

Triangles

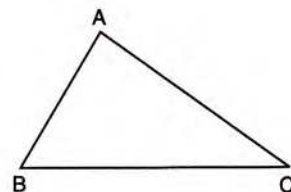
[Congruency in Triangles]

9.1 INTRODUCTION

A plane figure bounded by three straight line segments, is called a **triangle**. Every triangle has three vertices and three sides.

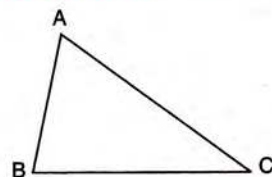
The adjoining figure shows a triangle ABC ($\triangle ABC$), whose three

- (i) vertices are A, B and C. (ii) sides are AB, BC and CA.



9.2 RELATION BETWEEN SIDES AND ANGLES OF TRIANGLES

1. *If all the sides of a triangle are of different lengths, its angles are also of different measures in such a way that, the greater side has greater angle opposite to it.*



In the given triangle ABC, sides AB, BC and AC are all of different lengths. Therefore, its angles i.e., $\angle A$, $\angle B$ and $\angle C$ are also of different measures.

Thus, in $\triangle ABC$,

$$AB \neq BC \neq AC$$

$$\Rightarrow \angle A \neq \angle B \neq \angle C$$

Also, according to the given figure, $AC > BC > AB$

$$\Rightarrow \text{Angle opposite to } AC > \text{angle opposite to } BC > \text{angle opposite to } AB$$

$$\Rightarrow \angle B > \angle A > \angle C$$

2. *Conversely, if all the angles of a triangle have different measures, its sides are also of different lengths in such a way that, the greater angle has greater side opposite to it.*

In the given figure,

$$\angle A \neq \angle B \neq \angle C$$

$$\Rightarrow AB \neq BC \neq AC$$

Also, according to the given figure, $\angle C > \angle A > \angle B$

$$\Rightarrow \text{Side opposite to } \angle C > \text{side opposite to } \angle A > \text{side opposite to } \angle B$$

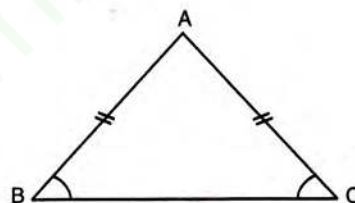
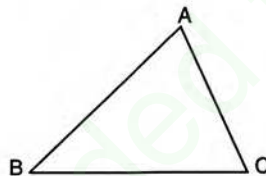
$$\Rightarrow AB > BC > AC$$

3. *If any two sides of a triangle are equal, the angles opposite to them are also equal. Conversely, if any two angles of a triangle are equal, the sides opposite to them are also equal.*

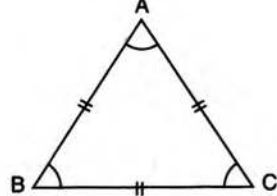
Thus, in triangle ABC,

$$(i) AB = AC \Rightarrow \angle B = \angle C$$

$$\text{and, } (ii) \angle B = \angle C \Rightarrow AB = AC$$



4. If all the sides of a triangle are equal, all its angles are also equal. Conversely, if all the angles of a triangle are equal, all its sides are also equal.



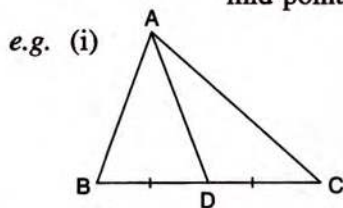
Thus, in triangle ABC,

$$(i) AB = BC = AC \Rightarrow \angle A = \angle B = \angle C$$

$$\text{and, } (ii) \angle A = \angle B = \angle C \Rightarrow AB = BC = AC$$

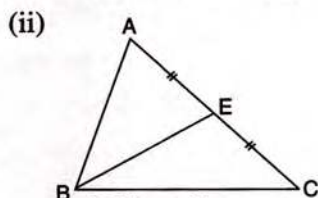
9.3 SOME IMPORTANT TERMS

1. Median : The **median** of a triangle, corresponding to any side, is the line joining the mid-point of that side with the opposite vertex.



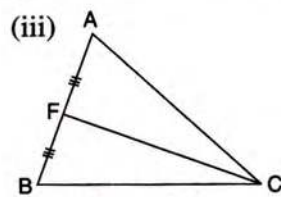
(AD is median

corresponding to side BC)



(BE is median

corresponding to side AC)



(CF is median

corresponding to side AB)

1. A triangle has three medians and all the three medians are always concurrent *i.e.*, they intersect each other at one point only.

2. The point of intersection of the medians is called the **centroid** of the triangle.

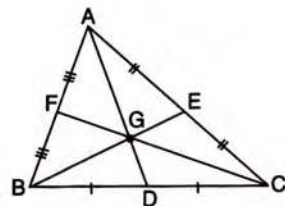
In the figure, G is the centroid of $\triangle ABC$.

3. Also, the centroid of a triangle divides each median in the ratio 2 : 1.

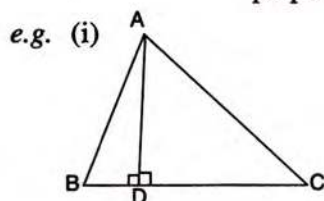
i.e., in the given figure :

$$AG : GD = 2 : 1; BG : GE = 2 : 1$$

$$\text{and } CG : GF = 2 : 1.$$

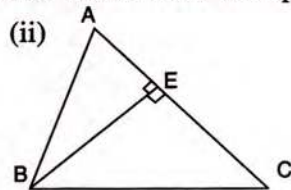


2. Altitude : An **altitude** of a triangle, corresponding to any side, is the length of the perpendicular drawn from the opposite vertex to that side.



