

Expansions

POINTS TO REMEMBER

1. (i) $(a + b)^2 = a^2 + b^2 + 2ab$

(iii) $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

(v) $(a + b)(a - b) = (a^2 - b^2)$

(vii) $(a - b)^2 = (a + b)^2 - 4ab$

(ii) $(a - b)^2 = a^2 + b^2 - 2ab$

(iv) $(a + b)^2 - (a - b)^2 = 4ab$

(vi) $(a + b)^2 = (a - b)^2 + 4ab$

2. (i) $\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$

(iii) $\left(a + \frac{1}{a}\right)\left(a - \frac{1}{a}\right) = \left(a^2 - \frac{1}{a^2}\right)$

(v) $\left(a + \frac{1}{a}\right)^2 - \left(a - \frac{1}{a}\right)^2 = 4$

(vii) $\left(a - \frac{1}{a}\right)^2 = \left(a + \frac{1}{a}\right)^2 - 4$

(ii) $\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$

(iv) $\left(a + \frac{1}{a}\right)^2 + \left(a - \frac{1}{a}\right)^2 = 2\left(a^2 + \frac{1}{a^2}\right)$

(vi) $\left(a + \frac{1}{a}\right)^2 = \left(a - \frac{1}{a}\right)^2 + 4$

3. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

4. (i) $(x + a)(x + b) = x^2 + (a + b)x + ab$

(ii) $(x + a)(x - b) = x^2 + (a - b)x - ab$

(iii) $(x - a)(x + b) = x^2 - (a - b)x - ab$

(iv) $(x - a)(x - b) = x^2 - (a + b)x + ab$

Note $(x + a)(x + b) = x^2 + (a + b)x + ab$

$= x^2 + (\text{Algebraic sum of 2nd terms})x + (\text{Product of second terms}).$

5. $(a + b)^3 = a^3 + b^3 + 3ab(a + b) = a^3 + 3a^2b + 3ab^2 + b^3$ and $\left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right)$

6. $(a - b)^3 = a^3 - b^3 - 3ab(a - b) = a^3 - 3a^2b + 3ab^2 - b^3$ and $\left(a - \frac{1}{a}\right)^3 = a^3 - \frac{1}{a^3} - 3\left(a - \frac{1}{a}\right)$

7. If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

EXERCISE 4 (A)

Using the standard formulae, expand each of the following (Q. No. 1 to 13) :

Q. 1. (i) $(4a + 9)^2$

(ii) $(3x + 10y)^2$

(iii) $(\sqrt{2}m + \sqrt{3}n)^2$.

Sol. (i) $(4a + 9)^2 = (4a)^2 + (9)^2 + 2 \times 4a \times 9$
 $= 16a^2 + 81 + 72a$

(ii) $(3x + 10y)^2$
 $= (3x)^2 + (10y)^2 + 2 \times 3x \times 10y$
 $= 9x^2 + 100y^2 + 60xy$

(iii) $(\sqrt{2}m + \sqrt{3}n)^2 = (\sqrt{2}m)^2 + (\sqrt{3}n)^2$
 $+ 2 \times \sqrt{2}m \times \sqrt{3}n$
 $= 2m^2 + 3n^2 + 2\sqrt{6}mn$ **Ans.**

Q. 2. (i) $(2a^2 + 3b)^2$

(ii) $(3x^2y + z)^2$

(iii) $\left(2x + \frac{1}{3x}\right)^2$.

Sol. (i) $(2a^2 + 3b)^2$
 $= (2a^2)^2 + (3b)^2 + 2 \times 2a^2 \times 3b$
 $= 4a^4 + 9b^2 + 12a^2b$

(ii) $(3x^2y + z)^2$
 $= (3x^2y)^2 + (z)^2 + 2 \times 3x^2y \times z$
 $= 9x^4y^2 + z^2 + 6x^2yz$

(iii) $\left(2x + \frac{1}{3x}\right)^2$
 $= (2x)^2 + \left(\frac{1}{3x}\right)^2 + 2 \times 2x \times \frac{1}{3x}$
 $= 4x^2 + \frac{1}{9x^2} + \frac{4}{3}$ **Ans.**

Q. 3. (i) $\left(\frac{2}{5}x + \frac{5}{6}y\right)^2$

(ii) $\left(\frac{x}{3} + \frac{6}{x}\right)^2$

(iii) $\left(6 + \frac{5}{x}\right)^2$.

Sol. (i) $\left(\frac{2}{5}x + \frac{5}{6}y\right)^2 = \left(\frac{2}{5}x\right)^2 + \left(\frac{5}{6}y\right)^2$
 $+ 2 \times \frac{2}{5}x \times \frac{5}{6}y$

$= \frac{4}{25}x^2 + \frac{25}{36}y^2 + \frac{2}{3}xy$

(ii) $\left(\frac{x}{3} + \frac{6}{x}\right)^2 = \left(\frac{x}{3}\right)^2 + \left(\frac{6}{x}\right)^2 + 2 \times \frac{x}{3} \times \frac{6}{x}$
 $= \frac{x^2}{9} + \frac{36}{x^2} + 4$

(iii) $\left(6 + \frac{5}{x}\right)^2 = (6)^2 + \left(\frac{5}{x}\right)^2 + 2 \times 6 \times \frac{5}{x}$
 $= 36 + \frac{25}{x^2} + \frac{60}{x}$ **Ans.**

Q. 4. (i) $(5x - 3y)^2$

(ii) $(3a - 7b)^2$

(iii) $\left(\frac{1}{2}x - \frac{3}{2}y\right)^2$.

Sol. (i) $(5x - 3y)^2$
 $= (5x)^2 + (3y)^2 - 2 \times 5x \times 3y$
 $= 25x^2 + 9y^2 - 30xy$

(ii) $(3a - 7b)^2$
 $= (3a)^2 + (7b)^2 - 2 \times 3a \times 7b$
 $= 9a^2 + 49b^2 - 42ab$

(iii) $\left(\frac{1}{2}x - \frac{3}{2}y\right)^2 = \left(\frac{1}{2}x\right)^2 + \left(\frac{3}{2}y\right)^2$
 $- 2 \times \frac{1}{2}x \times \frac{3}{2}y$
 $= \frac{1}{4}x^2 + \frac{9}{4}y^2 - \frac{3}{2}xy$ **Ans.**

Q. 5. (i) $\left(a^2 - \frac{b}{2}\right)^2$ (ii) $\left(\frac{3a}{2b} - \frac{2b}{3a}\right)^2$

(iii) $\left(5x - \frac{2}{3x}\right)^2$.

$$\text{Sol. (i)} \left(a^2 - \frac{b}{2}\right)^2 = (a^2)^2 + \left(\frac{b}{2}\right)^2 - 2 \times a^2 \times \frac{b}{2}$$

$$= a^4 + \frac{b^2}{4} - a^2 b$$

$$\text{(ii)} \left(\frac{3a}{2b} - \frac{2b}{3a}\right)^2 = \left(\frac{3a}{2b}\right)^2 + \left(\frac{2b}{3a}\right)^2 - 2 \times \frac{3a}{2b} \times \frac{2b}{3a}$$

$$= \frac{9a^2}{4b^2} + \frac{4b^2}{9a^2} - 2$$

$$\text{(iii)} \left(5x - \frac{2}{3x}\right)^2 = (5x)^2 + \left(\frac{2}{3x}\right)^2 - 2 \times 5x \times \frac{2}{3x}$$

$$= 25x^2 + \frac{4}{9x^2} - \frac{20}{3} \text{ Ans.}$$

$$\text{Q. 6. (i)} (a + 2b + 3c)^2$$

$$\text{(ii)} (3x + 5y - 2z)^2$$

$$\text{(iii)} (2x - 3y + 7z)^2$$

$$\text{Sol. (i)} (a + 2b + 3c)^2$$

$$= (a)^2 + (2b)^2 + (3c)^2 + 2 \times a \times 2b + 2 \times 2b \times 3c + 2 \times 3c \times a$$

