

Solids

[Surface Area and Volume of 3-D Solids]

21.1 INTRODUCTION

- 1. Solid: Anything which occupies space and has a definite shape is called a solid.
- 2. Volume: The space occupied by a solid is called its volume.
 - (a) The capacity of a container = Its internal volume.
 - (b) The volume of material in a hollow body = Its external volume Its internal volume.
- 3. Surface area of a solid: The sum of the areas of all the surfaces of a solid is called its surface area or its total surface area.

Since, a solid has three dimensions i.e. length, breadth and height, a solid is a three-dimensional (3-D) figure.

In the same way:

- (i) area is a 2-dimensional (2-D) **figure** as it has two dimensions *i.e.* length and breadth.
- (ii) perimeter is a single dimensional (1-D) figure as it has only one dimension *i.e.* length.

21.2 CUBOID

A rectangular solid which has six faces, each of which is a rectangle, is called a cuboid.

For a cuboid:

1. Volume = length \times breadth \times height

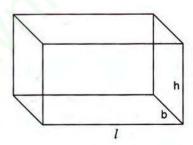
$$= l \times b \times h$$

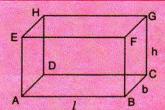
- 2. Total surface area = $2(l \times b + b \times h + h \times l)$
- 3. Lateral surface area of a cuboid = The sum of areas of the four walls (vertical faces) of the cuboid.

$$= 2(l+b) \times h$$

4. Length of diagonal = $\sqrt{l^2 + b^2 + h^2}$

The length of the longest rod that can be placed in a rectangular box or in a room = its diagonal.





Every cuboid has four diagonals

In the figure, given above,

diagonals are:

AG, BH, CE and DF.

such that, AG = BH = CE = DF

Hence, the diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$

 $BD^2 = l^2 + b^2$

 \Rightarrow BD = $\sqrt{l^2 + b^2}$

G In right triangle ABD,

In right triangle DBH,

$$BH^{2} = \left(\sqrt{l^{2} + b^{2}}\right)^{2} + h^{2}$$
$$= l^{2} + b^{2} + h^{2}$$

$$\Rightarrow$$
 BH = $\sqrt{I^2 + h^2 + h^2}$

CUBE

A rectangular solid, in which each face is a square, is called a cube.

For a square:

If length of each edge = a unit.

$$\Rightarrow \qquad l = b = h = a \text{ (edge)}$$

1. Volume =
$$l \times b \times h$$

= $a \times a \times a = a^3 = (edge)^3$

2. Total surface area =
$$6a^2$$

3. Lateral surface area =
$$4a^2$$

4. Length of diagonal =
$$\sqrt{a^2 + a^2 + a^2} = a\sqrt{3}$$

COST OF AN ARTICLE

Multiply known quantity of the article with its rate to get the cost of the article.

- (a) We buy petrol by its volume:
 - :. Cost of petrol bought = The quantity of perol bought × Rate of petrol
- (b) We buy land by its area:
 - :. Cost of land bought = The area of land bought × Rate of land
- (c) We buy cloth by its length:
 - :. Cost of cloth bought = Length of cloth bought × Rate of cloth

$$Cost = Rate \times Quantity$$

0 The outer dimensions of a closed wooden box are 22 cm, 15 cm and 10 cm. Thickness of the wood is 1 cm. Find the cost of wood required to make the box, if 1cm³ of wood costs ₹ 7.50.

Solution:

External volume of box = $22 \times 15 \times 10 \text{ cm}^3 = 3300 \text{ cm}^3$

Since, external dimensions are 22 cm, 15 cm and 10 cm; and thickness of the wood is 1 cm

:. Internal dimensions =
$$(22-2\times1)$$
 cm, $(15-2\times1)$ cm and $(10-2\times1)$ cm = 20 cm, 13 cm and 8 cm.

Hence, internal volume of the box = $20 \times 13 \times 8 \text{ cm}^3 = 2080 \text{ cm}^3$

and volume of wood in the box = $3300 \text{ cm}^3 - 2080 \text{ cm}^3 = 1220 \text{ cm}^3$

Ans.

A cube of a metal of 5 cm edge is melted and casted into a cuboid whose base is 2.50 cm × 0.50 cm. Find the height of the cuboid.

Also, find the surface areas of cube and cuboid.

