UNIT 1 – PURE ARITHMETIC

1

Rational and Irrational Numbers

EXERCISE 1(A)

Q. 1. Without actual division find which of the following fractions are terminating decimal :

(*i*) $\frac{9}{25}$ (*ii*) $\frac{7}{12}$

(*iii*) $\frac{13}{16}$ (*iv*) $\frac{25}{128}$

(*v*) $\frac{9}{50}$ (*vi*) $\frac{121}{125}$

(*vii*) $\frac{19}{55}$ (*viii*) $\frac{37}{78}$

(*ix*) $\frac{23}{80}$ (*x*) $\frac{19}{30}$

Sol. $\because$ A rational number is terminating if its denominator has prime factors only 2 or 5 or both

(*i*) In $\frac{9}{25}$, the prime factors of 25 are 5, 5

$\therefore$ It is terminating decimal

(*ii*) In $\frac{7}{12}$, the prime factors of 12 are 2, 2, 3

$\therefore$ It is not terminating decimal.

(*iii*) In $\frac{13}{16}$, the prime factors of 16 are 2, 2, 2, 2

$\therefore$ It is terminating decimal.

(*iv*) In $\frac{25}{128}$, the prime factors of 128 are 2, 2, 2, 2, 2, 2, 2.

$\therefore$ It is terminating decimal.

(*v*) In $\frac{9}{50}$, the prime factors of 50 are 2, 5, 5.

$\therefore$ It is terminating decimal.

(*vi*) In $\frac{121}{125}$, the prime factors of 125 are 5, 5, 5

$\therefore$ It is terminating decimal.

(*vii*) $\frac{19}{55}$, the prime factors of 55 are 5, 11

$\therefore$ It is not terminating decimal.

(*viii*) In $\frac{37}{78}$, the prime factors of 78 are 2, 3, 13

$\therefore$ It is not terminating decimal.

(*ix*) In $\frac{23}{80}$, the prime factors of 80 are 2, 2, 2, 2, 5

$\therefore$ It is terminating decimal

(*x*) In $\frac{19}{30}$, the prime factors of 30 are 2, 3, 5

$\therefore$ It is not terminating decimal. **Ans.**

Q. 2. Convert each of the following decimals into a vulgar fraction in its lowest terms :

(*i*) 0·65 (*ii*) 1·08

(*iii*) 0·075 (*iv*) 2·016

(*v*) 1·732

Sol. (*i*) $0{\cdot}65 = \frac{65}{100} = \frac{65 \div 5}{100 \div 5} = \frac{13}{20}$

(*ii*) $1{\cdot}08 = \dfrac{108}{100} = \dfrac{108 \div 4}{100 \div 4} = \dfrac{27}{25}$

(*iii*) $0{\cdot}075 = \dfrac{75}{1000} = \dfrac{75 \div 25}{1000 \div 25} = \dfrac{3}{40}$

(*iv*) $2{\cdot}016 = \dfrac{2016}{1000} = \dfrac{2016 \div 8}{1000 \div 8} = \dfrac{252}{125}$

(*v*) $1{\cdot}732 = \dfrac{1732}{1000} = \dfrac{1732 \div 4}{1000 \div 4} = \dfrac{433}{250}$

Ans.

Q. 3. Convert each of the following into a decimal :

(*i*) $\dfrac{1}{8}$ (*ii*) $\dfrac{3}{32}$

(*iii*) $\dfrac{44}{9}$ (*iv*) $\dfrac{11}{24}$

(*v*) $\dfrac{12}{13}$ (*vi*) $\dfrac{27}{44}$

(*vii*) $2\dfrac{5}{12}$ (*viii*) $1\dfrac{31}{55}$

Sol. (*i*) $\dfrac{1}{8} = 0{\cdot}125$

0·125
8) 1·000
8
20
16
40
40
×

(*ii*) $\dfrac{3}{32} = 0{\cdot}09375$

0·09375
32) 3·00000
288
120
96
240
224
160
160
×

(*iii*) $\dfrac{44}{9} = 4{\cdot}888....$

$= 4{\cdot}\overline{8}$

4·888....
9) 44·000
36
80
72
80
72
80
72
8

(*iv*) $\dfrac{11}{24} = 0{\cdot}45833....$

$= 0{\cdot}458\overline{3}$

0·45833...
24) 11·00000
96
140
120
200
192
80
72
80
72
8

(*v*) $\dfrac{12}{13} = 0{\cdot}923076923076......$

$= 0{\cdot}\overline{923076}$

$$
\begin{array}{r}
0{\cdot}923076923076\ldots \\
13\,\overline{)\,12{\cdot}000000000000} \\
117 \\ \hline
30 \\
26 \\ \hline
40 \\
39 \\ \hline
100 \\
91 \\ \hline
90 \\
78 \\ \hline
120 \\
117 \\ \hline
30 \\
26 \\ \hline
40 \\
39 \\ \hline
100 \\
91 \\ \hline
90 \\
78 \\ \hline
12
\end{array}
$$

(*vi*) $\frac{27}{44} = 0{\cdot}61363636\ldots$

$= 0{\cdot}61\overline{36}$

$$
\begin{array}{r}
0{\cdot}61363636\ldots \\
44\,\overline{)\,27{\cdot}00000000} \\
264 \\ \hline
60 \\
44 \\ \hline
160 \\
132 \\ \hline
280 \\
264 \\ \hline
160 \\
132 \\ \hline
280 \\
264 \\ \hline
160 \\
132 \\ \hline
280 \\
264 \\ \hline
16
\end{array}
$$

(*vii*) $2\frac{5}{12} = \frac{29}{12} = 2{\cdot}41666\ldots$

$= 2{\cdot}41\overline{6}$

$$
\begin{array}{r}
2{\cdot}41666 \\
12\,\overline{)\,29{\cdot}00000} \\
24 \\ \hline
50 \\
48 \\ \hline
20 \\
12 \\ \hline
80 \\
72 \\ \hline
80 \\
72 \\ \hline
80 \\
72 \\ \hline
8
\end{array}
$$

(*viii*) $1\frac{31}{55} = \frac{86}{55} = 1{\cdot}5636363\ldots$

$= 1{\cdot}56\overline{3}$ or $= 1{\cdot}5\overline{63}$

$$
\begin{array}{r}
1{\cdot}5636363 \\
55\,\overline{)\,86{\cdot}0000000} \\
55 \\ \hline
310 \\
275 \\ \hline
350 \\
330 \\ \hline
200 \\
165 \\ \hline
350 \\
330 \\ \hline
200 \\
165 \\ \hline
350 \\
330 \\ \hline
200 \\
165 \\ \hline
35
\end{array}
$$

