26 Co-ordinate Geometry

26.1 INTRODUCTION

Co-ordinate Geometry is the branch of mathematics in which a pair of two numbers, called **co-ordinates**, is used to represent the position of a point with respect to two mutually perpendicular number lines called **co-ordinate axes**.

The **location of points** comes under the heading **co-ordinate** and their relations, with respect to different figures, come under the heading **geometry.**

Together, the location of the points and their relationship with different geometrical figures is called **Co-ordinate Geometry**.

26.2 DEPENDENT AND INDEPENDENT VARIABLES

In linear equations of the form : $3x + 4y = 5$, $x - 3y + 8 = 0$, $y = mx + c$, $x = 5y - 8$, etc., the letters 'x' and 'y' are called **variables**.

1. If a linear equation in x and y is expressed with y as **the subject of formula** (equation); y is called the **dependent variable** and x is called the **independent variable.** In each of the following equations; y is **dependent** variable and x is **independent** variable.

   (i) $y = 3x - 6$ (ii) $y = 5 - \frac{x}{4}$ (iii) $y = 2\,(3x - 5) + 7$

2. If a linear equation in x and y is expressed with x as **the subject of formula** (equation); x is called the **dependent variable** and y is called **the independent variable.** In each of the following equations; x is the **dependent** variable and y is the **independent variable.**

   (i) $x = 5y + 7$ (ii) $x = 5\,(5y + 8) - 10$ (iii) $x = 7 - \frac{2y}{3}$

In equation $y = 4x + 9$; the value of y depends on the value of x, so y is said to be dependent variable and x is said to be independent variable.

In the same way, in equation $x = 3y - 5$; the value of x depends on the value of y, so x is said to be dependent variable and y is said to be independent variable.

1 Express the equation $4x - 5y + 20 = 0$ in the form so that :

(i) x is dependent variable and y is independent variable.

(ii) y is dependent variable and x is independent variable.

Solution :

(i) $\mathbf{4x - 5y + 20 = 0} \Rightarrow 4x = 5y - 20$

$\Rightarrow x = \frac{5}{4}y - 5$ **Ans.**

(ii) $4x - 5y + 20 = 0 \Rightarrow \quad -5y = -4x - 20$

$\Rightarrow \quad 5y = 4x + 20$

$\Rightarrow \quad y = \frac{4}{5}x + 4$ **Ans.**

26.3 ORDERED PAIR

An ordered pair means, a pair of two objects taken in a specific order.

In relation to co-ordinate geometry, an ordered pair means, a pair of two numbers in which the order is important and necessary.

1. To form an ordered pair, the numbers are written in specific order, separated by a comma, and enclosed in small brackets.

   Each of the following represents an ordered pair :

   $(5, 7), (-6, 8), (0, 0), (0, -6), (5, 0), (3\frac{1}{2}, -2)$, etc.
2. In the ordered pair (a, b); a is called its **first component** and b is called its **second component.**
3. Ordered pairs (5, 7) and (7, 5) are different *i.e.* $(5, 7) \neq (7, 5)$.
4. If two ordered pairs are equal; their corresponding components are equal

   i.e. $(a, b) = (c, d) \Rightarrow a = c$ and $b = d$.
5. An ordered pair can have both of its components equal *i.e.* an ordered pair can be of the form : $(5, 5), (-6, -6), (0, 0)$, etc.

2 Find the values of x and y, if :

(i) $(x, 4) = (-7, y)$

(ii) $(x - 3, 6) = (4, x + y)$

Solution :

Two ordered pairs are equal

$\Rightarrow$ Their first components are equal and their second components are separately equal.

(i) $(x, 4) = (-7, y)$

$\Rightarrow \quad x = -7$ and $y = 4$ **Ans.**

(ii) $(x - 3, 6) = (4, x + y)$

$\Rightarrow \quad x - 3 = 4$ and $6 = x + y$

$\Rightarrow \quad x = 7$ and $6 = 7 + y$

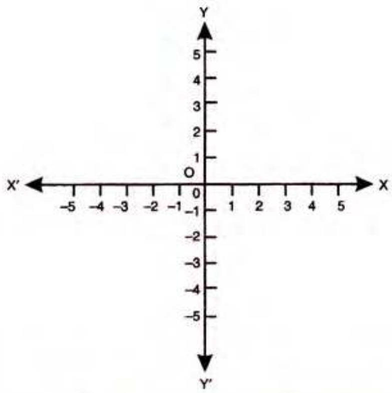
or $\quad x = 7$ and $y = -1$ **Ans.**

26.4 CARTESIAN PLANE

A cartesian (or a co-ordinate) plane consists of two mutually perpendicular number lines intersecting each other at their zeros.

The adjoining figure shows a cartesian plane consisting of two mutually perpendicular number lines XOX′ and YOY′ intersecting each other at their zero 0.

1. The horizontal number line XOX′ is called the **x-axis.**
2. The vertical number line YOY′ is called the **y-axis.**
3. The point of intersection 'O' is called the origin which is zero for both the axes.



The system consisting of the x-axis, the y-axis and the origin is also called cartesian co-ordinate system. The x-axis and the y-axis together are called co-ordinate axes.

