

8

Logarithms

8.1 INTRODUCTION

Logarithms are used to make the long and complicated calculations easy.

Consider $3^4 = 81$, this is the exponential form of representing relation between three numbers 3, 4 and 81. Now the same relation between 3, 4 and 81 can be written as

$\log_3 81 = 4$ (read as : logarithm of 81 at base 3 is 4).

Thus :

$$3^4 = 81 \Leftrightarrow \log_3 81 = 4$$

Definition : If a , b and c are three real numbers such that $a \neq 1$ and $a^b = c$ then b is called logarithm of c at the base a and is written as $\log_a c = b$; read as log of c at the base a is b .

$$a^b = c \Leftrightarrow \log_a c = b$$

8.2 INTERCHANGING

(Logarithmic form vis-à-vis exponential form)

$a^b = c$ is called the **exponential form**

and, $\log_a c = b$ is called the **logarithmic form**.

i.e., (i)

$$2^{-3} = 0.125$$

[Exponential form]

$\Rightarrow$ log of 0.125 to the base 2 = -3

i.e.,

$$\log_2 0.125 = -3$$

[Logarithmic form]

(ii)

$$\log_{64} 8 = \frac{1}{2}$$

[Logarithmic form]

$\Rightarrow$ log of 8 to the base 64 = $\frac{1}{2}$

i.e.

$$(64)^{\frac{1}{2}} = 8$$

[Exponential form] and so on.

Similarly :

If x is positive;

$$(iii) x^0 = 1 \Rightarrow \log_x 1 = 0 \quad \text{i.e. log of 1 to the base } x = 0$$

In general; the logarithm of 1 to any base is zero.

$$\text{i.e. } \log_5 1 = 0; \log_{10} 1 = 0; \log_a 1 = 0 \quad \text{and so on.}$$

$$(iv) x^1 = x \Rightarrow \log_x x = 1 \quad \text{i.e. log } x \text{ to the base } x = 1$$

In general, the logarithm of any number to the same base is always one.

$$\text{i.e. } \log_5 5 = 1; \log_{10} 10 = 1; \log_a a = 1 \quad \text{and so on.}$$

1 Find : (i) the logarithm of 1000 to the base 10.

(ii) the logarithm of $\frac{1}{9}$ to the base 3.

Solution :

$$\begin{aligned}\text{(i)} \quad \text{Let } \log_{10} 1000 &= x \Rightarrow 10^x = 1000 \\ &\Rightarrow 10^x = 10^3 \Rightarrow x = 3\end{aligned}$$

$$\therefore \log_{10} 1000 = 3$$

Ans.

$$\begin{aligned}\text{(ii)} \quad \text{Let } \log_3 \frac{1}{9} &= x \Rightarrow 3^x = \frac{1}{9} \\ &\Rightarrow 3^x = 3^{-2} \Rightarrow x = -2\end{aligned}$$

$$\therefore \log_3 \frac{1}{9} = -2$$

Ans.

2 Find x , if : (i) $\log_2 x = -2$ (ii) $\log_4(x+3) = 2$ (iii) $\log_x 64 = \frac{3}{2}$

Solution :

$$\text{(i)} \quad \log_2 x = -2 \Rightarrow 2^{-2} = x$$

$$\Rightarrow x = \frac{1}{4}$$

Ans.

$$\text{(ii)} \quad \log_4(x+3) = 2 \Rightarrow 4^2 = x+3$$

$$\Rightarrow x = 16 - 3 = 13$$

Ans.

$$\text{(iii)} \quad \log_x 64 = \frac{3}{2} \Rightarrow x^{\frac{3}{2}} = 64$$

$$\Rightarrow x = (64)^{\frac{2}{3}} = (2^6)^{\frac{2}{3}} = 2^4 = 16$$

Ans.

