UNIT 6 – MENSURATION

20

Perimeter and Area of Plane figures

POINTS TO REMEMBER

1. **Perimeter :** *The perimeter of a plane figure is the length of its boundary, i.e., the sum of its sides. The unit of perimeter is the same as the unit of length.*

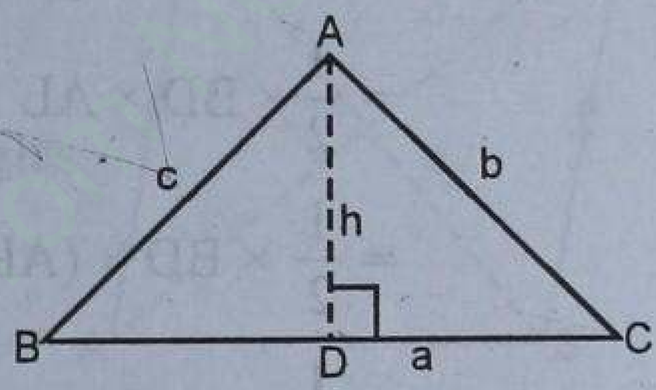
2. **Area :** *The area of a plane figure is the measure of the surface enclosed by its boundary, i.e., the surface enclosed by its sides.*

   It is measured in square units such as **square centimetres** or **square metres, written as cm^2** or **m^2** respectively.

3. **PERIMETER AND AREA OF TRIANGLES**

A. Area of a Triangle = $\frac{1}{2}$ × Base × Corresponding Height.

Any side of the traiangel may be taken as base and the length of perpendicular from the opposite vertex to the base is the corresponding height.



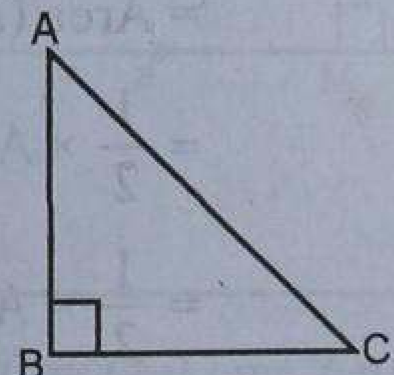
In given figure, Area of Δ ABC = ($\frac{1}{2}$ × BC × AD) sq. units

Perimeter = $(a + b + c)$ units

B Hero's Formula : Let *a, b, c,* be the lengths of the sides of a triangle and let $s = \frac{1}{2}(a + b + c)$, called semi-perimeter of the triangle. Then.

Area of the Triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ sq. units.

C. For a Right-Angled Δ ABC in which $\angle B = 90°$, we have :



(*i*) $AC^2 = AB^2 + BC^2$ (Pythagoras Theorem)

(*ii*) Area of Δ ABC = $\frac{1}{2}$ × (Product of sides containing the right angle)

$$= \left(\frac{1}{2} \times BC \times AB\right) \text{ sq. units}$$

(*iii*) Perimeter = (Sum of three sides) units

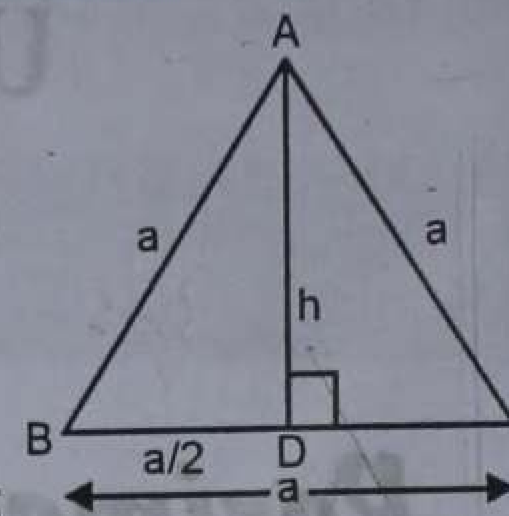
D. For An Equilateral Triangles of Side *a*, *we have :*

(*i*) Height = $\left(\frac{\sqrt{3}}{2}a\right)$ *units.*

$$\left[h = \sqrt{a^2 - \left(\frac{a}{2}\right)^2}\right]$$

(*ii*) **Area** = $\left(\dfrac{\sqrt{3}}{4}a^2\right)$ *sq. units*

$$\left[\because A = \frac{1}{2} \times a \times h\right]$$



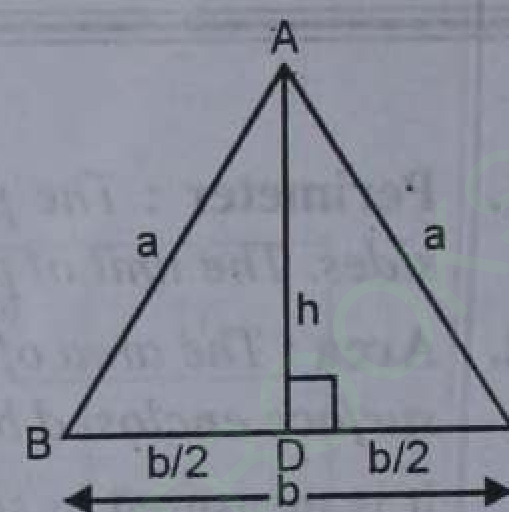
(*iii*) **Perimeter** = **3*a* *units*.**

E. For an Isosceles ΔABC in which AB = AC = *a* and BC = *b*, *we have* ;

(*i*) **Height** = $\dfrac{\sqrt{4a^2 - b^2}}{2}$ **units.** $\left[\because h = \sqrt{a^2 - \left(\dfrac{b}{2}\right)^2}\right]$

(*ii*) **Area** = $\left(\dfrac{1}{4}b\sqrt{4a^2 - b^2}\right)$ *sq. units* $\left[\because A = \dfrac{1}{2} \times b \times h\right]$

(*iii*) **Perimeter = (2*a* + *b*) *units*.**



4. PERIMETER AND AREA OF QUADRILATERALS

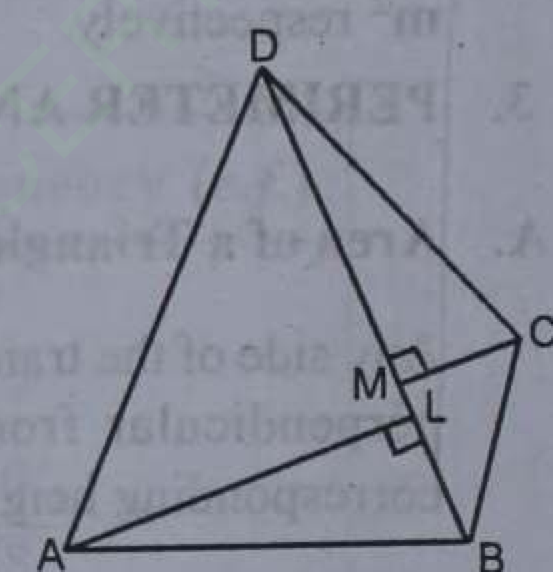
A. Area of Quadrilateral when one diagonal and perpendiculars from remaining vertices to the diagonal are given

(*i*) **Area of quad. ABCD** = area (Δ ABD) + area (Δ BCD)

$$= \frac{1}{2} \times BD \times AL + \frac{1}{2} \times BD \times CM$$

$$= \frac{1}{2} \times BD \times (AL + CM)$$

$$= \frac{1}{2} \times \textit{one diagonal} \times \textit{sum of lengths of perpendiculars on it from remaining vertices.}$$



(*ii*) **Perimeter** = sum of four sides units

B. Area of a Quadrilateral whose Diagonals Intersect At Right Angles

Let the diagonals AC and BD of quad. ABCD intersect at O at right angles. Then,

Area of quad. ABCD.

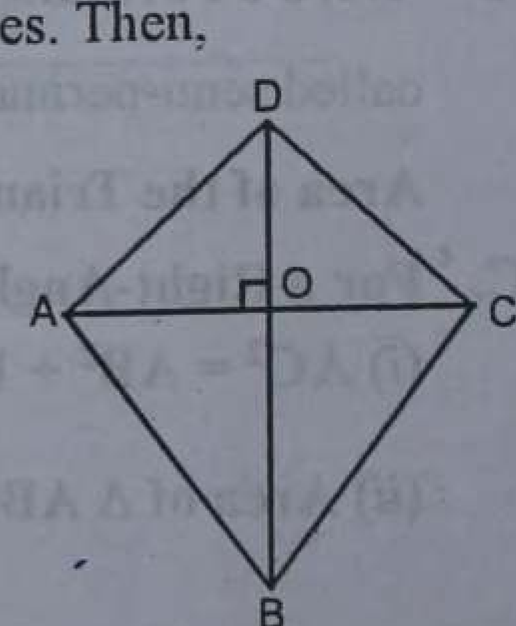
$$= \text{Area } (\Delta \text{ ABC}) + \text{Area } (\Delta \text{ ACD})$$

$$= \frac{1}{2} \times AC \times BO + \frac{1}{2} \times AC \times OD$$

$$= \frac{1}{2} \times AC \times (BO + OD)$$

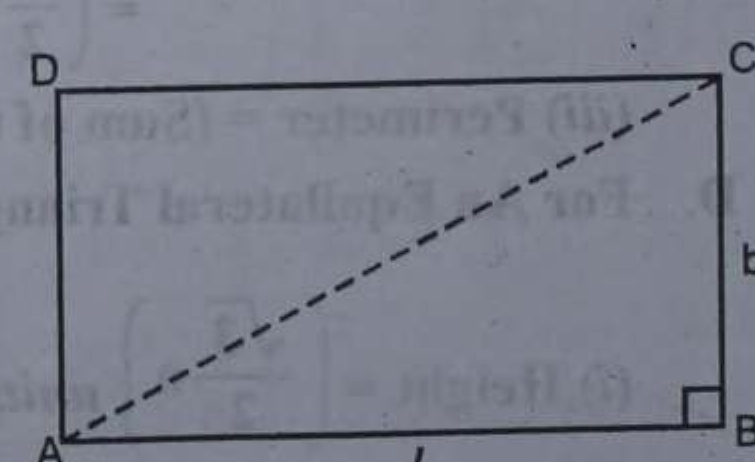
$$= \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times \textbf{(Product of its diagonals) square units.}$$