Q. 4. Express $\frac{15}{56}$ as a decimal correct to four decimal places.

Sol. $\frac{15}{56} = 0{\cdot}26785$

$= 0{\cdot}2679$

$$\begin{array}{r|l} & 0{\cdot}26785 \\ 56 & 15{\cdot}00000 \\ & 112 \\ \hline & 380 \\ & 336 \\ \hline & 440 \\ & 392 \\ \hline & 480 \\ & 448 \\ \hline & 320 \\ & 280 \\ \hline & 40 \end{array}$$

Q. 5. Express $\frac{13}{34}$ as a decimal correct to three decimal places.

Sol. $\frac{13}{34} = 0{\cdot}3823$

$= 0{\cdot}382$

$$\begin{array}{r|l} & 0{\cdot}3823 \\ 34 & 13{\cdot}0000 \\ & 102 \\ \hline & 280 \\ & 272 \\ \hline & 80 \\ & 68 \\ \hline & 120 \\ & 102 \\ \hline & 18 \end{array}$$

Q. 6. Express each of the following as a vulgar fractions :

(*i*) $0{\cdot}\overline{6}$ (*ii*) $0{\cdot}\overline{43}$

(*iii*) $1{\cdot}\overline{3}$ (*iv*) $0{\cdot}1\overline{2}$

(*v*) $0{\cdot}1\overline{36}$ (*vi*) $0{\cdot}5\overline{7}$

Sol. (*i*) Let $x = 0{\cdot}\overline{6} = 0{\cdot}6666$ ...(*i*)

then $10x = 6{\cdot}6666$ ...(*ii*)

Subtracting (*i*) from (*ii*)

$$9x = 6 \Rightarrow x = \frac{6}{9} = \frac{2}{3}$$

$\therefore$ Required fraction $= \frac{2}{3}$

(*ii*) Let $x = 0{\cdot}\overline{43} = 0{\cdot}43434343$ ...(*i*)

then $100x = 43{\cdot}43434343$ ...(*ii*)

Subtracting (*i*) from (*ii*)

$$99x = 43 \Rightarrow x = \frac{43}{99}$$

$\therefore$ Required fraction $= \frac{43}{99}$

(*iii*) Let $x = 1{\cdot}\overline{3} = 1{\cdot}3333$ ...(*i*)

then $10x = 13{\cdot}3333$ ...(*ii*)

Subtracting (*i*) from (*ii*)

$$9x = 12 \Rightarrow x = \frac{12}{9} = \frac{4}{3}$$

$\therefore$ Required fraction $= \frac{4}{3}$

(*iv*) Let $x = 0{\cdot}1\overline{2} = 0{\cdot}12222...$ ...(*i*)

then $10x = 1{\cdot}\overline{2} = 1{\cdot}2222$ ...(*ii*)

and $100x = 12{\cdot}2222$ ...(*iii*)

Subtracting (*ii*) from (*iii*)

$$90x = 11 \Rightarrow x = \frac{11}{90}$$

$\therefore$ Required fraction $= \frac{11}{90}$

(*v*) Let $x = 0{\cdot}1\overline{36} = 0{\cdot}136363636$ ...(*i*)

then $10x = 1{\cdot}36363636$ ...(*ii*)

and $1000x = 136{\cdot}36363636$ ...(*iii*)

Subtracting (*ii*) from (*iii*)

$$990x = 135 \Rightarrow x = \frac{135}{990} = \frac{3}{22}$$

$\therefore$ Required fraction $= \frac{3}{22}$

(*vi*) Let $x = 0{\cdot}5\overline{7} = 0{\cdot}57777$ ...(*i*)

then $10x = 5{\cdot}7777$ ...(*ii*)

and $100x = 57{\cdot}7777$ ...(*iii*)

Subtracting (*ii*) from (*iii*)

$$90x = 52 \Rightarrow x = \frac{52}{90} = \frac{26}{45}$$

$\therefore$ Required fraction $= \frac{26}{45}$ **Ans.**

Q. 7. Express $3\cdot\overline{146}$ as a vulgar fraction.

Sol. Let $x = 3\cdot\overline{146} = 3\cdot146146146146...$ ...(*i*)

then $1000x = 3146\cdot146146146146$ ...(*ii*)

Subtracting (*i*) from (*ii*)

$$999x = 3143 \Rightarrow x = \frac{3143}{999}$$ **Ans.**

Q. 8. Express $4\cdot3\overline{24}$ as a vulgar fraction.

Sol. Let $x = 4\cdot3\overline{24} = 4\cdot324242424$ ...(*i*)

then $10x = 43\cdot24242424$ ...(*ii*)

and $1000x = 4324\cdot24242424$ ...(*iii*)

Subtracting (*ii*) from (*iii*)

$$990x = 4324 - 43 = 4281$$

$$x = \frac{4281}{990} = \frac{1427}{330}$$ **Ans.**

EXERCISE 1(B)

Q. 1. Write the additive inverse of

(*i*) 5 (*ii*) – 7

(*iii*) $\frac{5}{9}$ (*iv*) $\frac{-3}{17}$

(*v*) 0

Sol. Additive inverse

(*i*) of 5 is – 5, (*ii*) of – 7 is 7

(*iii*) of $\frac{5}{9}$ is $\frac{-5}{9}$ (*iv*) of $\frac{-3}{17}$ is $\frac{3}{17}$

(*v*) of 0 is 0.

Q. 2. Write the multiplicative inverse of :

(*i*) 9 (*ii*) – 1

(*iii*) $\frac{11}{16}$ (*iv*) $5\frac{1}{4}$

(*v*) $\frac{-2}{3}$

Sol. Multiplicative inverse

(*i*) of 9 is $\frac{1}{9}$

(*ii*) of – 1 is – 1

(*iii*) of $\frac{11}{16}$ is $\frac{16}{11}$

(*iv*) of $5\frac{1}{4}$ or $\frac{21}{4}$ is $\frac{4}{21}$ and

(*v*) of $\frac{-2}{3}$ is $\frac{-3}{2}$.

Q. 3. Insert a rational number between $\frac{3}{5}$ and $\frac{7}{9}$.

Sol. We know that if a and b are two rational numbers, then between these two numbers one rational number will be $\frac{(a+b)}{2}$.