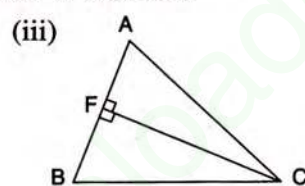
(AD is altitude

corresponding to side BC)



(BE is altitude

corresponding to side AC)



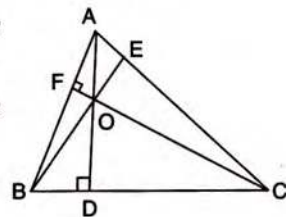
(CF is altitude

corresponding to side AB)

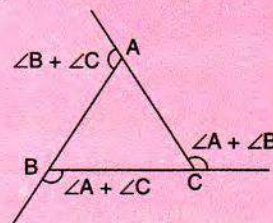
1. A triangle has three altitudes and all the three altitudes are *always concurrent i.e.*, they intersect each other at one point only.

2. The point of intersection of the altitudes of a triangle is called the **orthocentre**.

In the given figure, O is the orthocentre of the triangle ABC.



1. The sum of the angles of a triangle is equal to two right angles i.e. 180° .
2. If one side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.
3. As shown, in triangle ABC,
 - (i) $\angle A + \angle B + \angle C = 2 \text{ right angles} = 180^\circ$
 - (ii) Exterior angle at A = $\angle B + \angle C$.
 - (iii) Exterior angle at B = $\angle A + \angle C$.
 - (iv) Exterior angle at C = $\angle A + \angle B$.



Prove the following corollaries :

- Corollary 1 :** If one side of a triangle is produced, the exterior angle so formed is greater than each of the interior opposite angles.
- Corollary 2 :** A triangle cannot have more than one right angle.
- Corollary 3 :** A triangle cannot have more than one obtuse angle.
- Corollary 4 :** In a right angled triangle, the sum of the other two angles (acute angles) is 90° .
- Corollary 5 :** In every triangle, at least two angles are acute.
- Corollary 6 :** If two angles of a triangle are equal to two angles of any other triangle, each to each, then the third angles of both the triangles are also equal.

9.4 CONGRUENT TRIANGLES

Two triangles are said to be **congruent** to each other, if on placing one over the other, they exactly coincide.

In fact, two triangles are congruent, if they have exactly the *same shape* and the *same size*. i.e., all the angles and all the sides of one triangle are equal to the corresponding angles and the corresponding sides of the other triangle each to each.

Triangles with same shape means : Angles of one triangle are equal to angles of other triangle each to each.

Triangles with same size means : Sides of one triangle are equal to sides of other triangle each to each.

The given figure shows two triangles ABC and DEF such that :

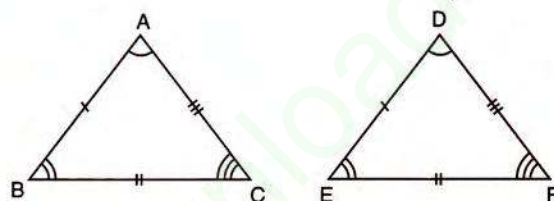
(i) $\angle A = \angle D$; $\angle B = \angle E$ and $\angle C = \angle F$.

(ii) $AB = DE$; $BC = EF$ and $AC = DF$.

$\therefore \Delta ABC$ is congruent to ΔDEF

and we write : $\Delta ABC \cong \Delta DEF$.

The symbol $\cong$ is read as "is congruent to".



1. Congruent figures (triangles) always coincide by *superposition* i.e. by placing one figure over the other.
2. In congruent triangles, the sides and the angles that *coincide* by *superposition* are called *corresponding sides* and *corresponding angles*.
3. The corresponding sides lie *opposite* to the *equal angles* and corresponding angles lie *opposite* to the *equal sides*.

In the figure alongside, $\triangle ABC \cong \triangle EFD$.

Since, $\angle A = \angle E$, therefore the side opposite to $\angle A$ and the side opposite to $\angle E$ are corresponding sides i.e., BC and DF are corresponding sides.

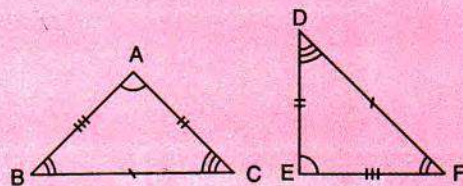
Similarly, AB and EF are corresponding sides as $\angle C = \angle D$.

Also, AC and DE are corresponding sides.

Conversely, as side AB = side EF, therefore, angles opposite to these sides i.e. $\angle C$ and $\angle D$ are the corresponding angles and so on.

4. Corresponding Parts of Congruent Triangles are also Congruent.

Abbreviated as : **C.P.C.T.C.**



9.5 CONDITIONS FOR CONGRUENCY OF TRIANGLES

1. If three sides of one triangle are equal to three sides of the other triangle, each to each, the triangles are congruent. **Abbreviated as : S.S.S.**

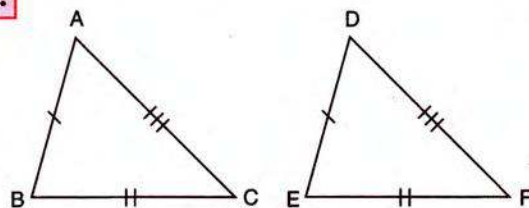
In the figure alongside, $AB = DE$,

$BC = EF$

and,

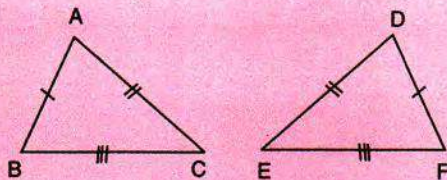
$AC = DF$

$\therefore \triangle ABC \cong \triangle DEF$. [By S.S.S.].