$$= a^2 + 4b^2 + 9c^2 + 4ab + 12bc + 6ca$$

$$\text{(ii)} (3x + 5y - 2z)^2 = (3x)^2 + (5y)^2$$

$$+ (2z)^2 + 2 \times 3x \times 5y - 2 \times 5y \times 2z - 2 \times 2z \times 3x$$

$$= 9x^2 + 25y^2 + 4z^2 + 30xy - 20yz - 12zx$$

$$\text{(iii)} (2x - 3y + 7z)^2 = (2x)^2 + (3y)^2$$

$$+ (7z)^2 - 2 \times 2x \times 3y - 2 \times 3y \times 7z + 2 \times 7z \times 2x$$

$$= 4x^2 + 9y^2 + 49z^2 - 12xy - 42yz + 28zx \text{ Ans.}$$

$$\text{Q. 7. (i)} (6 - 2y + 4z)^2$$

$$\text{(ii)} (4x - 3y + z)^2$$

$$\text{(iii)} (7 - 2x - 3y)^2$$

$$\text{Sol. (i)} (6 - 2y + 4z)^2 = (6)^2 + (2y)^2$$

$$+ (4z)^2 - 2 \times 6 \times 2y - 2 \times 2y \times 4z + 2 \times 4z \times 6$$

$$= 36 + 4y^2 + 16z^2 - 24y - 16yz + 48z$$

$$\text{(ii)} (4x - 3y + z)^2 = (4x)^2 + (3y)^2 + (z)^2$$

$$- 2 \times 4x \times 3y - 2 \times 3y \times z + 2 \times z \times 4x$$

$$= 16x^2 + 9y^2 + z^2 - 24xy - 6yz + 8zx$$

$$\text{(iii)} (7 - 2x - 3y)^2 = (7)^2 + (2x)^2 + (3y)^2$$

$$- 2 \times 7 \times 2x + 2 \times 2x \times 3y - 2 \times 3y \times 7$$

$$= 49 + 4x^2 + 9y^2 - 28x + 12xy - 42y$$

Ans.

$$\text{Q. 8. (i)} \left(\frac{a}{2} + \frac{b}{3} + \frac{c}{4}\right)^2$$

$$\text{(ii)} \left(\frac{2x}{3} + \frac{3}{2y} - 2\right)^2$$

$$\text{(iii)} \left(2x + \frac{3}{x} - 1\right)^2$$

$$\text{Sol. (i)} \left(\frac{a}{2} + \frac{b}{3} + \frac{c}{4}\right)^2$$

$$= \left(\frac{a}{2}\right)^2 + \left(\frac{b}{3}\right)^2 + \left(\frac{c}{4}\right)^2$$

$$+ 2 \times \frac{a}{2} \times \frac{b}{3} + 2 \times \frac{b}{3} \times \frac{c}{4} + 2 \times \frac{c}{4} \times \frac{a}{2}$$

$$= \frac{a^2}{4} + \frac{b^2}{9} + \frac{c^2}{16} + \frac{1}{3}ab + \frac{1}{6}bc + \frac{1}{4}ca$$

$$\text{(ii)} \left(\frac{2x}{3} + \frac{3}{2y} - 2\right)^2$$

$$= \left(\frac{2x}{3}\right)^2 + \left(\frac{3}{2y}\right)^2 + (2)^2$$

$$+ 2 \times \frac{2x}{3} \times \frac{3}{2y} - 2 \times \frac{3}{2y} \times 2 - 2 \times 2 \times \frac{2x}{3}$$

$$= \frac{4}{9}x^2 + \frac{9}{4y^2} + 4 + 2 \cdot \frac{x}{y} - \frac{6}{y} - \frac{8}{3}x$$

$$\Rightarrow \frac{4x^2}{9} + \frac{9}{4y^2} + 4 + 2 \cdot \frac{x}{y} - \frac{6}{y} - \frac{8x}{3}$$

$$(iii) \left(2x + \frac{3}{x} - 1\right)^2 = (2x)^2 + \left(\frac{3}{x}\right)^2 + (1)^2$$

$$+ 2 \times 2x \times \frac{3}{x} - 2 \times \frac{3}{x} \times 1 - 2 \times 1 \times 2x$$

$$= 4x^2 + \frac{9}{x^2} + 1 + 12 - \frac{6}{x} - 4x$$

$$= 4x^2 + \frac{9}{x^2} + 13 - \frac{6}{x} - 4x \text{ Ans.}$$

Q. 9. (i) $(x+7)(x+4)$

(ii) $(a+13)(a-8)$

(iii) $(y-6)(y-4)$

Sol. (i) $(x+7)(x+4)$

$$= (x)^2 + (7+4)x + 7 \times 4$$

$$= x^2 + 11x + 28$$

(ii) $(a+13)(a-8)$

$$= a^2 + (13-8)a + 13 \times (-8)$$

$$= a^2 + 5a - 104$$

(iii) $(y-6)(y-4)$

$$= y^2 + (-6-4)y + (-6)(-4)$$

$$= y^2 - 10y + 24 \text{ Ans.}$$

Q. 10. (i) $(9+2x)(9-3x)$

(ii) $(5x-4y)(5x+3y)$

(iii) $(3-7a)(3+4a)$

Sol. (i) $(9+2x)(9-3x)$

$$= (9)^2 + (2x-3x) \times 9 + 2x \times (-3x)$$

$$= 81 - 9x - 6x^2$$

(ii) $(5x-4y)(5x+3y)$

$$= (5x)^2 + (-4y+3y) \times 5x$$

$$+ (-4y)(3y)$$

$$= 25x^2 - 5xy - 12y^2$$

(iii) $(3-7a)(3+4a)$

$$= (3)^2 + (-7a+4a) \times 3 + (-7a)(4a)$$

$$= 9 - 9a - 28a^2 \text{ Ans.}$$

Q. 11. (i) $(3a+2b)(3a-2b)$

(ii) $\left(5x + \frac{1}{5x}\right)\left(5x - \frac{1}{5x}\right)$

(iii) $\left(2x^2 + \frac{3}{x^2}\right)\left(2x^2 - \frac{3}{x^2}\right)$

Sol. (i) $(3a+2b)(3a-2b)$

$$= (3a)^2 - (2b)^2 = 9a^2 - 4b^2$$

$$\{\because (a+b)(a-b) = a^2 - b^2\}$$

(ii) $\left(5x + \frac{1}{5x}\right)\left(5x - \frac{1}{5x}\right)$

$$= (5x)^2 - \left(\frac{1}{5x}\right)^2$$

$$\{\because (a+b)(a-b) = a^2 - b^2\}$$

$$= 25x^2 - \frac{1}{25x^2} \text{ Ans.}$$

(iii) $\left(2x^2 + \frac{3}{x^2}\right)\left(2x^2 - \frac{3}{x^2}\right)$

$$= (2x^2)^2 - \left(\frac{3}{x^2}\right)^2$$

$$\{\because (a+b)(a-b) = a^2 - b^2\}$$

$$= 4x^4 - \frac{9}{x^4} \text{ Ans.}$$

Q. 12. (i) $(2-x)(2+x)(4+x^2)$

(ii) $(x+y)(x-y)(x^2+y^2)$

Sol. (i) $(2-x)(2+x)(4+x^2)$

$$= \{(2)^2 - (x)^2\}(4+x^2)$$

$$\{\because (a+b)(a-b) = a^2 - b^2\}$$

$$= (4-x^2)(4+x^2)$$

$$= (4)^2 - (x^2)^2 = 16 - x^4 \text{ Ans.}$$

(ii) $(x+y)(x-y)(x^2+y^2)$

$$= \{(x)^2 - (y)^2\}(x^2+y^2)$$

$$= (x^2-y^2)(x^2+y^2)$$

$$= (x^2)^2 - (y^2)^2 = x^4 - y^4 \text{ Ans.}$$

Q. 13. (i) $(x-2)(x-3)(x+4)$

(ii) $(x-5)(2x-1)(2x+3)$

Sol. (i) $(x-2)(x-3)(x+4)$
 $= \{x^2 + (-2-3)x + (-2)(-3)\}(x+4)$
 $= (x^2 - 5x + 6)(x+4)$
 $= x^3 - 5x^2 + 6x + 4x^2 - 20x + 24$
 $= x^3 - x^2 - 14x + 24$ **Ans.**