$\therefore$ Rational number between $\frac{3}{5}$ and $\frac{7}{9}$

will be $= \frac{1}{2}\left(\frac{3}{5}+\frac{7}{9}\right)$

$$= \frac{1}{2}\left(\frac{27+35}{45}\right) = \frac{1}{2} \times \frac{62}{45} = \frac{31}{45}$$

$\therefore \frac{3}{5} < \frac{31}{45} < \frac{7}{9}$ **Ans.**

Q. 4. Insert two rational number between

(*i*) 2 and 3 (*ii*) $\frac{1}{3}$ and $\frac{2}{5}$ (*iii*) $\frac{3}{4}$ and $1\frac{1}{5}$

Sol. (*i*) 2 and 3

$$2 < 3 \Rightarrow 2 < \frac{1}{2}(2+3) < 3$$

$$\Rightarrow 2 < \frac{5}{2} < 3$$

$$\Rightarrow 2 < \frac{5}{2} < \frac{1}{2}\left(\frac{5}{2}+3\right) < 3$$

$$\Rightarrow 2 < \frac{5}{2} < \frac{1}{2}\left[\frac{11}{2}\right] < 3$$

$$\Rightarrow 2 < \frac{5}{2} < \frac{11}{4} < 3$$

$\therefore$ Required rational numbers are $\frac{5}{2}$ and $\frac{11}{4}$

(*ii*) $\frac{1}{3}$ and $\frac{2}{5}$

$$\frac{1}{3}<\frac{1}{2}\left[\frac{1}{3}+\frac{2}{5}\right]<\frac{2}{5}$$

$$\Rightarrow \frac{1}{3}<\frac{1}{2}\left[\frac{5+6}{15}\right]<\frac{2}{5}$$

$$\Rightarrow \frac{1}{3}<\frac{1}{2}\left[\frac{11}{15}\right]<\frac{2}{5} \Rightarrow \frac{1}{3}<\frac{11}{30}<\frac{2}{5}$$

$$\Rightarrow \frac{1}{3}<\frac{11}{30}<\frac{1}{2}\left[\frac{11}{30}+\frac{2}{5}\right]<\frac{2}{5}$$

$$\Rightarrow \frac{1}{3}<\frac{11}{30}<\frac{1}{2}\left[\frac{11+12}{30}\right]<\frac{2}{5}$$

$$\Rightarrow \frac{1}{3}<\frac{11}{30}<\frac{23}{60}<\frac{2}{5}$$

$\therefore$ $\frac{11}{30}$ and $\frac{23}{60}$ are two rational numbers.

(*iii*) $\frac{3}{4}$ and $1\frac{1}{5} \Rightarrow \frac{3}{4}$ and $\frac{6}{5}$

$$\frac{3}{4}<\frac{1}{2}\left[\frac{3}{4}+\frac{6}{5}\right]<\frac{6}{5}$$

$$\Rightarrow \frac{3}{4}<\frac{1}{2}\left[\frac{15+24}{20}\right]<\frac{6}{5}$$

$$\Rightarrow \frac{3}{4}<\frac{1}{2}\left[\frac{39}{20}\right]<\frac{6}{5} \Rightarrow \frac{3}{4}<\frac{39}{40}<\frac{6}{5}$$

$$\Rightarrow \frac{3}{4}<\frac{39}{40}<\frac{1}{2}\left[\frac{39}{40}+\frac{6}{5}\right]<\frac{6}{5}$$

$$\Rightarrow \frac{3}{4}<\frac{39}{40}<\frac{1}{2}\left[\frac{39+48}{40}\right]<\frac{6}{5}$$

$$\Rightarrow \frac{3}{4}<\frac{39}{40}<\frac{1}{2}\left[\frac{87}{40}\right]<\frac{6}{5}$$

$$\Rightarrow \frac{3}{4}<\frac{39}{40}<\frac{87}{80}<\frac{6}{5}$$

$\therefore$ $\frac{39}{40}$ and $\frac{87}{80}$ are two rational numbers.

Q. 5. Insert three rational numbers between :

(*i*) 4 and 5 (*ii*) $\frac{1}{2}$ and $\frac{3}{5}$ (*iii*) -1 and 1 (*iv*) $2\frac{1}{3}$ and $3\frac{2}{3}$

(*v*) $-\frac{1}{2}$ and $\frac{1}{3}$ (*vi*) $-\frac{1}{3}$ and $\frac{1}{4}$

Sol. (*i*) Between 4 and 5, three rational numbers,

$$4<\frac{1}{2}\left(\frac{4+5}{1}\right)<5 \Rightarrow 4<\frac{9}{2}<5$$

Again $4<\frac{1}{2}\left(4+\frac{9}{2}\right)<\frac{9}{2}<\frac{1}{2}\left(\frac{9}{2}+5\right)<5$

$$=4<\frac{17}{4}<\frac{9}{2}<\frac{19}{4}<5$$

$\therefore$ Three rational numbers are $\frac{17}{4}, \frac{9}{2}, \frac{19}{4}$

(*ii*) Between $\frac{1}{2}$ and $\frac{3}{5}$, three rational numbers

$$\frac{1}{2}<\frac{1}{2}\left(\frac{1}{2}+\frac{3}{5}\right)<\frac{3}{5}$$

$$\Rightarrow \frac{1}{2}<\frac{11}{20}<\frac{3}{5}$$

$$\Rightarrow \frac{1}{2}<\frac{1}{2}\left(\frac{1}{2}+\frac{11}{20}\right)<\frac{11}{20}<\frac{1}{2}\left(\frac{11}{20}+\frac{3}{5}\right)<\frac{3}{5}$$

$$\Rightarrow \frac{1}{2}<\frac{21}{40}<\frac{11}{20}<\frac{23}{40}<\frac{3}{5}$$

$\therefore$ Three rational numbers are $\frac{21}{40}, \frac{11}{20}, \frac{23}{40}$

(*iii*) Between -1 and 1, three rational numbers are

$$-1<\frac{1}{2}(-1+1)<1 \Rightarrow -1<0<1$$

$$\Rightarrow -1<\frac{1}{2}(-1+0)<0<\frac{1}{2}(0+1)<1$$

$$\Rightarrow -1<-\frac{1}{2}<0<\frac{1}{2}<1$$

$\therefore$ Three rational numbers are $-\frac{1}{2}, 0, \frac{1}{2}$

(*iv*) Between $2\frac{1}{3}$ and $3\frac{2}{3}$, three rational numbers are between $\frac{7}{3}$ and $\frac{11}{3}$

$$\frac{7}{3} < \frac{1}{2}\left(\frac{7}{3}+\frac{11}{3}\right) < \frac{11}{3} \Rightarrow \frac{7}{3} < 3 < \frac{11}{3}$$

$$\Rightarrow \frac{7}{3} < \frac{1}{2}\left(\frac{7}{3}+\frac{3}{1}\right) < 3 < \frac{1}{2}\left(3+\frac{11}{3}\right) < \frac{11}{3}$$

$$\Rightarrow \frac{7}{3} < \frac{8}{3} < 3 < \frac{10}{3} < \frac{11}{3}$$

$\therefore$ Three rational numbers are

$\frac{8}{3}, 3, \frac{10}{3}$.