EXERCISE 8(A)

1. Express each of the following in *logarithmic form* :

$$\text{(i)} 5^3 = 125 \quad \text{(ii)} 3^{-2} = \frac{1}{9}$$

$$\text{(iii)} 10^{-3} = 0.001 \quad \text{(iv)} (81)^{\frac{3}{4}} = 27$$

2. Express each of the following in *exponential form* :

$$\text{(i)} \log_8 0.125 = -1 \quad \text{(ii)} \log_{10} 0.01 = -2$$

$$\text{(iii)} \log_a A = x \quad \text{(iv)} \log_{10} 1 = 0$$

3. Solve for x : $\log_{10} x = -2$.

4. Find the logarithm of :

$$\text{(i)} 100 \text{ to the base } 10$$

$$\text{(ii)} 0.1 \text{ to the base } 10$$

$$\text{(iii)} 0.001 \text{ to the base } 10$$

$$\text{(iv)} 32 \text{ to the base } 4$$

$$\text{(v)} 0.125 \text{ to the base } 2$$

$$\text{(vi)} \frac{1}{16} \text{ to the base } 4$$

$$\text{(vii)} 27 \text{ to the base } 9$$

$$\text{(viii)} \frac{1}{81} \text{ to the base } 27$$

5. State, true or false :

$$\text{(i)} \text{ If } \log_{10} x = a, \text{ then } 10^x = a.$$

$$\text{(ii)} \text{ If } x^y = z, \text{ then } y = \log_z x.$$

$$\text{(iii)} \log_2 8 = 3 \text{ and } \log_8 2 = \frac{1}{3}.$$

6. Find x , if :

$$\text{(i)} \log_3 x = 0 \quad \text{(ii)} \log_x 2 = -1$$

$$\text{(iii)} \log_9 243 = x \quad \text{(iv)} \log_5 (x-7) = 1$$

$$\text{(v)} \log_4 32 = x - 4 \quad \text{(vi)} \log_7 (2x^2 - 1) = 2$$

7. Evaluate :

$$\text{(i)} \log_{10} 0.01$$

$$\text{(ii)} \log_2 (1 \div 8)$$

(iii) $\log_5 1$ (iv) $\log_5 125$

(v) $\log_{16} 8$ (vi) $\log_{0.5} 16$

8. If $\log_a m = n$, express a^{n-1} in terms of a and m .

$$\begin{aligned}\log_a m = n &\Rightarrow a^n = m \\ \Rightarrow a^{n-1} &= \frac{a^n}{a} = \frac{m}{a}\end{aligned}$$

9. Given $\log_2 x = m$ and $\log_5 y = n$.

(i) Express 2^{m-3} in terms of x .

(ii) Express 5^{3n+2} in terms of y .

10. If $\log_2 x = a$ and $\log_3 y = a$, write 72^a in terms of x and y .

11. Solve for x : $\log(x-1) + \log(x+1) = \log_2 1$.

12. If $\log(x^2 - 21) = 2$, show that $x = \pm 11$.

8.3 LAWS OF LOGARITHM WITH USE

First Law (Product Law) :

The *logarithm of a product at any non-zero base* is equal to the *sum of the logarithms of its factors at the same base*.

i.e. $\log_a (m \times n) = \log_a m + \log_a n$

$\log_x (m \times n \times p) = \log_x m + \log_x n + \log_x p$ and so on.

Remember : $\log_a (m + n) \neq \log_a m + \log_a n$

Second Law (Quotient Law) :

The *logarithm of a fraction at any non-zero base* is equal to the *difference between the logarithm of the numerator minus the logarithm of the denominator, both at the same base*.

i.e. $\log_a \frac{m}{n} = \log_a m - \log_a n$

Remember : $\frac{\log_a m}{\log_a n} \neq \log_a m - \log_a n$. Also, $\log_a (m - n) \neq \log_a m - \log_a n$

Third Law (Power Law) :

The *logarithm of a power of a number at any non-zero base* is equal to the *logarithm of the number (at the same base) multiplied by the power*.

i.e. $\log_a (m)^n = n \log_a m$

Corollary : Since $\sqrt[n]{m} = m^{\frac{1}{n}}$

$\therefore \log_a \sqrt[n]{m} = \log_a m^{\frac{1}{n}} = \frac{1}{n} \log_a m$

1. Logarithms to the **base 10** are known as **common logarithms**.

2. If **no base is given**, the **base** is always taken as **10**,

i.e. $\log 8 = \log_{10} 8$; $\log a = \log_{10} a$; $\log 10 = \log_{10} 10$ and so on.