C. For a Rectangle with length = *l units* and *Breadth* = *b units*, we have

(*i*) **Perimeter** = 2 (Length + Breadth) = **2 (*l* + *b*) units.**



(*ii*) **Area =** (Length × Breadth) = ($l \times b$) ***sq. units.***

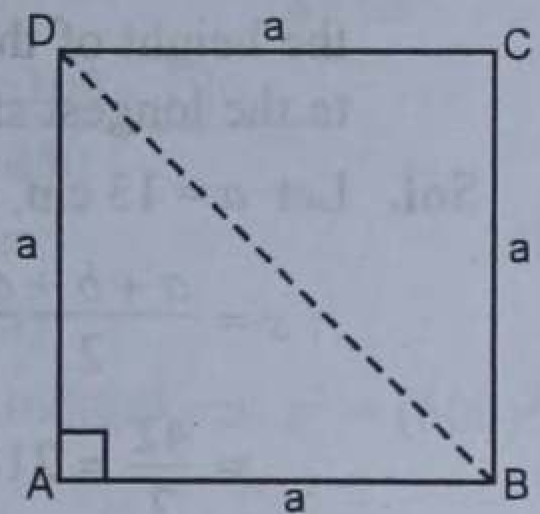
(*iii*) **Diagonal =** $\sqrt{l^2 + b^2}$ ***units.***

D. For a square with side *a* units, we have :

(*i*) **Perimeter = (4 × side) = 4 *a* units.**

(*ii*) **Area =** (side)^2 = a^2 ***sq. units.***

(*iii*) **Area =** $\frac{1}{2}$ **× (Diagonal)^2** ***sq. units.***



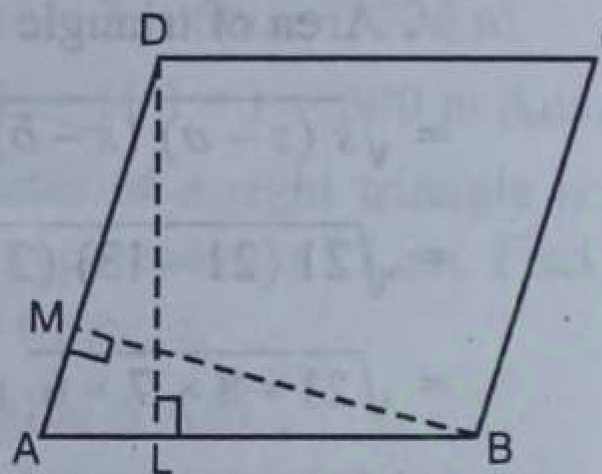
(*iv*) **Diagonal =** $\sqrt{2}$ ***a units*** **=** $\sqrt{2 \times \text{Area}}$ **units..**

E. Area of a Parallelogram = Base × Height.

(*i*) Area of ||gm ABCD = AB × DL

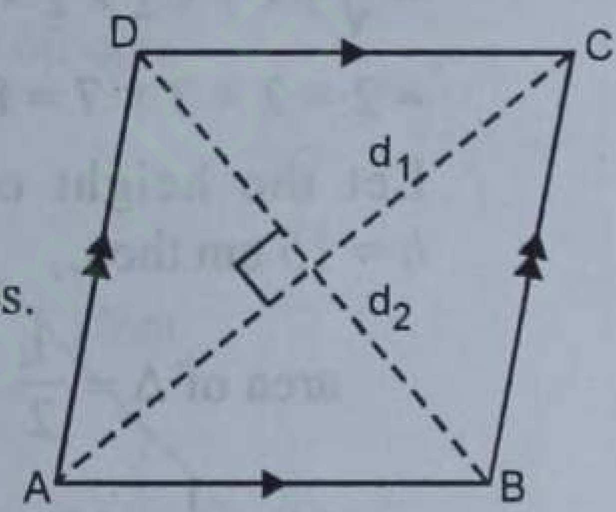
= AD × BM.

(*ii*) Perimeter = 2 (AB + BC) units



F. Area of Rhombus = $\frac{1}{2}$ **× Product of its diagonals.**

$$= \left(\frac{1}{2} \times d_1 \times d_2\right) \textit{sq. units.}$$



Remark. The diagonals of a rhombus bisect each other at right angles.

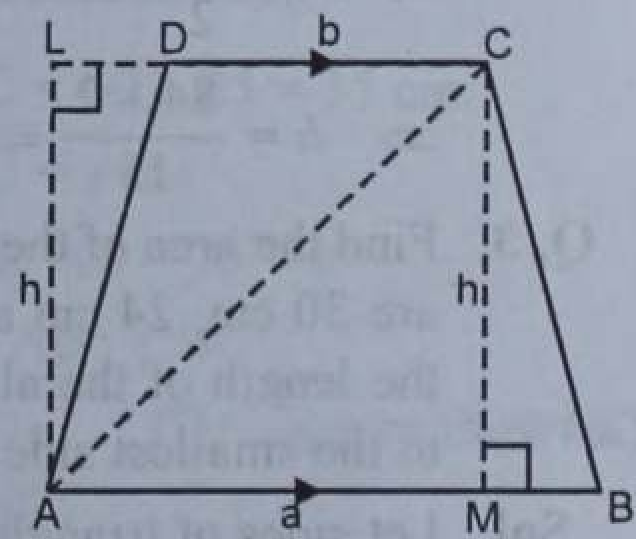
G. (i) Area of Trap. ABCD = Area of ΔABC + Area of ΔACD

$$= \frac{1}{2} \times AB \times h + \frac{1}{2} \times CD \times h$$

$$= \frac{1}{2} \times (AB + CD) \times h$$

$$= \frac{1}{2} \times \textit{(Sum of parallel sides)}$$

× (***Distance between them***) **sq. units.**



(*ii*) **Perimeter** = sum of four sides units

Note : $\sqrt{2}$ = 1·414 or 1·41

$\sqrt{3}$ = 1·732 or 1·73

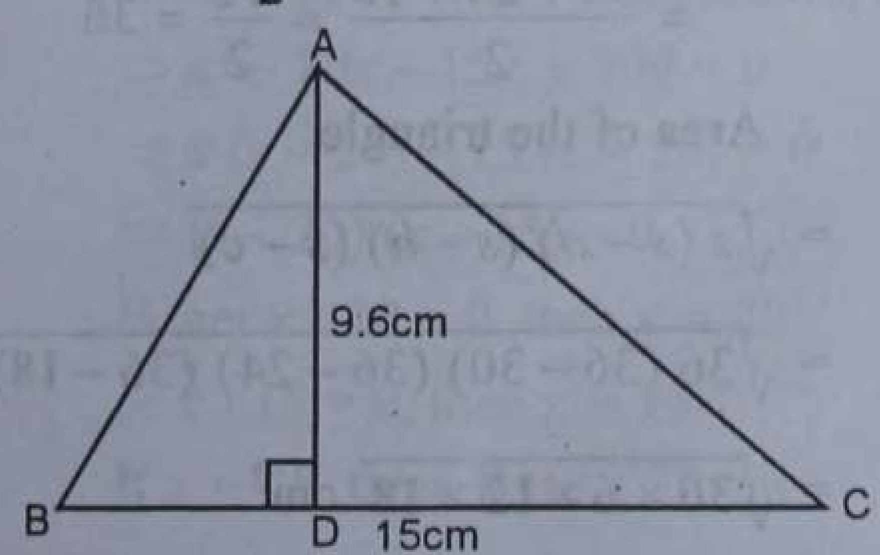
EXERCISE 20 (A)

Q. 1. Find the area of a triangle whose base is 15 cm and the corresponding height is 9·6 cm.

Sol. Base BC of triangle ABC = 15 cm and its altitude AD = 9·6 cm

∴ Area of Δ ABC = $\frac{1}{2}$ base × altitude

$$= \frac{1}{2} \times 15 \times 9{\cdot}6 = 72{\cdot}0 \text{ cm}^2 \textbf{ Ans.}$$



Q. 2. Find the area of the triangle whose sides are 13 cm, 14 cm and 15 cm. Also find the height of the triangle corresponding to the longest side.

Sol. Let $a = 13$ cm, $b = 14$ cm and $c = 15$ cm

$$\therefore s = \frac{a+b+c}{2} = \frac{13+14+15}{2}$$

$$= \frac{42}{2} = 21\text{cm}.$$

$\therefore$ Area of triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{21(21-13)(21-14)(21-15)}$$

$$= \sqrt{21 \times 8 \times 7 \times 6}\ \text{cm}^2$$

$$= \sqrt{3 \times 7 \times 2 \times 2 \times 2 \times 7 \times 2 \times 3}$$

$$= 2 \times 2 \times 3 \times 7 = 84\ \text{cm}^2$$

Let the height on the longest side $h = 15$ cm then,

$$\text{area of } \Delta = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\Rightarrow 84 = \frac{1}{2} \times 15 \times h$$

$$\Rightarrow \quad h = \frac{84 \times 2}{15} = \frac{56}{5} = 11{\cdot}2 \text{ cm. } \textbf{Ans.}$$

Q. 3. Find the area of the triangle whose sides are 30 cm, 24 cm and 18 cm. Also find the length of the altitude corresponding to the smallest side of the triangle.

Sol. Let sides of triangle are $a = 30$ cm, $b = 24$ cm and $c = 18$ cm

and $s = \dfrac{a+b+c}{2}$

$$= \frac{30+24+18}{2} = \frac{72}{2} = 36$$

$\therefore$ Area of the triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{36(36-30)(36-24)(36-18)}$$

$$= \sqrt{36 \times 6 \times 12 \times 18}\ \text{cm}^2$$

$$= \sqrt{2 \times 2 \times 3 \times 3 \times 2 \times 3 \times 3 \times 2 \times 2 \times 2 \times 3 \times 3}\ \text{cm}^2$$

$$= 2 \times 3 \times 2 \times 3 \times 2 \times 3 = 216\ \text{cm}^2$$

Let the length of altitude on the smallest side of 18 cm $= h$.

Then area of triangle

$$= \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$\Rightarrow 216 = \frac{1}{2} \times 18 \times h$$

$$\Rightarrow 9h = 216 \Rightarrow h = \frac{216}{9} = 24$$

$\therefore$ Altitude = 24 cm. **Ans.**

Q. 4. The lengths of the sides of a triangle are in the ratio 3 : 4 : 5 and its perimeter is 144 cm. Find the area of the triangle.

Sol. Ratio in sides = 3 : 4 : 5 and

Perimeter = 144 cm

Let the sides are $3x$, $4x$ and $5x$

$$\therefore 3x + 4x + 5x = 144 \Rightarrow 12x = 144$$

$$\Rightarrow \quad x = \frac{144}{12} = 12$$

$\therefore$ Sides are 3×12, 4×12, 5×12 *i.e.* 36 cm, 48 cm and 60 cm.

$$\therefore s = \frac{\text{sum of sides}}{2} = \frac{144}{2} = 72$$

$\therefore$ Area of the triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{72(72-36)(72-48)(72-60)}$$

$$= \sqrt{72 \times 36 \times 24 \times 12}\ \text{cm}^2$$

$$= \sqrt{36 \times 2 \times 36 \times 2 \times 12 \times 12}\ \text{cm}^2$$

$$= 2 \times 36 \times 12\ \text{cm}^2 = 864\ \text{cm}^2\ \textbf{Ans.}$$

Q.5. The perimeter of triangular field is 540 m and its sides are in the ratio 25 : 17 : 12. Find the area of the triangle. Also, find the cost of cultivating the field at Rs. 24·60 per 100 m².