(*v*) Between $\frac{-1}{2}$ and $\frac{1}{3}$, three rational numbers are, $\frac{-1}{2} < \frac{1}{2}\left(\frac{-1}{2}+\frac{1}{3}\right) < \frac{1}{3}$

$$\Rightarrow \frac{-1}{2} < \frac{-1}{12} < \frac{1}{3}$$

$$\Rightarrow -\frac{1}{2} < \frac{1}{2}\left(-\frac{1}{2}-\frac{1}{12}\right)$$

$$< -\frac{1}{12} < \frac{1}{2}\left(-\frac{1}{12}+\frac{1}{3}\right) < \frac{1}{3}$$

$$\Rightarrow \frac{-1}{2} < \frac{-7}{24} < \frac{-1}{12} < \frac{1}{8} < \frac{1}{3}$$

$\therefore$ Three rational numbers are

$\frac{-7}{24}, \frac{-1}{12}, \frac{1}{8}$

(*vi*) Between $-\frac{1}{3}$ and $\frac{1}{4}$, three rational numbers are

$$-\frac{1}{3} < \frac{1}{2}\left(-\frac{1}{3}+\frac{1}{4}\right) < \frac{1}{4}$$

$$\Rightarrow -\frac{1}{3} < -\frac{1}{24} < \frac{1}{4}$$

$$\Rightarrow -\frac{1}{3} < \frac{1}{2}\left(-\frac{1}{3}-\frac{1}{24}\right)$$

$$< -\frac{1}{24} < \frac{1}{2}\left(-\frac{1}{24}+\frac{1}{4}\right) < \frac{1}{4}$$

$$\Rightarrow -\frac{1}{3} < -\frac{3}{16} < -\frac{1}{24} < \frac{5}{48} < \frac{1}{4}$$

$\therefore$ Three rational numbers are

$-\frac{3}{16}, -\frac{1}{24}, \frac{5}{48}$

Q. 6. Insert four rational numbers between 4 and 4·5.

Sol. We know that if a and b are two rational numbers, then a rational number between a and b will be $\frac{a+b}{2}$

$\therefore$ One rational number between 4 and 4·5

$$= \frac{4+4\cdot5}{2} = \frac{8\cdot5}{2} = 4\cdot25$$

Second number between 4 and 4·25

$$= \frac{4+4\cdot25}{2} = \frac{8\cdot25}{2} = 4\cdot125$$

Third number between 4 and 4·125

$$= \frac{4+4\cdot125}{2} = \frac{8\cdot125}{2} = 4\cdot0625$$

Fourth number between 4·25 and 4·5

$$= \frac{4\cdot25+4\cdot50}{2} = \frac{8\cdot75}{2} = 4\cdot375$$

Hence four rational numbers between 4 and 4·5 are 4·0625, 4·125, 4·25 and 4·375 **Ans.**

Q. 7. Insert six rational numbers between 3 and 4.

Sol. Six rational numbers between 3 and 4 can be

3·1, 3·2, 3·3, 3·4, 3·5, 3·6 **Ans.**

Q. 8. Insert ten rational numbers between $\frac{1}{8}$ and $\frac{2}{3}$.

Sol. Ten rational numbers can be between $\frac{1}{8}$ and $\frac{2}{3}$

(LCM of 8 and 3 = 24)

$$\frac{1}{8} = \frac{1\times3}{8\times3} = \frac{3}{24}$$

and $\frac{2}{3} = \frac{2}{3}\times\frac{8}{8} = \frac{16}{24}$

$\therefore$ The numbers will be

$$\frac{4}{24}, \frac{5}{24}, \frac{6}{24}, \frac{7}{24}, \frac{8}{24}, \frac{9}{24}, \frac{10}{24}, \frac{11}{24}, \frac{12}{24}$$

and $\frac{13}{24}$ **Ans.**

Q. 9. Show that the negative of a rational number is rational.

Sol. Let $x = \frac{p}{q}$ where $\frac{p}{q}$ is a rational number, p and q are integers and $q \neq 0$

$\therefore -x = \frac{-p}{q}$.

But $\frac{-p}{q}$ is the additive inverse of $\frac{p}{q}$ which is a rational number.

$\therefore -x$ is a rational.

Hence ngative of a rational number is also a rational.

EXERCISE 1(C)

Q. 1. Classify the rational and irrational numbers from the following :

(i) 5 (ii) $\frac{9}{14}$

(iii) $\sqrt{3}$ (iv) π

(v) $3 \cdot \dot{1}416$ (vi) $\sqrt{4}$

(vii) $-\sqrt{5}$ (viii) $\sqrt[3]{8}$

(ix) $\sqrt[3]{3}$ (x) $2\sqrt{6}$

(xi) $0 \cdot \overline{36}$ (xii) $0 \cdot 202202220...$

(xiii) $\frac{2}{\sqrt{3}}$ (xiv) $\frac{22}{7}$

Sol. (i) 5 (ii) $\frac{9}{14}$

(v) $3 \cdot 1416$ (vi) $\sqrt{4}$

(viii) $\sqrt[3]{8}$ (xi) $0 \cdot \overline{36}$

and (xiv) $\frac{22}{7}$ are rational numbers

(iii) $\sqrt{3}$, (iv) π, (vii) $-\sqrt{5}$,

(ix) $\sqrt[3]{3}$, (x) $2\sqrt{6}$,

(xii) $0 \cdot 202202220...$

(xiii) $\frac{2}{\sqrt{3}}$ are irrational numbers.