26.5 CO-ORDINATES OF POINTS

The position of each point in a co-ordinate plane is determined by means of an *ordered pair* (a pair of numbers) with reference to the co-ordinate axes as stated below :

(i) Starting from the origin O, measure the distance of the point along x-axis. This distance is called *x-co-ordinate* or ***abscissa*** of the point.

(ii) Starting from the origin O, measure the distance of the point along the y-axis. This distance is called the *y-co-ordinate* or ***ordinate*** of the point.

Thus, the co-ordinates of the point

= Position of the point with reference to co-ordinate axes.

= (abscissa, ordinate).

In stating the co-ordinates of a point, the *abscissa preceeds the ordinate* and both are enclosed in a small bracket after being separated by a comma.

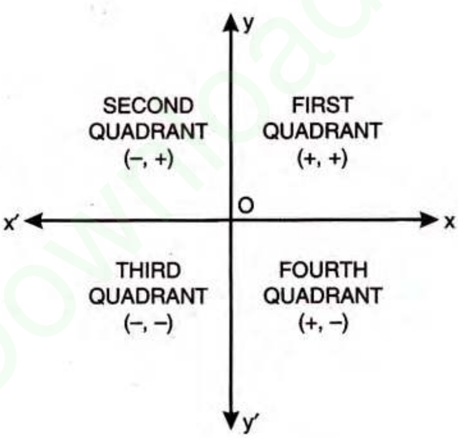
e.g. if the abscissa of a point is x and its ordinate is y, its co-ordinates = (x, y).

26.6 QUADRANTS AND SIGN CONVENTION

1. Quadrants :

As shown in the adjoining diagram, the co-ordinate axes divide a co-ordinate plane into four parts, which are known as *quadrants*. Each point in the plane is located either in one of the quadrants or on one of the axes.

Starting from OX in the anti-clockwise direction; **XOY** is called the **first quadrant, YOX′** is called the **second quadrant, X′OY′** is called the **third quadrant** and **Y′OX** is called **the fourth quadrant.**



2. Sign Convention :

It is clear from the figure (given on the previous page); the co-ordinate axes divide a plane into four quadrants. Also :

(i) in the *first quadrant, XOY,* the *abscissa* and the *ordinate* both are *positive*

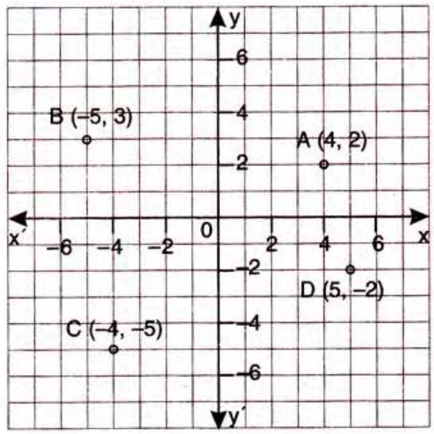
(ii) in the *second quadrant, X′OY,* the *abscissa* is *negative* and the *ordinate* is *positive*

(iii) in the *third quadrant, X′OY′,* the *abscissa* and the *ordinate* both are *negative* and

(iv) in the *fourth quadrant, XOY′,* the *abscissa* is *positive* and the *ordinate* is *negative.*

26.7 PLOTTING OF POINTS

3 Plot the points A (4, 2), B (–5, 3), C (– 4, –5) and D (5, –2).



Solution **:**

On a graph paper, draw the co-ordinate axes XOX′ and YOY′ intersecting at origin O. With proper scale, mark the numbers on the two co-ordinate axes.

For plotting any point; two steps are to be adopted. *e.g.* to plot point A (4, 2).

Step 1 :

Starting from the origin O, move 4 units along the positive direction of the x-axis *i.e.* to the right of the origin O.

Step 2 :

Now, from there, move 2 units up (*i.e.* parallel to positive direction of the y-axis) and place a dot at the point reached. Label this point as **A (4, 2).**

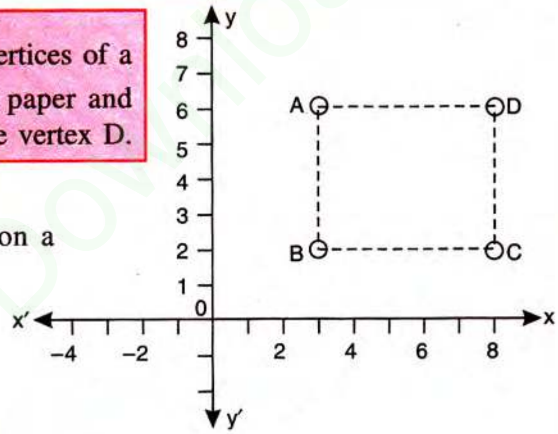
Similarly, plot the other points B (–5, 3), C (– 4, –5) and D (5, – 2)

1. The co-ordinates of the origin = (0,0)
2. *For a point on the x-axis,* its ordinate is always zero and so the co-ordinates of a point on x-axis is of the form $(x, 0)$.

   e.g. (7, 0), (3, 0), (0, 0), (– 4, 0), (– 8, 0), etc.
3. *For a point on the y-axis;* its abscissa is always zero and so the co-ordinates of a point on y-axis is of the form $(0, y)$.

   e.g. (0, 8), (0, 3), (0, 0), (0, –2), (0, –5), etc.

