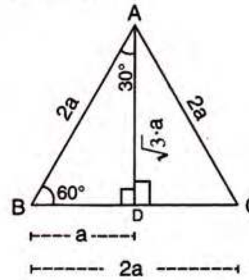
23 Trigonometrical Ratios of Standard Angles

[Including Evaluation of an Expression Involving Trigonometric Ratios]

23.1 TRIGONOMETRICAL RATIOS OF ANGLES 30° AND 60°

Let ABC be an equilateral triangle with side '$2a$' and AD is perpendicular to BC.



Clearly, $BD = \dfrac{BC}{2} = a$

In Δ ABD, $AD^2 = AB^2 - BD^2$ [Using Pythagoras Theorem]

$= (2a)^2 - a^2 = 3a^2$

$\therefore AD = \sqrt{3}\,.\,a$

Since, each angle of an equilateral triangle is 60°

$\therefore \quad \angle B = 60°$ and

$\angle BAD = 180° - (60° + 90°) = 30°$

$\therefore$ In Δ ABD,

$\sin 60° = \dfrac{\text{perp.}}{\text{hyp.}} = \dfrac{AD}{AB} = \dfrac{\sqrt{3}\cdot a}{2a} = \dfrac{\sqrt{3}}{2}$; $\sin 30° = \dfrac{\text{perp.}}{\text{hyp.}} = \dfrac{BD}{AB} = \dfrac{a}{2a} = \dfrac{1}{2}$

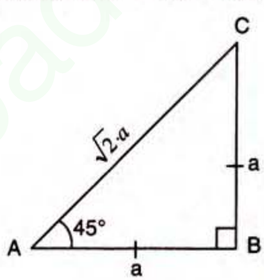
$\cos 60° = \dfrac{\text{base}}{\text{hyp.}} = \dfrac{BD}{AB} = \dfrac{a}{2a} = \dfrac{1}{2}$; $\cos 30° = \dfrac{\text{base}}{\text{hyp.}} = \dfrac{AD}{AB} = \dfrac{\sqrt{3}.a}{2a} = \dfrac{\sqrt{3}}{2}$

$\tan 60° = \dfrac{\text{perp.}}{\text{base}} = \dfrac{AD}{BD} = \dfrac{\sqrt{3}\cdot a}{a} = \sqrt{3}$; $\tan 30° = \dfrac{\text{perp.}}{\text{base}} = \dfrac{BD}{AD} = \dfrac{a}{\sqrt{3}\cdot a} = \dfrac{1}{\sqrt{3}}$

and so on.

23.2 TRIGONOMETRICAL RATIOS OF ANGLE 45°

The adjoining figure shows a right-angled isosceles triangle in which $\angle B = 90°$ and $AB = BC = a$.



Clearly, $\angle A = 45°$

and, $AC = \sqrt{2}.a$ [Since, $AC^2 = AB^2 + BC^2$]

$\therefore \sin 45° = \dfrac{\text{perp.}}{\text{hyp.}} = \dfrac{BC}{AC} = \dfrac{a}{\sqrt{2}\cdot a} = \dfrac{1}{\sqrt{2}}$;

$\cos 45° = \dfrac{\text{base}}{\text{hyp.}} = \dfrac{AB}{AC} = \dfrac{a}{\sqrt{2}\cdot a} = \dfrac{1}{\sqrt{2}}$

$\tan 45° = \dfrac{\text{perp.}}{\text{base}} = \dfrac{BC}{AB} = \dfrac{a}{a} = 1$ and so on.

Also, remember that :

$\sin 0° = 0$; $\cos 0° = 1$

$\sin 90° = 1$; $\cos 90° = 0$

Therefore, for standard angles of 0°, 30°, 45°, 60° and 90°, we have :

Angle→	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞ (not defined)
cot	∞ (not defined)	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞ (not defined)
cosec	∞ (not defined)	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

1. It can be seen from the table, given above, that as the angle increases from 0° to 90°;

(i) value of sin increases from 0 to 1

(ii) value of cos decreases from 1 to 0

(iii) value of tan increases from 0 to ∞ and so on.

