x) = f(g(x))$
- Step-by-step methods to check injectivity and surjectivity using algebraic proofs and graphical analysis

Complete Solutions

Question 1

QUESTION

Show that the function $f : \mathbb{R}^* \rightarrow \mathbb{R}^*$ defined by $f(x) = (1)/x$ is one-one and onto, where \mathbb{R}^* is the set of all non-zero real numbers. Is the result true, if the domain \mathbb{R}^* is replaced by \mathbb{N} with co-domain being same as \mathbb{R}^* ?

SOLUTION

This question asks us to prove that the function is both one-one (injective) and onto (surjective) when defined from the set of non-zero real numbers to itself. It also asks us to consider what happens when the domain is changed to the set of natural numbers.

Step 1: Proving is one-one when

To prove is one-one, we need to show that if , then for all .

Let . Then, .

Multiplying both sides by (which is allowed since are non-zero), we get .

Therefore, is one-one.

Step 2: Proving is onto when

To prove is onto, we need to show that for every , there exists an such that .

Let . We want to find an such that , i.e., .

Solving for , we get . Since , is a non-zero real number, and therefore is also a non-zero real number. Thus, .

Therefore, is onto.

Step 3: Considering

Now, let's consider the case where the domain is replaced by , so .

In this case, is still one-one because if , then , which implies . Since , the argument holds.

However, is *not* onto. For example, consider . We need to find an such that , i.e., . This gives , but . Therefore, there exists no natural number such that .

Final Answer: No.

ANSWER

No.

Question 2

QUESTION

Check the injectivity and surjectivity of the following functions:

1. $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x^2$.
2. $f : \mathbb{Z} \rightarrow \mathbb{Z}$ given by $f(x) = x^2$.
3. $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^2$.
4. $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x^3$.
5. $f : \mathbb{Z} \rightarrow \mathbb{Z}$ given by $f(x) = x^3$.

SOLUTION

We need to check the injectivity and surjectivity of the given functions.

(i) given by .

Step 1: Check for injectivity (one-to-one)

If , then . Taking the square root, we get . Since the domain is (natural numbers), we only consider the positive value. Thus, . Therefore, the function is injective.

Step 2: Check for surjectivity (onto)

For the function to be surjective, every element in the codomain () must have a pre-image in the domain (). Consider the number 2 in the codomain. There is no natural number such that . Therefore, the function is not surjective.

Final Answer: Injective but not surjective.

(ii) given by .

Step 1: Check for injectivity

Consider and . Since but , the function is not injective.

Step 2: Check for surjectivity

Consider the number 2 in the codomain (). There is no integer such that . Therefore, the function is not surjective.

Final Answer: Neither injective nor surjective.

(iii) given by .

Step 1: Check for injectivity

Similar to (ii), and . Since but , the function is not injective.

Step 2: Check for surjectivity

Consider the number -1 in the codomain (\mathbb{R}) . There is no real number x such that $x^3 = -1$. Therefore, the function is not surjective.

Final Answer: Neither injective nor surjective.

(iv) given by $f(x) = x^3$.

Step 1: Check for injectivity

If $f(x) = f(y)$, then $x^3 = y^3$. Taking the cube root, we get $x = y$. Therefore, the function is injective.

Step 2: Check for surjectivity

Consider the number 2 in the codomain (\mathbb{N}) . There is no natural number x such that $x^3 = 2$. Therefore, the function is not surjective.

Final Answer: Injective but not surjective.

(v) given by $f(x) = x^3$.

Step 1: Check for injectivity

If $f(x) = f(y)$, then $x^3 = y^3$. Taking the cube root, we get $x = y$. Therefore, the function is injective.

Step 2: Check for surjectivity

Consider the number 2 in the codomain (\mathbb{Z}) . There is no integer x such that $x^3 = 2$. Therefore, the function is not surjective.

Final Answer: Injective but not surjective.

ANSWER

- (i) Injective but not surjective.
- (ii) Neither injective nor surjective.
- (iii) Neither injective nor surjective.
- (iv) Injective but not surjective.
- (v) Injective but not surjective.

Question 3

QUESTION

Prove that the Greatest Integer Function $f : \mathbb{R} \rightarrow \mathbb{R}$, given by $f(x) = [x]$, is neither one-one nor onto, where $[x]$ denotes the greatest integer less than or equal to x .

SOLUTION

We need to prove that the greatest integer function is neither one-one (injective) nor onto (surjective).

Step 1: Understanding the Greatest Integer Function

The greatest integer function, denoted by $[x]$, returns the largest integer less than or equal to x . For example, $[1.5] = 1$, $[2.9] = 2$, and $[3] = 3$.