3. $\log_{10} 1 = 0$; $\log_{10} 10 = 1$;

$\log_{10} 100 = 2$

$[\log_{10} 100 = \log_{10} 10^2 = 2 \log_{10} 10 = 2 \times 1 = 2]$

Similarly, $\log_{10} 1000 = 3$; $\log_{10} 10000 = 4$ and so on.

8.4 EXPANSION OF EXPRESSIONS WITH THE HELP OF LAWS OF LOGARITHM

Let $y = \frac{a^4 \times b^2}{c^3} \Rightarrow \log y = \log \frac{a^4 \times b^2}{c^3}$

i.e.

$\log y = \log(a^4 \times b^2) - \log c^3$

$[\because \log \frac{m}{n} = \log m - \log n]$

$$= \log a^4 + \log b^2 - \log c^3 \quad [\because \log m \times n = \log m + \log n]$$

$$= 4 \log a + 2 \log b - 3 \log c \quad [\because \log m^n = n \log m]$$

$\therefore$ $\log y = 4 \log a + 2 \log b - 3 \log c$ is the logarithmic expansion of the

$$\text{given expression } y = \frac{a^4 \times b^2}{c^3}$$

Similarly,

$$\begin{aligned} m &= \frac{3^x}{5^y \times 8^z} \Rightarrow \log m = \log 3^x - \log (5^y \times 8^z) \\ &= x \log 3 - [\log 5^y + \log 8^z] \\ &= x \log 3 - y \log 5 - z \log 8 \\ \Rightarrow \log m &= x \log 3 - y \log 5 - z \log 8 \end{aligned}$$

Conversely :

$$\begin{aligned} \log V &= \log \pi + 2 \log r + \log h - \log 3 \\ \Rightarrow \log V &= \log \pi + \log r^2 + \log h - \log 3 \\ &= \log \frac{\pi r^2 h}{3} \Rightarrow V = \frac{\pi r^2 h}{3} \end{aligned}$$

3 Express $\log_{10} \sqrt[5]{108}$ in terms of $\log_{10} 2$ and $\log_{10} 3$.

Solution :

$$\begin{aligned} \log_{10} \sqrt[5]{108} &= \log_{10} (108)^{\frac{1}{5}} & [\sqrt[n]{m} &= m^{\frac{1}{n}}] \\ &= \frac{1}{5} \log_{10} 108 & [\log_{10} n^m &= m \log_{10} n] \\ &= \frac{1}{5} \log_{10} (2^2 \times 3^3) & [108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3] \\ &= \frac{1}{5} [\log_{10} 2^2 + \log_{10} 3^3] & [\log_{10} m \times n &= \log_{10} m + \log_{10} n] \\ &= \frac{1}{5} [2\log_{10} 2 + 3\log_{10} 3] \end{aligned}$$

Ans.

4 Express as a single logarithm : $2 + \frac{1}{2} \log_{10} 9 - 2 \log_{10} 5$

Solution :

$$\begin{aligned} &= \log_{10} 100 + \log_{10} 9^{\frac{1}{2}} - \log_{10} 5^2 & [\log_{10} 100 = 2; \log 9 = \log 3^2 \text{ and } 2 \log 5 = \log 5^2] \\ &= \log_{10} 100 + \log_{10} 3 - \log_{10} 25 \\ &= \log_{10} \frac{100 \times 3}{25} = \log_{10} 12 & [\log a + \log b - \log c = \log \frac{a \times b}{c}] \end{aligned}$$

Ans.

5 Find x , if : (i) $\log_{10}(x+5) = 1$.

(ii) $\log_{10}(x+1) + \log_{10}(x-1) = \log_{10} 11 + 2 \log_{10} 3$

Solution :

$$(i) \Rightarrow \log_{10}(x+5) = \log_{10} 10$$

$$\Rightarrow x+5 = 10 \Rightarrow x = 5$$

Ans.

$$(ii) \Rightarrow \log_{10}(x+1)(x-1) = \log_{10} 11 + \log_{10} 3^2$$

$$\Rightarrow \log(x^2 - 1) = \log(11 \times 9)$$

$$\Rightarrow x^2 - 1 = 99$$

$$\therefore x^2 = 100 \text{ and, } x = 10$$

Ans.