2. If $x = y = 45°$ or, $x + y = 90°$ then, $\sin x = \cos y$; $\tan x = \cot y$ and $\sec x = \text{cosec } y$.

For example : $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$ $[x = y = 45°]$

and also $\sin 30° = \cos 60° = \frac{1}{2}$ $[x + y = 90°]$

3. Whatever be the measure of angle A;

(i) $\sin^2 A + \cos^2 A = 1$

(ii) $\sec^2 A - \tan^2 A = 1$

(iii) $\text{cosec}^2 A - \cot^2 A = 1$

For example : If A = 30°; $\mathbf{\sin^2 A + \cos^2 A}$ $= \sin^2 30° + \cos^2 30°$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{1}{4} + \frac{3}{4} = \mathbf{1}$$

Similarly, $\mathbf{\sec^2 A - \tan^2 A}$ $= \sec^2 30° - \tan^2 30°$

$$= \left(\frac{2}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= \frac{4}{3} - \frac{1}{3} = 1$$

1. $\sin(A + B) \neq \sin A + \sin B$
2. $\cos(A + B) \neq \cos A + \cos B$
3. $\tan(A + B) \neq \tan A + \tan B$
4. $\sin(A - B) \neq \sin A - \sin B$ and so on.

1 Evaluate :

(i) $\sin^2 30° - 2\cos^3 60° + 3\tan^4 45°$

(ii) $(\cos 0° + \sin 45° + \sin 30°)(\sin 90° - \cos 45° + \cos 60°)$

Solution :

(i) $\sin^2 30° - 2\cos^3 60° + 3\tan^4 45°$

$$= \left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)^3 + 3(1)^4$$

$$= \frac{1}{4} - 2 \times \frac{1}{8} + 3 \times 1 = \frac{1}{4} - \frac{1}{4} + 3 = 3 \qquad \textbf{Ans.}$$

(ii) $(\cos 0° + \sin 45° + \sin 30°)(\sin 90° - \cos 45° + \cos 60°)$

$$= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right)\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right)$$

$$= \left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right)\left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right)$$

$$= \left(\frac{3}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{9}{4} - \frac{1}{2} = 1\frac{3}{4} \qquad \textbf{Ans.}$$

2 Find the value of :

$$\frac{\sin 30° - \sin 90° + 2\cos 0°}{\tan 30° \times \tan 60°}$$

Solution :

$$\frac{\sin 30° - \sin 90° + 2\cos 0°}{\tan 30° \times \tan 60°} = \frac{\frac{1}{2} - 1 + 2 \times 1}{\frac{1}{\sqrt{3}} \times \sqrt{3}}$$

$$= \frac{\frac{1}{2} - 1 + 2}{1}$$

$$= \frac{3}{2} = 1\frac{1}{2} \qquad \textbf{Ans.}$$

3 If A = 60°, verify that :

(i) $\sin^2 A + \cos^2 A = 1$ (ii) $\sec^2 A - \tan^2 A = 1$

Solution :

(i) $\sin^2 A + \cos^2 A = \sin^2 60° + \cos^2 60° = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = 1$

(ii) $\sec^2 A - \tan^2 A = \sec^2 60° - \tan^2 60° = (2)^2 - \left(\sqrt{3}\right)^2 = 4 - 3 = 1$

4 If $x = 15°$, evaluate : $8 \sin 2x \cos 4x \sin 6x$

Solution :

$$
\begin{aligned}
8 \sin 2x \cos 4x \sin 6x &= 8 \sin(2 \times 15°) \cos(4 \times 15°) \sin(6 \times 15°) \\
&= 8 \sin 30° \cos 60° \sin 90° \\
&= 8 \times \frac{1}{2} \times \frac{1}{2} \times 1 = 2
\end{aligned}
$$

Ans.