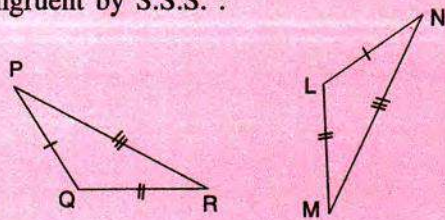
In each of the following figures, triangles are congruent by S.S.S. :

(i)



$[\triangle ABC \cong \triangle DFE]$

(ii)



$[\triangle PQR \cong \triangle NLM]$

2. If two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, the triangles are congruent.

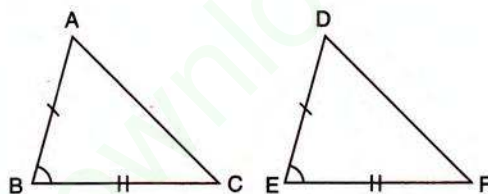
Abbreviated as : **S.A.S.**

In the figure alongside,

$AB = DE$;

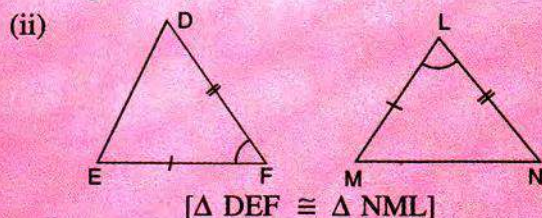
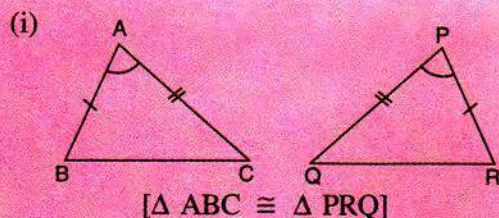
$BC = EF$ and $\angle B = \angle E$.

$\therefore \triangle ABC \cong \triangle DEF$ [By S.A.S.]



Triangles will be congruent only when the equal angles are the included angles.

In each of the following figures, triangles are congruent by S.A.S. :



3. If two angles and the included side of one triangle are equal to two angles and the included side of the other triangle, the triangles are congruent. **Abbreviated as : A.S.A.**

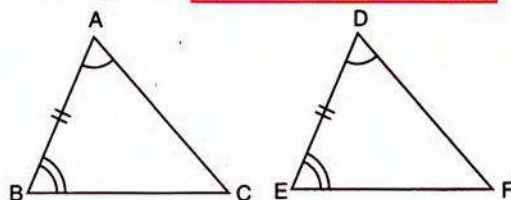
In the figure alongside, $\angle A = \angle D$,

$$\angle B = \angle E$$

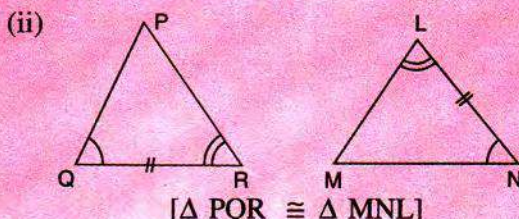
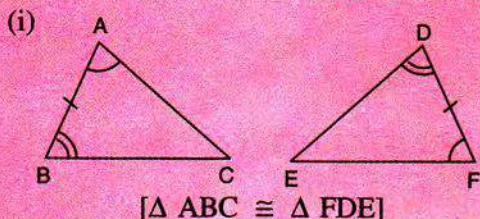
and,

$$AB = DE$$

$$\therefore \Delta ABC \cong \Delta DEF. \text{ [By A.S.A.]}$$



In each of the following figures, triangles are congruent by A.S.A. :



4. If two angles and one side of one triangle are equal to two angles and the corresponding side of the other triangle, the triangles are congruent. **Abbreviated as : A.A.S.**

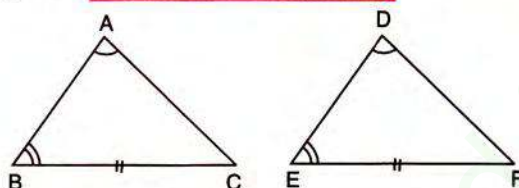
In the figure alongside, $\angle A = \angle D$,

$$\angle B = \angle E$$

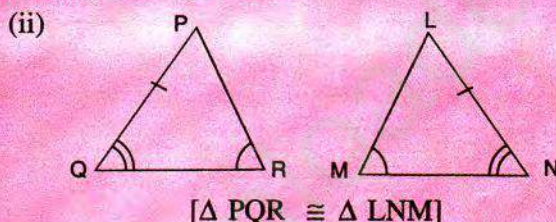
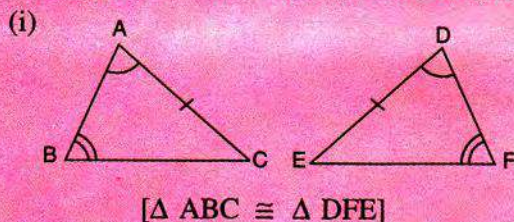
and,

$$BC = EF$$

$$\therefore \Delta ABC \cong \Delta DEF. \text{ [By A.A.S.]}$$



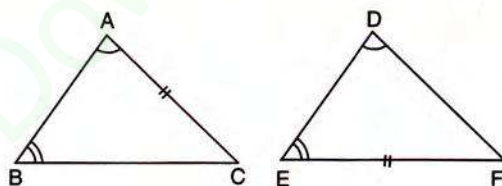
In each of the following figures, triangles are congruent by A.A.S. :



Precaution :

While using the axiom A.A.S., the two equal sides must be the corresponding sides, that is, the sides must be opposite to equal angles of the two triangles under consideration.