6 If $\log 2 = 0.3010$ and $\log 3 = 0.4771$, find the value of :

(i) $\log 6$ (ii) $\log 5$ (iii) $\log \sqrt{24}$

Solution :

$$(i) \log 6 = \log 2 \times 3 = \log 2 + \log 3 = 0.3010 + 0.4771 = 0.7781$$

Ans.

$$(ii) \log 5 = \log \frac{10}{2} = \log 10 - \log 2 = 1 - 0.3010 = 0.6990 \quad \text{Ans.} \quad [\because \log 10 = 1]$$

$$\begin{aligned} (iii) \log \sqrt{24} &= \log (24)^{\frac{1}{2}} = \frac{1}{2} \log (2^3 \times 3) & [24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3] \\ &= \frac{1}{2} [3 \times \log 2 + \log 3] \\ &= \frac{1}{2} [3 \times 0.3010 + 0.4771] = 0.69005 \end{aligned}$$

Ans.

EXERCISE 8(B)

1. Express in terms of $\log 2$ and $\log 3$:

(i) $\log 36$ (ii) $\log 144$

(iii) $\log 4.5$ (iv) $\log \frac{26}{51} - \log \frac{91}{119}$

(v) $\log \frac{75}{16} - 2 \log \frac{5}{9} + \log \frac{32}{243}$

2. Express each of the following in a form free from logarithm :

(i) $2 \log x - \log y = 1$

(ii) $2 \log x + 3 \log y = \log a$

(iii) $a \log x - b \log y = 2 \log 3$

3. Evaluate each of the following without using tables :

(i) $\log 5 + \log 8 - 2 \log 2$

(ii) $\log_{10} 8 + \log_{10} 25 + 2 \log_{10} 3 - \log_{10} 18$

(iii) $\log 4 + \frac{1}{3} \log 125 - \frac{1}{5} \log 32$

4. Prove that :

$$2 \log \frac{15}{18} - \log \frac{25}{162} + \log \frac{4}{9} = \log 2.$$

5. Find x , if :

$$x - \log 48 + 3 \log 2 = \frac{1}{3} \log 125 - \log 3.$$

6. Express $\log_{10} 2 + 1$ in the form of $\log_{10} x$.

7. Solve for x :

(i) $\log_{10}(x-10) = 1$

(ii) $\log(x^2 - 21) = 2$

(iii) $\log(x-2) + \log(x+2) = \log 5$

(iv) $\log(x+5) + \log(x-5) = 4 \log 2 + 2 \log 3$

8. Solve for x :

(i) $\frac{\log 81}{\log 27} = x$ (ii) $\frac{\log 128}{\log 32} = x$

(iii) $\frac{\log 64}{\log 8} = \log x$ (iv) $\frac{\log 225}{\log 15} = \log x$

$$(i) \frac{\log 81}{\log 27} = x$$

$$\Rightarrow x = \frac{\log 3^4}{\log 3^3} = \frac{4 \log 3}{3 \log 3} = \frac{4}{3} \quad \text{Ans.}$$

9. Given $\log x = m + n$ and $\log y = m - n$, express the value of $\log \frac{10x}{y^2}$ in terms of m and n .

10. State, true or false :

(i) $\log 1 \times \log 1000 = 0$

(ii) $\frac{\log x}{\log y} = \log x - \log y$

(iii) If $\frac{\log 25}{\log 5} = \log x$, then $x = 2$

(iv) $\log x \times \log y = \log x + \log y$

11. If $\log_{10} 2 = a$ and $\log_{10} 3 = b$; express each of the following in terms of 'a' and 'b' :

(i) $\log 12$ (ii) $\log 2.25$ (iii) $\log 2\frac{1}{4}$

(iv) $\log 5.4$ (v) $\log 60$ (vi) $\log 3\frac{1}{8}$

12. If $\log 2 = 0.3010$ and $\log 3 = 0.4771$; find the value of :

(i) $\log 12$ (ii) $\log 1.2$

(iii) $\log 3.6$ (iv) $\log 15$

(v) $\log 25$ (vi) $\frac{2}{3} \log 8$

13. Given $2 \log_{10} x + 1 = \log_{10} 250$, find :

(i) x (ii) $\log_{10} 2x$

14. Given $3 \log x + \frac{1}{2} \log y = 2$, express y in term of x .

15. If $x = (100)^a$, $y = (10000)^b$ and $z = (10)^c$, find

$\log \frac{10\sqrt{y}}{x^2 z^3}$ in terms of a , b and c .