EXERCISE 23(A)

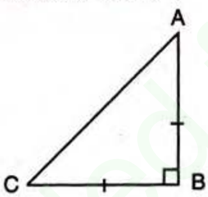
1. Find the value of :
   (i) $\sin 30° \cos 30°$
   (ii) $\tan 30° \tan 60°$
   (iii) $\cos^2 60° + \sin^2 30°$
   (iv) $\text{cosec}^2 60° - \tan^2 30°$
   (v) $\sin^2 30° + \cos^2 30° + \cot^2 45°$
   (vi) $\cos^2 60° + \sec^2 30° + \tan^2 45°$

2. Find the value of :
   (i) $\tan^2 30° + \tan^2 45° + \tan^2 60°$
   (ii) $\dfrac{\tan 45°}{\text{cosec } 30°} + \dfrac{\sec 60°}{\cot 45°} - \dfrac{5 \sin 90°}{2 \cos 0°}$
   (iii) $3 \sin^2 30° + 2 \tan^2 60° - 5 \cos^2 45°$

3. Prove that :
   (i) $\sin 60° \cos 30° + \cos 60°. \sin 30° = 1$
   (ii) $\cos 30°. \cos 60° - \sin 30°. \sin 60° = 0$
   (iii) $\text{cosec}^2 45° - \cot^2 45° = 1$
   (iv) $\cos^2 30° - \sin^2 30° = \cos 60°$
   (v) $\left(\dfrac{\tan 60° + 1}{\tan 60° - 1}\right)^2 = \dfrac{1 + \cos 30°}{1 - \cos 30°}$
   (vi) $3 \text{ cosec}^2 60° - 2 \cot^2 30° + \sec^2 45° = 0.$

4. Prove that :
   (i) $\sin (2 \times 30°) = \dfrac{2 \tan 30°}{1 + \tan^2 30°}$
   (ii) $\cos (2 \times 30°) = \dfrac{1 - \tan^2 30°}{1 + \tan^2 30°}$
   (iii) $\tan (2 \times 30°) = \dfrac{2 \tan 30°}{1 - \tan^2 30°}$

5. ABC is an isosceles right-angled triangle. Assuming AB = BC = x, find the value of each of the following trigonometric ratios :
   (i) $\sin 45°$
   (ii) $\cos 45°$
   (iii) $\tan 45°$



6. Prove that :
   (i) $\sin 60° = 2 \sin 30° \cos 30°$.
   (ii) $4 (\sin^4 30° + \cos^4 60°) - 3 (\cos^2 45° - \sin^2 90°) = 2$

7. (i) If $\sin x = \cos x$ and x is acute, state the value of x.
   (ii) If $\sec A = \text{cosec } A$ and $0° \leq A \leq 90°$, state the value of A.
   (iii) If $\tan \theta = \cot \theta$ and $0° \leq \theta \leq 90°$, state the value of θ.
   (iv) If $\sin x = \cos y$; write the relation between x and y, if both the angles x and y are acute.

8. (i) If $\sin x = \cos y$, then $x + y = 45°$; write true or false.

(ii) sec θ. cot θ = cosec θ; write true or false.

(iii) For any angle θ, state the value of : $\sin^2\theta + \cos^2\theta$.

9. State for any acute angle θ whether :

(i) sin θ increases or decreases as θ increases.

(ii) cos θ increases or decreases as θ increases.

(iii) tan θ increases or decreases as θ decreases.

10. If $\sqrt{3} = 1{\cdot}732$, find (correct to two decimal places) the value of each of the following :

(i) $\sin 60°$ (ii) $\dfrac{2}{\tan 30°}$

11. Evaluate :

(i) $\dfrac{\cos 3A - 2\cos 4A}{\sin 3A + 2\sin 4A}$, when A = 15°.

(ii) $\dfrac{3\sin 3B + 2\cos(2B + 5°)}{2\cos 3B - \sin(2B - 10°)}$; when B = 20°.

5 If A = 60° and B = 30°; prove that :

$$\sin(A - B) = \sin A\cos B - \cos A\sin B$$

Solution :

L.H.S. $= \sin(60° - 30°)$ [L.H.S. = Left hand side]

$= \sin 30° = \dfrac{1}{2}$

R.H.S. $= \sin 60°\cos 30° - \cos 60°\sin 30°$ [R.H.S. = Right hand side]

$= \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{3}}{2} - \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{3}{4} - \dfrac{1}{4} = \dfrac{1}{2}$

∴ **L.H.S. = R.H.S.**

Hence Proved.