4 A (3, 6), B (3, 2) and C (8, 2) are the vertices of a rectangle. Plot these points on a graph paper and then use it to find the co-ordinates of the vertex D.



Solution **:**

After plotting the given points A, B and C on a graph paper; join A with B and B with C.

Complete the rectangle ABCD.

Now, read the co-ordinates of D.

As is clear from the graph; **D = (8, 6)** **Ans.**

5 Find the co-ordinates of the point whose abscissa is the solution of the first quadrant and the ordinate is the solution of the second equation.
$0{\cdot}5x - 3 = -\,0{\cdot}25\,x$ and $8 - 0{\cdot}2\,(y + 3) = 3y + 1$

Solution :

$$0{\cdot}5\,x - 3 = -\,0{\cdot}25x \Rightarrow 0{\cdot}5x + 0{\cdot}25x = 3$$
$$\Rightarrow 0{\cdot}75x = 3$$
$$\Rightarrow x = \frac{3}{0{\cdot}75} = \frac{3\times100}{75} = 4$$
$$8 - 0{\cdot}2\,(y + 3) = 3y + 1 \Rightarrow 8 - 0{\cdot}2y - 0{\cdot}6 = 3y + 1$$
$$\Rightarrow -0{\cdot}2y - 3y = 1 + 0{\cdot}6 - 8$$
$$\Rightarrow -\,3{\cdot}2y = -\,6{\cdot}4 \Rightarrow y = 2$$

$\therefore$ **The co-ordinates of the point = (4, 2)** **Ans.**

EXERCISE 26(A)

1. For each equation given below; name the dependent and independent variables.

   (i) $y = \frac{4}{3}x - 7$

   (ii) $x = 9y + 4$

   (iii) $x = \frac{5y+3}{2}$

   (iv) $y = \frac{1}{7}\,(6x + 5)$

2. Plot the following points on the same graph paper :

   (i) (8, 7) (ii) (3, 6)
   (iii) (0, 4) (iv) (0, – 4)
   (v) (3, – 2) (vi) (– 2, 5)
   (vii) (– 3, 0) (viii) (5, 0)
   (ix) (– 4, – 3)

3. Find the values of x and y if :

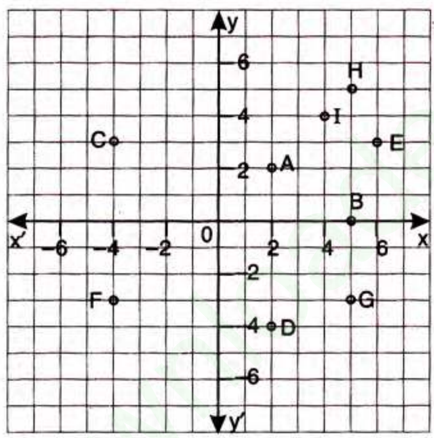
   (i) $(x - 1,\ y + 3) = (4,\ 4)$

   (ii) $(3x + 1,\ 2y - 7) = (9,\ -\,9)$

   (iii) $(5x - 3y,\ y - 3x) = (4,\ -\,4)$

4. Use the graph given alongside, to find the co-ordinates of the point (s) satisfying the given condition :

   (i) the abscissa is 2.
   (ii) the ordinate is 0.
   (iii) the ordinate is 3.
   (iv) the ordinate is – 4.
   (v) the abscissa is 5.
   (vi) the abscissa is equal to the ordinate.
   (vii) the ordinate is half of the abscissa.



5. State, true or false :

   (i) The ordinate of a point is its x-co-ordinate.
   (ii) The origin is in the first quadrant.
   (iii) The y-axis is the vertical number line.
   (iv) Every point is located in one of the four quadrants.

(v) If the ordinate of a point is equal to its abscissa; the point lies either in the first quadrant or in the second quadrant.

(vi) The origin (0, 0) lies on the x-axis.

(vii) The point (a, b) lies on the y-axis if $b = 0$.

6. In each of the following, find the co-ordinates of the point whose abscissa is the solution of the first equation and ordinate is the solution of the second equation :

   (i) $3 - 2x = 7;\ 2y + 1 = 10 - 2\frac{1}{2}y.$

   (ii) $\frac{2a}{3} - 1 = \frac{a}{2};\ \frac{15 - 4b}{7} = \frac{2b - 1}{3}.$

   (iii) $5x - (5 - x) = \frac{1}{2}(3 - x);\ 4 - 3y = \frac{4 + y}{3}$

7. In each of the following, the co-ordinates of the three vertices of a rectangle ABCD are given. By plotting the given points; find, in each case, the co-ordinates of the fourth vertex :

   (i) A (2, 0), B (8, 0) and C (8, 4).

   (ii) A (4, 2), B (–2, 2) and D (4, –2).

   (iii) A (– 4, – 6), C (6, 0) and D (– 4, 0)

   (iv) B (10, 4), C (0, 4) and D (0, – 2).

8. A (– 2, 2), B (8, 2) and C (4, – 4) are the vertices of a parallelogram ABCD. By plotting the given points on a graph paper; find the co-ordinates of the fourth vertex D.