16. If $3(\log 5 - \log 3) - (\log 5 - 2 \log 6) = 2 - \log x$, find x .

7 If $\log_{10} 4 = 0.6020$; find the value of :

(i) $\log_{10} 8$

(ii) $\log_{10} 2.5$

Solution :

If $\log_{10} 4 = 0.6020 \Rightarrow 2 \log 2 = 0.6020$

$[\because \log 4 = \log 2^2 = 2 \log 2]$

$\Rightarrow \log 2 = \frac{0.6020}{2} = 0.3010$

(i) $\log_{10} 8 = \log_{10} 2^3$
 $= 3 \log 2 = 3 \times 0.3010 = 0.9030$

Ans.

(ii) $\log_{10} 2.5 = \log \frac{25}{10} = \log \frac{5}{2} = \log \frac{10}{2 \times 2}$
 $= \log 10 - 2 \log 2 = 1 - 2 \times 0.3010 = 0.3980$

Ans.

8 Given $\log_{10} x = a$ and $\log_{10} y = b$.

(i) Write down 10^{a-1} in terms of x .

(ii) Write down 10^{2b} in terms of y .

(iii) If $\log_{10} P = 2a - b$; express P in terms of x and y .

Solution :

(i) $\log_{10} x = a \Rightarrow 10^a = x$

$\therefore 10^{a-1} = \frac{10^a}{10^1} = \frac{x}{10}$

Ans.

$$(ii) \quad \log_{10} y = b \quad \Rightarrow \quad 10^b = y$$

$$\therefore \quad 10^{2b} = (10^b)^2 = y^2$$

Ans.

$$(iii) \quad \log_{10} P = 2a - b$$

$$\Rightarrow \quad \log_{10} P = 2\log_{10} x - \log_{10} y$$

$$\Rightarrow \quad \log P = \log x^2 - \log y \Rightarrow \log P = \log \frac{x^2}{y} \quad \therefore P = \frac{x^2}{y}$$

Ans.

EXERCISE 8(C)

1. If $\log_{10} 8 = 0.90$; find the value of :

(i) $\log_{10} 4$ (ii) $\log \sqrt{32}$

(iii) $\log 0.125$

2. If $\log 27 = 1.431$, find the value of :

(i) $\log 9$ (ii) $\log 300$

3. If $\log_{10} a = b$, find 10^{3b-2} in terms of a .

4. If $\log_5 x = y$, find 5^{2y+3} in terms of x .

5. Given: $\log_3 m = x$ and $\log_3 n = y$.

(i) Express 3^{2x-3} in terms of m .

(ii) Write down $3^{1-2y+3x}$ in terms of m and n .

(iii) If $2 \log_3 A = 5x - 3y$; find A in terms of m and n .

6. Simplify :

(i) $\log (a)^3 - \log a$ (ii) $\log (a)^3 + \log a$

7. If $\log (a + b) = \log a + \log b$, find a in terms of b .

8. Prove that :

(i) $(\log a)^2 - (\log b)^2 = \log \left(\frac{a}{b} \right) \cdot \log (ab)$

(ii) If $a \log b + b \log a - 1 = 0$,
then $b^a \cdot a^b = 10$

9. (i) If $\log (a + 1) = \log (4a - 3) - \log 3$;
find a .

(ii) If $2 \log y - \log x - 3 = 0$,
express x in terms of y .

(iii) Prove that : $\log_{10} 125 = 3(1 - \log_{10} 2)$.

10. Given $\log x = 2m - n$, $\log y = n - 2m$ and
 $\log z = 3m - 2n$, find in terms of m and n ,

the value of $\log \frac{x^2 y^3}{z^4}$.