6 If A = 30°, then prove that :

$$\cos 3A = 4\cos^3 A - 3\cos A$$

Solution :

L.H.S. $= \cos 3A$

$= \cos(3 \times 30°) = \cos 90° = 0$

R.H.S. $= 4\cos^3 A - 3\cos 30°$

$= 4\cos^3 30° - 3\cos 30°$

$= 4\left(\dfrac{\sqrt{3}}{2}\right)^3 - 3\left(\dfrac{\sqrt{3}}{2}\right)$

$= 4 \times \dfrac{3.\sqrt{3}}{8} - \dfrac{3.\sqrt{3}}{2} = 0$

∴ **L.H.S. = R.H.S.**

Hence Proved.

1. Given A = 60° and B = 30°, prove that :
   (i) $\sin (A + B) = \sin A \cos B + \cos A \sin B$
   (ii) $\cos (A + B) = \cos A \cos B - \sin A \sin B$
   (iii) $\cos (A - B) = \cos A \cos B + \sin A \sin B$
   (iv) $\tan (A - B) = \dfrac{\tan A - \tan B}{1 + \tan A . \tan B}$
2. If A = 30°, then prove that :
   (i) $\sin 2A = 2 \sin A \cos A = \dfrac{2 \tan A}{1 + \tan^2 A}$
   (ii) $\cos 2A = \cos^2 A - \sin^2 A$
   $= \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$
   (iii) $2 \cos^2 A - 1 = 1 - 2 \sin^2 A$
   (iv) $\sin 3A = 3 \sin A - 4 \sin^3 A$
3. If A = B = 45°, show that :
   (i) $\sin (A - B) = \sin A \cos B - \cos A \sin B$
   (ii) $\cos (A + B) = \cos A \cos B - \sin A \sin B$
4. If A = 30°; show that :
   (i) $\sin 3A$
   $= 4 \sin A \sin (60° - A) \sin (60° + A)$
   (ii) $(\sin A - \cos A)^2 = 1 - \sin 2A$
   (iii) $\cos 2A = \cos^4 A - \sin^4 A$
   (iv) $\dfrac{1 - \cos 2A}{\sin 2A} = \tan A$
   (v) $\dfrac{1 + \sin 2A + \cos 2A}{\sin A + \cos A} = 2 \cos A.$
   (vi) $4 \cos A \cos (60° - A) \cdot \cos (60° + A)$
   $= \cos 3A$
   (vii) $\dfrac{\cos^3 A - \cos 3A}{\cos A} + \dfrac{\sin^3 A + \sin 3A}{\sin A} = 3$

23.3 SOLVING A TRIGONOMETRIC EQUATION

To solve a trigonometric equation means, **to find the value of the unknown angle** that satisfies the given equation.

7 Find A, if :

(i) $\sin 2A = 1$ (ii) $2 \cos 3A = 1$ (iii) $(\sec A - 2)(\tan 3A - 1) = 0$

Solution :

(i) $\sin 2A = 1 \Rightarrow \sin 2A = \sin 90°$ [Since, $\sin 90° = 1$]

$\therefore\ 2A = 90°$ and $A = 45°$ **Ans.**

(ii) $2 \cos 3A = 1 \Rightarrow \cos 3A = \dfrac{1}{2}$ [Since, $\cos 60° = \dfrac{1}{2}$]

$\Rightarrow \cos 3A = \cos 60°$

$\therefore\ 3A = 60°$ and $A = 20°$ **Ans.**

(iii) $(\sec A - 2)(\tan 3A - 1) = 0$

$\Rightarrow \sec A - 2 = 0$ or $\tan 3A - 1 = 0$

$\Rightarrow \sec A = 2$ and $\tan 3A = 1 = \tan 45°$

$\Rightarrow \sec A = \sec 60°$ and $3A = 45°$

$\therefore\ A = 60°$ and $A = 15°$ **Ans.**

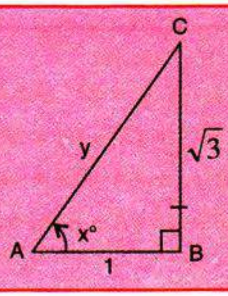
8 From the adjoining figure, find :

(i) $\tan x°$

(ii) $x°$

(iii) $\cos x°$

(iv) use $\sin x°$ to find y.