   Also, from the same graph, state the co-ordinates of the mid-points of the sides AB and CD.

9. A (– 2, 4), C (4, 10) and D (– 2, 10) are the vertices of a square ABCD. Use the graphical method to find the co-ordinates of the fourth vertex B. Also, find :

   (i) the co-ordinates of the mid-point of BC;

   (ii) the co-ordinates of the mid-point of CD and

   (iii) the co-ordinates of the point of intersection of the diagonals of the square ABCD.

10. By plotting the following points on the same graph paper, check whether they are collinear or not :

    (i) (3, 5), (1, 1) and (0, –1)

    (ii) (– 2, –1), (– 1, – 4) and (– 4, 1)

11. Plot the point A (5, – 7). From point A, draw AM perpendicular to x-axis and AN perpendicular to y-axis. Write the co-ordinates of points M and N.

12. In square ABCD; A = (3, 4), B = (– 2, 4) and C = (–2, –1). By plotting these points on a graph paper, find the co-ordinates of vertex D. Also, find the area of the square.

13. In rectangle OABC; point O is the origin, OA = 10 units along x-axis and AB = 8 units. Find the co-ordinates of vertices A, B and C.

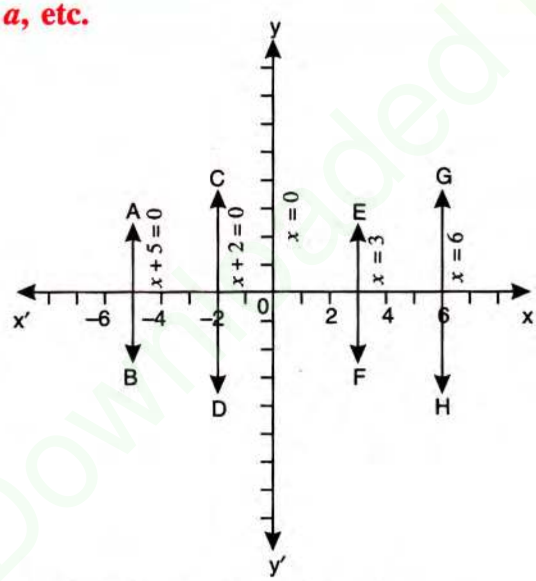
26.8 GRAPHS OF $x = 0$, $y = 0$, $x = a$, $y = a$, etc.

1. $\boldsymbol{x = 0}$ is the equation of the **y–axis** as the value of 'x' for every point (x, y) on the y-axis is '0'.

For example :

Points (0, 7), (0, 0), (0, –8), (0, 15) are all on the y-axis since for each of these points; the value of the abscissa, $x = 0$.

2. $\boldsymbol{x = a}$ is the equation of a **line parallel to the y-axis** and at a **distance of 'a' units from it.**



For example :

In the given figure;

(i) AB is parallel to the y-axis and is at a distance of '–5' units from the y-axis

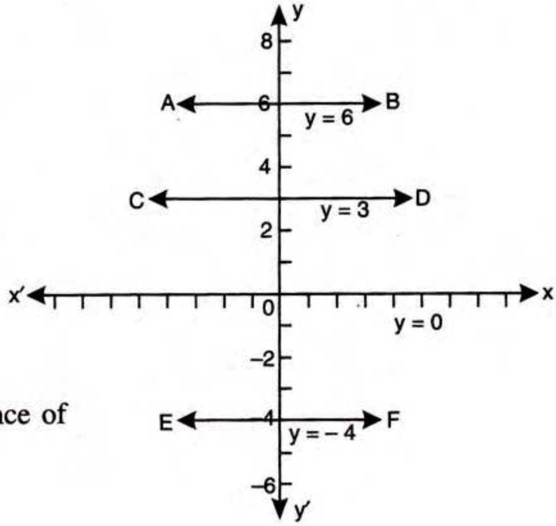
$\Rightarrow$ equation of **AB** : $\boldsymbol{x = -5}$ *i.e.* $\boldsymbol{x + 5 = 0}$.

(ii) Equation of **CD** is $x = -2$ *i.e.,* $x + 2 = 0$;

(iii) Equation of **EF** is $x = 3$

(iv) Equation of **GH** is $x = 6$ and so on.

3. $y = 0$ is the equation of the ***x*-axis;** as the value of 'y' for every point (x, y) on the x-axis is '0'.



For example :

Points (8, 0), (0, 0), (–7, 0), (15, 0), etc. are all on the x-axis since for each of these points, the value of the ordinate, $y = 0$.

4. $y = a$ is the equation of a **line parallel to *x*-axis** and **at a distance of '*a*' units from it.**

For example :

In the given figure :

(i) AB is parallel to the x-axis and is at a distance of 6 units from the x-axis

$\Rightarrow$ equation of **AB :** $y = 6$.

(ii) Equation of **CD** is $y = 3$.

(iii) Equation of EF is $y = -4$ *i.e.* $y + 4 = 0$.

6 Draw the graph of each of the following equations :

(i) $y = 3$ (ii) $y + 5 = 0$ (iii) $x = 4$ (iv) $x + 6 = 0$

Solution :

(i) The graph of $y = 3$ is the straight line AB which is parallel to the x-axis at a distance of 3 units from it.