Solution:

Here, volume of cuboid formed = volume of cube melted

i.e.
$$2.50 \times 0.50 \times h = (5)^3$$

$$\mathbf{h} = \frac{5 \times 5 \times 5}{2 \cdot 50 \times 0.50} \text{ cm}$$

Ans.

Surface area of cube $= 6a^2$

$$= 6(5)^2 \text{ cm}^2 = 150 \text{ cm}^2$$

Ans.

Surface area of cuboid = $2(l \cdot b + b \cdot h + h \cdot l)$

$$= 2 (2.50 \times 0.50 + 0.50 \times 100 + 100 \times 2.50) \text{ cm}^2$$

$$= 602.50 \text{ cm}^2$$

Ans.

- A small indoor green house (herbarium) is made entirely of glass panes (including base) held together with tape. The dimensions of the green house are 40 cm × 30 cm × 25 cm. Find:
 - (i) the area of the glass used.
 - (ii) the length of the tape required.

Solution:

(i) Since, the green house is in the shape of a cuboid and its length (l) = 40 cm, breadth (b) = 30 cm and height (h) = 25 cm

.. Area of the glass used =
$$2(lb + bh + hl)$$

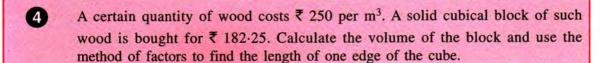
= $2(40 \times 30 + 30 \times 25 + 25 \times 40)$ cm²

Ans.

(ii) Length of the tape used = Perimeter of the top + Perimeter of the bottom + Four vertical edges

$$= 2(l+b) + 2(l+b) + 4h$$

$$= 4(40 + 30) \text{ cm} + 4 \times 25 \text{ cm} = 380 \text{ cm}^2$$



Solution:

Volume of cubical block × Rate = Cost

⇒ Volume of cubical block =
$$\frac{182 \cdot 25}{250}$$
 m³ [: Volume = $\frac{\text{Cost}}{\text{Rate}}$] = 0.729 m³ Ans.

$$\therefore \qquad (edge)^3 = 0.729 \qquad [\because Volume of cube = (edge)^3]$$
$$= \frac{729}{1000} = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 5 \times 5 \times 5}$$

$$\therefore \qquad \text{Edge of the cube} = \frac{3 \times 3}{2 \times 5} \text{ m} = 0.9 \text{ m}$$
Ans

Three cubes, each with 8 cm edge, are joined end to end. Find the total surface

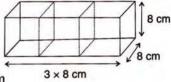
6 Three cubes, each with 8 cm edge, are joined end to end. Find the total surface area of the resulting cuboid.

Solution :

As is clear from the adjoining figure;

the length of the resulting cuboid = 3×8 cm = 24 cm

its width = 8 cm and its height = 8 cm



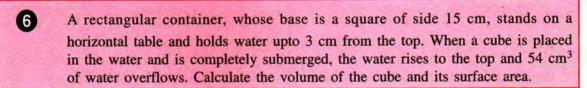
Ans.

i.e. l = 24 cm, b = 8 cm and h = 8 cm

... The total surface area of the resulting cuboid

=
$$2(l \times b + b \times h + h \times l)$$

= $2(24 \times 8 + 8 \times 8 + 8 \times 24)$ cm² = 896 cm² Ans.



Solution:

The volume of the cube submerged

= volume of water that fills 3 cm height of the container + volume of the water that overflows

$$= 15 \times 15 \times 3 \text{ cm}^3 + 54 \text{ cm}^3 = 729 \text{ cm}^3$$
 Ans

If side of the cube submerged = x cm

its volume =
$$x^3$$
 cm³
 $x^3 = 729 = 9 \times 9$

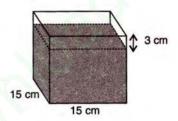
$$x^3 = 729 = 9 \times 9 \times 9$$

$$\Rightarrow \qquad x = 9$$

$$\therefore$$
 The side of the cube = 9 cm

And, its surface area =
$$6 \times (\text{side})^2$$

= $6 \times 9 \times 9 \text{ cm}^2 = 486 \text{ cm}^2$



0

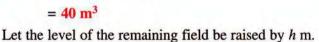
A field is 15 m long and 12 m broad. At one corner of this field a rectangular well of dimensions $8 \text{ m} \times 2.5 \text{ m} \times 2 \text{ m}$ is dug and the dug-out soil is spread evenly over the the rest of the field. Find the rise in the level of the rest of the field.

Solution :

The volume of the soil removed

= volume of the well dug.