Solution :

(i) $\tan x° = \frac{\text{perp.}}{\text{base}} = \frac{\sqrt{3}}{1} = \sqrt{3}$ **Ans.**

(ii) Since, $\tan x° = \sqrt{3}$

and $\tan 60° = \sqrt{3}$ $\therefore$ $x° = 60°$ **Ans.**

(iii) $\cos x° = \cos 60° = \frac{1}{2}$ **Ans.**

(iv) $\sin x° = \sin 60°$ [Since, $x° = 60°$]

$\Rightarrow \frac{\sqrt{3}}{y} = \frac{\sqrt{3}}{2}$ [since, $\sin x° = \frac{\text{perp.}}{\text{hyp.}} = \frac{\sqrt{3}}{y}$ and $\sin 60° = \frac{\sqrt{3}}{2}$]

$\Rightarrow \quad y = 2$ **Ans.**

9 *If* $4 \sin^2 x° - 3 = 0$ and $x°$ is an acute angle; find :

(i) $\sin x°$ (ii) $x°$

Solution :

(i) $4 \sin^2 x° - 3 = 0 \Rightarrow \sin^2 x° = \frac{3}{4}$

$\Rightarrow \sin x° = \frac{\sqrt{3}}{2}$ **Ans.**

(ii) $\sin x° = \frac{\sqrt{3}}{2} \Rightarrow \sin x° = \sin 60°$ $\left[\sin 60° = \frac{\sqrt{3}}{2}\right]$

$\Rightarrow x = 60°$ **Ans.**

10 Find the magnitude of angle A, if :

(i) $4 \sin A \sin 2A + 1 - 2 \sin 2A = 2 \sin A$

(ii) $2 \sin^2 A - 3 \sin A + 1 = 0$

(iii) $3 \cot^2 (x - 5°) = 1$

(iv) $\sin^2 2x + \sin^2 60° = 1$

Solution :

(i) $4 \sin A \sin 2A + 1 - 2 \sin 2A = 2 \sin A$

$\Rightarrow$ $4 \sin A \sin 2A - 2 \sin 2A - 2 \sin A + 1 = 0$

$\Rightarrow$ $2 \sin 2A(2 \sin A - 1) - 1(2 \sin A - 1) = 0$

$\Rightarrow$ $(2 \sin A - 1)(2 \sin 2A - 1) = 0$

$\Rightarrow$ $2 \sin A - 1 = 0$ or $2 \sin 2A - 1 = 0$

$2 \sin A - 1 = 0 \Rightarrow \sin A = \frac{1}{2}$ $\Rightarrow \sin A = \sin 30°$ $\Rightarrow A = 30°$

$2 \sin 2A - 1 = 0 \Rightarrow \sin 2A = \frac{1}{2}$ $\Rightarrow \sin 2A = \sin 30°$ $\Rightarrow 2A = 30°$ $\Rightarrow A = 15°$

$\therefore$ **Angle A = 30° or 15°** **Ans.**

(ii) $2 \sin^2 A - 3 \sin A + 1 = 0$

$\Rightarrow$ $2 \sin^2 A - 2 \sin A - \sin A + 1 = 0$

$\Rightarrow$ $2 \sin A(\sin A - 1) - (\sin A - 1) = 0$

$\Rightarrow$ $(\sin A - 1)(2 \sin A - 1) = 0$

$\Rightarrow$ $\sin A - 1 = 0$ or $2 \sin A - 1 = 0$

$\sin A - 1 = 0 \Rightarrow \sin A = 1$ $\Rightarrow \sin A = \sin 90°$ $\Rightarrow A = 90°$