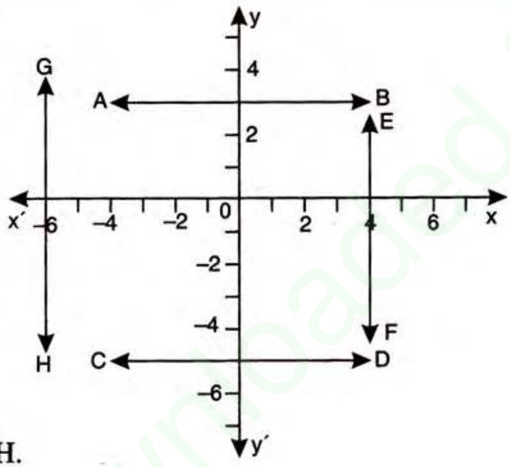
(ii) Since, $y + 5 = 0 \Rightarrow y = -5$.

$\therefore$ The graph is the straight line CD which is parallel to the x-axis at a distance of –5 units from it.

(iii) The graph of $x = 4$ is the straight line EF which is parallel to the y-axis at a distance of 4 units from it.

(iv) Since, $x + 6 = 0 \Rightarrow x = -6$.

$\therefore$ The graph of $x + 6 = 0$ is the straight line GH.



26.9 GRAPHING A LINEAR EQUATION

If the graph of an equation is a straight line, the equation is called a *linear equation.*

To draw the graph of a linear equation :

(i) plot a few points, which satisfy the given equation;

(ii) draw a straight line passing through these points.

Type 1 :

When the given linear equation is of the form $y = mx$.

7 Draw the graph of $y = -2x$.

Solution :

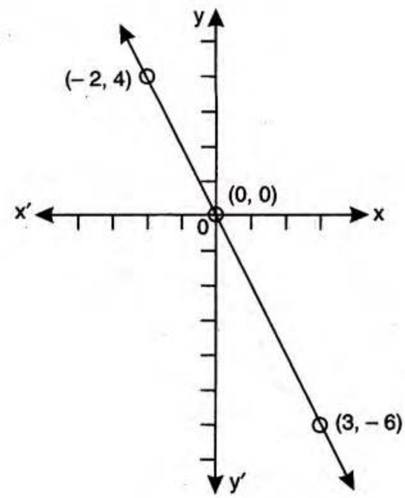
Step 1 :

Give at least three suitable values to the variable x and find the corresponding values of y.

Let $x = 0$, then $y = -2 \times 0 = 0$

Let $x = 3$, then $y = -2 \times 3 = -6$

Let $x = -2$, then $y = -2 \times -2 = 4$



Step 2 :

Make a table (as given below) for the different pairs of the values of x and y :

x	0	3	-2
y	0	-6	4

Step 3 :

3. Plot the points, from the table, on a graph paper and then draw a straight line passing through the points plotted on the graph.

Type 2 :

When the equation is of the form $y = mx + c$; where c is a rational but not zero.

8 Draw the graph of $y = 3x - 4$.

Solution :

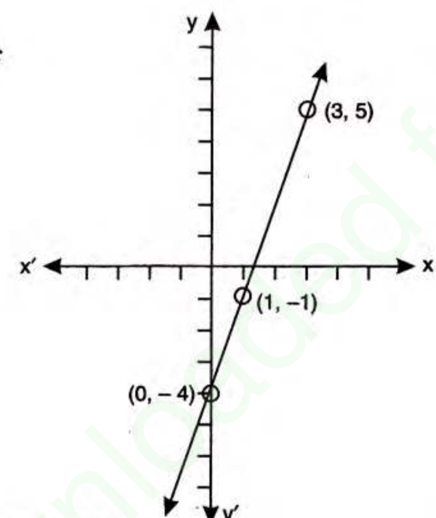
When $x = 1$, $y = 3 \times 1 - 4 = -1$

When $x = 3$, $y = 3 \times 3 - 4 = 5$

When $x = 0$, $y = 3 \times 0 - 4 = -4$.

$\therefore$ The table for x and y is :

x	1	3	0
y	-1	5	-4



The required graph (straight line) will be as drawn alongside.

9 Draw the graph of $y = -2x + \frac{3}{2}$.

Solution :

When $x = 2$, $y = -2 \times 2 + \frac{3}{2} = -\frac{5}{2}$

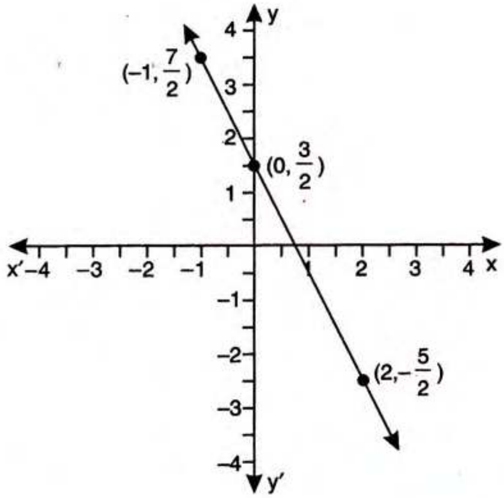
When $x = -1$, $y = 2 + \frac{3}{2} = \frac{7}{2}$

When $x = 0$, $y = 0 + \frac{3}{2} = \frac{3}{2}$

The table for x and y is :

x	2	-1	0
y	$\frac{-5}{2}$	$\frac{7}{2}$	$\frac{3}{2}$

The required graph (straight line) will be as drawn alongside.