11. Given $\log_x 25 - \log_x 5 = 2 - \log_x \frac{1}{125}$; find x .

8.5 MORE ABOUT LOGARITHMS

1. Since, $2^3 = 8 \Rightarrow \log_2 8 = 3$

Also, $2^3 = 8 \Rightarrow 8^{\frac{1}{3}} = 2 \Rightarrow \log_8 2 = \frac{1}{3}$

Thus, $\log_2 8 = 3$ and $\log_8 2 = \frac{1}{3} \Rightarrow \log_2 8 = \frac{1}{\log_8 2}$

In the same way,

$$5^4 = 625 \Rightarrow \log_5 625 = 4$$

and, $5^4 = 625 \Rightarrow 625^{\frac{1}{4}} = 5 \Rightarrow \log_{625} 5 = \frac{1}{4}$

$\therefore \log_5 625 = 4$ and $\log_{625} 5 = \frac{1}{4} \Rightarrow \log_5 625 = \frac{1}{\log_{625} 5}$

Thus, if a and b are two positive numbers

$$\log_b a = \frac{1}{\log_a b} \text{ and } \log_a b = \frac{1}{\log_b a}.$$

2. Since, $\log_b a = \frac{1}{\log_a b} \Rightarrow \log_b a \times \log_a b = 1$

$$\Rightarrow \text{(i) } \log_5 3 \times \log_3 5 = 1 \quad \text{(ii) } \log_8 12 \times \log_{12} 8 = 1$$

$$\text{(iii) } \log_{18} 35 \times \log_{35} 18 = 1 \quad \text{and so on.}$$

3. Since, log of a number at the same base is 1 (one)

$$\log_a a = 1 \Rightarrow x \log_a a = x \quad [\text{On multiplying both the sides by } x]$$

$$\Rightarrow \log_a a^x = x$$

$$\Rightarrow \text{(i) } \log_2 2^5 = 5 \quad \text{(ii) } \log_5 5^8 = 8$$

$$\text{(iii) } \log_8 8^4 = 4 \quad \text{and so on.}$$

4. $\log_b a = \frac{\log_x a}{\log_x b}$, where a , b and x all are positive.

For example :

$$\begin{aligned} \log_{100} 1000 &= \frac{\log_{10} 1000}{\log_{10} 100} \\ &= \frac{\log_{10} 10^3}{\log_{10} 10^2} = \frac{3 \log_{10} 10}{2 \log_{10} 10} = \frac{3 \times 1}{2 \times 1} = \frac{3}{2} \end{aligned}$$

9 Evaluate :

$$\text{(i) } \log_{125} 625 - \log_{16} 64 \quad \text{(ii) } \log_{16} 32 - \log_{25} 125 + \log_9 27.$$

Solution :

$$\begin{aligned} \text{(i) } \log_{125} 625 - \log_{16} 64 &= \frac{\log_{10} 625}{\log_{10} 125} - \frac{\log_{10} 64}{\log_{10} 16} & \left[\because \log_n m = \frac{\log_a m}{\log_a n} \right] \\ &= \frac{\log 5^4}{\log 5^3} - \frac{\log 2^6}{\log 2^4} & \left[\because \log_{10} x = \log x \right] \\ &= \frac{4 \log 5}{3 \log 5} - \frac{6 \log 2}{4 \log 2} = \frac{4}{3} - \frac{3}{2} = -\frac{1}{6} & \text{Ans.} \end{aligned}$$

$$\text{(ii) } \log_{16} 32 - \log_{25} 125 + \log_9 27$$

$$= \frac{\log 32}{\log 16} - \frac{\log 125}{\log 25} + \frac{\log 27}{\log 9}$$

$$\begin{aligned}
 &= \frac{\log 2^5}{\log 2^4} - \frac{\log 5^3}{\log 5^2} + \frac{\log 3^3}{\log 3^2} \\
 &= \frac{5 \log 2}{4 \log 2} - \frac{3 \log 5}{2 \log 5} + \frac{3 \log 3}{2 \log 3} \\
 &= \frac{5}{4} - \frac{3}{2} + \frac{3}{2} = \frac{5}{4} = 1\frac{1}{4}
 \end{aligned}$$

Ans.