 $= 8 \times 2.5 \times 2 \text{ m}^3$



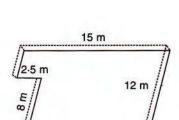
Since, the area of the remaining field = $(15 \times 12 - 8 \times 2.5)$ m²

 $= 160 \text{ m}^2$

And, volume of the soil spread = Area of the remaining field x rise in level

 $\Rightarrow 40 \text{ m}^3 = 160 \text{ m}^2 \times h$

 $h = \frac{40}{160} \text{ m} = \frac{1}{4} \text{ m}$



15 m

12 m

2.5 m

Ans.

.. The rise in level of the rest of the field = 25 cm

The sum of the length, breadth and height of a cuboid is 19 cm and the length of its diagonal is 11 cm. Find the surface area of the cuboid.

Solution:

 \Rightarrow

Given,
$$l + b + h = 19 \text{ cm}$$
 and $\sqrt{l^2 + b^2 + h^2} = 11 \text{ cm}$

i.e.
$$l+b+h=19 \text{ cm}$$
 and $l^2+b^2+h^2=121$

Required to find:

Surface area of the cuboid = 2(l.b + b.h + h.l)

Since,
$$(l+b+h)^2 = l^2 + b^2 + h^2 + 2(lb+bh+hl)$$

$$\Rightarrow 19^2 = 121 + 2(lb + bh + hl)$$

$$\Rightarrow \qquad 361 = 121 + 2(lb + bh + hl)$$

Surface area =
$$2(lb + bh + hl)$$

= $(361 - 121) \text{ cm}^2 = 240 \text{ cm}^2$

Ans.

How many bricks, (each measuring 20 cm × 16 cm × 8 cm), will be required to build a wall 30 m long, 30 cm thick and 5 m high, with a provision of 2 doors, each measuring 2.5 m × 1.2 m. It is given that one-ninth of the wall is occupied by the cement and the sand mixture.

Solution :

Area of the wall (exculding doors) =
$$30 \times 5$$
 m² – $2 \times (2.5 \times 1.2)$ m²
= 150 m² – 6 m² = 144 m²

Volume of the wall = area of wall into its thickness

=
$$144 \times \frac{30}{100}$$
 m³ = $144 \times \frac{30}{100} \times 100 \times 100 \times 100$ cm³
= 43200000 cm³

Space occupied by the cement and the sand mixture

$$=\frac{1}{9} \times 43200000 \text{ cm}^3 = 4800000 \text{ cm}^3$$

⇒ Volume of the wall occupied by the bricks

$$= 43200000 \text{ cm}^3 - 4800000 \text{ cm}^3 = 38400000 \text{ cm}^3$$

Since, volume of each brick = $20 \times 16 \times 8 \text{ cm}^3$

Number of bricks required =
$$\frac{\text{Volume of the wall to be occupied by the bricks}}{\text{Volume of each brick}}$$
$$= \frac{38400000}{20 \times 16 \times 8} = 15000$$

- 1. The length, breadth and height of a rectangular solid are in the ratio 5: 4: 2. If the total surface area is 1216 cm², find the length, the breadth and the height of the solid.
- 2. The volume of a cube is 729 cm³. Find its total surface area.
- 3. The dimensions of a Cinema Hall are 100 m, 60 m and 15 m. How many persons can sit in the hall, if each requires 150 m³ of air?
- 4. 75 persons can sleep in a room 25 m by 9.6 m. If each person requires 16 m³ of air; find the height of the room.
- The edges of three cubes of metal are 3 cm,
 4 cm and 5 cm. They are melted and formed into a single cube. Find the edge of the new cube.
- 6. Three cubes, whose edges are x cm, 8 cm and 10 cm respectively, are melted and recasted into a single cube of edge 12 cm. Find 'x'.
- 7. Three equal cubes are placed adjacently in a row. Find the ratio of the total surface area of the resulting cuboid to that of the sum of the total surface areas of the three cubes.

8. The cost of papering the four walls of a room at 75 paise per square metre is ₹ 240. The height of the room is 5 metres. Find the length and the breadth of the room, if they are in the ratio 5:3.