$2 \sin A - 1 = 0 \Rightarrow \sin A = \frac{1}{2}$ $\Rightarrow \sin A = \sin 30°$ $\Rightarrow A = 30°$

$\therefore$ **Angle A = 90° or 30°** **Ans.**

(iii) $3 \cot^2(x - 5°) = 1 \Rightarrow \cot^2(x - 5°) = \frac{1}{3}$

$\Rightarrow \cot(x - 5°) = \frac{1}{\sqrt{3}} = \cot 60°$

$\Rightarrow x - 5° = 60°$ *i.e.* $x = 65°$ **Ans.**

(iv) $\sin^2 2x + \sin^2 60° = 1 \Rightarrow \sin^2 2x + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$

$\Rightarrow \sin^2 2x = 1 - \frac{3}{4} = \frac{1}{4}$

$\Rightarrow \sin 2x = \frac{1}{2} = \sin 30°$

$\Rightarrow 2x = 30°$ *i.e.* $x = 15°$ **Ans.**

11 Find acute angles A and B, if $\sin(A + B) = \cos(A - B) = \frac{\sqrt{3}}{2}$

Solution :

$\sin(A + B) = \frac{\sqrt{3}}{2} \Rightarrow \sin(A + B) = \sin 60° \Rightarrow A + B = 60°$ I

$\cos(A - B) = \frac{\sqrt{3}}{2} \Rightarrow \cos(A - B) = \cos 30° \Rightarrow A - B = 30°$ II

On solving equations I and II, we get $A = 45°$ and $B = 15°$ **Ans.**

12 If $\tan(A + B) = \sqrt{3}$ and $\sqrt{3}\tan(A - B) = 1$; find the angles A and B.

Solution :

$$\tan(A + B) = \sqrt{3} = \tan 60^\circ \quad \Rightarrow \quad A + B = 60^\circ$$

$$\sqrt{3}\tan(A - B) = 1 \quad \Rightarrow \quad \tan(A - B) = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\Rightarrow \quad A - B = 30^\circ$$

On solving equation $A + B = 60^\circ$ and $A - B = 30^\circ$

We get $\mathbf{A = 45^\circ}$ and $\mathbf{B = 15^\circ}$ **Ans.**

13 If $\sqrt{3}\tan 2\theta = 3$ and $0^\circ < \theta \leq 90^\circ$; find the value of $3\sqrt{3}\cos\theta + 2\sin\theta - 6\tan^2\theta$.

Solution :

$$\sqrt{3}\tan 2\theta = 3 \quad \text{i.e.} \quad \tan 2\theta = \frac{3}{\sqrt{3}} = \sqrt{3} = \tan 60^\circ$$

$$\Rightarrow \quad 2\theta = 60^\circ \quad \text{and} \quad \theta = 30^\circ$$

$\therefore$ $\mathbf{3\sqrt{3}\cos\theta + 2\sin\theta - 6\tan^2\theta}$

$$= 3\sqrt{3}\cos 30^\circ + 2\sin 30^\circ - 6\tan^2 30^\circ$$

$$= 3\sqrt{3} \times \frac{\sqrt{3}}{2} + 2 \times \frac{1}{2} - 6 \times \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= \frac{9}{2} + 1 - 2 = \frac{9+2-4}{2} = \frac{7}{2} = 3\frac{1}{2}$$ **Ans.**

14 Solve for θ $(0^\circ < \theta < 90^\circ)$:

(i) $\sin^2\theta - \frac{1}{2}\sin\theta = 0$ (ii) $2\sin^2\theta - 2\cos\theta = \frac{1}{2}$

(iii) $\tan^2\theta + 3 = 3\sec\theta$.