In the figure, given alongside, the equal sides are AC and EF. And these two sides (i.e., AC



and $\angle F$) are not opposite to equal angles.

$\therefore \triangle ABC$ and $\triangle DEF$ are not congruent by A.A.S.

The A.A.S. axiom is another form of axiom A.S.A.

In the figure, given alongside :

$$\angle A = \angle D, \angle B = \angle E$$

and, $AC = DF \Rightarrow \triangle ABC \equiv \triangle DEF$ by A.A.S.

Since, the sum of the angles of every triangle is 180° , therefore, if any two angles of a triangle are equal to two angles of another triangle, each to each, then their third angles will also be equal.

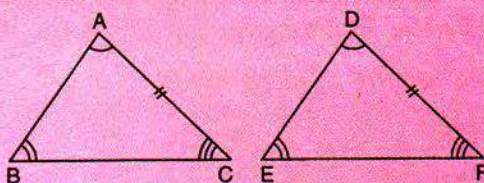
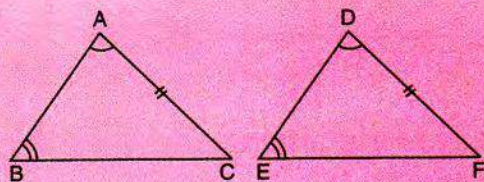
$\therefore$ In triangles ABC and DEF ;

$$\angle A = \angle D \text{ and } \angle B = \angle E \Rightarrow \angle C = \angle F.$$

Reconsidering the given figure, now we have :

$$\angle A = \angle D, \angle C = \angle F \text{ and } AC = DF$$

$\Rightarrow \triangle ABC \equiv \triangle DEF$ by A.S.A.



5. Two right-angled triangles are congruent, if the hypotenuse and one side of one triangle are equal to the hypotenuse and corresponding side of the other triangle. **Abbreviated as : R.H.S.**

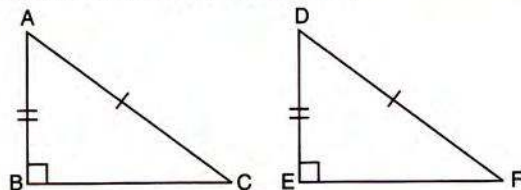
The given figure shows two right-angled triangles ABC and DEF such that :

$$\angle B = \angle E = 90^\circ;$$

$$AC = DF$$

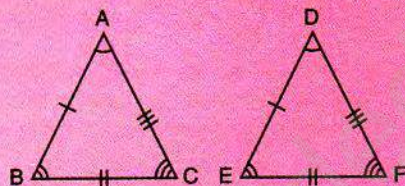
and, $AB = DE$

$\therefore \triangle ABC \equiv \triangle DEF$. [By R.H.S.]



Important :

- The given figure shows two congruent triangles ABC and DEF such that vertex A corresponds to vertex D (as, $\angle A = \angle D$); vertex B corresponds to vertex E (as, $\angle B = \angle E$) and, vertex C corresponds to vertex F (as, $\angle C = \angle F$).



We write : $\triangle ABC \equiv \triangle DEF$ and not $\triangle ABC \equiv \triangle DFE$ or $\triangle BAC \equiv \triangle DEF$, etc.

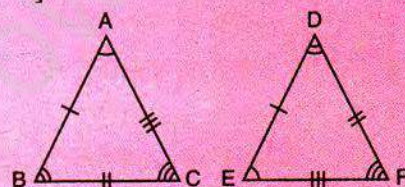
In fact, the order of vertices of two congruent triangles must be written in such a way that the corresponding vertices occupy the same position.

Thus, triangle ABC is congruent to triangle DEF

$$\Rightarrow \triangle ABC \equiv \triangle DEF \quad [A \leftrightarrow D, B \leftrightarrow E \text{ and } C \leftrightarrow F]$$

- The adjoining figure shows two congruent triangles such that, the corresponding vertices are: $A \leftrightarrow E$; $B \leftrightarrow D$ and $C \leftrightarrow F$.

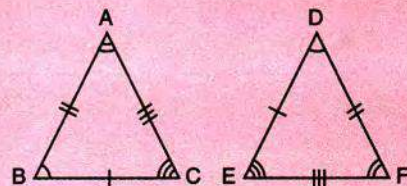
$$\therefore \triangle ABC \equiv \triangle EDF$$



3. In the adjoining figure, the corresponding vertices are : A and F, B and D and C and E.

$$\therefore \triangle ABC \cong \triangle FDE$$

4. The same procedure is used for **Similar Triangles**.



- 1** P is any point in the angle ABC such that the perpendiculars drawn from P on AB and BC are equal. Prove that BP bisects angle ABC.

Solution :

According to the given statement, the figure will be as shown alongside. In the figure, $PM \perp AB$ and $PN \perp BC$ such that $PM = PN$.

Construction : Join P and B.