Q. 2. If U

$$= \left\{-8, \sqrt{25}, \frac{-3}{5}, \sqrt{8}, 0, \pi, \sqrt[3]{5}, 2 \cdot \overline{4}, -\sqrt{3}\right\}$$

then describe the set of (i) rationals in U (ii) irrationals in U.

Sol. Set of rationals Q

$$= \left\{-8, \sqrt{25}, \frac{-3}{5}, 0, \pi, 2 \cdot \overline{4}\right\}$$

and set of irrational

$$= \{\sqrt{8}, \pi, \sqrt[3]{5}, -\sqrt{3}\}$$ **Ans.**

Q. 3. State, giving reason, whether the given number is rational or irrational :

(i) $(3+\sqrt{5})$ (ii) $(-1+\sqrt{3})$

(iii) $5\sqrt{6}$ (iv) $-\sqrt{7}$

(v) $\frac{\sqrt{6}}{4}$ (vi) $\frac{3}{\sqrt{2}}$

(vii) $(3+\sqrt{3})(3-\sqrt{3})$

Sol. (i) $(3+\sqrt{5})$ is irrational because 3 is rational and $\sqrt{5}$ is irrational.

$\because$ Sum of rational and irrational is irrational.

(ii) $(-1+\sqrt{3})$ is irrational

$\because -1$ is rational and $\sqrt{3}$ is irrational and sum of rational and irrational is also irrational.

(iii) $5\sqrt{6}$ is irrational because product of rational and irrational is irrational. Here 5 is rational and $\sqrt{6}$ is irrational.

(iv) $-\sqrt{7}$ is irrational.

(*v*) $\dfrac{\sqrt{6}}{4}$ is irrational

Because $\sqrt{6}$ is irrational and 4 is rational and quotient of irrational and rational is also irrational.

(*vi*) $\dfrac{3}{\sqrt{2}}$ is also irrational

because 3 is rational and $\sqrt{2}$ is irrational and quotient of rational and irrational is also irrational.

(*vii*) $(3+\sqrt{3})(3-\sqrt{3})$

$\because (3+\sqrt{3})$ and $(3-\sqrt{3})$ are irrational factors

and $(3+\sqrt{3})(3-\sqrt{3}) = (3)^2 - (\sqrt{3})^2$

$= 9 - 3 = 6$

$\therefore (3+\sqrt{3})(3-\sqrt{3})$ is rational

Q. 4. Write down the values of :

(*i*) $(2\sqrt{3})^2$ (*ii*) $\left(\dfrac{3}{2}\sqrt{2}\right)^2$

(*iii*) $(5+\sqrt{3})^2$ (*iv*) $(\sqrt{6}-3)^2$

(*v*) $(3+2\sqrt{5})^2$ (*vi*) $(\sqrt{5}+\sqrt{6})^2$

(*vii*) $\left(\dfrac{3}{2\sqrt{2}}\right)^2$

Sol. (*i*) $(2\sqrt{3})^2 = 2\sqrt{3} \times 2\sqrt{3} = 4 \times (\sqrt{3})^2$

$= 4 \times 3 = 12$

(*ii*) $\left(\dfrac{3}{2}\sqrt{2}\right)^2 = \dfrac{3}{2}\sqrt{2} \times \dfrac{3}{2}\sqrt{2}$

$= \dfrac{9}{4}(\sqrt{2})^2 = \dfrac{9}{4} \times 2 = \dfrac{9}{2}$

(*iii*) $(5+\sqrt{3})^2 = (5)^2 + (\sqrt{3})^2 + 2(5)(\sqrt{3})$

$= 5 + 3 + 10\sqrt{3}$

$\{\because (a+b)^2 = a^2 + b^2 + 2ab\}$

$= 8 + 10\sqrt{3}$

(*iv*) $(\sqrt{6}-3)^2$

$= (\sqrt{6})^2 + (3)^2 - 2 \times \sqrt{6} \times 3$

$[\because (a-b)^2 = a^2 + b^2 - 2ab]$

$= 6 + 9 - 6\sqrt{6} = 15 - 6\sqrt{6}$

(*v*) $(3+2\sqrt{5})^2$

$= (3)^2 + (2\sqrt{5})^2 + 2 \times 3 \times 2\sqrt{5}$

$[\because (a+b)^2 = a^2 + b^2 + 2ab]$

$= 9 + 4 \times 5 + 12\sqrt{5}$

$= 9 + 20 + 12\sqrt{5} = 29 + 12\sqrt{5}$

(*vi*) $(\sqrt{5}+\sqrt{6})^2$

$= (\sqrt{5})^2 + (\sqrt{6})^2 + 2 \times \sqrt{5} \times \sqrt{6}$

$= 5 + 6 + 2\sqrt{30}$

$\{\because (a+b)^2 = a^2 + b^2 + 2ab\}$

$= 11 + 2\sqrt{30}$

(*vii*) $\left(\dfrac{3}{2\sqrt{2}}\right)^2 = \dfrac{(3)^2}{(2\sqrt{2})^2} = \dfrac{9}{4 \times 2} = \dfrac{9}{8}$

Ans.

Q. 5. Show that each of the following numbers is irrational :

(*i*) $(2+\sqrt{5})^2$ (*ii*) $(3-\sqrt{3})^2$

(*iii*) $(\sqrt{5}+\sqrt{3})^2$ (*iv*) $\dfrac{6}{\sqrt{3}}$

Sol. (*i*) $(2+\sqrt{5})^2$

$= (2)^2 + (\sqrt{5})^2 + 2 \times 2\sqrt{5}$

$\{\because (a+b)^2 = a^2 + b^2 + 2ab\}$

$= 4 + 5 + 4\sqrt{5} = 9 + 4\sqrt{5}$

which is an irrational number.

(*ii*) $(3-\sqrt{3})^2$

$= (3)^2 + (\sqrt{3})^2 - 2 \times 3 \times \sqrt{3}$

$\{\because (a-b)^2 = a^2 + b^2 - 2ab\}$

$= 9 + 3 - 6\sqrt{3} = 12 - 6\sqrt{3}$

which is an irrational number.

(iii) $(\sqrt{5}+\sqrt{3})^2$

$= (\sqrt{5})^2 + (\sqrt{3})^2 + 2 \times \sqrt{5} \times \sqrt{3}$

$\{\because (a-b)^2 = a^2 + b^2 - 2ab\}$

$= 5 + 3 + 2\sqrt{15}$

$= 8 + 2\sqrt{15}$

which is an irrational number.