(ii) $(x-5)(2x-1)(2x+3)$
 $= (x-5)\{(2x)^2 + (-1+3)2x + (-1)(3)\}$
 $= (x-5)\{4x^2 + 4x - 3\}$
 $= 4x^3 + 4x^2 - 3x - 20x^2 - 20x + 15$
 $= 4x^3 - 16x^2 - 23x + 15$ **Ans.**

Q. 14. Simplify :

(i) $(a+b)^2 + (a-b)^2$

(ii) $(a+b)^2 - (a-b)^2$

(iii) $\left(x + \frac{1}{x}\right)^2 + \left(x - \frac{1}{x}\right)^2$

(iv) $\left(x + \frac{1}{x}\right)^2 - \left(x - \frac{1}{x}\right)^2$

(v) $\left(\frac{a}{2b} + \frac{2b}{a}\right)^2 - \left(\frac{2b}{a} - \frac{a}{2b}\right)^2$

(vi) $\left(3x - \frac{1}{3x}\right)^2 - \left(3x + \frac{1}{3x}\right)\left(3x - \frac{1}{3x}\right)$

(vii) $(5a+3b)^2 - (5a-3b)^2 - 60ab$

(viii) $(3x+1)^2 - (3x+2)(3x-1)$

Sol. (i) $(a+b)^2 + (a-b)^2$
 $= a^2 + b^2 + 2ab + a^2 + b^2 - 2ab$
 $= 2a^2 + 2b^2 = 2(a^2 + b^2)$ **Ans.**

(ii) $(a+b)^2 - (a-b)^2$
 $= (a^2 + b^2 + 2ab) - (a^2 + b^2 - 2ab)$
 $= a^2 + b^2 + 2ab - a^2 - b^2 + 2ab$
 $= 4ab$ **Ans.**

(iii) $\left(x + \frac{1}{x}\right)^2 + \left(x - \frac{1}{x}\right)^2$
 $= \left(x^2 + \frac{1}{x^2} + 2\right) + \left(x^2 + \frac{1}{x^2} - 2\right)$

$= x^2 + \frac{1}{x^2} + 2 + x^2 + \frac{1}{x^2} - 2$
 $= 2x^2 + \frac{2}{x^2} = 2\left(x^2 + \frac{1}{x^2}\right)$ **Ans.**

(iv) $\left(x + \frac{1}{x}\right)^2 - \left(x - \frac{1}{x}\right)^2$
 $= \left(x^2 + \frac{1}{x^2} + 2\right) - \left(x^2 + \frac{1}{x^2} - 2\right)$
 $= x^2 + \frac{1}{x^2} + 2 - x^2 - \frac{1}{x^2} + 2 = 4$ **Ans.**

(v) $\left(\frac{a}{2b} + \frac{2b}{a}\right)^2 - \left(\frac{2b}{a} - \frac{a}{2b}\right)^2$
 $= \left\{\left(\frac{a}{2b}\right)^2 + \left(\frac{2b}{a}\right)^2 + 2 \times \frac{a}{2b} \times \frac{2b}{a}\right\}$
 $- \left\{\left(\frac{2b}{a}\right)^2 + \left(\frac{a}{2b}\right)^2 - 2 \times \frac{2b}{a} \times \frac{a}{2b}\right\}$
 $= \left(\frac{a^2}{4b^2} + \frac{4b^2}{a^2} + 2\right) - \left(\frac{4b^2}{a^2} + \frac{a^2}{4b^2} - 2\right)$
 $= \frac{a^2}{4b^2} + \frac{4b^2}{a^2} + 2 - \frac{4b^2}{a^2} - \frac{a^2}{4b^2} + 2$
 $= 4$ **Ans.**

(vi) $\left(3x - \frac{1}{3x}\right)^2 - \left(3x + \frac{1}{3x}\right)\left(3x - \frac{1}{3x}\right)$
 $= \left(3x - \frac{1}{3x}\right)^2 - \left\{(3x)^2 - \left(\frac{1}{3x}\right)^2\right\}$
 $\{\because (a+b)(a-b) = a^2 - b^2\}$
 $= \left\{(3x)^2 + \left(\frac{1}{3x}\right)^2 - 2 \times 3x \times \frac{1}{3x}\right\}$
 $- \left\{9x^2 - \frac{1}{9x^2}\right\}$

$$= \left(9x^2 + \frac{1}{9x^2} - 2 \right) - \left(9x^2 - \frac{1}{9x^2} \right)$$

$$= 9x^2 + \frac{1}{9x^2} - 2 - 9x^2 + \frac{1}{9x^2}$$

$$= \frac{2}{9}x^2 - 2 = 2 \left(\frac{1}{9}x^2 - 1 \right) \text{ Ans.}$$

$$\begin{aligned} \text{(vii)} \quad & (5a + 3b)^2 - (5a - 3b)^2 - 60ab \\ &= \{(5a)^2 + (3b)^2 + 2 \times 5a \times 3b\} \\ &- \{(5a)^2 + (3b)^2 - 2 \times 5a \times 3b\} - 60ab \\ &= (25a^2 + 9b^2 + 30ab) \end{aligned}$$

$$\begin{aligned} &- (25a^2 + 9b^2 - 30ab) - 60ab \\ &= 25a^2 + 9b^2 + 30ab - 25a^2 - 9b^2 \\ &\quad + 30ab - 60ab \\ &= 0 \text{ Ans.} \end{aligned}$$

$$\begin{aligned} \text{(viii)} \quad & (3x + 1)^2 - (3x + 2)(3x - 1) \\ &= \{(3x)^2 + (1)^2 + 2 \times 3x \times 1\} \\ &- \{(3x)^2 + (2 - 1)3x + 2 \times (-1)\} \\ &= (9x^2 + 1 + 6x) - (9x^2 + 3x - 2) \\ &= 9x^2 + 1 + 6x - 9x^2 - 3x + 2 \\ &= 3x + 3 = 3(x + 1) \text{ Ans.} \end{aligned}$$

Q. 15. (i) If $a + b = 7$ and $ab = 10$, find the value of $(a - b)$.

(ii) If $x - y = 5$ and $xy = 24$, find the value of $(x + y)$.

$$\begin{aligned} \text{Sol. (i)} \quad & (a - b)^2 = (a + b)^2 - 4ab \\ &= (7)^2 - 4 \times 10 = 49 - 40 \\ &= 9 = (\pm 3)^2 \end{aligned}$$

$$\therefore a - b = \pm 3 \text{ Ans.}$$

$$\begin{aligned} \text{(ii)} \quad & (x + y)^2 = (x - y)^2 + 4xy \\ &= (5)^2 + 4 \times 24 = 25 + 96 = 121 \\ &= (\pm 11)^2 \end{aligned}$$

$$\therefore x + y = \pm 11 \text{ Ans.}$$

Q. 16. If $(3a + 4b) = 16$ and $ab = 4$, find the value of $(9a^2 + 16b^2)$.

$$\text{Sol. } (3a + 4b) = 16$$

Squaring both sides

$$(3a)^2 + (4b)^2 + 2 \times 3a \times 4b = (16)^2$$

$$\Rightarrow 9a^2 + 16b^2 + 24ab = 256$$

$$\Rightarrow 9a^2 + 16b^2 + 24 \times 4 = 256$$

$$\Rightarrow 9a^2 + 16b^2 + 96 = 256$$

$$\Rightarrow 9a^2 + 16b^2 = 256 - 96 = 160$$

$$\text{Hence } 9a^2 + 16b^2 = 160 \text{ Ans.}$$

Q. 17. If $(a + b) = 2$ and $(a - b) = 10$, find the values of :

(i) $(a^2 + b^2)$ (ii) ab .