Solution :

(i) $\mathbf{\sin^2\theta - \frac{1}{2}\sin\theta = 0}$

$$\Rightarrow \quad \sin\theta\left(\sin\theta - \frac{1}{2}\right) = 0$$

$$\Rightarrow \quad \sin\theta = 0 \quad \text{or} \quad \sin\theta - \frac{1}{2} = 0 \quad \text{i.e.} \quad \sin\theta = \frac{1}{2}$$

$$\Rightarrow \quad \sin\theta = \sin 0^\circ \quad \text{or} \quad \sin\theta = \sin 30^\circ$$

$$\Rightarrow \quad \theta = 0^\circ \quad \text{or} \quad \theta = 30^\circ$$ **Ans.**

(ii) $2 \sin^2 \theta - 2 \cos \theta = \frac{1}{2}$

$\Rightarrow \quad 4 \sin^2 \theta - 4 \cos \theta = 1$

$\Rightarrow \quad 4(1 - \cos^2 \theta) - 4 \cos \theta = 1$

$\Rightarrow \quad 4 - 4 \cos^2 \theta - 4 \cos \theta = 1$

i.e. $\quad 4 \cos^2 \theta + 4 \cos \theta - 3 = 0$

$\Rightarrow \quad 4 \cos^2 \theta + 6 \cos \theta - 2 \cos \theta - 3 = 0$

i.e. $\quad 2 \cos \theta (2 \cos \theta + 3) - 1 (2 \cos \theta + 3) = 0$

$\Rightarrow \quad (2 \cos \theta + 3)(2 \cos \theta - 1) = 0$

i.e. $\quad 2 \cos \theta + 3 = 0$ or $2 \cos \theta - 1 = 0$

i.e. $\quad \cos \theta = -\frac{3}{2}$ or $\cos \theta = \frac{1}{2} = \cos 60°$

> For every value of angle θ;
> $\sin^2 \theta + \cos^2 \theta = 1$
> $\Rightarrow \quad \sin^2 \theta = 1 - \cos^2 \theta$

But $\cos \theta = -\frac{3}{2}$ is not possible for $0° < \theta < 90°$

$\therefore \quad \theta = 60°$ **Ans.**

> For each value of angle θ, between 0° and 90°, the values of sin θ and cos θ are always between 0 and 1.

(iii) $\tan^2 \theta + 3 = 3 \sec \theta$

$\Rightarrow \quad \sec^2 \theta - 1 + 3 = 3 \sec \theta$

$\Rightarrow \quad \sec^2 \theta - 3 \sec \theta + 2 = 0$

$\Rightarrow \quad \sec^2 \theta - 2 \sec \theta - \sec \theta + 2 = 0$

$\Rightarrow \quad \sec \theta (\sec \theta - 2) - 1(\sec \theta - 2) = 0$

$\Rightarrow \quad (\sec \theta - 2)(\sec \theta - 1) = 0$

$\Rightarrow \quad \sec \theta - 2 = 0$ or $\sec \theta - 1 = 0$

$\Rightarrow \quad \sec \theta = 2$ or $\sec \theta = 1$

i.e. $\quad \sec \theta = \sec 60°$ or $\sec \theta = \sec 0°$

$\Rightarrow \quad \theta = 60°$ [As θ lies between 0° and 90°] **Ans.**

> For every value of angle θ;
> $1 + \tan^2 \theta = \sec^2 \theta$
> $\Rightarrow \quad \tan^2 \theta = \sec^2 \theta - 1$

EXERCISE 23(C)

1. Solve the following equations for A, if :
   (i) $2 \sin A = 1$ (ii) $2 \cos 2A = 1$
   (iii) $\sin 3A = \frac{\sqrt{3}}{2}$ (iv) $\sec 2A = 2$
   (v) $\sqrt{3} \tan A = 1$ (vi) $\tan 3A = 1$
   (vii) $2 \sin 3A = 1$ (viii) $\sqrt{3} \cot 2A = 1$

2. Calculate the value of A, if :
   (i) $(\sin A - 1)(2 \cos A - 1) = 0$
   (ii) $(\tan A - 1)(\text{cosec } 3A - 1) = 0$
   (iii) $(\sec 2A - 1)(\text{cosec } 3A - 1) = 0$
   (iv) $\cos 3A . (2 \sin 2A - 1) = 0$
   (v) $(\text{cosec } 2A - 2)(\cot 3A - 1) = 0$