Sol. Perimeter of the field = 540 m.

Ratio of sides = 25 : 17 : 12

Let sides be $25x$, $17x$, $12x$

Then $25x + 17x + 12x = 540$

$\Rightarrow \quad 54x = 540 \quad \Rightarrow \quad x = 10$

$\therefore$ Sides are 25×10, 17×10, 12×10

$= 250$ m, 170 m, 120 m.

$$\therefore s = \frac{\text{sum of sides}}{2} = \frac{540}{2} = 270$$

$\therefore$ Area of the triangular field

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{270(270-250)(270-170)(270-120)}$$

$$= \sqrt{270 \times 20 \times 100 \times 150}$$

$$= \sqrt{3 \times 3 \times 3 \times 10 \times 2 \times 10 \times 2 \times 5 \times 10 \times 3 \times 5 \times 10}$$

$$= \sqrt{2 \times 3 \times 3 \times 5 \times 10 \times 10} \text{ m}^2$$

$= 9000 \text{ m}^2$

Cost of cultivating the field

$= \text{Rs.}24{\cdot}60$ per 100m^2

$$\therefore \text{ Total cost} = \text{Rs. } \frac{9000 \times 24 \cdot 60}{100}$$

$$= \text{Rs. } \frac{9000 \times 2460}{100 \times 100} = \text{Rs. } 2214 \textbf{ Ans.}$$

Q. 6. The base of a triangular field is twice its altitude. If the cost of cultivating the field at Rs. 14·50 per 100 m² is Rs. 52,200, find its base and altitude.

Sol. Let altitude of the triangle $= x$

Then base $= 2x$

$$\therefore \quad \text{Area} = \frac{1}{2} \text{ base} \times \text{altitude}$$

$$\Rightarrow \quad \frac{1}{2} \times 2x \times x = x^2 \qquad (i)$$

Total cost of cultivative of the field

$= \text{Rs. } 52{,}200$

Rate of cultivating

$= \text{Rs. } 14{\cdot}50$ per 100 m^2

$$\therefore \text{ Area of the field} = \frac{52200 \times 100}{14 \cdot 50}$$

$$= \frac{52200 \times 100 \times 100}{1450} m^2$$

$$= 360000 \text{ m}^2 \qquad ...(ii)$$

From (i) and (ii)

$\therefore \quad x^2 = 360000 \Rightarrow x^2 = (600)^2$

$\Rightarrow x = 600$

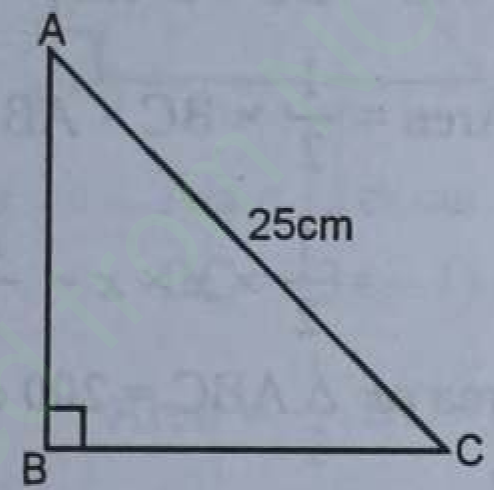
Hence Base $= 2 \times 600 = 1200$ m

and altitude $= 600 \times 1 = 600$ m **Ans.**

Q. 7. The perimeter of a right triangle is 60 cm and its hypotenuse is 25 cm. Find the area of the triangle.

Sol. In right angled Δ ABC,

$\angle B = 90°$, hypotenuse AC = 25cm and perimeter = 60 cm.



$\therefore$ AB + BC $= 60 - 25 = 35$ cm

Let base BC $= x$ cm

Then altitude AB $= 35 - x$

But $AC^2 = AB^2 + BC^2$

(Pythagoras theorem)

$\Rightarrow (25)^2 = (35 - x)^2 + x^2$

$\Rightarrow 625 = 1225 - 70x + x^2 + x^2$

$\Rightarrow 2x^2 - 70x + 1225 - 625 = 0$

$\Rightarrow 2x^2 - 70x + 600 = 0$

$\Rightarrow x^2 - 35x + 300 = 0$ (Dividing by 2)

$\Rightarrow x^2 - 20x - 15x + 300 = 0$

$\Rightarrow x(x - 20) - 15(x - 20) = 0$

$\Rightarrow (x - 20)(x - 15) = 0$

Either $x - 20 = 0$, then $x = 20$

or $x - 15 = 0$, then $x = 15$

If $x = 20$, then, Base = 20 cm

and altitude = 35 − 20 = 15 cm

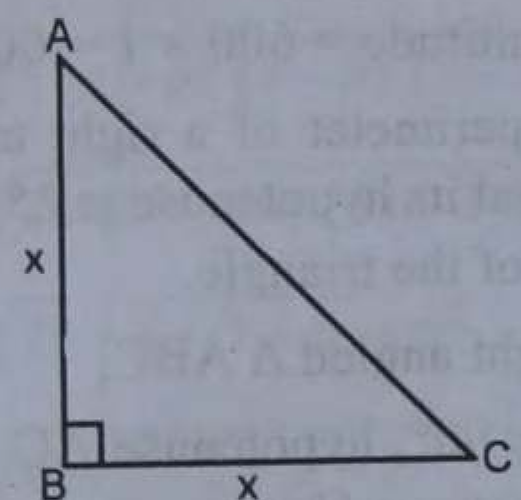
If $x = 15$, then, Base = 15 cm

and altitude = 35 − 15 = 20 cm

Hence sides are 15cm, 20 cm. **Ans.**

Q. 8. Find the length of hypotenuse of an isosceles right angled triangle having an area of 200 cm^2 (Take $\sqrt{2} = 1{\cdot}414$).

Sol. In right angled isosceles triangle ABC, $\angle B = 90°$ and AB = BC



Let AB = BC = x *cm*

$$\therefore \quad \text{Area} = \frac{1}{2} \times BC \times AB$$

$$= \frac{1}{2} \times x \times x = \frac{x^2}{2}$$

But area of Δ ABC = 200 cm^2

$$\therefore \frac{x^2}{2} = 200 \Rightarrow x^2 = 400 = (20)^2$$

$$\therefore \quad x = 20$$

Now hypotenuse $AC = \sqrt{AB^2 + BC^2}$

$$= \sqrt{(20)^2 + (20)^2}$$

$$= \sqrt{400 + 400} = \sqrt{800}$$

$$= \sqrt{2 \times 400}$$

$$= 20 \times \sqrt{2} \text{ cm}$$

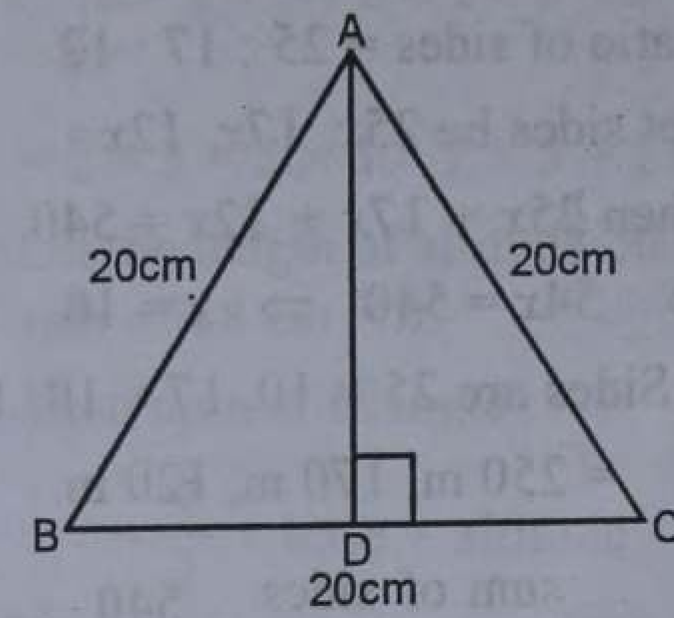
$$= 20 \times 1{\cdot}414 \text{ cm}$$

$$= 28{\cdot}280 \text{ cm}$$

$= 28{\cdot}28$ cm **Ans.**

Q. 9. Calculate the area and the height of an equilateral triangle whose perimeter is 60 cm.

Sol. Perimeter of Δ ABC = 60 cm



$$\therefore \text{ Each side} = \frac{60}{3} = 20\text{cm}.$$

$$\text{Now altitude AD} = \frac{\sqrt{3}}{2} a$$

$$= \frac{1{\cdot}732}{2} \times 20 = 17{\cdot}32 \text{ cm}.$$

and area of ΔABC

$$= \frac{\sqrt{3}}{4} a^2 = \frac{1{\cdot}732}{4} (20^2) \text{ cm}^2$$

$= \frac{1{\cdot}732}{4} \times 400 = 173{\cdot}2$ cm^2 **Ans.**

Q. 10. Find the perimeter and area of an equilateral triangle whose height is 12 cm Write your answers, correct to two decimal places.

Sol. Height of an equilateral triangle = 12 cm

Let side of equilateral triangle = a

$$\text{then height } (h) = \frac{\sqrt{3}}{2} a$$

$$\therefore \frac{\sqrt{3}}{2} a = 12 \Rightarrow a = \frac{12 \times 2}{\sqrt{3}} \text{ cm}$$

$$\Rightarrow \quad a = \frac{24 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$= \frac{24 \times \sqrt{3}}{3} = 8\sqrt{3} \text{ cm}$$

Now perimeter of the triangle = $3a$

$$= 3 \times 8\sqrt{3} \text{ cm}$$

$$= 24\,(1{\cdot}732) \text{ cm}$$

$$= 41{\cdot}568 \text{ cm}$$

$= 41{\cdot}57$ cm

and area of the triangle $= \frac{\sqrt{3}}{4} a^2$

$= \frac{1{\cdot}732}{4} \times (8\sqrt{3})^2 \text{ cm}^2$

$= \frac{1{\cdot}732}{4} \times 64 \times 3 \text{ cm}^2$

$= 0{\cdot}433 \times 192 \text{ cm}^2$

$= 83{\cdot}136 \text{ cm}^2$

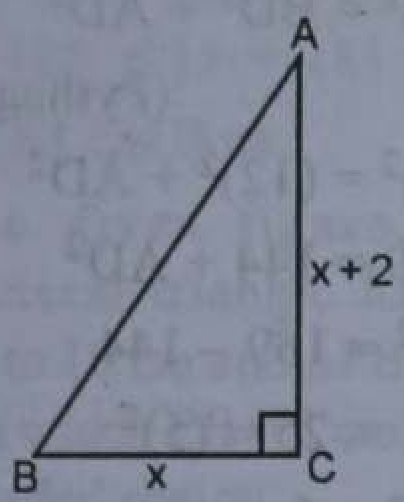
$= 83{\cdot}14 \text{ cm}^2$ **Ans.**

Q. 11. The lengths of two sides of a right triangle containing the right angle differ by 2cm. If the area of the triangle is 24cm², find the perimeter of the triangle.