Sol. $a + b = 2$ and $a - b = 10$

$$\begin{aligned} \text{(i)} \quad & 2(a^2 + b^2) = (a + b)^2 + (a - b)^2 \\ &= (2)^2 + (10)^2 = 4 + 100 = 104 \end{aligned}$$

$$\therefore a^2 + b^2 = \frac{104}{2} = 52 \text{ Ans.}$$

$$\begin{aligned} \text{(ii)} \quad & 4ab = (a + b)^2 - (a - b)^2 \\ &= (2)^2 - (10)^2 = 4 - 100 = -96 \end{aligned}$$

$$\therefore ab = \frac{-96}{4} = -24 \text{ Ans.}$$

Q. 18. If $(a - b) = 0.9$ and $ab = 0.36$, find the values of (i) $(a + b)$ (ii) $(a^2 - b^2)$.

Sol. $a - b = 0.9$ and $ab = 0.36$

Now

$$\begin{aligned} \text{(i)} \quad & (a + b)^2 = (a - b)^2 + 4ab \\ &= (0.9)^2 + 4 \times 0.36 \\ &= 0.81 + 1.44 = 2.25 = (\pm 1.5)^2 \end{aligned}$$

$$\therefore a + b = \pm 1.5$$

$$\begin{aligned} \text{(ii)} \quad & a^2 - b^2 = (a + b)(a - b) \\ &= \pm 1.5 \times 0.9 = \pm 1.35 \text{ Ans.} \end{aligned}$$

Q. 19. If $\left(x + \frac{1}{x}\right) = 5$, find the values of

(i) $\left(x^2 + \frac{1}{x^2}\right)$ (ii) $\left(x^4 + \frac{1}{x^4}\right)$.

$$\text{Sol. } x + \frac{1}{x} = 5$$

Squaring both sides,

$$\text{(i)} \quad \left(x + \frac{1}{x}\right)^2 = (5)^2 \Rightarrow x^2 + \frac{1}{x^2} + 2 = 25$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 25 - 2 = 23$$

$$\therefore x^2 + \frac{1}{x^2} = 23 \text{ Ans.}$$

Again squaring both sides,

$$\begin{aligned} (ii) \left(x^2 + \frac{1}{x^2}\right)^2 &= (23)^2 \\ \Rightarrow (x^2)^2 + \frac{(1)^2}{(x^2)^2} + 2 \times x \times \frac{1}{x} &= 529 \\ \Rightarrow x^4 + \frac{1}{x^4} + 2 &= 529 \\ \Rightarrow x^4 + \frac{1}{x^4} &= 529 - 2 = 527 \text{ Ans.} \end{aligned}$$

Q. 20. If $\left(x - \frac{1}{x}\right) = 4$, find the values of

$$(i) \left(x^2 + \frac{1}{x^2}\right) \quad (ii) \left(x^4 + \frac{1}{x^4}\right).$$

$$\text{Sol. (i) } x - \frac{1}{x} = 4 \Rightarrow \left(x - \frac{1}{x}\right)^2 = 4^2$$

Squaring both sides,

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} = 16$$

$$x^2 + \frac{1}{x^2} - 2 = 16$$

$$x^2 + \frac{1}{x^2} = 16 + 2 = 18$$

$$\therefore x^2 + \frac{1}{x^2} = 18$$

(ii) Again squaring both sides,

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (18)^2$$

$$\Rightarrow x^4 + \frac{1}{x^4} + 2 \times x^2 \times \frac{1}{x^2} = 324$$

$$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 324$$

$$\Rightarrow x^4 + \frac{1}{x^4} = 324 - 2 = 322$$

$$\therefore x^4 + \frac{1}{x^4} = 322 \text{ Ans.}$$

Q. 21. If $x - 2 = \frac{1}{3x}$ find the values of

$$(i) \left(x^2 + \frac{1}{9x^2}\right) \quad (ii) \left(x^4 + \frac{1}{81x^4}\right).$$

$$\text{Sol. } x - 2 = \frac{1}{3x} \Rightarrow x - \frac{1}{3x} = 2,$$

Squaring both sides,

$$(i) \left(x - \frac{1}{3x}\right)^2 = (2)^2$$

$$\Rightarrow x^2 + \frac{1}{9x^2} - 2 \times x \times \frac{1}{3x} = 4$$

$$\Rightarrow x^2 + \frac{1}{9x^2} - \frac{2}{3} = 4$$

$$\Rightarrow x^2 + \frac{1}{9x^2} = 4 + \frac{2}{3} = \frac{14}{3}$$

$$\therefore x^2 + \frac{1}{9x^2} = \frac{14}{3}$$

(ii) Again squaring both sides,

$$\left(x^2 + \frac{1}{9x^2}\right)^2 = \left(\frac{14}{3}\right)^2$$

$$\Rightarrow (x^2)^2 + \left[\frac{1}{9x^2}\right]^2 + 2 \times x^2 \times \frac{1}{9x^2} = \frac{196}{9}$$

$$\Rightarrow x^4 + \frac{1}{81x^4} + \frac{2}{9} = \frac{196}{9}$$

$$\Rightarrow x^4 + \frac{1}{81x^4} = \frac{196}{9} - \frac{2}{9} = \frac{194}{9} \text{ Ans.}$$

Q. 22. If $\left(x + \frac{1}{x}\right) = 6$, find the values of

$$(i) \left(x - \frac{1}{x}\right) \quad (ii) \left(x^2 - \frac{1}{x^2}\right).$$

$$\text{Sol. } x + \frac{1}{x} = 6$$

$$\begin{aligned} (i) \left(x - \frac{1}{x}\right)^2 &= \left(x + \frac{1}{x}\right)^2 - 4 \\ &= (6)^2 - 4 = 36 - 4 = 32 \end{aligned}$$

$$\begin{aligned} \therefore x - \frac{1}{x} &= \pm \sqrt{32} = \pm \sqrt{16 \times 2} \\ &= \pm 4\sqrt{2} \end{aligned}$$

$$(ii) \quad x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right) \\ = 6 \times (\pm 4\sqrt{2}) = \pm 24\sqrt{2} \quad \text{Ans.}$$

Q. 23. If $\left(x - \frac{1}{x}\right) = 8$, find the values of

$$(i) \left(x + \frac{1}{x}\right) \quad (ii) \left(x^2 - \frac{1}{x^2}\right).$$

Sol. $x - \frac{1}{x} = 8$

$$(i) \quad \left(x + \frac{1}{x}\right)^2 = \left(x - \frac{1}{x}\right)^2 + 4 \\ = (8)^2 + 4 = 64 + 4 = 68 \\ = 4 \times 17$$

$$\therefore x + \frac{1}{x} = \pm \sqrt{4 \times 17} = \pm 2\sqrt{17}.$$

$$(ii) \quad x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right) \\ = \pm 2\sqrt{17} \times 8 = \pm 16\sqrt{17} \quad \text{Ans.}$$

Q. 24. If $\left(x^2 + \frac{1}{x^2}\right) = 7$, find the values of

$$(i) \left(x + \frac{1}{x}\right) \quad (ii) \left(x - \frac{1}{x}\right)$$

$$(iii) \left(2x^2 - \frac{2}{x^2}\right).$$

Sol. (i) $x^2 + \frac{1}{x^2} = 7$

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \\ = 7 + 2 = 9 = (\pm 3)^2$$

$$\therefore x + \frac{1}{x} = \pm 3$$

$$(ii) \quad \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2 \\ = 7 - 2 = 5 = (\pm\sqrt{5})^2$$

$$\therefore x - \frac{1}{x} = \pm\sqrt{5} \quad \text{Ans.}$$

$$(iii) \quad 2x^2 - \frac{2}{x^2} = 2 \left(x^2 - \frac{1}{x^2}\right) \\ = 2 \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right) \\ = 2(\pm 3)(\pm\sqrt{5}) = \pm 6\sqrt{5} \quad \text{Ans.}$$