(iv) $\dfrac{6}{\sqrt{3}} = \dfrac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \dfrac{6\sqrt{3}}{3} = 2\sqrt{3}$

which is an irrational number.

Q. 6. Show that $\sqrt{5}$ is not a rational number.

Sol. Let $\sqrt{5}$ is a rational number and let $\sqrt{5} = \dfrac{p}{q}$.

where p and q have no common factor and $q \neq 0$.

Squaring both sides,

$$5 = \frac{p^2}{q^2} \Rightarrow p^2 = 5q^2 \quad ...(i)$$

$\because p^2$ is a multiple of 5

$\therefore p$ is also or multiple of 5.

and let $p = 5m$, then

$p^2 = 25m^2 \quad ...(ii)$

From (i) and (ii)

$5q^2 = 25m^2$

$\Rightarrow \quad q^2 = 5m^2$

$\therefore \; q^2$ is multiple of 5

$\Rightarrow q$ is multiple of 5

$\therefore \; p$ and q are both multiple of 5

But it is against hypothesis that p and q have no common factor.

Hence $\sqrt{5}$ is not a rational number.

Q. 7. List three distinct irrational numbers.

Sol. We can write infinite number of irrational numbers, such as

(i) 0·1011001110001111....

(ii) 0·30330333033330....

(iii) 0·200220002220000222....

Q. 8. Show that :

(i) $(\sqrt{3}+\sqrt{7})$ is an irrational number

(ii) $(\sqrt{3}+\sqrt{5})$ is an irrational number.

Sol. (i) Let $(\sqrt{3}+\sqrt{7})$ is a rational number.

On squaring $(\sqrt{3}+\sqrt{7})^2$ is also a rational number.

$\therefore (\sqrt{3}+\sqrt{7})^2$

$= (\sqrt{3})^2 + (\sqrt{7})^2 + 2\sqrt{3} \times \sqrt{7}$

$= 3 + 7 + 2\sqrt{21} = 10 + 2\sqrt{21}$

which is an irrational number

$\therefore$ It is against our supposition

Hence $\sqrt{3}+\sqrt{7}$ is an irrational.

(ii) Let $(\sqrt{3}+\sqrt{5})$ is a rational number.

On squaring, $(\sqrt{3}+\sqrt{5})^2$ is also a rational

$\Rightarrow (\sqrt{3})^2 + (\sqrt{5})^2 + 2\sqrt{3} \times \sqrt{5}$

$= 3 + 5 + 2\sqrt{15} = 8 + 2\sqrt{15}$

It is irrational number.

which is against our supposition

Hence $\sqrt{3}+\sqrt{5}$ is an irrational number.

Q. 9. Insert two irrational number between $\sqrt{14}$ and $\sqrt{19}$.

Sol. Two irrational numbers between $\sqrt{14}$ and $\sqrt{19}$ can be $\sqrt{15}$, $\sqrt{17}$

$\therefore \sqrt{14} < \sqrt{15} < \sqrt{17} < \sqrt{19}$

Q. 10. Insert three irrational numbers between $\sqrt{2}$ and $\sqrt{7}$.

Sol. Three irrational number between $\sqrt{2}$ and $\sqrt{7}$ can be $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$

$\therefore \sqrt{2} < \sqrt{3} < \sqrt{5} < \sqrt{6}$

Q. 11. Insert two irrational numbers between 2 and 3.

Sol. Two irrational numbers between 2 and 3 can be

$2 \cdot 1010010001....,\ 2 \cdot 2202200220002....$

$\therefore 2 < 2 \cdot 1010010001...$

$< 2 \cdot 2202200220002... < 3$

Q. 12. Insert three irrational numbers between 0 and 1.

Sol. Three irrational numbers between 0 and 1 can be

$0 < 0 \cdot 1011001110001111...$

$< 0 \cdot 1010011000111...$

$< 0 \cdot 202002000200002... < 1$

Q. 13. State in each case, whether true or false :

(*i*) The sum of two rationals is a rational.

(*ii*) The sum of two irrationals is an irrational.

(*iii*) The product of two rationals is a rational.

(*iv*) The product of two irrationals is an irrational.

(*v*) The sum of a rational and an irrational is an irrational.

(*vi*) The product of a rational and an irrational is a rational.

Sol. (*i*) True (*ii*) False

(*iii*) True (*iv*) False

(*v*) True (*vi*) False

Q. 14. Rationalise the denominator of :

(*i*) $\frac{1}{(3-\sqrt{5})}$ (*ii*) $\frac{6}{(\sqrt{5}+\sqrt{2})}$

(*iii*) $\frac{1}{(2\sqrt{5}-\sqrt{3})}$ (*iv*) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

Sol. (*i*) $\frac{1}{3-\sqrt{5}}$

In order to rationalise the denominator, multiply by the rationalising factor $3+\sqrt{5}$,

we get, $\frac{1}{3-\sqrt{5}} = \frac{3+\sqrt{5}}{(3-\sqrt{5})(3+\sqrt{5})}$

$= \frac{3+\sqrt{5}}{(3)^2-(\sqrt{5})^2}$

$\{\because (a+b)(a-b) = a^2-b^2\}$

$= \frac{3+\sqrt{5}}{9-5} = \frac{3+\sqrt{5}}{4}$

(*ii*) $\frac{6}{\sqrt{5}+\sqrt{2}}$

In order to rationalise the denominator. Multiply by the rationalising factor $\sqrt{5}-\sqrt{2}$

we get, $\frac{6(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$

$= \frac{6(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2}$

$\{\because (a+b)(a-b) = a^2-b^2\}$

$= \frac{6(\sqrt{5}-\sqrt{2})}{5-2} = \frac{6(\sqrt{5}-\sqrt{2})}{3}$

$= 2(\sqrt{5}-\sqrt{2})$

(*iii*) $\frac{1}{2\sqrt{5}-\sqrt{3}}$

In order to rationalise the denominator, multiply with the rationalising factor $2\sqrt{5}+\sqrt{3}$, we get

$\frac{1}{2\sqrt{5}-\sqrt{3}} = \frac{1(2\sqrt{5}+\sqrt{3})}{(2\sqrt{5}-\sqrt{3})(2\sqrt{5}+\sqrt{3})}$