Sol. In right Δ ABC, $\angle C = 90°$

Let BC = x cm

Then AC = $x + 2$ cm.



$\therefore$ Area $= \frac{1}{2} \times BC \times AC$

$\Rightarrow 24 = \frac{1}{2} x (x + 2)$

$\Rightarrow 48 = x^2 + 2x$

$\Rightarrow x^2 + 2x - 48 = 0$

$\Rightarrow x^2 + 8x - 6x - 48 = 0$

$\Rightarrow x (x + 8) - 6 (x + 8) = 0$

$\Rightarrow (x + 8) (x - 6) = 0$

Either $x + 8 = 0$, then $x = -8$ which is not possible

or $x - 6 = 0$, then $x = 6$

$\therefore$ BC = 6 cm and AC = 6 + 2 = 8 cm.

But $AB^2 = BC^2 + AC^2$

$= (6)^2 + (8)^2 = 36 + 64$

$= 100 = (10)^2$

$\therefore$ AB = 10 cm

Now perimeter of the triangle

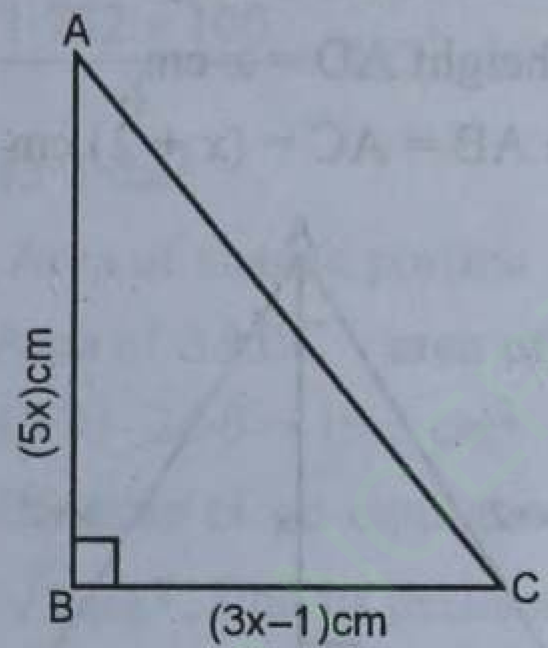
= AB + BC + CA

= 10 + 6 + 8 cm = 24 cm **Ans.**

Q. 12. The sides of a right–angled triangle containing the right angle are $(5x)$ cm and $(3x - 1)$ cm. If its area is 60 cm², find its perimeter.

Sol. In right Δ ABC, $\angle B = 90°$

Area of triangle = 60 cm²



This AB = $(5x)$ cm

and BC = $(3x - 1)$ cm

$\therefore$ Area $= \frac{1}{2}$ BC × AB

$\Rightarrow 60 = \frac{1}{2} \times (3x - 1) \times 5x$

$120 = 15x^2 - 5x$

$\Rightarrow 15x^2 - 5x - 120 = 0$

$\Rightarrow 3x^2 - x - 24 = 0$

$\Rightarrow 3x^2 - 9x + 8x - 24 = 0$

$\Rightarrow 3x (x - 3) + 8 (x - 3) = 0$

$\Rightarrow (x - 3) (3x + 8) = 0$

Either $x - 3 = 0$, then $x = 3$

or $3x + 8 = 0$,

then $x = -\frac{8}{3}$ which is not possible

$\therefore$ AB = $5x = 5 \times 3 = 15$ cm and

BC = $3x - 1 = 3 \times 3 - 1 = 9 - 1 = 8$ cm

But $AC^2 = AB^2 + BC^2$

(Pythagoras theorem)

$= (15)^2 + (8)^2 = 225 + 64$
$= 289 = (17)^2$

$\therefore$ AC = 17 cm.

Now perimeter of the triangle

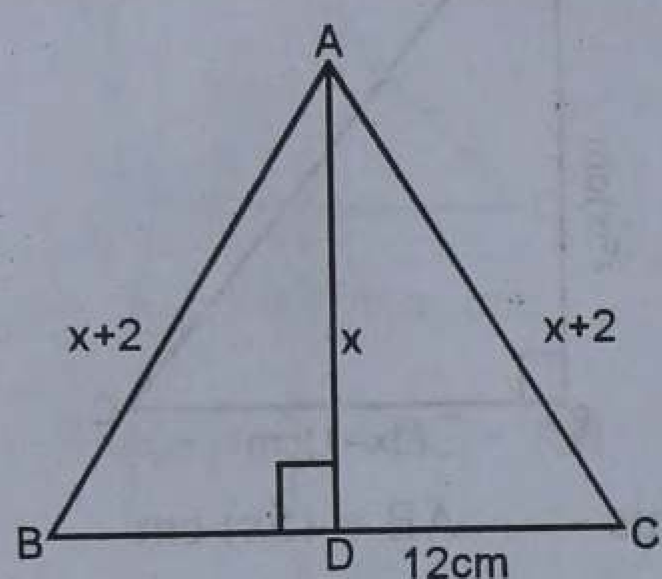
AB + BC + CA

$= 15 + 8 + 17 = 40$ cm **Ans.**

Q. 13. Each of the equal sides of an isosceles triangle is 2 cm more than its height and the base of the triangle is 12 cm. Find the area of the triangle.

Sol. Let height AD = x cm

then AB = AC = $(x + 2)$ cm



Base BC = 12 cm

$\because$ AD $\perp$ BC

$\therefore \quad BD = DC = \frac{12}{2} = 6\,\text{cm}$

$\therefore$ In right ΔABD,

$AB^2 = BD^2 + AD^2$

$\Rightarrow \quad (x+2)^2 = (6)^2 + x^2$

$\Rightarrow x^2 + 4x + 4 = 36 + x^2$

$\Rightarrow x^2 + 4x - x^2 = 36 - 4$

$\Rightarrow 4x = 32 \quad \Rightarrow \quad x = \frac{32}{4} = 8$

$\therefore$ AD = 8 cm

Now Area of ΔABC

$= \frac{1}{2} \times \text{Base} \times \text{Altitude}$

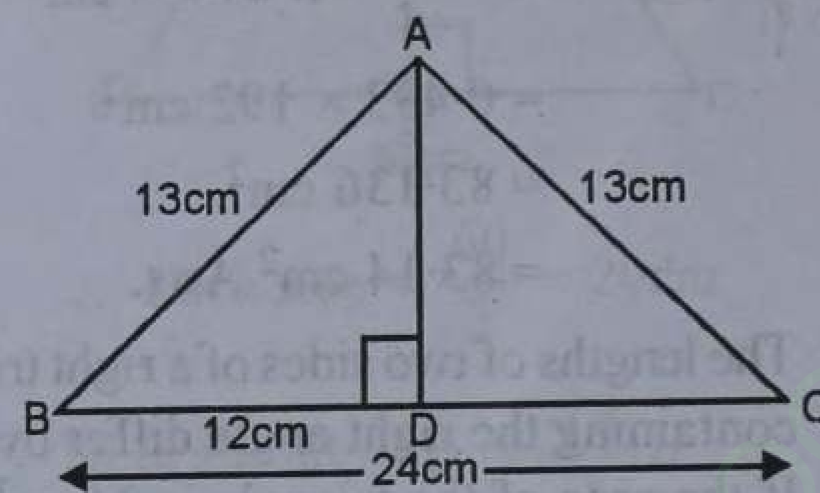
$= \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 12 \times 8\ \text{cm}^2 = 48\ \text{cm}^2$ **Ans.**

Q. 14. Find the area of an isosceles triangle, each of whose equal sides is 13 cm and base 24 cm.

Sol. ΔABC is an isosceles triangle in which AB = AC = 13 cm and BC = 24 cm.

AD $\perp$ BC which bisects BC in D.



$\therefore$ BD = DC

$= \frac{1}{2} BC = \frac{1}{2} \times 24 = 12\,\text{cm}$

Now in right ΔABD,

$AB^2 = BD^2 + AD^2$

(Pythagoras Theorem)

$\Rightarrow (13)^2 = (12)^2 + AD^2$

$\Rightarrow 169 = 144 + AD^2$

$\Rightarrow AD^2 = 169 - 144$

$= 25 = (5)^2$

$\therefore \quad$ AD = 5 cm

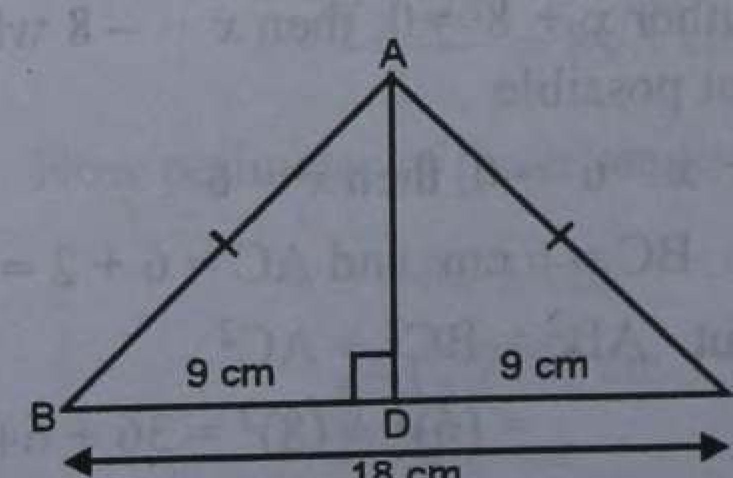
Now area of $\Delta ABC = \frac{1}{2} BC \times AD$

$= \frac{1}{2} \times 24 \times 5 = 60\ \text{cm}^2$ **Ans.**

Q. 15. The base of an isosceles triangle is 18 cm and its area is 108 cm^2. Find its perimeter.

Sol. Area of isosceles ΔABC = 108 cm^2

Base BC = 18 cm



Let AD be its height,

Then, $AD = \frac{\text{Area} \times 2}{\text{Base}}$

$= \frac{108 \times 2}{18} = 12$ cm

In ΔABC, $AD \perp BC$

$\therefore$ D is mid-point of BC

$\therefore BD = DC = \frac{18}{2} = 9$ cm

$\therefore AB^2 = BD^2 + AD^2$

(Pythagoras Theorem)

$= (9)^2 + (12)^2 = 81 + 144$

$= 225 = (15)^2$

$\therefore$ AB = 15 cm.