- 9. The area of a playground is 3650 m². Find the cost of covering it with gravel 1·2 cm deep, if the gravel costs ₹ 6·40 per cubic metre.
- 10. A square plate of side 'x' cm is 8 mm thick. If its volume is 2880 cm³; find the value of x.
- 11. The external dimensions of a closed wooden box are 27 cm, 19 cm and 11 cm. If the thickness of the wood in the box is 1.5 cm; find:
 - (i) volume of the wood in the box;
 - (ii) the cost of the box, if wood costs ₹ 1.20 per cm³;
 - (iii) number of 4 cm cubes that could be placed into the box.
- 12. A tank 20 m long, 12 m wide and 8 m deep is to be made of iron sheet. It is open at the top. Determine the cost of iron-sheet, at the rate of ₹ 12.50 per metre, if the sheet is 2.5 m wide.

Area of sheet = Surface area of the tank

⇒ Length of the sheet × its width

= Area of 4 walls of the tank + Area of its base

⇒ Length of the sheet × 2.5 m

 $= 2(20 + 12) \times 8 \text{ m}^2$ $+ 20 \times 12 \text{ m}^2$

⇒ Length of the sheet = 300.8 m

And, cost of the sheet = 300.8×712.50

= ₹ 3,760

Ans.

- 13. A closed rectangular box is made of wood of 1.5 cm thickness. The exterior length and breadth are respectively 78 cm and 19 cm, and the capacity of the box is 15 cubic decimetres. Calculate the exterior height of the box.
- 14. The square on the diagonal of a cube has an area of 1875 sq. cm. Calculate:
 - (i) the side of the cube.
 - (ii) the total surface area of the cube.

If the side of the cube = a cm

 \Rightarrow The length of its diagonal = $a\sqrt{3}$ cm

And, $(a\sqrt{3})^2 = 1875 \Rightarrow a = 25$ cm

15. A hollow square-shaped tube open at both ends is made of iron. The internal square is of 5 cm side and the length of the tube is 8 cm. There are 192 cm³ of iron in this tube. Find its thickness.

Let thickness of the tube = x cm

- :. Side of the external square = (5 + 2x) cm
- : Ext. vol. of the tube
 - its internal vol.

= vol. of iron in the tube

 \Rightarrow (5 + 2x) (5 + 2x) × 8 - 5 × 5 × 8 = 192.

8 cm

16. Four identical cubes are joined end to end to form a cuboid. If the total surface area of the resulting cuboid is 648 cm²; find the length of edge of each cube.

Also, find the ratio between the surface area of the resulting cuboid and the surface area of a cube.

21.5 CROSS-SECTION

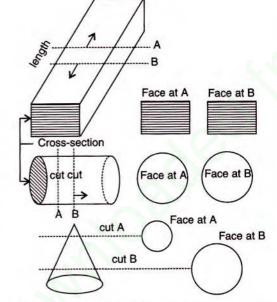
The cut, which is made through a solid perpendicular to its length (or, height), is called its *cross-section*.

A solid is said to have *uniform cross-section*, if this perpendicular cut is of the same shape and size at each point of its length (or height).

e.g.

(i) When a cuboid is cut through points A and B perpendicular to its length; the faces obtained (as the cross-section) at both the points are of same shape and size Therefore, a cuboid has uniform cross-section.

Similarly, a right-circular cylinder has uniform cross-section.



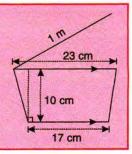
(ii) The cross-section of a right-circular cone is not uniform; since, the cuts made give faces of same shape; but not of same size.

When a body has uniform cross-section, its:

- 1. Volume = Area of cross-section × length
- 2. Surface area (excluding cross-section) = Perimeter of cross-section × length.



The adjoining figure shows a solid of uniform cross-section which is a trapezium in shape. If the length of the solid is 1m, find its volume.



Solution:

Area of cross-section of the solid =
$$\frac{1}{2}$$
 (23 + 17) × 10cm²
= 200 cm²

Volume of solid = Area of cross-section × length
=
$$200 \times 100 \text{ cm}^3$$
 [length = $1\text{m} = 100 \text{ cm}$]
= $20,000 \text{ cm}^3$

21.5 FLOW OF WATER (OR ANY OTHER LIQUID)

If water is flowing through a pipe or a canal, etc. of uniform cross-section; the volume of water that flows in unit time = Area of cross-section \times speed of flow of water.



How many litres of water flows out of a pipe of cross-section area 5 cm² in 1 minute, if the speed of the water in the pipe is 30 cm/s? [1 litre = 1000 cm³]

Solution:

Volume of water flowing in 1 sec. =
$$5 \times 30 \text{ cm}^3$$

= 150 cm^3
 \Rightarrow Volume of water flowing in 1 min. = $150 \times 60 \text{ cm}^3$
= $9000 \text{ cm}^3 = 9 \text{ litres}$

Ans.