Step 2: Proving that the function is not one-one

A function is one-one if different elements in the domain have different images in the codomain. In other words, if $x \neq y$, then $f(x) \neq f(y)$. Equivalently, if $f(x) = f(y)$, then $x = y$.

Consider $x = 1.5$ and $y = 2.5$. Then, $f(1.5) = 1$ and $f(2.5) = 2$. So, $f(1.5) \neq f(2.5)$. Therefore, the function is not one-one.

Step 3: Proving that the function is not onto

A function is onto if every element in the codomain has a pre-image in the domain. In other words, for every y in the codomain, there exists an x in the domain such that $f(x) = y$.

The codomain of the function is \mathbb{R} (the set of real numbers). Let's consider a real number $y = 1.5$ in the codomain, say y . We need to find an x in the domain such that $f(x) = 1.5$. However, the greatest integer function only returns integer values. Therefore, there is no x in the domain such that $f(x) = 1.5$. This means the function is not onto.

Step 4: Conclusion

Since the greatest integer function is neither one-one nor onto, the proof is complete.

ANSWER

Neither one-one nor onto.

Question 4

QUESTION

Show that the Modulus Function $f : \mathbb{R} \rightarrow \mathbb{R}$, given by $f(x) = |x|$, is neither one-one nor onto, where $|x|$ is x if x is positive or 0 and $|x|$ is $-x$ if x is negative.

SOLUTION

We need to show that the modulus function is neither one-one (injective) nor onto (surjective) when defined from \mathbb{R} to \mathbb{R} .

Step 1: Check for one-one (injectivity)

A function is one-one if different elements in the domain have different images in the codomain. In other words, if $x \neq y$, then $f(x) \neq f(y)$. Let's consider two different real numbers, say x and y .

Then, $f(x) = |x|$ and $f(y) = |y|$. Since $x \neq y$, but $|x| = |y|$, the function is not one-one.

Step 2: Check for onto (surjectivity)

A function is onto if every element in the codomain has a pre-image in the domain. In other words, for every y in the codomain, there exists an x in the domain such that $f(x) = y$. The modulus function, $f(x) = |x|$, always returns a non-negative value. Therefore, the range of f is \mathbb{R}^+ .

However, the codomain is given as \mathbb{R} , which includes all real numbers, both positive and negative. Consider a negative real number, say -1 . There is no real number x such that $|x| = -1$. This is because the absolute value of any real number is always non-negative.

Since there exists an element in the codomain (e.g., -1) that does not have a pre-image in the domain, the function is not onto.

Final Answer:

The modulus function is neither one-one nor onto.

ANSWER

Neither one-one nor onto.

Question 5

QUESTION

Show that the Signum Function $f : \mathbb{R} \rightarrow \mathbb{R}$, given by

$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

is neither one-one nor onto.

SOLUTION

We need to show that the Signum function, defined as is neither one-one (injective) nor onto (surjective).

Step 1: Prove that the function is not one-one.

A function is one-one if different elements in the domain have different images in the codomain. In other words, if $x \neq y$, then $f(x) \neq f(y)$.

Consider $x = 1$ and $y = 2$. Both are greater than 0.

Then, $f(1) = 1$ and $f(2) = 1$. So, $f(1) = f(2)$, but $1 \neq 2$.

Therefore, the function is not one-one.

Step 2: Prove that the function is not onto.

A function is onto if every element in the codomain has a pre-image in the domain. In other words, for every y in the codomain, there exists an x in the domain such that $f(x) = y$.

The codomain of the function is \mathbb{R} (the set of all real numbers).

However, the range of the function is only $\{-1, 0, 1\}$. This is because the function can only output these three values, regardless of the input.

Consider the number 2 in the codomain. There is no x in the domain such that $f(x) = 2$. The same is true for any real number other than $-1, 0,$ and 1 .

Therefore, the function is not onto.

Final Answer: The Signum function is neither one-one nor onto.

ANSWER

Neither one-one nor onto.

Question 6

QUESTION

Let $A = \{1,2,3\}$, $B = \{4,5,6,7\}$ and let $f = \{(1,4), (2,5), (3,6)\}$ be a function from A to B . Show that f is one-one.

SOLUTION

We are given two sets, A and B , and a function f from A to B . We need to show that f is one-one (injective).

Step 1: Understand the definition of a one-one function

A function is said to be one-one (or injective) if different elements in the domain map to different elements in the codomain. Mathematically, if $x \neq y$, then $f(x) \neq f(y)$. Equivalently, if $f(x) = f(y)$, then $x = y$.