$= \frac{2\sqrt{5}+\sqrt{3}}{(2\sqrt{5})^2-(\sqrt{3})^2}$

$= \frac{2\sqrt{5}+\sqrt{3}}{20-3} = \frac{2\sqrt{5}+\sqrt{3}}{17}$

(*iv*) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

In order to rationalise the denominator, multiply with the rationalising factor

$(\sqrt{3}+\sqrt{2})$, we get

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$

$$= \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$= \frac{3+2+2\sqrt{3}\times\sqrt{2}}{3-2}$$

$$= \frac{5+2\sqrt{6}}{1} = 5+2\sqrt{6} \text{ **Ans.**}$$

Q. 15. If $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a+b\sqrt{3}$, find the value of a and b.

Sol. $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a+b\sqrt{3}$

$$\Rightarrow \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = a+b\sqrt{3}$$

$$\Rightarrow \frac{3+1-2\sqrt{3}\times 1}{3-1} = a+b\sqrt{3}$$

$$\Rightarrow \frac{4-2\sqrt{3}}{2} = a+b\sqrt{3}$$

$$\Rightarrow 2-\sqrt{3} = a+b\sqrt{3}$$

Comparing both sides,

$a=2$ and $b=-1$ **Ans.**

Q. 16. If $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a+b\sqrt{2}$, find the value of a and b.

Sol. $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a+b\sqrt{2}$

$$\Rightarrow \frac{(3+\sqrt{2})(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = a+b\sqrt{2}$$

$$\Rightarrow \frac{(3+\sqrt{2})^2}{(3)^2-(\sqrt{2})^2} = a+b\sqrt{2}$$

$$\Rightarrow \frac{9+2+2\times3\sqrt{2}}{9-2} = a+b\sqrt{2}$$

$$\Rightarrow \frac{11+6\sqrt{2}}{7} = a+b\sqrt{2}$$

$$\Rightarrow \frac{11}{7}+\frac{6}{7}\sqrt{2} = a+b\sqrt{2}$$

Comparing both sides,

$$a=\frac{11}{7},\ b=\frac{6}{7}$$

Q. 17. Rationalise the denominator of $\frac{10}{(2\sqrt{2}+\sqrt{3})}$.

Sol. $\frac{10}{2\sqrt{2}+\sqrt{3}}$

In order to rationalise the denominator, multiply by the rationalising factor $2\sqrt{2}-\sqrt{3}$, we get

$$\frac{10}{2\sqrt{2}+\sqrt{3}} = \frac{10(2\sqrt{2}-\sqrt{3})}{(2\sqrt{2}+\sqrt{3})(2\sqrt{2}-\sqrt{3})}$$

$$= \frac{10(2\sqrt{2}-\sqrt{3})}{(2\sqrt{2})^2-(\sqrt{3})^2} = \frac{10(2\sqrt{2}-\sqrt{3})}{8-3}$$

$$= \frac{10(2\sqrt{2}-\sqrt{3})}{5} = 2(2\sqrt{2}-\sqrt{3})$$

Q. 18. Simplify :

(*i*) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

(*ii*) $\frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}$

Sol. (*i*) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$= \frac{(\sqrt{5}+\sqrt{3})^2+(\sqrt{5}-\sqrt{3})^2}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$$

$$= \frac{5+3+2\sqrt{5}\times\sqrt{3}+5+3-2\sqrt{5}\times\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$= \frac{8+2\sqrt{15}+8-2\sqrt{15}}{5-3} = \frac{16}{2} = 8 \text{ **Ans.**}$$

(ii) $\dfrac{7+3\sqrt{5}}{3+\sqrt{5}} - \dfrac{7-3\sqrt{5}}{3-\sqrt{5}}$

$$= \frac{(7+3\sqrt{5})(3-\sqrt{5})-(7-3\sqrt{5})(3+\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$$

$$= \frac{21-7\sqrt{5}+9\sqrt{5}-15-21-7\sqrt{5}+9\sqrt{5}+15}{(3)^2-(\sqrt{5})^2}$$

$$= \frac{21+2\sqrt{5}-15-21+2\sqrt{5}+15}{9-5}$$

$$= \frac{4\sqrt{5}}{4} = \sqrt{5} \text{ Ans.}$$

Q. 19. Rationalise the denominator and simplify :

(i) $\dfrac{1}{1+\sqrt{5}+\sqrt{3}}$ (ii) $\dfrac{1}{\sqrt{6}+\sqrt{5}-\sqrt{11}}$.

Sol. (i) $\dfrac{1}{1+\sqrt{5}+\sqrt{3}}$

$$= \frac{1}{1+(\sqrt{5}+\sqrt{3})} \times \frac{1-(\sqrt{5}+\sqrt{3})}{1-(\sqrt{5}+\sqrt{3})}$$

$$= \frac{1-(\sqrt{5}+\sqrt{3})}{(1)^2-(\sqrt{5}+\sqrt{3})^2} = \frac{1-\sqrt{5}-\sqrt{3}}{1-(5+3+2\sqrt{15})}$$

$$= \frac{1-\sqrt{5}-\sqrt{3}}{1-8-2\sqrt{15}} = \frac{1-\sqrt{5}-\sqrt{3}}{-7-2\sqrt{15}}$$

$$= \frac{\sqrt{5}+\sqrt{3}-1}{7+2\sqrt{15}} \quad \text{(Dividing by } -1)$$

$$= \frac{(\sqrt{5}+\sqrt{3}-1)(7-2\sqrt{15})}{(7+2\sqrt{15})(7-2\sqrt{15})}$$

$$= \frac{7\sqrt{5}+7\sqrt{3}-7-2\sqrt{75}-2\sqrt{45}+2\sqrt{15}}{49-4(15)}$$

$$= \frac{7\sqrt{5}+7\sqrt{3}-7-2\sqrt{25\times 3}-2\sqrt{9\times 5}+2\sqrt{15}}{49-60}$$

$$= \frac{7\sqrt{5}+7\sqrt{3}-7-2\times 5\sqrt{3}-2\times 3\sqrt{5}+2\sqrt{15}}{-11}$$

$$= \frac{7\sqrt{5}+7\sqrt{3}-7-10\sqrt{3}-6\sqrt{5}+2\sqrt{15}}{-11}$$

$$= \frac{\sqrt{5}-3\sqrt{3}-7+2\sqrt{15}}{-11}$$

$$= \frac{-(7-\sqrt{5}+3\sqrt{3}-2\sqrt{15})}{-11}$$

$$= \frac{7-\sqrt{5}+3\sqrt{3}-2\sqrt{15}}{11} \text{ Ans.}$$

(ii) $\dfrac{1}{\sqrt{6}+\sqrt{5}-\sqrt{11}}$

$$= \frac{1\,[(\sqrt{6}+\sqrt{5})+\sqrt{11}]}{[(\sqrt{6}+\sqrt{5})-\sqrt{11}]\,[(\sqrt{6}+\sqrt{5})+\sqrt{11}]}$$