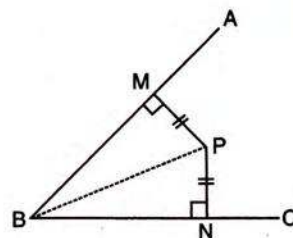
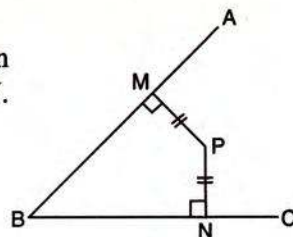
Proof :

Statement :

Reason :

In $\triangle PMB$ and $\triangle PNB$:

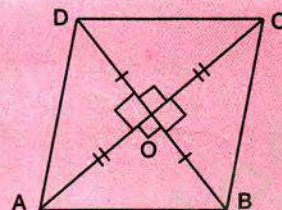
- | | |
|--|--------------------|
| (i) $PM = PN$ | [Given] |
| (ii) $\angle PMB = \angle PNB$ | [Each 90°] |
| (iii) $PB = PB$ | [Common] |
| $\therefore \triangle PMB \cong \triangle PNB$ | [R.H.S.] |
| $\Rightarrow \angle PBM = \angle PBN$ | [C.P.C.T.C.] |
| $\Rightarrow$ BP bisects angle ABC. | |



Hence proved.

- 2** If the diagonals of a quadrilateral bisect each other at right angle; prove that the quadrilateral is a rhombus.

A rhombus has all its four sides equal.



Solution :

Given : A quadrilateral ABCD, in which diagonals AC and BD bisect each other at 90° .

To Prove : ABCD is a rhombus i.e., $AB = BC = CD = DA$.

Proof :

Statement :

Reason :

In triangles AOB and COB :

- | | |
|--|---|
| 1. $OA = OC$ | [Given, diagonals bisect each other] |
| 2. $OB = OB$ | [Common] |
| 3. $\angle AOB = \angle BOC$ | [Given, diagonals bisect each other at 90°] |
| $\therefore \triangle AOB \cong \triangle COB$ | [S.A.S.] |
| $\therefore AB = BC$ ----- I | [Corresponding parts of congruent Δ s] |

Similarly, by proving that $\triangle BOC \cong \triangle DOC$ and $\triangle COD \cong \triangle AOD$; we get :

$$BC = CD \text{ and } CD = DA$$

----- II

$$\therefore AB = BC = CD = DA$$

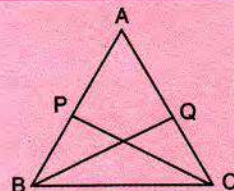
[Combining I and II]

i.e. **ABCD is a rhombus.**

Hence Proved.

- 3** In triangle ABC, the sides AB and AC are equal. If P is a point on AB and Q is a point on AC such that $AP = AQ$; prove that :

- (i) triangle APC and AQB are congruent.
- (ii) triangle BPC and CQB are congruent.



Solution :

Given : A triangle ABC, in which $AB = AC$ and $AP = AQ$.

(i) **Proof :**

Statement :

In triangles APC and AQB :

1. $AC = AB$
2. $AP = AQ$
3. $\angle CAP = \angle BAQ$

$$\therefore \triangle APC \cong \triangle AQB.$$

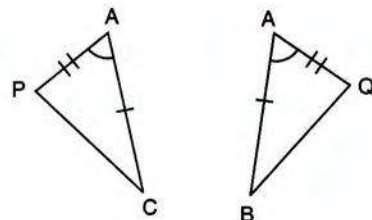
Reason :

[Given]

[Given]

[Common]

[S.A.S.]



Hence Proved.

(ii) **Proof :**

Statement :

Since; $AB = AC$ and $AP = AQ$

Reason :

[Given]

$$\therefore AB - AP = AC - AQ$$

$$\text{i.e. } BP = CQ.$$

In triangles BPC and CQB :

1. $BP = CQ$
2. $CP = BQ$
3. $BC = BC$

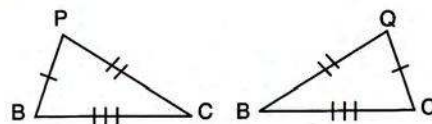
$$\therefore \triangle BPC \cong \triangle CQB$$

[From above]

[Corresponding sides of $\cong$ Δ s APC and AQB]

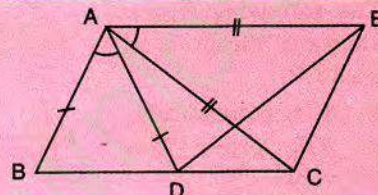
[Common]

[S.S.S.]



Hence Proved.

- 4** In the given figure, $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$. Show that : $BC = DE$.



Solution :

$$\angle BAD = \angle EAC$$

[Given]

$$\Rightarrow \angle BAD + \angle DAC = \angle EAC + \angle DAC \quad [\text{Adding } \angle DAC \text{ on both the sides}]$$

$$\Rightarrow \angle BAC = \angle DAE$$

$$AB = AD$$

and, $AC = AE$

$$\Rightarrow \triangle BAC \cong \triangle DAE$$

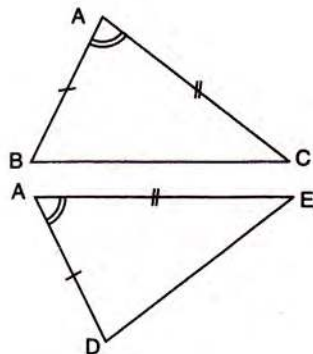
$$\Rightarrow \mathbf{BC = DE}$$

[Given]

[Given]

[By S.A.S.]

[C.P.C.T.C.]