Step 2: Analyze the given function

The function is defined as a set of ordered pairs: $f = \{(1,4), (2,5), (3,6)\}$. This means:

- $f(1) = 4$
- $f(2) = 5$
- $f(3) = 6$

Step 3: Check if different elements in map to different elements in

We can see that:

- For $x=1$ and $y=2$, and $f(x) \neq f(y)$. Since $1 \neq 2$, we have $f(1) \neq f(2)$.
- For $x=1$ and $y=3$, and $f(x) \neq f(y)$. Since $1 \neq 3$, we have $f(1) \neq f(3)$.
- For $x=2$ and $y=3$, and $f(x) \neq f(y)$. Since $2 \neq 3$, we have $f(2) \neq f(3)$.

Alternatively, we can directly observe that each element in the domain maps to a unique element in the codomain. No two elements in A map to the same element in B .

Step 4: Conclusion

Since different elements in A map to different elements in B , the function is one-one.

ANSWER

One-one.

Question 7

QUESTION

In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

1. $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3 - 4x$.
2. $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 1 + x^2$.

SOLUTION

We need to determine if the given functions are one-one (injective), onto (surjective), or bijective (both one-one and onto). We will analyze each case separately.

(i) defined by .

Step 1: Check for one-one (injectivity)

Assume for some . We need to show that .

If , then .

Subtracting 3 from both sides, we get .

Dividing both sides by -4, we get .

Since implies , the function is one-one.

Step 2: Check for onto (surjectivity)

Let be an arbitrary element in the codomain. We need to find an such that .

We want to solve for .

Rearranging the equation, we get .

Dividing by 4, we get .

Since , is also a real number. Thus, for every in the codomain, there exists an in the domain such that .

Therefore, the function is onto.

Step 3: Conclusion for (i)

Since the function is both one-one and onto, it is bijective.

(ii) defined by .

Step 1: Check for one-one (injectivity)

Consider and . Then and .

So, but . Therefore, the function is not one-one.

Step 2: Check for onto (surjectivity)

We need to check if for every y , there exists an x such that $f(x) = y$.

Consider $y = 2$. We want to find an x such that $f(x) = 2$, which means $x^2 = 2$.

However, there is no real number x such that $x^2 = 2$. Therefore, the function is not onto.

Step 3: Conclusion for (ii)

Since the function is neither one-one nor onto, it is not bijective.

ANSWER

(i) One-one and onto.

(ii) Neither one-one nor onto.

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Question 8

QUESTION

Let A and B be sets. Show that $f : A \times B \rightarrow B \times A$ such that $f(a,b) = (b,a)$ is a bijective function.

SOLUTION

We are given a function defined as $f : A \times B \rightarrow B \times A$, and we need to show that this function is bijective, meaning it is both one-one (injective) and onto (surjective).

Step 1: Prove that f is one-one (injective)

To prove that f is one-one, we need to show that if $f(a,b) = f(c,d)$, then $(a,b) = (c,d)$.

Suppose $f(a,b) = f(c,d)$. This means $(b,a) = (d,c)$.

For two ordered pairs to be equal, their corresponding components must be equal. Therefore, $b = d$ and $a = c$.

Thus, $(a,b) = (c,d)$. This proves that f is one-one.

Step 2: Prove that f is onto (surjective)

To prove that f is onto, we need to show that for every element $(c,d) \in B \times A$, there exists an element $(a,b) \in A \times B$ such that $f(a,b) = (c,d)$.

Let (c,d) be an arbitrary element in $B \times A$. Since $a \in A$ and $b \in B$, the ordered pair (a,b) exists in $A \times B$.

Now, consider (a,b) . By the definition of f , we have $f(a,b) = (b,a)$.

Thus, for every $(c,d) \in B \times A$, there exists $(a,b) \in A \times B$ such that $f(a,b) = (c,d)$. This proves that f is onto.

Step 3: Conclusion

Since f is both one-one and onto, it is a bijective function.

ANSWER

Bijjective (one-one and onto).

Question 9

QUESTION

Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be defined by

$$f(n) = \begin{cases} (n+1)/2, & \text{if } n \text{ is odd} \\ n/2, & \text{if } n \text{ is even} \end{cases}$$

for all $n \in \mathbb{N}$. State whether the function f is bijective. Justify your answer.

SOLUTION

We are given a function defined piecewise and asked to determine if it is bijective. A function is bijective if it is both injective (one-to-one) and surjective (onto).