Q. 25. If $\left(x^2 + \frac{1}{25x^2}\right) = 9\frac{2}{5}$, find the value of $\left(x - \frac{1}{5x}\right).$

Sol. $x^2 + \frac{1}{25x^2} = 9\frac{2}{5} = \frac{47}{5}$

$$\left(x - \frac{1}{5x}\right)^2 = x^2 + \frac{1}{25x^2} - 2 \times x \times \frac{1}{5x} \\ = x^2 + \frac{1}{25x^2} - \frac{2}{5} \\ = \frac{47}{5} - \frac{2}{5} = \frac{45}{5} = 9 = (\pm 3)^2$$

$$\therefore x - \frac{1}{5x} = \pm 3 \quad \text{Ans.}$$

Q. 26. If $(a^2 - 4a - 1) = 0$ and $a \neq 0$, find the value of:

$$(i) \left(a - \frac{1}{a}\right) \quad (ii) \left(a + \frac{1}{a}\right)$$

$$(iii) \left(a^2 - \frac{1}{a^2}\right) \quad (iv) \left(a^2 + \frac{1}{a^2}\right).$$

Sol. $a^2 - 4a - 1 = 0$

Dividing by a ($\because a \neq 0$)

$$\therefore a - 4 - \frac{1}{a} = 0$$

$$(i) \quad a - \frac{1}{a} - 4 = 0 \Rightarrow a - \frac{1}{a} = 4$$

$$\therefore a - \frac{1}{a} = 4$$

$$(ii) \quad \left(a + \frac{1}{a}\right)^2 = \left(a - \frac{1}{a}\right)^2 + 4 \\ = (4)^2 + 4 = 16 + 4 = 20 = 4 \times 5$$

$$\therefore a + \frac{1}{a} = \pm \sqrt{4 \times 5} = \pm 2\sqrt{5}$$

$$(iii) \quad a^2 - \frac{1}{a^2} = \left(a + \frac{1}{a}\right) \left(a - \frac{1}{a}\right) \\ = \pm 2\sqrt{5} \times 4 = \pm 8\sqrt{5}$$

$$(iv) \quad a^2 + \frac{1}{a^2} = \left(a - \frac{1}{a}\right)^2 + 2 \\ = (4)^2 + 2 = 16 + 2 = 18 \text{ Ans.}$$

Q. 27. If $a = \frac{1}{(a-5)}$ where $a \neq 5$ and $a \neq 0$, find the values of :

$$(i) \left(a - \frac{1}{a}\right) \quad (ii) \left(a + \frac{1}{a}\right)$$

$$(iii) \left(a^2 - \frac{1}{a^2}\right) \quad (iv) \left(a^2 + \frac{1}{a^2}\right)$$

$$\text{Sol. } a = \frac{1}{a-5} \Rightarrow a-5 = \frac{1}{a} \\ \Rightarrow a - \frac{1}{a} = 5$$

$$(i) \therefore a - \frac{1}{a} = 5$$

$$(ii) \left(a + \frac{1}{a}\right)^2 = \left(a - \frac{1}{a}\right)^2 + 4 = (5)^2 + 4 \\ = 25 + 4 = 29$$

$$\therefore a + \frac{1}{a} = \pm \sqrt{29}$$

$$(iii) \quad a^2 - \frac{1}{a^2} = \left(a + \frac{1}{a}\right) \left(a - \frac{1}{a}\right) \\ = \pm \sqrt{29} \times 5 = \pm 5\sqrt{29}$$

$$(iv) \quad a^2 + \frac{1}{a^2} = \left(a - \frac{1}{a}\right)^2 + 2 \\ = (5)^2 + 2 = 25 + 2 \Rightarrow 27$$

$$\Rightarrow \therefore a^2 + \frac{1}{a^2} = \sqrt{27} = \sqrt{9 \times 3} \\ = 3\sqrt{3} \text{ Ans.}$$

Q. 28. Using $(a+b)^2 = (a^2 + b^2 + 2ab)$ evaluate :

$$(i) (137)^2 \quad (ii) (1008)^2$$

$$(iii) (11.6)^2$$

$$\text{Sol. } (i) (137)^2 = (130 + 7)^2 \\ = (130)^2 + (7)^2 + 2 \times 130 \times 7 \\ = 16900 + 49 + 1820 = 18769$$

$$(ii) (1008)^2 = (1000 + 8)^2 \\ = (1000)^2 + (8)^2 + 2 \times 1000 \times 8 \\ = 1000000 + 64 + 16000 \\ = 1016064$$

$$(iii) (11.6)^2 = (11 + 0.6)^2 \\ = (11)^2 + (0.6)^2 + 2 \times 11 \times 0.6 \\ = 121 + 0.36 + 13.2 \\ = 134.56 \text{ Ans.}$$

Q. 29. Using $(a-b)^2 = (a^2 + b^2 - 2ab)$, evaluate :

$$(i) (97)^2 \quad (ii) (992)^2$$

$$(iii) (9.98)^2$$

$$\text{Sol. } (i) (97)^2 = (100 - 3)^2 \\ = (100)^2 + (3)^2 - 2 \times 100 \times 3 \\ = 10000 + 9 - 600 = 10009 - 600 \\ = 9409$$

$$(ii) (992)^2 = (1000 - 8)^2 \\ = (1000)^2 + (8)^2 - 2 \times 1000 \times 8 \\ = 1000000 + 64 - 16000 \\ = 1000064 - 16000 = 984064$$

$$(iii) (9.98)^2 = (10 - 0.02)^2 \\ = (10)^2 + (0.02)^2 - 2 \times 10 \times 0.02 \\ = 100 + 0.0004 - 0.4 \\ = 99.6004 \text{ Ans.}$$