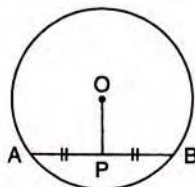
EXERCISE 9(A)

1. Which of the following pairs of triangles are congruent? In each case, state the condition of congruency :

- In $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $BC = EF$ and $\angle B = \angle E$.
- In $\triangle ABC$ and $\triangle DEF$, $\angle B = \angle E = 90^\circ$; $AC = DF$ and $BC = EF$.
- In $\triangle ABC$ and $\triangle QRP$, $AB = QR$, $\angle B = \angle R$ and $\angle C = \angle P$.
- In $\triangle ABC$ and $\triangle PQR$, $AB = PQ$, $AC = PR$ and $BC = QR$.
- In $\triangle ABC$ and $\triangle PQR$, $BC = QR$, $\angle A = 90^\circ$, $\angle C = \angle R = 40^\circ$ and $\angle Q = 50^\circ$.

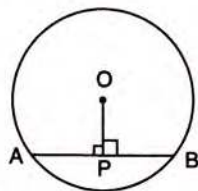
2. The given figure shows a circle with centre O. P is mid-point of chord AB.

Show that OP is perpendicular to AB.



3. The following figure shows a circle with centre O.

If OP is perpendicular to AB, prove that $AP = BP$.



4. In a triangle ABC, D is mid-point of BC; AD is produced upto E so that $DE = AD$.

Prove that :

- $\triangle ABD$ and $\triangle ECD$ are congruent.
- $AB = EC$.
- AB is parallel to EC.

5. A triangle ABC has $\angle B = \angle C$.

Prove that :

- the perpendiculars from the mid-point of BC to AB and AC are equal.
- the perpendiculars from B and C to the opposite sides are equal.

6. The perpendicular bisectors of the sides of a triangle ABC meet at I.

Prove that : $IA = IB = IC$.

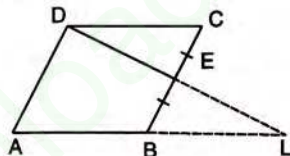
7. A line segment AB is bisected at point P and through point P another line segment PQ, which is perpendicular to AB, is drawn. Show that : $QA = QB$.

8. If AP bisects angle BAC and M is any point on AP, prove that the perpendiculars drawn from M to AB and AC are equal.

9. From the given diagram, in which ABCD is a parallelogram, ABL is a line segment and E is mid point of BC.

Prove that :

- $\triangle DCE \cong \triangle LBE$
- $AB = BL$.
- $AL = 2DC$



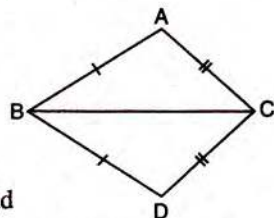
10. In the given figure, $AB = DB$ and $AC = DC$.

If $\angle ABD = 58^\circ$,

$\angle DBC = (2x - 4)^\circ$,

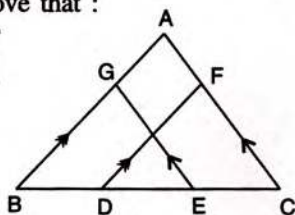
$\angle ACB = y + 15^\circ$ and

$\angle DCB = 63^\circ$; find the values of x and y.



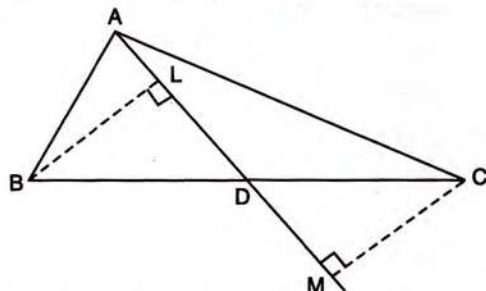
11. In the given figure : $AB \parallel FD$, $AC \parallel GE$ and $BD = CE$; prove that :

- (i) $BG = DF$
(ii) $CF = EG$



12. In a triangle ABC , $AB = AC$. Show that the altitude AD is median also.

13. In the following figure, $BL = CM$.

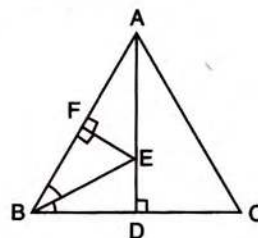


Prove that AD is a median of triangle ABC .

14. In the following figure, $AB = AC$ and AD is perpendicular to BC . BE bisects angle B and EF is perpendicular to AB .

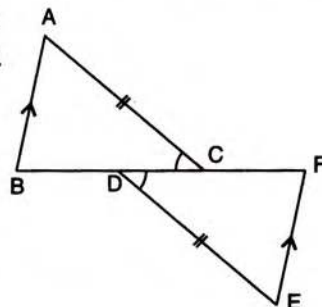
Prove that :

- (i) $BD = CD$
(ii) $ED = EF$



15. Use the information in the given figure to prove :

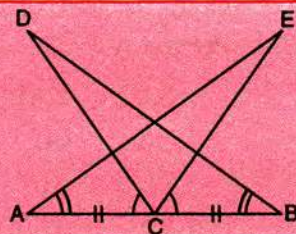
- (i) $AB = FE$
(ii) $BD = CF$



5

Use the information given in the adjoining figure, to prove :

- (i) $\triangle DBC \cong \triangle EAC$
(ii) $DC = EC$



Solution :

- (i) Let $\angle ACD = \angle BCE = x$

$$\therefore \angle ACE = \angle DCE + \angle ACD = \angle DCE + x \dots\dots\dots (i)$$

$$\text{and, } \angle BCD = \angle DCE + \angle BCE = \angle DCE + x \dots\dots\dots (ii)$$

From (i) and (ii), we get : $\angle ACE = \angle BCD$

Now, in $\triangle DBC$ and $\triangle EAC$,

$$\angle ACE = \angle BCD \quad [\text{Proved above}]$$

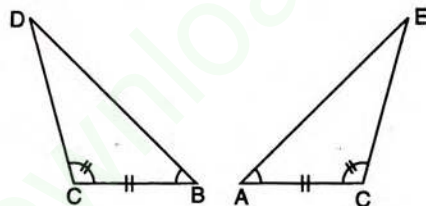
$$BC = AC \quad [\text{Given}]$$

$$\angle CBD = \angle EAC \quad [\text{Given}]$$

$$\therefore \triangle DBC \cong \triangle EAC \quad [\text{By A.S.A.}]$$

- (ii) Since, $\triangle DBC \cong \triangle EAC$

$$\Rightarrow DC = EC.$$



Hence proved.