But, AC = AB = 15 cm.

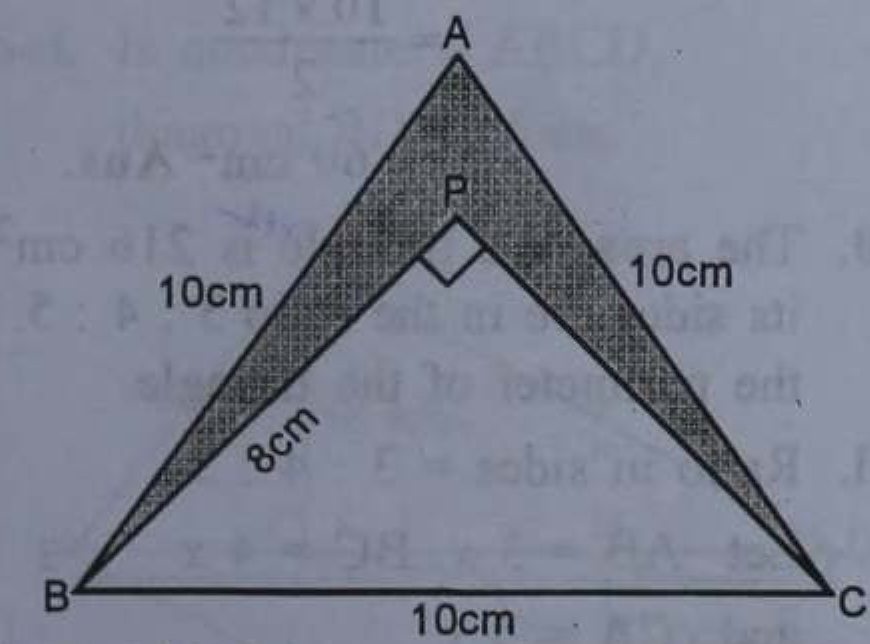
Now, perimeter of ΔABC

= AB + AC + BC

= (15 + 15 + 18) cm

= 48 cm **Ans.**

Q. 16. In the given figure, ΔABC is an equilateral triangle having each side equal to 10 cm and ΔPBC is right angled at P in which PB = 8 cm. Find the area of the shaded region.



Sol. ΔABC is an equilateral triangle whose each side = 10 cm

ΔBPC is right angled triangle in which $\angle P = 90^\circ$ and PB = 8 cm

In right ΔPBC, $BC^2 = PB^2 + PC^2$

(Pythagoras Theorem)

$\Rightarrow (10)^2 = (8)^2 + PC^2$

$\Rightarrow 100 = 64 + PC^2$

$\Rightarrow PC^2 = 100 - 64 = 36 = (6)^2$

$\therefore$ PC = 6 cm

Now area of $\Delta PBC = \frac{1}{2} \times PB \times PC$

$= \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$

and area of ΔABC

$= \frac{\sqrt{3}}{4}a^2 = \frac{1{\cdot}732}{4} \times (10)^2 \text{ cm}^2$

$= \frac{1{\cdot}732 \times 100}{4} = 433 \times 100 \text{ cm}^2$

$= 43{\cdot}3 \text{ cm}^2$

$\therefore$ Area of shaded portion

= Area of ΔABC – area of ΔPBC

$= 43{\cdot}3 - 24{\cdot}0 = 19{\cdot}3 \text{ cm}^2$ **Ans.**

Q. 17. If the area of an equilateral triangle is $81\sqrt{3} \text{ cm}^2$, find its perimeter.

Sol. Let each side of equilateral triangle = a

$\therefore$ Area $= \frac{\sqrt{3}}{4}a^2$

$\Rightarrow \frac{\sqrt{3}}{4}a^2 = 8\sqrt{3}$

$\Rightarrow a^2 = \frac{81 \times 4\sqrt{3}}{\sqrt{3}}$

$\Rightarrow a^2 = 324 = (18)^2$

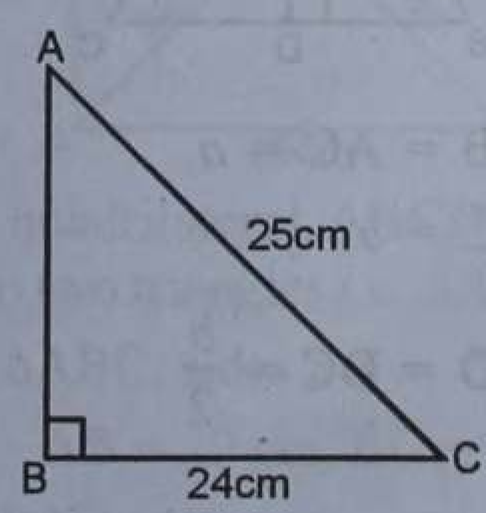
$\therefore a = 18$ cm

Now perimeter $= 3a = 3 \times 18$

= 54 cm **Ans.**

Q. 18. The base of right – angled triangle is 24 cm and its hypotenuse is 25 cm. Find the area of the triangle.

Sol.



In ΔABC, $\angle B = 90^\circ$

$BC = 24$ cm

and $AC = 25$ cm

But $AC^2 = AB^2 + BC^2$

(Pythagoras Theorem)

$\Rightarrow (25)^2 = AB^2 + (24)^2$

$\Rightarrow \quad 625 = AB^2 + 576$

$\Rightarrow \quad AB^2 = 625 - 576$

$= 49 = (7)^2$

$\therefore \quad AB = 7$ cm

Now area of ΔABC

$$= \frac{1}{2} \text{Base} \times \text{Altitude}$$

$$= \frac{1}{2} \times BC \times AB$$

$$= \frac{1}{2} \times 24 \times 7 \text{ cm}^2$$

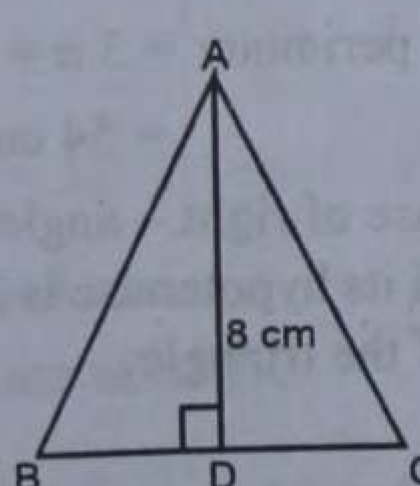
$= 84$ cm^2 **Ans.**

Q. 19. The altitude drawn to the base of an isosceles triangle is 8 cm and the perimeter is 32 cm. Find the area of the triangle.

Sol. ΔABC is an isosceles triangle in which $AB = AC$ and $AD \perp BC$

Now, perimeter = 32 cm

and $AD = 8$ cm



Let $AB = AC = a$

and $BC = b$

$$\therefore \quad BD = DC = \frac{b}{2}$$

Now, in right ΔABD,

$AB^2 = BD^2 + AD^2$

$$\Rightarrow a^2 = \frac{b^2}{4} + (8)^2$$

$\Rightarrow 4a^2 = b^2 + 256$

$\Rightarrow 4a^2 - b^2 = 256$

$\Rightarrow (2a + b)(2a - b) = 256 \quad$ ...(*i*)

But, $AB + AC + BC = 32$

$\Rightarrow a + a + b = 32$

$\Rightarrow 2a + b = 32 \quad$ ...(*ii*)

Dividing (*i*) by (*ii*),

$$2a - b = \frac{256}{32} = 8 \quad \text{...}(iii)$$

Adding (*ii*) and (*iii*),

$4a = 40$

$\Rightarrow \quad a = 10$

Subtracting (*iii*) from (*ii*),

$2b = 24$

$\Rightarrow \quad b = 12$

$$\text{Now, area of } \Delta ABC = \frac{BC \times AD}{2}$$

$$= \frac{10 \times 12}{2}$$

$= 60$ cm^2 **Ans.**

Q. 20. The area of a triangle is 216 cm^2 and its sides are in the ratio 3 : 4 : 5. Find the perimeter of the triangle.

Sol. Ratio in sides = 3 : 4 : 5

Let $AB = 3x$, $BC = 4x$

and $CA = 5x$

$$\therefore \quad s = \frac{AB + BC + CA}{2}$$

$$= \frac{3x + 4x + 5x}{2}$$

$$= \frac{12x}{2} = 6x$$

$\therefore$ Area of ΔABC,

$$= \sqrt{6x(6x-3x)(6x-4x)(6x-5x)}$$

$$= \sqrt{6x \times 3x \times 2x \times x}$$

$$= \sqrt{36x^4}$$

$$= 6x^2$$

But, area of triangle = 216 cm^2

$\therefore \quad 6x^2 = 216$

$\Rightarrow \quad x^2 = \frac{216}{6} = 36 = (6)^2$

$\therefore \quad x = 6$

$\therefore$ Sides are 3×6, 4×6, 5×6 or 18, 24, 30 cm.

and perimeter = Sum of sides

$= (18 + 24 + 30)$ cm

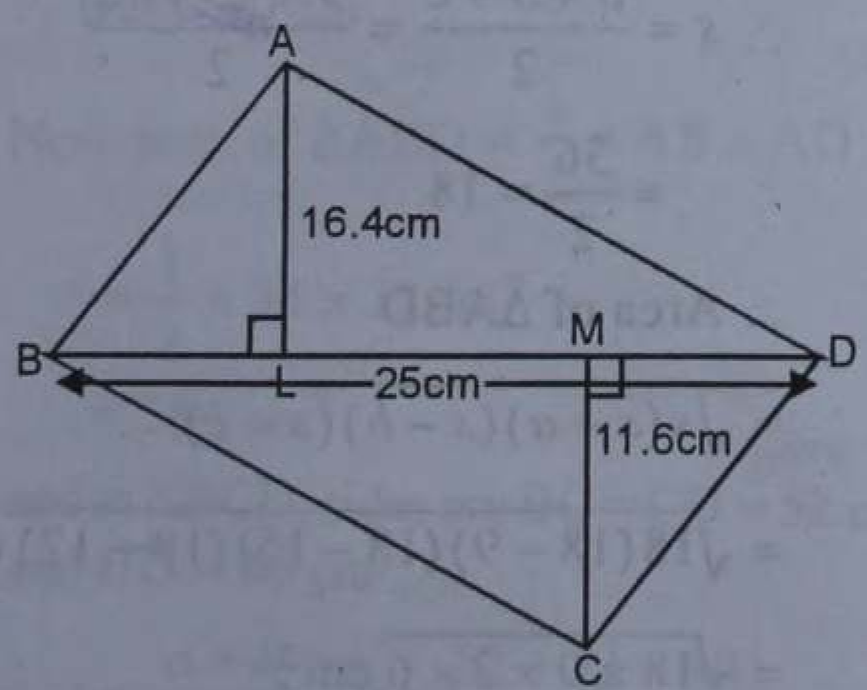
$= 72$ cm **Ans.**

EXERCISE 20 (B)

Q. 1. Find the area of a quadrilateral one of whose diagonals is 25 cm long and the lengths of perpendiculars from the other two vertices one 16·4 cm and 11·6 cm respectively.