Step 1: Check for injectivity (one-to-one)

To check if f is injective, we need to see if $f(a) = f(b)$ implies $a = b$ for all $a, b \in \mathbb{N}$. Let's consider two cases:

Case 1: Let a (odd) and b (even). Then,

$f(a) = (a+1)/2$ and $f(b) = b/2$. So, $(a+1)/2 = b/2$, but $a \neq b$. This shows that f is not injective.

Step 2: Check for surjectivity (onto)

To check if f is surjective, we need to see if for every $m \in \mathbb{N}$, there exists an n such that $f(n) = m$.

Let m be even. If we choose $n = 2m$, then $f(n) = m$. So, for any even m , we can find an even number n such that $f(n) = m$. This suggests that f might be surjective.

Alternatively, if we choose m to be odd, then $m = (a+1)/2$ for some a . So, for any odd m , we can find an odd number $n = 2m - 1$ such that $f(n) = m$.

Step 3: Conclusion

Since we found that f is not injective, the function is not bijective. Although it appears to be surjective, the lack of injectivity is enough to conclude that f is not bijective.

Final Answer: No.

ANSWER

No.

Question 10

QUESTION

Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f : A \rightarrow B$ defined by $f(x) = (x - 2)/(x - 3)$. Is f one-one and onto? Justify your answer.

SOLUTION

We are given two sets, A and B , and a function defined by f . We need to determine if f is one-one (injective) and onto (surjective) and justify our answer.

Step 1: Check for one-one (injectivity)

To prove f is one-one, we need to show that if $f(x) = f(y)$, then $x = y$ for all $x, y \in A$.

Let $f(x) = f(y)$. Then,

Cross-multiplying, we get:

Expanding both sides:

Simplifying, we have:

Therefore, f is one-one.

Step 2: Check for onto (surjectivity)

To prove f is onto, we need to show that for every $y \in B$, there exists an $x \in A$ such that $f(x) = y$.

Let $f(x) = y$. We need to solve for x in terms of y .

Since $y \neq 1$, so the denominator $x - 3$ is well-defined. Also, we need to check if $x \neq 3$. If $x = 3$, then

This is a contradiction, so $x \neq 3$. Therefore, $x \in A$.

Thus, for every $y \in B$, there exists an $x \in A$ such that $f(x) = y$, so f is onto.

Final Answer: Yes, f is one-one and onto.

ANSWER

Yes.

Question 11

QUESTION

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = x^4$. Choose the correct answer.

- (A) f is one-one onto
- (B) f is many-one onto
- (C) f is one-one but not onto
- (D) f is neither one-one nor onto.

SOLUTION

We are given a function defined by and need to determine if it is one-one (injective) and/or onto (surjective).

Step 1: Check if the function is one-one (injective)

A function is one-one if different elements in the domain map to different elements in the codomain. In other words, if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$. Let's test this:

Suppose $x_1 \neq x_2$. Then $x_1^4 = x_2^4$.

Taking the fourth root of both sides, we get $x_1 = x_2$.

This means that x_1 can be equal to x_2 . For example, 1 and -1 . Since $1^4 = (-1)^4$, the function is not one-one.

Step 2: Check if the function is onto (surjective)

A function is onto if every element in the codomain has a corresponding element in the domain. In other words, for every y in the codomain, there exists an x in the domain such that $f(x) = y$.

Since the codomain is (all real numbers), we need to check if for every real number y , there exists a real number x such that $x^4 = y$.

Consider $y = -1$. We need to find an x such that $x^4 = -1$. However, for any real number x , x^4 is always non-negative (i.e., ≥ 0). Therefore, there is no real number x such that $x^4 = -1$. This means the function is not onto.

Step 3: Conclusion

Since the function is neither one-one nor onto, the correct answer is (D).

ANSWER

D

Question 12

QUESTION

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = 3x$. Choose the correct answer.

- (A) f is one-one onto
- (B) f is many-one onto
- (C) f is one-one but not onto
- (D) f is neither one-one nor onto.

SOLUTION

We are given a function defined by and we need to determine if it is one-one (injective) and onto (surjective).

Step 1: Check for one-one (injectivity)

To prove is one-one, we need to show that if , then for all .

Let . Then, .

Dividing both sides by 3, we get .

Since implies , the function is one-one.

Step 2: Check for onto (surjectivity)

To prove is onto, we need to show that for every (the codomain), there exists an (the domain) such that .

Let be an arbitrary element in the codomain. We need to find an such that .

We have . So, we want to solve for .

Dividing both sides by 3, we get .

Since and 3 is a non-zero real number, is also a real number. Thus, .

Therefore, for every , there exists an such that .

This means the function is onto.

Step 3: Conclusion

Since is both one-one and onto, the correct answer is (A).