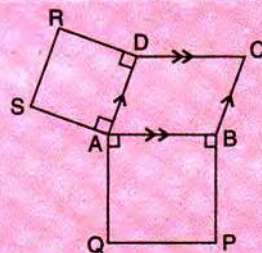
Hence proved.

6

The given figure shows a parallelogram ABCD. Squares ABPQ and ADRS are drawn on sides AB and AD of the parallelogram ABCD.

Prove that :

- (i) $\angle SAQ = \angle ABC$.
(ii) $SQ = AC$.



Solution :

- (i) Let $\angle ABC = x$

Since, adjacent angles of a parallelogram are supplementary, $\angle DAB + \angle ABC = 180^\circ$

$$\text{i.e. } \angle DAB + x = 180^\circ \Rightarrow \angle DAB = 180^\circ - x$$

Consider the angles around the point A :

$$\angle SAQ + \angle SAD + \angle QAB + \angle DAB = 360^\circ$$

$$\Rightarrow \angle SAQ + 90^\circ + 90^\circ + 180^\circ - x = 360^\circ$$

$$\Rightarrow \angle SAQ = x = \angle ABC.$$

Hence proved.

- (ii) Since, sides of a square are equal, therefore in square ADRS, $SA = AD$.

Since, opposite sides of a parallelogram are equal, therefore in parallelogram ABCD, $AD = BC$.

Combining the two results, we get : $SA = BC$.

Consider the $\triangle SAQ$ and $\triangle ABC$:

$$\angle SAQ = \angle ABC \quad [\text{Proved in part (i)}]$$

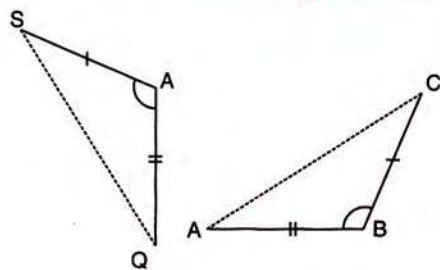
$$SA = BC \quad [\text{Proved above}]$$

$$AQ = AB \quad [\text{Sides of the square ABPQ}]$$

$$\therefore \triangle SAQ \cong \triangle CBA \quad [\text{By S.A.S.}]$$

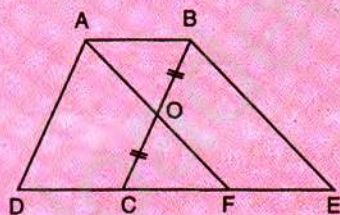
$$\Rightarrow SQ = AC. \quad [\text{By C.P.C.T.C.}]$$

Hence proved.



7

In the given figure, ABCD and ABEF are parallelograms. If O is the mid-point of BC, show that : $DC = CF = FE$.



Solution :

In $\triangle ABO$ and $\triangle FCO$,

$$OB = OC$$

[Given]

$$\angle AOB = \angle FOC$$

[Vertically opposite angles]

$$\angle OBA = \angle OCF$$

[Alternate angles as $AB \parallel CF$ and BC is transversal]

$$\Rightarrow \triangle ABO \cong \triangle FCO$$

[By A.S.A.]

$$\Rightarrow AB = CF$$

[By C.P.C.T.C.]

Also, $AB = DC$

and, $AB = FE$

[Opposite sides of parallelogram ABEF]

$\therefore DC = CF = FE$

Hence Proved

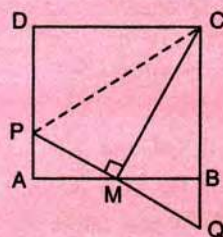
8

In the given figure, ABCD is a square. M is the mid-point of AB and PQ is perpendicular to CM. CB produced meets PQ at point Q.

Prove that :

(i) $PA = BQ$

(ii) $CP = AB + PA$.



Solution :

(i) In $\triangle PAM$ and $\triangle QBM$,

$\angle A = \angle ABC$

$= \angle MBQ$

$\angle AMP = \angle BMQ$

$AM = BM$

$\Rightarrow \triangle PAM \cong \triangle QBM$

$\Rightarrow PA = BQ$

[Each angle of a square is 90°]

$[\angle MBQ = 180^\circ - \angle ABC = 180^\circ - 90^\circ = 90^\circ]$

[Vertically opposite angles]

[M is mid-point of AB]

[By A.S.A.]

[By C.P.C.T.C.]

Hence Proved

(ii) $\triangle PAM \cong \triangle QBM$

[Proved above]

$\Rightarrow PM = QM$

[By C.P.C.T.C.]

$\angle CMP = \angle CMQ = 90^\circ$

and, $CM = CM$

[Common]

$\Rightarrow \triangle PCM \cong \triangle QCM$

[By S.A.S.]

$\Rightarrow CP = CQ$

[By C.P.C.T.C.]