Sol. In quadrilateral ABCD,

diagonal BD = 25 cm



AL $\perp$ BD and CM $\perp$ BD

AL = 16·4 cm, CM = 11·6

$\therefore$ Area of quadrilateral ABCD

$$= \frac{1}{2}(AL + CM) \times BD$$

$$= \frac{1}{2}(16 \cdot 4 + 11 \cdot 6) \times 25 \text{ cm}^2$$

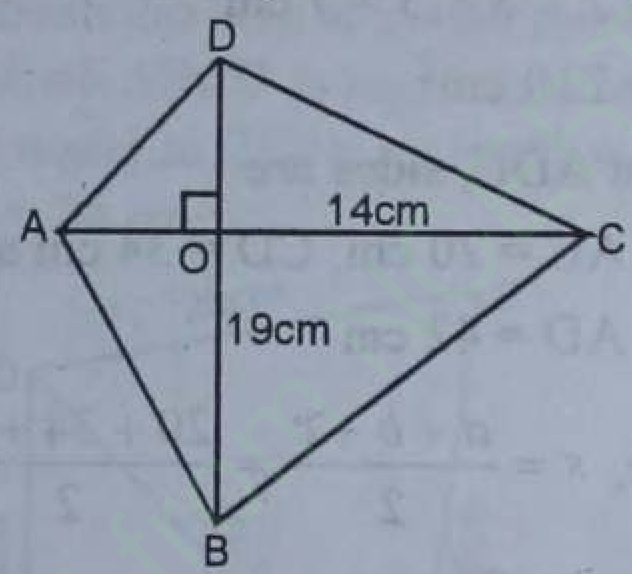
$$= \frac{1}{2} \times 28 \times 25 = 350 \text{ cm}^2 \textbf{ Ans.}$$

Q. 2. The diagonals of a quadrilateral intersect each other at right angles. If the lengths of these diagonals be 14 cm and 19 cm respectively, find the area of the quadrilateral.

Sol. In quadrilateral ABCD,

diagonals AC and BD intersect each other at O at right angles, and

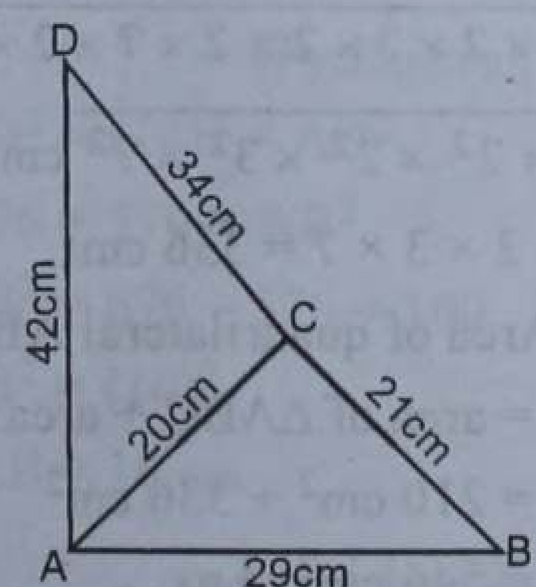
AC = 14 cm, BD = 19 cm



$\therefore$ Area $= \frac{1}{2} AC \times BD$

$$= \frac{1}{2} \times 14 \times 19 \text{ cm}^2 = 133 \text{ cm}^2 \textbf{ Ans.}$$

Q. 3. Find the area of a quadrilateral ABCD in which AB = 29 cm, BC = 21 cm, AC = 20 cm, CD = 34 cm and DA = 42 cm.



Sol. In quadrilateral ABCD, AC divides it into two triangles *i.e.* ΔABC and ΔADC.

In ΔABC, sides are

AB = 29 cm, BC = 21 cm

and AC = 20 cm

$\therefore s = \frac{a+b+c}{2}$

$= \frac{29+21+20}{2} = \frac{70}{2} = 35$

$\therefore$ Area of ΔABC

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{35(35-29)(35-21)(35-20)}$

$= \sqrt{35 \times 6 \times 14 \times 15}\ cm^2$

$= \sqrt{5 \times 7 \times 2 \times 3 \times 2 \times 7 \times 3 \times 5}$

$= \sqrt{2^2 \times 3^2 \times 5^2 \times 7^2}$

$= 2 \times 3 \times 5 \times 7\ cm^2$

$= 210\ cm^2$

In ADC, sides are

AC = 20 cm, CD = 34 cm and

AD = 42 cm

$\therefore s = \frac{a+b+c}{2} = \frac{20+34+42}{2}$

$= \frac{96}{2} = 48$

$\therefore$ Area of ΔADC

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{48(48-20)(48-34)(48-42)}\ cm^2$

$= \sqrt{48 \times 28 \times 14 \times 6}\ cm^2$

$= \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 2 \times 2 \times 7 \times 2 \times 7 \times 2 \times 3}$

$= \sqrt{2^2 \times 2^2 \times 2^2 \times 2^2 \times 3^2 \times 7^2}\ cm^2$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 336\ cm^2$

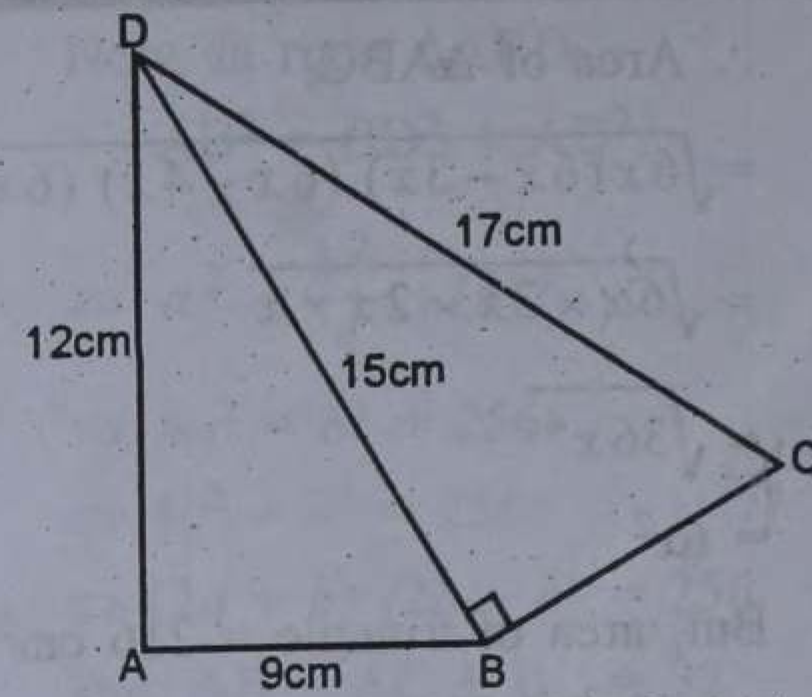
$\therefore$ Area of quadrilateral ABCD

= area of ΔABC + area of ΔADC

$= 210\ cm^2 + 336\ cm^2$

$= 546\ cm^2$ **Ans.**

Q. 4. Find the perimeter and area of quadrilateral ABCD in which AB = 9 cm, AD = 12 cm, BD = 15 cm, CD = 17 cm and $\angle CBD = 90°$.



Sol. In quadrilateral ABCD, diagonal BD divides it into two triangles.

In ΔBCD, $\angle CBD = 90°$

$\therefore AC^2 = AB^2 + BC^2$

(Pythagoras Theorem)

$\Rightarrow (17)^2 = (15)^2 + BC^2$

$\Rightarrow 289 = 225 + BC^2$

$\Rightarrow BC^2 = 289 - 225 = 64 = (8)^2$

$\therefore$ BC = 8 cm

Now area of $\Delta DBC = \frac{1}{2} \times BC \times BD$

$= \frac{1}{2} \times 8 \times 15 = 60\ cm^2$

and in ΔABD, sides are

AB = 9 cm, BD = 15 cm and

AD = 12 cm

$\therefore s = \frac{a+b+c}{2} = \frac{9+15+12}{2}$

$= \frac{36}{2} = 18$

$\therefore$ Area of ΔABD

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18(18-9)(18-15)(18-12)}\ cm^2$

$= \sqrt{18 \times 9 \times 3 \times 6}\ cm^2$

$= \sqrt{2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 2}\ cm^2$

$= \sqrt{2^2 \times 3^2 \times 3^2 \times 3^2}\ cm^2$

$= 2 \times 3 \times 3 \times 3 = 54\ cm^2$

$\therefore$ Area of quadrilateral ABCD

= area of ΔDBC + area of ΔABD

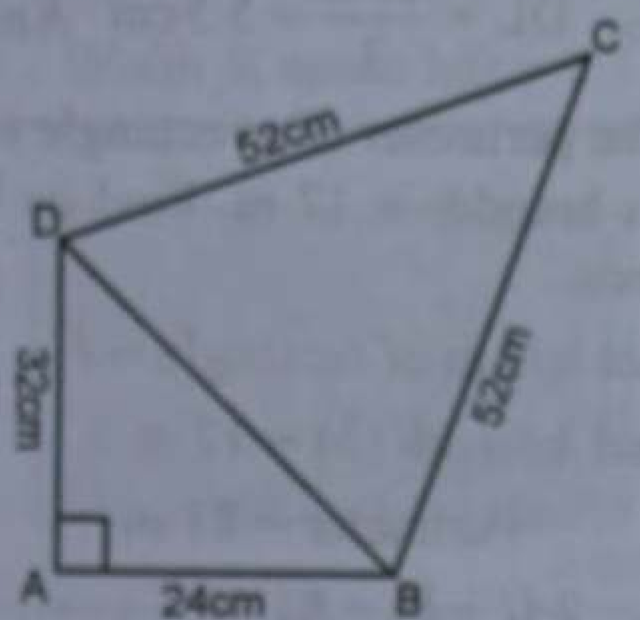
$= 60\ cm^2 + 54\ cm^2 = 114\ cm^2$

and perimeter = AB + BC + CD + DA

$= (9 + 8 + 17 + 12)\ cm^2$

$= 46\ cm^2$ **Ans.**

Q. 5. Calculate the area of quadrilateral ABCD in which : AB = 24 cm, AD = 32 cm, $\angle BAD = 90°$ and BC = CD = 52 cm.



Sol. In quadrilateral ABCD, diagonal BD divides it into two triangles ΔABD and ΔBCD.