10 Draw the graph of the equation $3x + 2y - 5 = 0$. Use this graph to find :

(i) x_1, the value of x, when $y = 7$.

(ii) y_1, the value of y, when $x = 3$.

Solution :

$$3x + 2y - 5 = 0$$

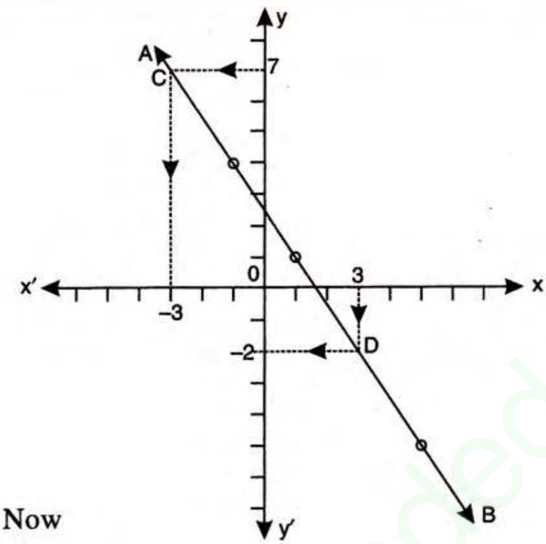
$$\Rightarrow \quad 2y = -3x + 5$$

$$\Rightarrow \quad y = \frac{-3x + 5}{2}$$

When $x = 1, y = \frac{-3+5}{2} = 1$

When $x = 5, y = \frac{-15+5}{2} = -5$

When $x = -1$, $y = \frac{3+5}{2} = 4$



Plot the points (1, 1), (5, – 5) and (–1, 4). Now draw the required straight line AB.

(i) To find x_1, the value of x, when $y = 7$:

Through the point $y = 7$, draw a horizontal straight line which meets the line AB at point C.

Through point C, draw a vertical line which meets the x-axis at $x = -3$.

Thus, the value of x, when $y = 7$, is -3 *i.e.* $\mathbf{x_1 = -3.}$ **Ans**

(ii) Through the point $x = 3$, draw a vertical line which meets the line AB at point D. Now, through point D, draw a horizontal line which meets the y-axis at $y = -2$.

Thus, the value of y, when $x = 3$, is -2 *i.e.* $\mathbf{y_1 = -2.}$ **Ans.**

EXERCISE 26(B)

1. Draw the graph for each linear equation given below :

(i) $x = 3$ (ii) $x + 3 = 0$

(iii) $x - 5 = 0$ (iv) $2x - 7 = 0$

(v) $y = 4$ (vi) $y + 6 = 0$

(vii) $y - 2 = 0$ (viii) $3y + 5 = 0$

(ix) $2y - 5 = 0$ (x) $y = 0$

(xi) $x = 0$

2. Draw the graph for each linear equation given below :

(i) $y = 3x$ (ii) $y = -x$

(iii) $y = -2x$ (iv) $y = x$

(v) $5x + y = 0$ (vi) $x + 2y = 0$

(vii) $4x - y = 0$ (viii) $3x + 2y = 0$

(ix) $x = -2y$

3. Draw the graph for each linear equation given below :

(i) $y = 2x + 3$ (ii) $y = \frac{2}{3}x - 1$

(iii) $y = -x + 4$ (iv) $y = 4x - \frac{5}{2}$

(v) $y = \frac{3}{2}x + \frac{2}{3}$ (vi) $2x - 3y = 4$

(vii) $\frac{x-1}{3} - \frac{y+2}{2} = 0$ (viii) $x - 3 = \frac{2}{5}(y + 1)$

(ix) $x + 5y + 2 = 0$

4. Draw the graph for each equation given below :

(i) $3x + 2y = 6$ (ii) $2x - 5y = 10$

(iii) $\frac{1}{2}x + \frac{2}{3}y = 5$ (iv) $\frac{2x-1}{3} - \frac{y-2}{5} = 0$

In each case, find the co-ordinates of the points where the graph (line) drawn meets the co-ordinate axes.

5. For each linear equation, given above, draw the graph and then use the graph drawn (in each case) to find the area of a triangle enclosed by the graph and the co-ordinate axes :

(i) $3x - (5 - y) = 7$

(ii) $7 - 3(1 - y) = -5 + 2x$.

6. For each pair of linear equations given below, draw graphs and then state, whether the lines drawn are parallel or perpendicular to each other.

(i) $y = 3x - 1$, $y = 3x + 2$ (ii) $y = x - 3$, $y = -x + 5$

(iii) $2x - 3y = 6$, $\frac{x}{2} + \frac{y}{3} = 1$ (iv) $3x + 4y = 24$, $\frac{x}{4} + \frac{y}{3} = 1$

7. On the same graph paper, plot the graph of $y = x - 2$, $y = 2x + 1$ and $y = 4$ from $x = -4$ to 3.

8. On the same graph paper, plot the graphs of $y = 2x - 1$, $y = 2x$ and $y = 2x + 1$ from $x = -2$ to $x = 4$. Are the graphs (lines) drawn parallel to each other ?