Ans.



A rectangular tank is 25 m long and 9.5 m deep. If 600 cubic metres of water be drawn off the tank, the level of water in the tank goes down by 1.5 m. Calculate:

- (i) the width of the tank.
- (ii) the total volume of water which the tank can hold.

Solution:

(i) Volume of water taken out of tank = length of the tank × its width × fall in level of water \Rightarrow 600 m³ = 25 m × width of the tank × 1.5 m

$$\Rightarrow \qquad \text{The width of the tank} = \frac{600}{25 \times 1.5} \text{ m} = 16 \text{ m}$$
 Ans.

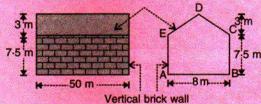
(ii) Total volume of water which tank can hold

= length of the tank × its width × its depth
=
$$25 \text{ m} \times 16 \text{ m} \times 9.5 \text{ m} = 3.800 \text{ m}^3$$
 Ans.

13

ABCDE is the end view of a factory shed which is 50 m long. The roofing of the shed consists of asbestos sheets as shown in the figure. The two ends of the shed are completely closed by brick walls.

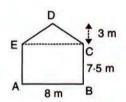
Slant asbestos sheet roofing



- (i) Calculate the total volume content of the shed.
- (ii) If the cost of asbestos sheet roofing is ₹ 25 per m² (sq. metre); find the cost of roofing.
- (iii) If the whole outside surface of the shed (including roofing) is to be painted, find the cost of painting it at ₹ 5 per m² (sq. metre).

Solution:

(i) Area of cross-section = Area of ABCDE = Area of rect. ABCE + Area of \triangle CDE = $8 \times 7.5 \text{ m}^2 + \frac{1}{2} \times 8 \times 3 \text{ cm}^2 = 72 \text{ m}^2$



- : Total volume contents of the shed
 - = Area of cross-section \times length = $72 \text{ m}^2 \times 50 \text{ m} = 3.600 \text{ m}^3$

Ans.

(ii) In ΔCDE, draw DP ⊥ CE.

Clearly,
$$EP = CP = \frac{8}{2} \text{ m} = 4 \text{ m}.$$

Using Pythagoras theorem,

We can find: DE = DC = 5 m.

- .. Area of asbestos sheet roofing = DE \times 50 m² + CD \times 50 m² = 5 \times 50 m² + 5 \times 50 m² = 500 m²
- ∴ The cost of asbestos roofing = Area of the sheet × Rate $= 500 \times ₹ 25 = ₹ 12,500$ Ans.
- (iii) The whole outside surface area of the shed

=
$$2 \times \text{Area}$$
 of cross-section + $2 \times \text{Area}$ of each side wall + Area of roofing
= $2 \times 72 \text{ m}^2 + 2 \times (50 \text{ m} \times 7.5 \text{ m}) + 500 \text{ m}^2$

$$= 144 \text{ m}^2 + 750 \text{ m}^2 + 500 \text{ m}^2 = 1,394 \text{ m}^2$$

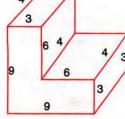
∴ The cost of painting =
$$1,394 \times ₹5 = ₹6,970$$

EXERCISE 21(B)

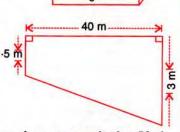
1. The following figure shows a solid of uniform cross-section. Find the volume of the solid.

All measurements are in centimetres.

Assume that all angles in the figure are right angles.



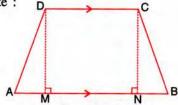
2. A swimming pool is 40 m long and 1.5 m wide. Its shallow and deep ends are



1.5 m and 3 m deep respectively. If the bottom of the pool slopes uniformly, find the amount of water in litres required to fill the pool.

3. The cross-section of a tunnel perpendicular to its length is a trapezium ABCD as shown in the following figure; also given that:

AM = BN; AB = 7 m; CD = 5 m. The height of the tunnel is 2.4 m. The tunnel is 40 m long. Calculate:



- (i) the cost of painting the internal surface of the tunnel (excluding the floor) at the rate of ₹ 5 per m² (sq. metre).
- (ii) the cost of paving the floor at the rate of ₹ 18 per m².
- 4. Water is discharged from a pipe of cross-section area 3.2 cm² at the speed of 5m/s. Calculate the volume of water discharged:
 - (i) in cm³ per sec.
 - (ii) in litres per minute.