In ΔABD, $\angle BAD = 90°$

$\therefore BD^2 = AD^2 + AB^2$

(Pythagoras Theorem)

$= (32)^2 + (24)^2 = 1024 + 576 = 1600$

$= (40)^2$

$\therefore BD = 40$ cm

Now area of $\Delta ABD = \frac{1}{2} \times AB \times AD$

$= \frac{1}{2} \times 24 \times 32\ cm^2$

$= 384\ cm^2$

and in ΔBCD, sides are BC = CD = 52 cm and BD = 40 cm

$\therefore s = \frac{a+b+c}{2} = \frac{52+52+40}{2}$

$= \frac{144}{2} = 72$

$\therefore$ Area of ΔBCD

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{72(72-52)(72-52)(72-40)}$

$= \sqrt{72 \times 20 \times 20 \times 32}$

$= \sqrt{2 \times 2 \times 2 \times 3 \times 3 \times 20 \times 20 \times 2 \times 2 \times 2 \times 2 \times 2}$

$= \sqrt{2^2 \times 2^2 \times 2^2 \times 2^2 \times 3^2 \times 20^2}\ cm^2$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 20\ cm^2$

$= 960\ cm^2$

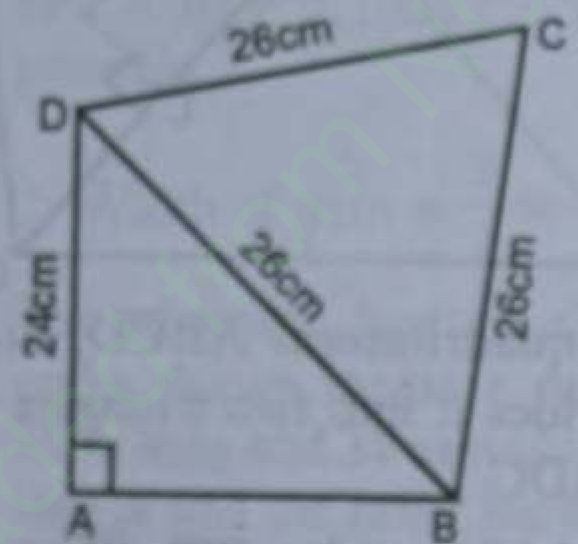
$\therefore$ Area of quadrilateral ABCD

= area of ΔABD + area of ΔBCD

$= 384\ cm^2 + 960\ cm^2$

$= 1344\ cm^2$ **Ans.**

Q. 6. Calculate the area of quadrilateral ABCD in which ΔBCD is equilateral with each side equal to 26 cm, $\angle BAD = 90°$ and AD = 24 cm.



Sol. In quadrilateral ABCD, diagonal BD divides into two triangles : ΔABD and ΔBCD.

In ΔABD, $\angle BAD = 90°$

$\therefore BD^2 = AD^2 + AB^2$

(Pythagoras Theorem)

$\Rightarrow (26)^2 = (24)^2 + AB^2$

$\Rightarrow 676 = 576 + AB^2$

$\Rightarrow AB^2 = 676 - 576 = 100$

$\Rightarrow AB^2 = (10)^2$

$\Rightarrow AB = 10$ cm

Now area of $\Delta ABD = \frac{1}{2} \times AB \times AD$

$= \frac{1}{2} \times 10 \times 24 = 120\ cm^2$

and area of equilateral ΔBCD whose each side is 26 cm

$= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{1.732}{4} \times (26)^2 \text{ cm}^2$

$= \frac{1.732}{4} \times 26 \times 26$

$= 0.433 \times 26 \times 26 \text{ cm}^2 = 292.71 \text{ cm}^2$

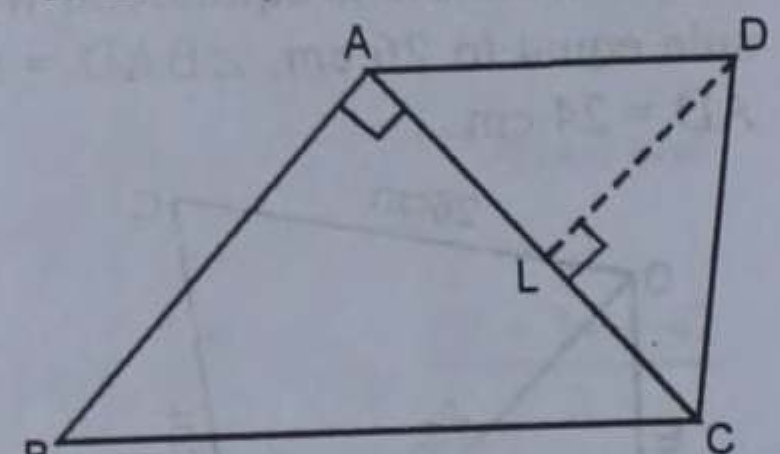
$\therefore$ Area of quadrilateral ABCD

$=$ area of ΔABD + area of ΔBCD

$= 120 \text{ cm}^2 + 292.71 \text{ cm}^2$

$= 412.71 \text{ cm}^2$ **Ans.**

Q. 7. In the adjoining figure, ΔABC is right angled at A, BC = 7·5 cm and AB = 4·5 cm. If the area of quadrilateral ABCD is 30 cm^2 and DL is the altitude of ΔDAC, calculate the length DL.



Sol. In quadrilateral ABCD, diagonal AC divides it into two triangles ΔABC and ΔADC.

In ΔABC, $\angle A = 90°$, BC = 7·5 cm and AB = 4·5 cm

But $BC^2 = AB^2 + AC^2$ (Pythagorus Theorem)

$\Rightarrow (7.5)^2 = (4.5)^2 + AC^2$

$\Rightarrow 56.25 = 20.25 + AC^2$

$\Rightarrow AC^2 = 56.25 - 20.25$

$\Rightarrow AC^2 = 36 = (6)^2$

$\therefore AC = 6$ cm

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times 4.5 \times 6 = 13.5 \text{ cm}^2$

But area of quadrilateral ABCD

$= 30 \text{ cm}^2$

$\therefore$ Area of $\Delta ADC = 30 - 13.5$

$= 16.5 \text{ cm}^2$

But area of ΔADC

$= \frac{1}{2} \times \text{Base} \times \text{Altitude}$

$= \frac{1}{2} \times AC \times DL$

$= \frac{1}{2} \times 6 \times DL = 3\,DL$

$\therefore 3 \cdot DL = 16.5$

$\Rightarrow DL = \frac{16.5}{3} = 5.5 \text{ cm}$ **Ans.**

Q. 8. The perimeter of a rectangle is 81 m and its breadth is 12 m. Find its length and area.

Sol. Let length of rectangle $= l$

and breadth $(b) = 12$ m

Perimeter = 81 m

$\therefore 2(l + b) = 81$

$\Rightarrow 2l + 2b = 81$

$\Rightarrow 2l + 2 \times 12 = 81$

$\Rightarrow 2l + 24 = 81$

$\Rightarrow 2l = 81 - 24 = 57$

$\therefore l = \frac{57}{2} = 28.5 \text{ m}$

Now area of the rectangle $= l \times b$

$= 28.5 \times 12 \text{ m}^2 = 342.0 \text{ m}^2$. **Ans.**

Q. 9. The perimeter of a rectangular field is $\frac{3}{5}$ km and its length is twice its breadth, find the area of the field in m^2.

Sol. Perimeter of rectangular field $= \frac{3}{5}$ km

$= \frac{3}{5} \times 1000 = 600 \text{ m}$

Let breadth of field $= x$ m

then length $= 2x$

$\therefore 2(2x + x) = 600$

$\Rightarrow 6x = 600$

$x = 100$

$\therefore$ Length $= 2x = 2 \times 100 = 200$ m

Breadth $= x = 100$ m

Area $= l \times b = 200 \times 100$

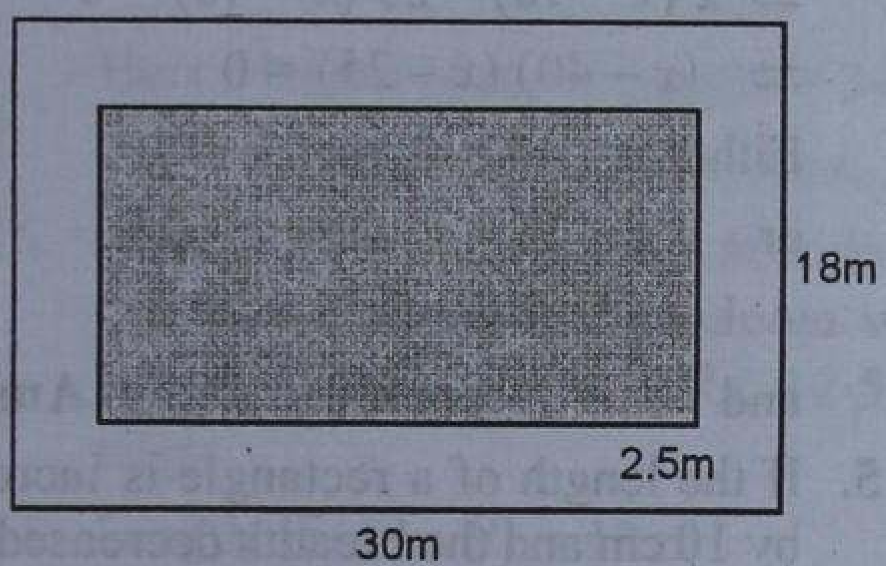
$= 20000 \text{ m}^2$ **Ans.**

Q. 10. A rectangular plot 30 m long and 18 m wide is to be covered with grass leaving 2·5 m all around it. Find the area to be laid with grass.

Sol. Length of rectangular plot $(l) = 30$ m

and width $(b) = 18$ m

Width of space left $= 2{\cdot}5$ m



$\therefore$ Inner length $(l_1) = 30 - 2 \times 2{\cdot}5$

$= 30 - 5 = 25$ m

and Inner breadth $(b_1) = 18 - 2 \times 2{\cdot}5$

$= 18 - 5 = 13$ m

$\therefore$ Area of grass area $= l_1 \times b_1$

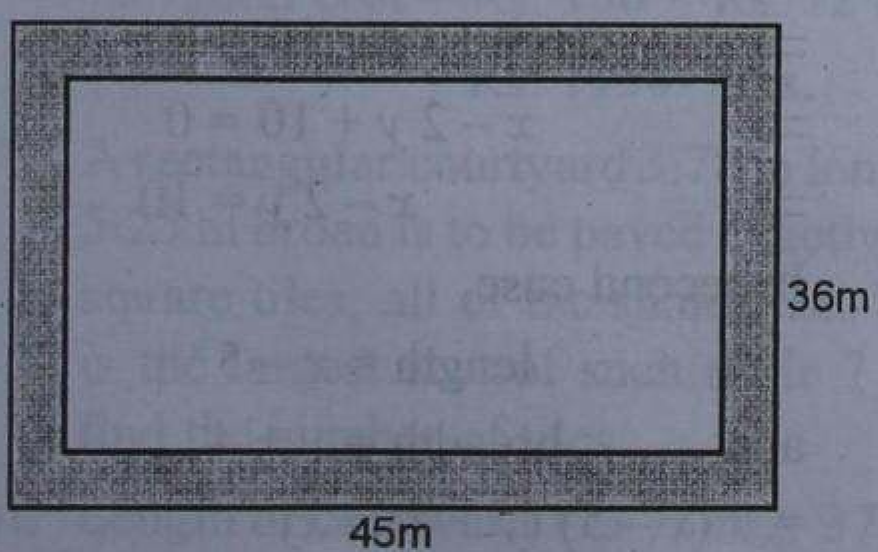
$= 25 \text{ m} \times 13 \text{ m} = 325 \text{ m}^2$ **Ans.**

Q. 11. A foot path of uniform width runs all around inside of a rectangular field 45 m long and 36 m wide. If the area of the path is 234 m², find the width of the path.