Q. 30. Fill in the blanks to make the given expression a perfect square :

$$(i) 16a^2 + 9b^2 + \dots\dots\dots$$

$$(ii) 25a^2 + 16b^2 - \dots\dots\dots$$

$$(iii) 4a^2 + 20ab + \dots\dots\dots$$

$$(iv) 9a^2 - 24ab + \dots\dots\dots$$

$$\text{Sol. } (i) 16a^2 + 9b^2 + \dots\dots\dots$$

$$= (4a)^2 + (3b)^2 + 2 \times 4a \times 3b$$

$$\{\because (a+b)^2 = a^2 + b^2 + 2ab\}$$

$$= 16a^2 + 9b^2 + 24ab$$

$$(ii) \quad 25a^2 + 16b^2 - \dots\dots\dots$$

$$= (5a)^2 + (4b)^2 - 2 \times 5a \times 4b$$

$$= 25a^2 + 16b^2 - 40ab$$

$$\{\because (a-b)^2 = a^2 + b^2 - 2ab\}$$

$$(iii) \quad 4a^2 + 20ab + \dots\dots\dots$$

$$= (2a)^2 + 2 \times 2a \times 5b + (5b)^2$$

$$= 4a^2 + 20ab + 25b^2$$

$$(iv) \quad 9a^2 - 24ab + \dots\dots\dots$$

$$= (3a)^2 - 2 \times 3a \times 4b + (4b)^2$$

$$= 9a^2 - 24ab + 16b^2 \quad \text{Ans.}$$

Q. 31. If $(a + b + c) = 14$ and $(a^2 + b^2 + c^2) = 74$, find the value of $(ab + bc + ca)$.

Sol. $a + b + c = 14$

Squaring both sides,

$$(a + b + c)^2 = (14)^2$$

$$\Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 196$$

$$\Rightarrow 2 + 2(ab + bc + ca) = 196$$

$$\Rightarrow 74(ab + bc + ca) = 196$$

$$\Rightarrow 2(ab + bc + ca) = 196 - 74$$

$$\Rightarrow 2(ab + bc + ca) = 122$$

$$\therefore ab + bc + ca = \frac{122}{2} = 61 \quad \text{Ans.}$$

Q. 32. If $(a + b + c) = 15$ and $(ab + bc + ca) = 74$, find the value of $(a^2 + b^2 + c^2)$.

Sol. $a + b + c = 15$

Squaring both sides,

$$(a + b + c)^2 = (15)^2$$

$$\Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 225$$

$$\Rightarrow a^2 + b^2 + c^2 + 2 \times 74 = 225$$

$$\Rightarrow a^2 + b^2 + c^2 + 148 = 225$$

$$\therefore a^2 + b^2 + c^2 = 225 - 148 = 77 \quad \text{Ans.}$$

Q. 33. If $(a^2 + b^2 + c^2) = 50$ and $(ab + bc + ca) = 47$, find the value of $(a + b + c)$.

Sol. We know that

$$(a + b + c)^2 = a^2 + b^2 + c^2$$

$$+ 2(ab + bc + ca)$$

$$= 50 + 2 \times 47 = 50 + 94$$

$$= 144 = (\pm 12)^2$$

$$\therefore a + b + c = \pm 12 \quad \text{Ans.}$$

Q. 34. If $(a^2 + b^2 + c^2) = 89$ and $(ab - bc - ca) = 16$, find the value of $(a + b - c)$.

Sol. We know that

$$(a + b - c)^2 = a^2 + b^2 + c^2$$

$$+ 2(ab - bc - ca)$$

$$= 89 + 2 \times 16 = 89 + 32$$

$$= 121 = (\pm 11)^2$$

$$\therefore a + b - c = \pm 11 \quad \text{Ans.}$$

EXERCISE 4 (B)

Q. 1. Expand :

$$(i) (3a + 5b)^3 \quad (ii) (2p - 3q)^3$$

$$(iii) \left(2x + \frac{1}{3x}\right)^3 \quad (iv) (3ab - 2c)^3$$

$$(v) \left(3a - \frac{1}{a}\right)^3 \quad (vi) \left(\frac{1}{2}x - \frac{2}{3}y\right)^3$$

Sol. (i) $(3a + 5b)^3 = (3a)^3 + 3 \times (3a)^2(5b)$
 $+ 3(3a)(5b)^2 + (5b)^3$
 $= 27a^3 + 135a^2b + 225ab^2 + 125b^3$
Ans.

(ii) $(2p - 3q)^3 = (2p)^3 - 3(2p)^2(3q)$
 $+ 3(2p)(3q)^2 - (3q)^3$
 $= 8p^3 - 36p^2q + 54pq^2 - 27q^3 \quad \text{Ans.}$

(iii) $\left(2x + \frac{1}{3x}\right)^3 = (2x)^3 + \left(\frac{1}{3x}\right)^3$
 $+ 3 \times 2x \times \frac{1}{3x} \left(2x + \frac{1}{3x}\right)$
 $= 8x^3 + \frac{1}{27x^3} + 2 \left(2x + \frac{1}{3x}\right)$
 $= 8x^3 + \frac{1}{27x^3} + 4x + \frac{2}{3x}$
 $= 8x^3 + 4x + \frac{2}{3x} + \frac{1}{27x^3} \quad \text{Ans.}$

(iv) $(3ab - 2c)^3 = (3ab)^3 - (2c)^3$
 $- 3 \times 3ab \times 2c(3ab - 2c)$

$$\begin{aligned}
 &= 27a^3b^3 - 8c^3 - 18abc(3ab - 2c) \\
 &= 27a^3b^3 - 8c^3 - 54a^2b^2c + 36abc^2 \\
 &= 27a^3b^3 - 54a^2b^2c + 36abc^2 - 8c^3
 \end{aligned}$$

Ans.

$$\begin{aligned}
 \text{(v)} \quad \left(3a - \frac{1}{a}\right)^3 &= (3a)^3 - \left(\frac{1}{a}\right)^3 \\
 &\quad - 3 \times 3a \times \frac{1}{a} \left(3a - \frac{1}{a}\right) \\
 &= 27a^3 - \frac{1}{a^3} - 9 \left(3a - \frac{1}{a}\right) \\
 &= 27a^3 - \frac{1}{a^3} - 27a + \frac{9}{a} \\
 &= 27a^3 - 27a + \frac{9}{a} - \frac{1}{a^3} \text{ Ans.}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad \left(\frac{1}{2}x - \frac{2}{3}y\right)^3 &= \left(\frac{1}{2}x\right)^3 - \left(\frac{2}{3}y\right)^3 \\
 &\quad - 3 \times \frac{1}{2}x \times \frac{2}{3}y \left(\frac{1}{2}x - \frac{2}{3}y\right) \\
 &= \frac{1}{8}x^3 - \frac{8}{27}y^3 - xy \left(\frac{1}{2}x - \frac{2}{3}y\right) \\
 &= \frac{1}{8}x^3 - \frac{8}{27}y^3 - \frac{1}{2}x^2y + \frac{2}{3}xy^2 \\
 &= \frac{1}{8}x^3 - \frac{1}{2}x^2y + \frac{2}{3}xy^2 - \frac{8}{27}y^3 \text{ Ans.}
 \end{aligned}$$

Q. 2. If $(4a + 3b) = 10$ and $ab = 2$, find the value of $(64a^3 + 27b^3)$.

Sol. $(4a + 3b) = 10$

Cubing both sides,

$$\begin{aligned}
 (4a + 3b)^3 &= (10)^3 \\
 \Rightarrow (4a)^3 + (3b)^3 + 3 \times 4a \times 3b(4a + 3b) &= 1000 \\
 \Rightarrow 64a^3 + 27b^3 + 36ab(4a + 3b) &= 1000 \\
 \Rightarrow 64a^3 + 27b^3 + 36 \times 2 \times 10 &= 1000 \\
 \Rightarrow 64a^3 + 27b^3 + 720 &= 1000 \\
 \therefore 64a^3 + 27b^3 &= 1000 - 720 = 280 \text{ Ans.}
 \end{aligned}$$

Q. 3. If $(3x - 2y) = 5$ and $xy = 6$, find the value of $(27x^3 - 8y^3)$.

Sol. $3x - 2y = 5$

Cubing both sides,

$$\begin{aligned}
 (3x - 2y)^3 &= (5)^3 \\
 \Rightarrow (3x)^3 - (2y)^3 - 3 \times 3x \times 2y(3x - 2y) &= 125 \\
 \Rightarrow 27x^3 - 8y^3 - 18xy(3x - 2y) &= 125 \\
 \Rightarrow 27x^3 - 8y^3 - 18 \times 6 \times 5 &= 125 \\
 \Rightarrow 27x^3 - 8y^3 - 540 &= 125 \\
 \therefore 27x^3 - 8y^3 &= 125 + 540 = 665 \text{ Ans.}
 \end{aligned}$$

Q. 4. If $(a + 3b) = 6$, show that

$$a^3 + 27b^3 + 54ab = 216.$$

Sol. $a + 3b = 6$

Cubing both sides,

$$\begin{aligned}
 (a + 3b)^3 &= (6)^3 \\
 \Rightarrow (a)^3 + (3b)^3 + 3 \times a \times 3b(a + 3b) &= 216 \\
 \Rightarrow a^3 + 27b^3 + 9ab(a + 3b) &= 216 \\
 \Rightarrow a^3 + 27b^3 + 9ab \times 6 &= 216 \\
 \Rightarrow a^3 + 27b^3 + 54ab &= 216 \\
 \text{Hence proved.}
 \end{aligned}$$

Q. 5. If $a + 2b + 3c = 0$, show that

$$a^3 + 8b^3 + 27c^3 = 18abc.$$

Sol. $a + 2b + 3c = 0$

$$a + 2b = -3c$$

Cubing both sides,

$$\begin{aligned}
 (a + 2b)^3 &= (-3c)^3 \\
 \Rightarrow (a)^3 + (2b)^3 + 3 \times a \times 2b(a + 2b) &= -27c^3 \\
 \Rightarrow a^3 + 8b^3 + 6ab(a + 2b) &= -27c^3 \\
 \Rightarrow a^3 + 8b^3 - 18abc &= -27c^3 \\
 \Rightarrow a^3 + 8b^3 + 27c^3 &= 18abc \\
 \text{Hence proved.}
 \end{aligned}$$

Q. 6. If $\left(x + \frac{1}{x}\right) = 3$,

find the value of $\left(x^3 + \frac{1}{x^3}\right)$.