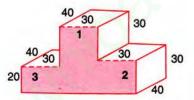
- 5. A hose-pipe of cross-section area 2 cm² delivers 1500 litres of water in 5 minutes. What is the speed of water in m/s through the pipe?
- 6. The cross-section of a piece of metal 4 m in length is shown below. Calculate:
 - oelow. Calculate:

 (i) the area of the cross-section; §

 (ii) the volume of [2]
 - (ii) the volume of the piece of metal in cubic centimetres.

If 1 cubic centimetre of the metal weighs 6.6 g, calculate the weight of the piece of metal to the nearest kg.

- 7. A rectangular water-tank measuring 80 cm x 60 cm x 60 cm is filled from a pipe of cross-sectional area 1.5 cm², the water emerging at 3.2 m/s. How long does it take to fill the tank?
- 8. A rectangular card-board sheet has length 32 cm and breadth 26 cm. Squares each of side 3 cm, are cut from the corners of the sheet and the sides are folded to make a rectangular container. Find the capacity of the container formed.
- 9. A swimming pool is 18 m long and 8 m wide. Its deep and shallow ends are 2 m and 1.2 m respectively. Find the capacity of the pool, assuming that the bottom of the pool slopes uniformly.
- 10. The following figure shows a closed victorystand whose dimensions are given in cm.



Find the volume and the surface area of the victory stand.

- 1. Each face of a cube has perimeter equal to 32 cm. Find its surface area and its volume.
- 2. A school auditorium is 40m long, 30 m broad and 12 m high. If each student requires 1.2 m² of the floor area; find the maximum number of students that can be accommodated in this auditorium. Also, find the volume of air available in the auditorium, for each student.

Max. no. of students =
$$\frac{40 \times 30}{1 \cdot 2}$$
 = 1000
And, air available for each student
= $\frac{40 \times 30 \times 12}{1000}$ m³.

- 3. The internal dimensions of a rectangular box are $12 \text{ cm} \times x \text{ cm} \times 9 \text{ cm}$. If the length of the longest rod that can be placed in this box is 17cm; find x.
- 4. The internal length, breadth and height of a box are 30 cm, 24 cm and 15 cm. Find the largest number of cubes which can be placed inside this box if the edge of each cube is

 (i) 3 cm (ii) 4 cm (iii) 5 cm
- (i) No. of cubes which can be placed along length = $\frac{30}{3}$ = 10. No. of cubes along the breadth = $\frac{24}{3}$ = 8, and no. of cubes along the height = $\frac{15}{3}$ = 5
- $\therefore \text{ The total no. of cubes placed} = 10 \times 8 \times 5 = 400$
- (ii) Cubes along length = $\frac{30}{4}$ = 7.5 = 7 (?), cubes along width = $\frac{24}{4}$ = 6 and cubes along height = $\frac{15}{4}$ = 3.75 = 3
 - $\therefore \text{ The total no. of cubes placed} = 7 \times 6 \times 3 = 126$

- 5. A rectangular field is 112 m long and 62 m broad. A cubical tank of edge 6 m is dug at each of the four corners of the field and the earth so removed is evenly spread on the remaining field. Find the rise in level.
- 6. When length of each side of a cube is increased by 3 cm, its volume is increased by 2457 cm³. Find its side. How much will its volume decrease, if length of each side of it is reduced by 20%?
- 7. A rectangular tank 30 cm × 20 cm × 12 cm contains water to a depth of 6 cm. A metal cube of side 10 cm is placed in the tank with its one face resting on the bottom of the tank. Find the volume of water, in litres, that must be poured in the tank so that the metal cube is just submerged in the water.
- The dimensions of a solid metallic cuboid are 72 cm × 30 cm × 75 cm. It is melted and recast into identical solid metal cubes with each of edge 6 cm. Find the number of cubes formed.

Also, find the cost of polishing the surfaces of all the cubes formed at the rate ₹ 150 per sq. m.

- 9. The dimensions of a car petrol tank are 50 cm × 32 cm × 24 cm, which is full of petrol. If car's average consumption is 15 km per litre, find the maximum distance that can be covered by the car.
- 10. The dimensions of a rectangular box are in the ratio 4:2:3. The difference between cost of covering it with paper at ₹ 12 per m² and with paper at the rate of 13.50 per m² is ₹ 1,248. Find the dimensions of the box.