$= CB + BQ$

$= AB + PA$

[$CB = AB =$ side of square ABCD and $BQ = PA$ (proved above)]

Hence Proved

EXERCISE 9(B)

1. On the sides AB and AC of triangle ABC, equilateral triangles ABD and ACE are drawn.

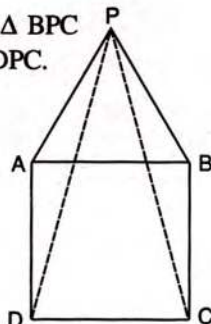
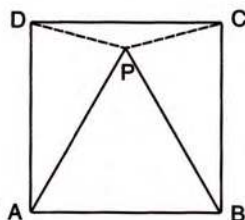
Prove that: (i) $\angle CAD = \angle BAE$ (ii) $CD = BE$.

2. In the following diagrams, ABCD is a square and APB is an equilateral triangle.

In each case,

(i) Prove that : $\triangle APD \cong \triangle BPC$

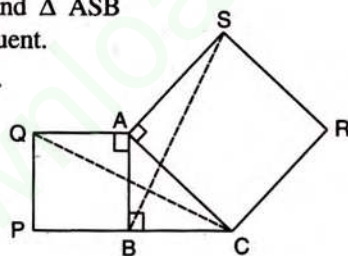
(ii) Find the angles of $\triangle DPC$.



3. In the figure, given below, triangle ABC is right-angled at B. ABPQ and ACRS are squares. Prove that :

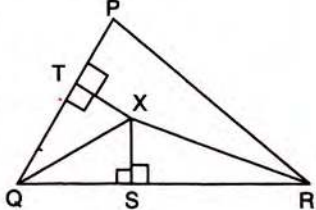
(i) $\triangle ACQ$ and $\triangle ASB$ are congruent.

(ii) $CQ = BS$.



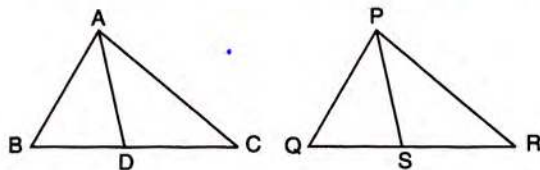
4. In a $\triangle ABC$, BD is the median to the side AC, BD is produced to E such that $BD = DE$. Prove that : AE is parallel to BC.

5. In the adjoining figure, QX and RX are the bisectors of the angles Q and R respectively of the triangle PQR.

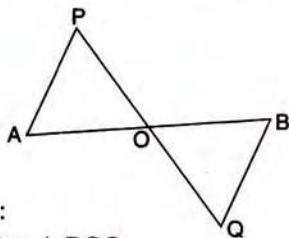


If $XS \perp QR$ and $XT \perp PQ$; prove that :

- (i) $\Delta XTQ \cong \Delta XSQ$
 - (ii) PX bisects angle P .
6. In the parallelogram ABCD, the angles A and C are obtuse. Points X and Y are taken on the diagonal BD such that the angles XAD and YCB are right angles.
- Prove that : $XA = YC$.
7. ABCD is a parallelogram. The sides AB and AD are produced to E and F respectively, such that $AB = BE$ and $AD = DF$.
- Prove that : $\Delta BEC \cong \Delta DCF$.
8. In the following figures, the sides AB and BC and the median AD of triangle ABC are respectively equal to the sides PQ and QR and median PS of the triangle PQR. Prove that ΔABC and ΔPQR are congruent.



9. In the following diagram, AP and BQ are equal and parallel to each other.

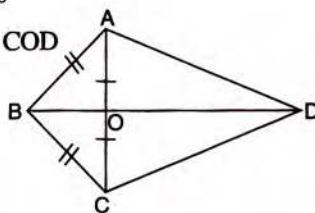


Prove that :

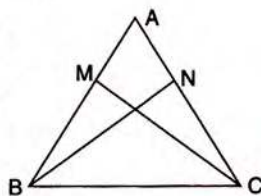
- (i) $\Delta AOP \cong \Delta BOQ$.
 - (ii) AB and PQ bisect each other.
10. In the following figure, $OA = OC$ and $AB = BC$.

Prove that :

- (i) $\angle AOB = 90^\circ$
- (ii) $\Delta AOD \cong \Delta COD$
- (iii) $AD = CD$

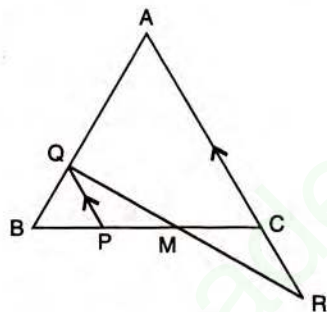


11. The following figure shows a triangle ABC in which $AB = AC$. M is a point on AB and N is a point on AC such that $BM = CN$.



Prove that :

- (i) $AM = AN$
 - (ii) $\Delta AMC \cong \Delta ANB$
 - (iii) $BN = CM$
 - (iv) $\Delta BMC \cong \Delta CNB$
12. In a triangle ABC, $AB = BC$, AD is perpendicular to side BC and CE is perpendicular to side AB. Prove that : $AD = CE$.
13. PQRS is a parallelogram. L and M are points on PQ and SR respectively such that $PL = MR$. Show that LM and QS bisect each other.
14. In the following figure, ABC is an equilateral triangle in which QP is parallel to AC. Side AC is produced upto point R so that $CR = BP$.



Prove that QR bisects PC.

Show that ΔQBP is equilateral

$\Rightarrow BP = PQ$, but $BP = CR$

$\Rightarrow PQ = CR \Rightarrow \Delta QPM \cong \Delta RCM$