Sol. $x + \frac{1}{x} = 3$

Cubing both sides,

$$\left(x + \frac{1}{x}\right)^3 = (3)^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x} \left(x + \frac{1}{x}\right) = 27$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 3 = 27$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 9 = 27$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 27 - 9 = 18$$

$$\text{Hence } x^3 + \frac{1}{x^3} = 18 \quad \text{Ans.}$$

Q. 7. If $\left(x - \frac{1}{x}\right) = 5$, find the value of $\left(x^3 - \frac{1}{x^3}\right)$.

$$\text{Sol. } x - \frac{1}{x} = 5$$

Cubing both sides,

$$\left(x - \frac{1}{x}\right)^3 = (5)^3$$

$$\Rightarrow x^3 - \frac{1}{x^3} - 3 \times x \times \frac{1}{x} \left(x - \frac{1}{x}\right) = 125$$

$$\Rightarrow x^3 - \frac{1}{x^3} - 3 \times 5 = 125$$

$$\Rightarrow x^3 - \frac{1}{x^3} - 15 = 125$$

$$\Rightarrow x^3 - \frac{1}{x^3} = 125 + 15 = 140$$

$$\text{Hence } x^3 - \frac{1}{x^3} = 140 \quad \text{Ans.}$$

Q. 8. If $\left(x - \frac{2}{x}\right) = 6$, find the value of $\left(x^3 - \frac{8}{x^3}\right)$.

$$\text{Sol. } x - \frac{2}{x} = 6$$

Cubing both sides,

$$\left(x - \frac{2}{x}\right)^3 = (6)^3$$

$$\Rightarrow x^3 - \frac{8}{x^3} - 3 \times x \times \frac{2}{x} \left(x - \frac{2}{x}\right) = 216$$

$$\Rightarrow x^3 - \frac{8}{x^3} - 6 \times 6 = 216$$

$$\Rightarrow x^3 - \frac{8}{x^3} - 36 = 216$$

$$\Rightarrow x^3 - \frac{8}{x^3} = 216 + 36 = 252$$

$$\therefore x^3 - \frac{8}{x^3} = 252 \quad \text{Ans.}$$

Q. 9. If $\left(x + \frac{1}{x}\right) = 4$, find the values of:

$$(i) \left(x^3 + \frac{1}{x^3}\right) \quad (ii) \left(x - \frac{1}{x}\right)$$

$$(iii) \left(x^3 - \frac{1}{x^3}\right)$$

$$\text{Sol. } x + \frac{1}{x} = 4$$

Cubing both sides,

$$(i) \left(x + \frac{1}{x}\right)^3 = (4)^3$$

$$= x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x} \left(x + \frac{1}{x}\right) = 64$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 4 = 64$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 12 = 64$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 64 - 12 = 52$$

$$\text{Hence } x^3 + \frac{1}{x^3} = 52.$$

$$(ii) \left(x - \frac{1}{x}\right)^2 = \left(x + \frac{1}{x}\right)^2 - 4$$

$$= (4)^2 - 4 = 16 - 4 = 12$$

$$\Rightarrow x - \frac{1}{x} = \pm \sqrt{12} = \pm \sqrt{4 \times 3}$$

$$\Rightarrow x - \frac{1}{x} = \pm 2\sqrt{3}$$

$$(iii) \left(x - \frac{1}{x}\right)^3 = (\pm 2\sqrt{3})^3$$

$$x^3 - \frac{1}{x^3} - 3x \times \frac{1}{x} \left(x - \frac{1}{x}\right) = \pm 8 \times 3\sqrt{3} = \pm 24\sqrt{3}$$

$$x^3 - \frac{1}{x^3} - 3 \times (\pm 2\sqrt{3}) = \pm 24\sqrt{3}$$

$$x^3 - \frac{1}{x^3} - \pm 6\sqrt{3} = \pm 24\sqrt{3}$$

$$\therefore x^3 - \frac{1}{x^3} = \pm 24\sqrt{3} \pm 6\sqrt{3}$$

$$= \pm 30\sqrt{3} \text{ Ans.}$$

Q. 10. If $\left(a^2 + \frac{1}{a^2}\right) = 23$, find the values of :

$$(i) \left(a + \frac{1}{a}\right) \quad (ii) \left(a^3 + \frac{1}{a^3}\right).$$

$$\text{Sol. } a^2 + \frac{1}{a^2} = 23$$

$$(i) \left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2 = 23 + 2 = 25$$

$$\therefore a + \frac{1}{a} = \pm \sqrt{25} = \pm 5$$

(ii) Cubing both sides,

$$\left(a + \frac{1}{a}\right)^3 = (\pm 5)^3$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3 \times a \times \frac{1}{a} \left(a + \frac{1}{a}\right) = \pm 125$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3 \times (\pm 5) = \pm 125$$

$$\Rightarrow a^3 + \frac{1}{a^3} \pm 15 = \pm 125$$

$$\Rightarrow a^3 + \frac{1}{a^3} = (\pm 125) - (\pm 15) = \pm 110.$$

Q. 11. If $\left(a - \frac{1}{a}\right) = \sqrt{5}$, find the values of :

$$(i) \left(a + \frac{1}{a}\right) \quad (ii) \left(a^3 + \frac{1}{a^3}\right).$$

$$\text{Sol. } a - \frac{1}{a} = \sqrt{5}$$

$$(i) \left(a + \frac{1}{a}\right)^2 = \left(a - \frac{1}{a}\right)^2 + 4$$

$$= (\sqrt{5})^2 + 4 = 5 + 4 = 9$$

$$\therefore a + \frac{1}{a} = \pm \sqrt{9} = \pm 3$$

Cubing both sides,

$$(ii) \left(a + \frac{1}{a}\right)^3 = (\pm 3)^3$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3 \times a \times \frac{1}{a} \left(a + \frac{1}{a}\right) = \pm 27$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3(\pm 3) = \pm 27$$

$$\Rightarrow a^3 + \frac{1}{a^3} \pm 9 = \pm 27$$

$$\therefore a^3 + \frac{1}{a^3} = \pm 27 - (\pm 9) = \pm 18$$

Q. 12. If $\left(a^2 + \frac{1}{a^2}\right) = 27$, find the values of

$$(i) \left(a - \frac{1}{a}\right) \quad (ii) \left(a^3 - \frac{1}{a^3}\right).$$

$$\text{Sol. } (i) \left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$$

$$\Rightarrow \left(a - \frac{1}{a}\right)^2 = 27 - 2 = 25$$

$$\therefore a - \frac{1}{a} = \pm \sqrt{25} = \pm 5$$

Cubing both sides,

$$(ii) \left(a - \frac{1}{a}\right)^3 = (\pm 5)^3$$

$$\Rightarrow a^3 - \frac{1}{a^3} - 3a \times \frac{1}{a} \left(a - \frac{1}{a} \right) = \pm 125$$

$$\Rightarrow a^3 - \frac{1}{a^3} - 3 \times (\pm 5) = \pm 125$$

$$\Rightarrow a^3 - \frac{1}{a^3} - (\pm 15) = \pm 125$$

$$\Rightarrow a^3 - \frac{1}{a^3} = \pm 125 + (\pm 15)$$

$$\therefore a^3 - \frac{1}{a^3} = \pm 140 \text{ Ans.}$$

Q. 13. If $\left(x^2 + \frac{1}{25x^2} \right) = 8\frac{3}{5}$, find the values, of

$$(i) \left(x + \frac{1}{5x} \right) \quad (ii) \left(x^3 + \frac{1}{125x^3} \right).$$