9. The graph of $3x + 2y = 6$ meets the x = axis at point P and the y-axis at point Q. Use the graphical method to find the co-ordinates of points P and Q.

10. Draw the graph of equation $x + 2y - 3 = 0$. From the graph, find :

(i) x_1, the value of x, when $y = 3$

(ii) x_2, the value of x, when $y = -2$.

11. Draw the graph of equation $3x - 4y = 12$. Use the graph drawn to find :

(i) y_1, the value of y, when $x = 4$

(ii) y_2, the value of y, when $x = 0$.

12. Draw the graph of equation $\frac{x}{4} + \frac{y}{5} = 1$. Use the graph drawn to find :

(i) x_1, the value of x, when $y = 10$

(ii) y_1, the value of y, when $x = 8$.

13. Use the graphical method to show that the straight lines given by the equations $x + y = 2$, $x - 2y = 5$ and $\frac{x}{3} + y = 0$ pass through the same point.

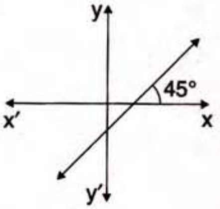
26.10 INCLINATION AND SLOPE

1. Inclination :

The angle which a straight line makes with the positive direction of x-axis (measured in the anti-clockwise direction) is called **inclination of the line**.

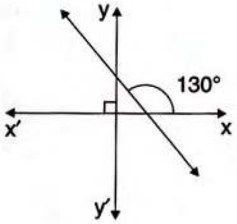
The inclination of a line is usually denoted by θ (theta).

(i)



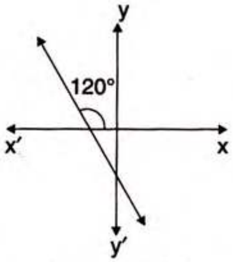
[Inclination, $\theta = 45°$]

(ii)



[$\theta = 130°$]

(iii)



[$\theta = 120°$]

1. For x-axis and every line parallel to x-axis, the inclination is zero *i.e.* $\theta = 0°$.
2. For y-axis and every line parallel to y-axis, the inclination is 90° *i.e.* $\theta = 90°$.

2. Slope (gradient) :

If θ is the inclination of a line; the slope of the line is $\tan\theta$ and is usually denoted by letter m.

$\therefore$ Slope $= m = \tan\theta$.

i.e. (i) If the inclination of a line is 30°, then $\theta = 30°$.

The slope (gradient) of the line $= m = \tan 30° = \dfrac{1}{\sqrt{3}}$

(ii) If the inclination of a line is 45°, then $\theta = 45°$.

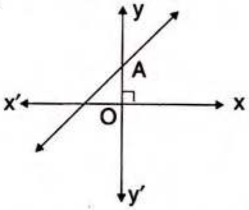
The gradient (slope) of the line $= m = \tan 45° = 1$.

1. ***For x-axis and every line parallel to x-axis,*** the inclination $\theta = 0°$.
   $\therefore$ Slope $(m) = \tan\theta = \tan 0° = 0$.
2. ***For y-axis and every line parallel to y-axis***, the inclination $\theta = 90°$.
   $\therefore$ Slope $(m) = \tan 90°$ = infinity (not defined).

26.11 Y-INTERCEPT

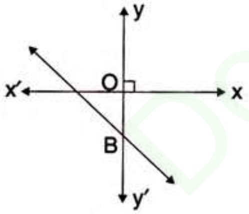
If a straight line meets y-axis at a point, the distance of this point from the origin is called ***y*-intercept** and is usually denoted by ***c***.

(i)



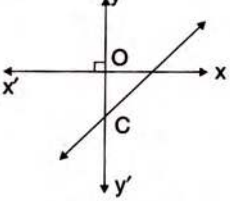
[y-intercept (c) = OA]

(ii)



[y-intercept = OB]

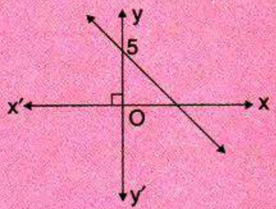
(iii)



[c = OC]

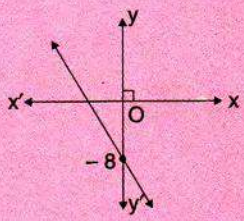
1. ***For x-axis***, y-intercept = 0
2. ***For every line parallel to y-axis;*** y-intercept = 0.
3. y-intercept is :
   (i) **positive,** if measured above the origin.
   (ii) **negative,** if measured below the origin.

(a)



[y-intercept (c) = 5]

(b)



[y-intercept (c) = –8]

26.12 FINDING THE SLOPE AND THE Y-INTERCEPT OF A GIVEN LINE

Steps :

1. Let the given line be $ax + by + c = 0$
2. Make y, the subject of the equation. For this :

$$ax + by + c = 0 \Rightarrow by = -ax - c$$

$$\Rightarrow y = \frac{-a}{b}x - \frac{c}{b}$$

3. The coefficient of x is the slope and the constant term is the y-intercept of the given line

$\therefore$ slope (m) = $\frac{-a}{b}$ and y-intercept (c) = $-\frac{c}{b}$.