Final Answer: (A) is one-one onto

Why other options are incorrect:

(B) is many-one onto: Incorrect because is one-one, not many-one.

(C) is one-one but not onto: Incorrect because is onto.

(D) is neither one-one nor onto: Incorrect because is both one-one and onto.

ANSWER

A

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Key Formulas

Important Formulas for Exercise 1.2

Formula / Concept	Description
Function	A function f from a set A (the domain) to a set B (the codomain) is a rule that assigns to each element x in A exactly one element y in B . It is denoted by $f: A \rightarrow B$.
One-to-one (Injective) Function	A function $f: A \rightarrow B$ is one-to-one if distinct elements in the domain have distinct images in the codomain. Mathematically, for every $x_1, x_2 \in A$, if $f(x_1) = f(x_2)$, then $x_1 = x_2$.
Many-one Function	A function $f: A \rightarrow B$ is many-one if it is not one-to-one. This means there exist at least two different elements in the domain A that have the same image in the codomain B .
Onto (Surjective) Function	A function $f: A \rightarrow B$ is onto if every element in the codomain B has at least one pre-image in the domain A . In other words, for every $y \in B$, there exists some $x \in A$ such that $f(x) = y$. This is equivalent to stating that the Range of the function is equal to its Codomain.
Into Function	A function $f: A \rightarrow B$ is an into function if it is not onto. This means there is at least one element in the codomain B which is not an image of any element in the domain A . The range is a proper subset of the codomain.
Bijjective Function	

Formula / Concept	Description
	A function is bijective if it is both one-to-one (injective) and onto (surjective). This creates a perfect one-to-one correspondence between the elements of the domain and the codomain.
Composition of Functions	Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be two functions. The composition of f and g , denoted by $g \circ f$, is a function $g \circ f: A \rightarrow C$ defined by $(g \circ f)(x) = g(f(x))$, for all $x \in A$.
Properties of Composition of Functions	<ul style="list-style-type: none"> • The composition of two one-to-one functions is one-to-one. • The composition of two onto functions is onto. • Consequently, the composition of two bijective functions is a bijective function. • Function composition is associative, i.e., $(f \circ g) \circ h = f \circ (g \circ h)$. • Function composition is generally not commutative, i.e., $f \circ g \neq g \circ f$.

🔗 Top FAQs

Q1. How many questions are in NCERT Solutions Class 12 Maths Chapter 1 Relations and Functions Exercise 1.2?

Exercise 1.2 of NCERT Solutions for Class 12 Maths Chapter 1 Relations and Functions contains exactly 12 questions. These questions cover various types of functions including one-one, onto, bijective functions and their properties. All 12 questions with step by step solutions are available for free PDF download for CBSE board exam 2025-26 preparation.

Q2. Where can I download free PDF of NCERT Solutions for Class 12 Maths Chapter 1 Relations and Functions Exercise 1.2?

You can download the free PDF of NCERT Solutions for Class 12 Maths Chapter 1 Relations and Functions Exercise 1.2 from the official NCERT website or various educational portals. These step by step solutions are updated according to the latest CBSE syllabus 2025-26 and include detailed explanations for all 12 questions covering types of relations and composition of functions.

Q3. How many marks does Relations and Functions Chapter 1 carry in CBSE Class 12 Maths board exam 2025-26?

Relations and Functions (Chapter 1) carries 4 marks weightage in the CBSE Class 12 Maths board exam 2025-26 as part of Unit I. Exercise 1.2 focuses on types of functions which is crucial for scoring these marks. Students should practice all questions from NCERT Solutions Class 12 Maths Chapter 1 Exercise 1.2 with step by step solutions for better exam preparation.

Q4. Which is the most difficult question in Exercise 1.2 of NCERT Solutions Class 12 Maths Chapter 1 Relations and Functions?

Questions 10, 11, and 12 in Exercise 1.2 of NCERT Solutions Class 12 Maths Chapter 1 Relations and Functions are considered the most challenging as they involve proving bijective nature and finding composition of functions. These questions require thorough understanding of one-one and onto concepts with step by step solutions approach recommended for CBSE board exam 2025-26 preparation.

Q5. What is Types of Relations covered in NCERT Solutions Class 12 Maths Chapter 1 Exercise 1.2?

Exercise 1.2 of NCERT Solutions for Class 12 Maths Chapter 1 primarily focuses on types of functions rather than types of relations, including one-one (injective), onto (surjective), and bijective functions. The exercise contains 12 questions with step by step solutions explaining how to identify and prove these function types, essential for CBSE board exam 2025-26 and available as free PDF download.


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