Sol. (i) $\left(x + \frac{1}{5x} \right)^2$

$$= x^2 + \frac{1}{25x^2} + 2 \times x \times \frac{1}{5x}$$

$$= x^2 + \frac{1}{25x^2} + \frac{2}{5}$$

$$= \frac{43}{5} + \frac{2}{5} = \frac{45}{5} = 9$$

$$\therefore x + \frac{1}{5x} = \pm \sqrt{9} = \pm 3$$

(ii) Cubing both sides,

$$\left(x + \frac{1}{5x} \right)^3 = (\pm 3)^3$$

$$(x^3) + \left(\frac{1}{5x} \right)^3 + 3 \times x \times \frac{1}{5x} \left(x + \frac{1}{5x} \right) = \pm 27$$

$$\Rightarrow x^3 + \frac{1}{125x^3} + \frac{3}{5} \times (\pm 3) = \pm 27$$

$$\Rightarrow x^3 + \frac{1}{125x^3} \pm \frac{9}{5} = \pm 27$$

$$\Rightarrow x^3 + \frac{1}{125x^3} = \pm 27 - \left(\pm \frac{9}{5} \right)$$

$$\Rightarrow x^3 + \frac{1}{125x^3} = \pm \frac{126}{5} = \pm 25\frac{1}{5} \text{ Ans.}$$

Q. 14. If $\left(x + \frac{1}{x} \right)^2 = 3$, show that

$$\left(x^3 + \frac{1}{x^3} \right) = 0.$$

Sol. $\left(x + \frac{1}{x} \right)^2 = 3 \quad \therefore x + \frac{1}{x} = \pm \sqrt{3}$

Cubing both sides,

$$\left(x + \frac{1}{x} \right)^3 = (\pm \sqrt{3})^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3x \times \frac{1}{x} \left(x + \frac{1}{x} \right) = \pm 3\sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times (\pm \sqrt{3}) = \pm 3\sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + (\pm 3\sqrt{3}) = \pm 3\sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} = \pm 3\sqrt{3} - (\pm 3\sqrt{3}) = 0$$

$$\text{Hence } x^3 + \frac{1}{x^3} = 0.$$

Q. 15. If $\frac{a}{b} = \frac{b}{c}$, prove that

$$(a + b + c)(a - b + c) = (a^2 + b^2 + c^2).$$

Sol. Let $\frac{a}{b} = \frac{b}{c} = k$, then

$$b = ck \text{ and } a = bk = ck \times k = ck^2$$

$$\text{L.H.S.} = (a + b + c)(a - b + c)$$

$$= (ck^2 + ck + c)(ck^2 - ck + c)$$

$$= c(k^2 + k + 1)c(k^2 - k + 1)$$

$$= c^2(k^2 + k + 1)(k^2 - k + 1)$$

$$= c^2(k^4 + k^2 + 1)$$

$$\text{R.H.S.} = a^2 + b^2 + c^2$$

$$= (ck^2)^2 + (ck)^2 + c^2$$

$$= c^2 k^4 + c^2 k^2 + c^2$$

$$= c^2(k^4 + k^2 + 1)$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

Second Method

$$\therefore \frac{a}{b} = \frac{b}{c}, \quad \therefore b^2 = ac$$

$$\begin{aligned} \text{Now L.H.S.} &= (a + b + c)(a - b + c) \\ &= [(a + c) + b][(a + c) - (b)] \\ &= (a + c)^2 - b^2 \\ &= (a + c)^2 - ac \quad (\because b^2 = ac) \\ &= a^2 + c^2 + 2ac - ac = a^2 + c^2 + ac \\ &= a^2 + c^2 + b^2 = a^2 + b^2 + c^2 = \text{R.H.S.} \end{aligned}$$

EXERCISE 4 (C)

Write down each of the following products :

Q. 1. $(x + 6)(x + 2)$

Sol. $(x + 6)(x + 2) = x^2 + (6 + 2)x + 6 \times 2$
 $= x^2 + 8x + 12$ **Ans.**

Q. 2. $(x + 8)(x - 3)$

Sol. $(x + 8)(x - 3) = x^2 + (8 - 3)x + 8 \times (-3)$
 $= x^2 + 5x - 24$ **Ans.**

Q. 3. $(x - 5)(x - 7)$

Sol. $(x - 5)(x - 7)$
 $= x^2 + (-5 - 7)x + (-5)(-7)$
 $= x^2 - 12x + 35$ **Ans.**

Q. 4. $(2 - x)(4 - x)$

Sol. $\{-(x - 2)\} \times \{-(x - 4)\}$
 $= (x - 2)(x - 4)$
 $= x^2 + (-2 - 4)x + (-2)(-4)$
 $= x^2 - 6x + 8 \Rightarrow 8 - 6x + x^2$ **Ans.**

Q. 5. $(y - 7)(y + 4)$

Sol. $(y - 7)(y + 4)$
 $= y^2 + (-7 + 4)y + (-7)(4)$
 $= y^2 - 3y - 28$ **Ans.**

Q. 6. $(ab + 3)(ab - 2)$

Sol. $(ab + 3)(ab - 2)$
 $= (ab)^2 + (3 - 2)ab + 3 \times (-2)$
 $= a^2 b^2 + ab - 6$ **Ans.**

Q. 7. $(5 - xy)(2 + xy)$

Sol. $(5 - xy)(2 + xy)$
 $= -(xy - 5)(xy + 2)$
 $= -\{(xy)^2 + (-5 + 2)xy + (-5) \times 2\}$

$$\begin{aligned} &= -[x^2 y^2 - 3xy - 10] \\ &= 10 + 3xy - x^2 y^2 \text{ **Ans.**} \end{aligned}$$

Q. 8. $(x^2 + 1)(x^2 + 2)$

Sol. Let $x^2 = y$, then
 $(y + 1)(y + 2) = y^2 + (1 + 2)y + 1 \times 2$
 $= y^2 + 3y + 2$
 $= (x^2)^2 + 3x^2 + 2$
 $= x^4 + 3x^2 + 2$ **Ans.**

Q. 9. $(3 - x^2)(5 + x^2)$

Sol. $(3 - x^2)(5 + x^2) = -(x^2 - 3)(x^2 + 5)$
 $= -\{(x^2)^2 + (-3 + 5)x^2 + (-3) \times 5\}$
 $= -\{x^4 + 2x^2 - 15\}$
 $= -x^4 - 2x^2 + 15$
 $= 15 - 2x^2 - x^4$ **Ans.**

Q. 10. $(6 - x)(x + 5)$

Sol. $(6 - x)(x + 5) = -(x - 6)(x + 5)$
 $= -\{x^2 + (-6 + 5)x + (-6) \times 5\}$
 $= -\{x^2 - x - 30\} = -x^2 + x + 30$
 $= 30 + x - x^2 = x - x^2 + 30$ **Ans.**

Q. 11. $(2x + 3)(3x + 5)$

Sol. $(2x + 3)(3x + 5)$
 $= 2x(3x + 5) + 3(3x + 5)$
 $= 6x^2 + 10x + 9x + 15$
 $= 6x^2 + 19x + 15$ **Ans.**

Q. 12. $(7m - 2)(4m + 3)$

Sol. $(7m - 2)(4m + 3)$
 $= 7m(4m + 3) - 2(4m + 3)$
 $= 28m^2 + 21m - 8m - 6$
 $= 28m^2 + 13m - 6$ **Ans.**

Q. 13. $(2y - 3)(3y - 5)$

Sol. $(2y - 3)(3y - 5)$
 $= 2y(3y - 5) - 3(3y - 5)$
 $= 6y^2 - 10y - 9y + 15$
 $= 6y^2 - 19y + 15$ **Ans.**

Q. 14. $(3a^2 - b^2)(2a^2 + 5b^2)$

Sol. $(3a^2 - b^2)(2a^2 + 5b^2)$
 $= 3a^2(2a^2 + 5b^2) - b^2(2a^2 + 5b^2)$
 $= 6a^4 + 15a^2 b^2 - 2a^2 b^2 - 5b^4$
 $= 6a^4 + 13a^2 b^2 - 5b^4$ **Ans.**