11 Find the slope and the y-intercept of the line :

(i) $2x - 3y + 5 = 0$ (ii) $2y + 5x = 7$ (iii) $2y - 5 = 0$

Solution :

(i) $\mathbf{2x - 3y + 5 = 0} \Rightarrow -3y = -2x - 5$

$$\Rightarrow 3y = 2x + 5$$

$$\Rightarrow y = \frac{2}{3}x + \frac{5}{3}$$

$\therefore$ **Slope = coefficient of x** = $\mathbf{\frac{2}{3}}$

And, **y-intercept** = constant term = $\mathbf{\frac{5}{3}}$ **Ans.**

(ii) $2y + 5x = 7 \Rightarrow 2y = -5x + 7$

$$\Rightarrow y = -\frac{5}{2}x + \frac{7}{2}$$

$\therefore$**Slope** = $\mathbf{-\frac{5}{2}}$ and ***y*-intercept** = $\mathbf{\frac{7}{2}}$ **Ans.**

(iii) $2y - 5 = 0 \Rightarrow 2y = 5 \Rightarrow y = \frac{5}{2}$

$$\Rightarrow y = 0 \times x + \frac{5}{2}$$

$\therefore$ **Slope = 0** and ***y*-intercept** $= \frac{5}{2}$ **Ans.**

Whenever an equation of a straight line is converted into the form $y = mx + c$; the slope of the line $= m$ and its y-intercept $= c$. ***Conversely***, if the **slope** of a line is $\boldsymbol{m}$ and its ***y*-intercept** is **c;** the equation of the line is $y = mx + c$.

12 Find the equation of a line whose :
(i) slope = –3 and y-intercept = 5 (ii) $m = 8$ and $c = -6$.

Solution :

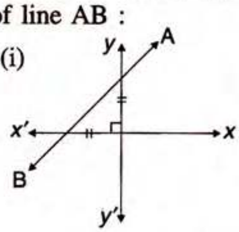
(i)
$$\text{slope} = -3 \Rightarrow m = -3$$
$$y\text{-intercept} = 5 \Rightarrow c = 5$$
$$\therefore \text{Equation is : } y = mx + c \Rightarrow y = -3x + 5 \Rightarrow 3x + y = 5$$ **Ans.**

(ii)
$$m = 8 \quad \text{and} \quad c = -6$$
$\Rightarrow$ Equation of the line is : $y = mx + c$ *i.e.* $y = 8x - 6$ **Ans.**

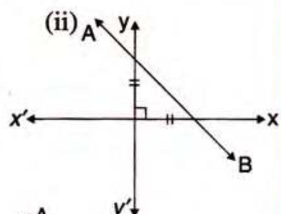
EXERCISE 26(C)

1. In each of the following, find the inclination of line AB :

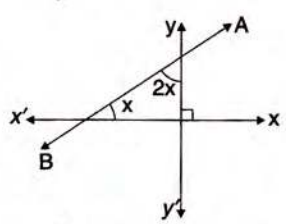
(i)



(ii)



(iii)



2. Write the inclination of a line which is :
   (i) parallel to x-axis.
   (ii) perpendicular to x-axis.
   (iii) parallel to y-axis.
   (iv) perpendicular to y-axis.

3. Write the slope of the line whose inclination is:
   (i) 0° (ii) 30° (iii) 45° (iv) 60°

4. Find the inclination of the line whose slope is:
   (i) 0 (ii) 1 (iii) $\sqrt{3}$ (iv) $\frac{1}{\sqrt{3}}$

(i) $m = 0 \Rightarrow \tan\theta = 0$
$\Rightarrow \tan\theta = \tan 0°$
$\Rightarrow \theta = 0°$
$\therefore$ Inclination = 0°

5. Write the slope of the line which is :
   (i) parallel to x-axis.
   (ii) perpendicular to x-axis.
   (iii) parallel to y-axis.
   (iv) perpendicular to y-axis.

6. For each of the equations given below, find the slope and the y-intercept :
   (i) $x + 3y + 5 = 0$ (ii) $3x - y - 8 = 0$
   (iii) $5x = 4y + 7$ (iv) $x = 5y - 4$
   (v) $y = 7x - 2$ (vi) $3y = 7$
   (vii) $4y + 9 = 0$

7. Find the equation of the line, whose :
   (i) slope = 2 and y-intercept = 3
   (ii) slope = 5 and y-intercept = – 8
   (iii) slope = – 4 and y-intercept = 2
   (iv) slope = – 3 and y-intercept = – 1
   (v) slope = 0 and y-intercept = –5
   (vi) slope = 0 and y-intercept = 0

8. Draw the line $3x + 4y = 12$ on a graph paper. From the graph paper, read the y-intercept of the line.

9. Draw the line $2x - 3y - 18 = 0$ on a graph paper. From the graph paper, read the y-intercept of the line.

10. Draw the graph of line $x + y = 5$. Use the graph paper drawn to find the inclination and the y-intercept of the line.