$$= \frac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{(\sqrt{6}+\sqrt{5})^2-(\sqrt{11})^2} = \frac{\sqrt{6}+\sqrt{5}-\sqrt{11}}{6+5+2\sqrt{30}-11}$$

$$= \frac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{2\sqrt{30}} = \frac{(\sqrt{6}+\sqrt{5}+\sqrt{11})\sqrt{30}}{2\sqrt{30}\times\sqrt{30}}$$

$$= \frac{\sqrt{180}+\sqrt{150}+\sqrt{330}}{2\times 30}$$

$$= \frac{\sqrt{36\times 5}+\sqrt{25\times 6}+\sqrt{330}}{60}$$

$$= \frac{6\sqrt{5}+5\sqrt{6}+\sqrt{330}}{60} \text{ Ans.}$$

EXERCISE 1(D)

Q. 1. Represent the following rational numbers on a real line :

(i) $\dfrac{2}{3}$ (ii) $-\dfrac{1}{4}$

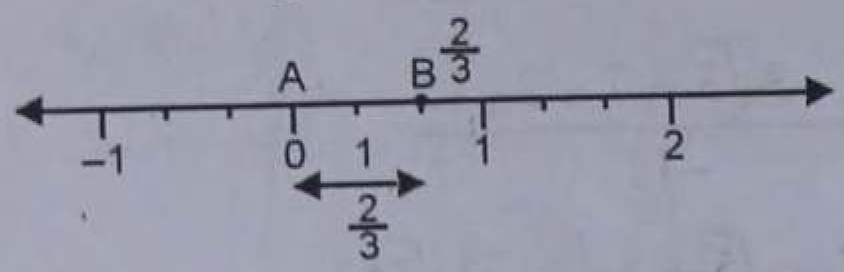
(iii) $1\dfrac{5}{6}$ (iv) $-2\dfrac{4}{5}$

Sol. (i) $\dfrac{2}{3}$

Draw a number line

Divide 0 – 1 with 3 small parts and take 2 small parts

$\therefore$ AB $= \dfrac{2}{3}$

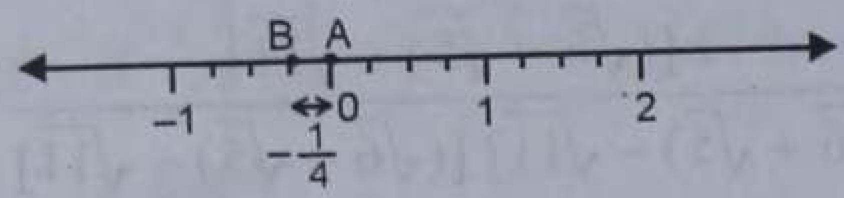


(*ii*) $-\dfrac{1}{4}$

Draw a number line

Divide 0 – 1 into 4 small parts and take one small part

$\therefore$ AB $= -\dfrac{1}{4}$

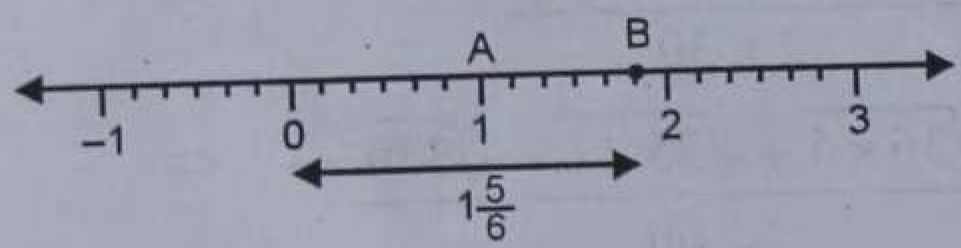


(*iii*) $1\dfrac{5}{6}$

Draw a number line

Divide 1 – 2 into 6 small parts and take 5 small parts

$\therefore$ AB $= 1 + \dfrac{5}{6} = 1\dfrac{5}{6}$

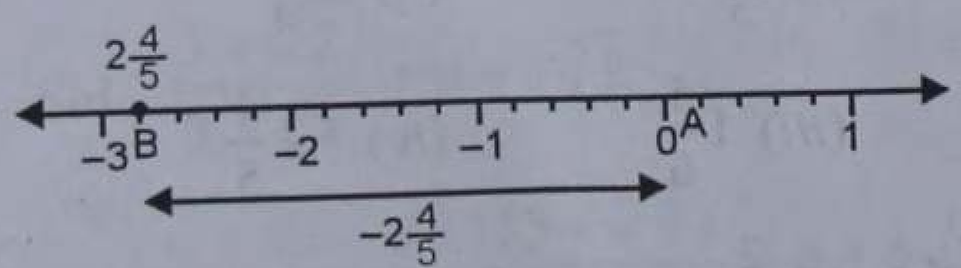


(*iv*) $-2\dfrac{4}{5}$

Draw a number line

Divide – 2 to – 3 into 5 small parts and take 4 small parts

$\therefore$ AB $= -2 + \dfrac{4}{5} = -2\dfrac{4}{5}$



Q. 2. What are rational numbers ? Give five examples of rational numbers.

Sol. The numbers of the form $\dfrac{p}{q}$ where p and q are integers and $q \neq 0$, are called rational numbers.

or $Q = \left\{\dfrac{p}{q},\ p \in z,\ q \in z,\ q \neq 0\right\}$

is a set of all rational numbers.

For example : $\dfrac{1}{2}, \dfrac{1}{3}, \dfrac{2}{3}, \dfrac{4}{5}, \dfrac{3}{5}$ etc. **Ans.**

Q. 3. What are irrational numbers ? Give five examples of irrational numbers.

Sol. A number which is not a rational number, is called an irrational numbers.

These numbers can be expressed as a non-terminating and non repeating decimals.

For examples : $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}$ etc.

or ·0·1001000100001.....,

0·34457323......

Q. 4. State whether true or false :

(*i*) Every real number is rational

(*ii*) Every real number is either rational or irrational

Sol. (*i*) False

(*ii*) True

(as every real number can be either rational or irrational) **Ans.**