3. If $2 \sin x° - 1 = 0$ and $x°$ is an acute angle; find:
   (i) $\sin x°$ (ii) $x°$ (iii) $\cos x°$ and $\tan x°$.

4. If $4\cos^2 x° - 1 = 0$ and $0 \le x° \le 90°$, find:

(i) $x°$ (ii) $\sin^2 x° + \cos^2 x°$

(iii) $\frac{1}{\cos^2 x°} - \tan^2 x°$

5. If $4\sin^2\theta - 1 = 0$ and angle θ is less than 90°, find the value of θ and hence the value of $\cos^2\theta + \tan^2\theta$.

6. If $\sin 3A = 1$ and $0 \le A \le 90°$, find :

(i) $\sin A$ (ii) $\cos 2A$

(iii) $\tan^2 A - \frac{1}{\cos^2 A}$

7. If $2\cos 2A = \sqrt{3}$ and A is acute, find :

(i) A (ii) $\sin 3A$

(iii) $\sin^2(75° - A) + \cos^2(45° + A)$

8. (i) If $\sin x + \cos y = 1$ and $x = 30°$, find the value of y.

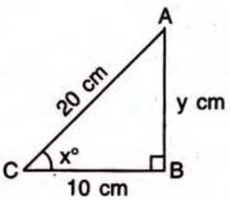
(ii) If $3\tan A - 5\cos B = \sqrt{3}$ and $B = 90°$, find the value of A.

9. From the given figure, find :

(i) $\cos x°$ (ii) $x°$

(iii) $\frac{1}{\tan^2 x°} - \frac{1}{\sin^2 x°}$

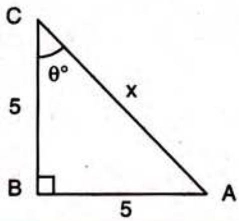
(iv) Use $\tan x°$, to find the value of y.



10. Use the given figure to find :

(i) $\tan\theta°$ (ii) $\theta°$ (iii) $\sin^2\theta° - \cos^2\theta°$

(iv) Use $\sin\theta°$ to find the value of x.



11. Find the magnitude of angle A, if :

(i) $2\sin A\cos A - \cos A - 2\sin A + 1 = 0$

(ii) $\tan A - 2\cos A\tan A + 2\cos A - 1 = 0$

(iii) $2\cos^2 A - 3\cos A + 1 = 0$

(iv) $2\tan 3A\cos 3A - \tan 3A + 1 = 2\cos 3A$

12. Solve for x :

(i) $2\cos 3x - 1 = 0$ (ii) $\cos\frac{x}{3} - 1 = 0$

(iii) $\sin(x + 10°) = \frac{1}{2}$ (iv) $\cos(2x - 30°) = 0$

(v) $2\cos(3x - 15°) = 1$ (vi) $\tan^2(x - 5°) = 3$

(vii) $3\tan^2(2x - 20°) = 1$

(viii) $\cos\left(\frac{x}{2} + 10°\right) = \frac{\sqrt{3}}{2}$

(ix) $\sin^2 x + \sin^2 30° = 1$

(x) $\cos^2 30° + \cos^2 x = 1$

(xi) $\cos^2 30° + \sin^2 2x = 1$

(xii) $\sin^2 60° + \cos^2(3x - 9°) = 1$

13. If $4\cos^2 x = 3$ and x is an acute angle; find the value of :

(i) x (ii) $\cos^2 x + \cot^2 x$

(iii) $\cos 3x$ (iv) $\sin 2x$

14. In Δ ABC, $\angle B = 90°$, AB = y units, BC = $\sqrt{3}$ units, AC = 2 units and angle A = $x°$, find :

(i) $\sin x°$ (ii) $x°$ (iii) $\tan x°$

(iv) use $\cos x°$ to find the value of y.

15. If $2\cos(A + B) = 2\sin(A - B) = 1$; find the values of A and B.