10 If $\frac{1}{\log_a x} + \frac{1}{\log_b x} = \frac{2}{\log_c x}$, prove that : $c^2 = ab$.

Solution :

Since, $\log_b a = \frac{1}{\log_a b} \Rightarrow \frac{1}{\log_a x} = \log_x a$, $\frac{1}{\log_b x} = \log_x b$ and $\frac{1}{\log_c x} = \log_x c$

$$\therefore \frac{1}{\log_a x} + \frac{1}{\log_b x} = \frac{2}{\log_c x} \Rightarrow \log_x a + \log_x b = 2 \log_x c$$

$$\Rightarrow \log_x ab = \log_x c^2 \Rightarrow ab = c^2$$

Hence Proved.

EXERCISE 8(D)

- If $\frac{3}{2} \log a + \frac{2}{3} \log b - 1 = 0$, find the value of $a^9.b^4$.
- If $x = 1 + \log 2 - \log 5$, $y = 2 \log 3$ and $z = \log a - \log 5$; find the value of a , if $x + y = 2z$.
- If $x = \log 0.6$; $y = \log 1.25$ and $z = \log 3 - 2 \log 2$, find the values of :
(i) $x + y - z$ (ii) 5^{x+y-z}
- If $a^2 = \log x$, $b^3 = \log y$ and $3a^2 - 2b^3 = 6 \log z$, express y in terms of x and z .
- If $\log \frac{a-b}{2} = \frac{1}{2} (\log a + \log b)$, show that:
 $a^2 + b^2 = 6ab$.
- If $a^2 + b^2 = 23ab$, show that :
 $\log \frac{a+b}{5} = \frac{1}{2} (\log a + \log b)$.
- If $m = \log 20$ and $n = \log 25$, find the value of x , so that : $2 \log (x - 4) = 2m - n$.
- Solve for x and y ; if $x > 0$ and $y > 0$:
 $\log xy = \log \frac{x}{y} + 2 \log 2 = 2$.
- Find x , if :
(i) $\log_x 625 = -4$ (ii) $\log_x (5x - 6) = 2$.
- If $p = \log 20$ and $q = \log 25$, find the value of x , if $2 \log (x + 1) = 2p - q$.
- If $\log_2 (x + y) = \log_3 (x - y) = \frac{\log 25}{\log 0.2}$, find the values of x and y .
- Given : $\frac{\log x}{\log y} = \frac{3}{2}$ and $\log (xy) = 5$; find the values of x and y .
- Given $\log_{10} x = 2a$ and $\log_{10} y = \frac{b}{2}$.
(i) Write 10^a in terms of x .
(ii) Write 10^{2b+1} in terms of y .
(iii) If $\log_{10} P = 3a - 2b$, express P in terms of x and y .
- Solve :
 $\log_5 (x + 1) - 1 = 1 + \log_5 (x - 1)$.
- Solve for x , if :
 $\log_x 49 - \log_x 7 + \log_x \frac{1}{343} + 2 = 0$.

16. If $a^2 = \log x$, $b^3 = \log y$ and $\frac{a^2}{2} - \frac{b^3}{3}$

$= \log c$, find c in terms of x and y .

17. Given $x = \log_{10} 12$, $y = \log_4 2 \times \log_{10} 9$ and $z = \log_{10} 0.4$, find :

(i) $x - y - z$ (ii) 13^{x-y-z}

18. Solve for x , $\log_x 15\sqrt{5} = 2 - \log_x 3\sqrt{5}$.

19. Evaluate :

(i) $\log_b a \times \log_c b \times \log_a c$

(ii) $\log_3 8 \div \log_9 16$

(iii) $\frac{\log_5 8}{\log_{25} 16 \times \log_{100} 10}$

20. Show that :

$$\log_a m \div \log_{ab} m = 1 + \log_a b$$

$$\begin{aligned} \log_a m \div \log_{ab} m &= \frac{\log_a m}{\log_{ab} m} \\ &= \frac{\log_m ab}{\log_m a} \left[\because \log_b a = \frac{1}{\log_a b} \right] \\ &= \log_a ab \left[\because \frac{\log_x a}{\log_x b} = \log_b a \right] \\ &= \log_a a + \log_a b \\ &= 1 + \log_a b \end{aligned}$$