Sol. Length of field $(l) = 45$ m

and breadth $(b) = 36$ m

Let width of inner path $= x$



$\therefore$ Inner length $(l_1) = 45 - 2 \times x$

$= (45 - 2x)$ m

and inner breadth $(b_1) = 36 - 2 \times x$

$= (36 - 2x)$ m

Area of path $= 234 \text{ m}^2$

$\therefore l.b - l_1 b_1 = 234$

$\Rightarrow 45 \times 36 - (45 - 2x)(36 - 2x) = 234$

$1620 - (1620 - 90x - 72x + 4x^2) = 234$

$\Rightarrow 1620 - (1620 - 162x + 4x^2) = 234$

$\Rightarrow 1620 - 1620 + 162x - 4x^2 = 234$

$\Rightarrow \quad -4x^2 + 162x = 234$

$\Rightarrow \quad 4x^2 - 162x + 234 = 0$

$\Rightarrow \quad 2x^2 - 81x + 117 = 0$

$\Rightarrow \; 2x^2 - 78x - 3x + 117 = 0$

$\Rightarrow 2x(x - 39) - 3(x - 39) = 0$

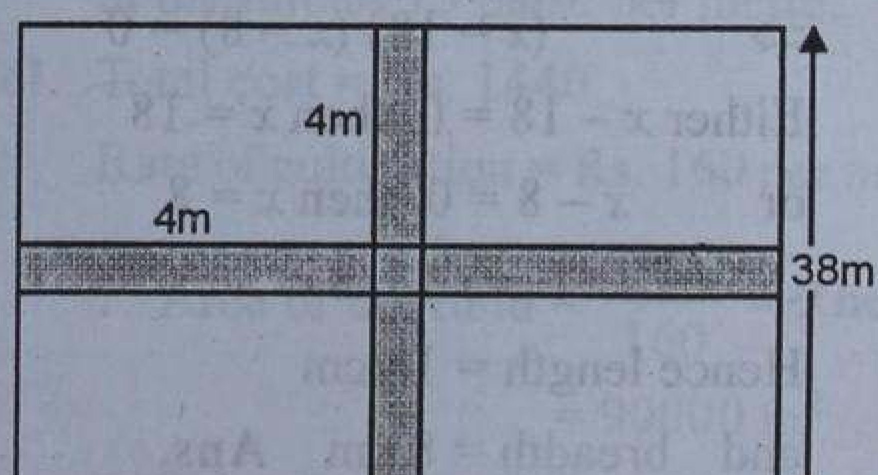
$\Rightarrow \; (2x - 3)(x - 39) = 0$

Either $2x - 3 = 0$ then $x = \dfrac{3}{2}$

or $x - 39 = 0$, then $x = 39$ which is not possible.

$\therefore$ Width of path $= \dfrac{3}{2} \text{ m} = 1{\cdot}5 \text{ m}$ **Ans.**

Q. 12. The adjoining diagram shows two cross paths drawn inside a rectangular field 45 m long and 38 m wide, one parallel to length and the other parallel to breadth. The width of each path is 4 m. Find the cost of gravelling the paths at Rs. 5·60 per m².



Sol. Length of rectangular field $(l) = 45$ m

and breadth $(b) = 38$ m

$\therefore$ Width of path $(a) = 4$ m

$\therefore$ Area of path $= l \times a + b \times a - a \times a$

$= (l + b)\, a - a^2$

$= (45 + 38) \times 4 - (4)^2 \text{ m}^2$

$= 83 \times 4 - 16 = 332 - 16 = 316 \text{ m}^2$

Rate of gravelling the path
$= \text{Rs. } 5{\cdot}60 \text{ per m}^2$
$\therefore$ Total cost $= \text{Rs. } 316 \times 5{\cdot}60$
$= \text{Rs. } 1769{\cdot}60$ **Ans.**

Q. 13. A rectangle of area 144 cm^2 has its length equal to x cm. Write down its breadth in terms of x. Given that its perimeter is 52 cm, write down an equation in x and solve it to determine the dimensions of the rectangle.

Sol. Area of rectangle $= 144 \text{ cm}^2$
Perimeter $= 52$ cm
Length of rectangle $= x$

$$\therefore \text{Breadth} = \frac{\text{Area}}{\text{Length}} = \frac{144}{x} \text{ cm}$$

$$\text{or Breadth} = \frac{\text{Perimeter} - 2 \text{ Length}}{2}$$

$$= \frac{52 - 2x}{2} = (26 - x) \text{ cm}$$

Now Area = length × breadth

$$\Rightarrow \quad 144 = x \times (26 - x)$$
$$\Rightarrow \quad 144 = 26x - x^2$$
$$\Rightarrow \quad x^2 - 26x + 144 = 0$$
$$\Rightarrow \quad x^2 - 18x - x + 144 = 0$$
$$\therefore \quad x(x - 18) - 8(x - 18) = 0$$
$$\Rightarrow \quad (x - 18)(x - 8) = 0$$

Either $x - 18 = 0$, then $x = 18$
or $x - 8 = 0$, then $x = 8$
$\therefore \quad x = 18, 8$
Hence length = 18 cm
and breadth = 8 cm **Ans.**

Q. 14. The perimeter of a rectangular plot is 130 m and its area is 1000 m^2. Take the length of the plot as x metres. Use the perimeter to write the value of breadth in terms of x. Use the values of length, breadth and area to write an equation in x. Solve the equation and calculate the length and breadth of the plot.

Sol. Area of rectangular field $= 1000 \text{ m}^2$
and perimeter $= 130$ m
Let length of the field $= x$ m

$$\therefore \text{Breadth} = \frac{130 - 2x}{2} = (65 - x)\, m$$

$$\therefore \quad \text{Area} = l \times b$$
$$\Rightarrow 1000 = x(65 - x)$$
$$\Rightarrow 1000 = 65x - x^2$$
$$\therefore \quad x^2 - 65x + 1000 = 0$$
$$\Rightarrow x^2 - 40x - 25x + 1000 = 0$$
$$\Rightarrow x(x - 40) - 25(x - 40) = 0$$
$$\Rightarrow \quad (x - 40)(x - 25) = 0$$

Either $x - 40 = 0$, then $x = 40$
or $x - 25 = 0$, then $x = 25$
$\therefore$ Length of the field = 40 m
and breadth = 25 m **Ans.**

Q. 15. If the length of a rectangle is increased by 10 cm and the breadth decreased by 5 cm, the area remains unchanged. If the length is decreased by 5 cm and the breadth is increased by 4 cm, even then the area remains unchanged. Find the dimensions of the rectangle.

Sol. Let length of a rectangle $= x$ cm
and breadth $= y$ cm
and area $= l \times b = xy \text{ cm}^2$
In first case
length $= (x + 10)$ cm
and breadth $= (y - 5)$ cm.

$$\therefore (x + 10)(y - 5) = xy$$
$$\Rightarrow xy - 5x + 10y - 50 = xy$$
$$\Rightarrow \quad -5x + 10y - 50 = 0$$
$$\Rightarrow \quad x - 2y + 10 = 0$$
$$\Rightarrow \quad x - 2y = 10 \quad \ldots(i)$$

In second case,
length $= x - 5$
and breadth $= y + 4$

$$\therefore \quad (x - 5)(y + 4) = xy$$
$$\Rightarrow xy + 4x - 5y - 20 = xy$$
$$\Rightarrow \quad 4x - 5y = 20 \quad \ldots(ii)$$

Multiply (i) by 5 and (ii) by 2,

$$5x - 10y = -50$$
$$8x - 10y = 40$$
$$- \quad + \quad -$$

Subtracting, $-3x = -90 \Rightarrow x = 30$

Substituting the value of x in (i)

$$30 - 2y = -10$$
$$\Rightarrow \quad -2y = -10 - 30 = -40$$
$$\therefore \quad y = \frac{-40}{-2} = 20$$

Hence length of the rectangle = 30 cm

and breadth = 20 cm **Ans.**

Q. 16. A room is 13 m long and 9 m wide. Find the cost of carpeting the room with a carpet 75 cm wide and Rs. 12·50 per metre.

Sol. Length of room (l) = 13 m

and width (b) = 9 m

$\therefore$ Area of the floor $= l \times b$

$= 13 \text{ m} \times 9 \text{ m}$

$= 117 \text{ m}^2$

Width of carpet $= 75 \text{ cm} = \frac{75}{100} = \frac{3}{4} \text{ m}$

$\therefore$ Length of carpet = Area ÷ Width

$= 117 \div \frac{3}{4} = \frac{117 \times 4}{3} \text{ m}$

$= 156$ m

Rate of carpet = Rs. 12·50 per m

$\therefore$ Total cost = Rs. 156 × Rs. 12·50

= Rs. 1950 **Ans.**

Q. 17. A rectangular courtyard 3·78 m long and 5·25 m broad is to be paved exactly with square tiles, all of the same size. What is the largest size of such a tile ? Also find the number of tiles.

Sol. Length of courtyard = 3·78 m = 378 cm

Width of courtyard = 5·25 m = 525 cm

$\therefore$ Largest size of the square tile used

= H.C.F. of 378 and 525

= 21 cm

$$\begin{array}{l} 378\,)\,525\,(\,1 \\ \quad\;\; 378 \\ \hline 147\,)\,378\,(\,2 \\ \quad\;\; 294 \\ \hline 84\,)\,147\,(\,1 \\ \quad\;\; 84 \\ \hline 63\,)\,84\,(\,1 \\ \quad\;\; 63 \\ \hline 21\,)\,63\,(\,3 \\ \quad\;\; 63 \\ \hline \quad\;\; \times \\ \hline \end{array}$$

Area of the courtyard $= l \times b$

$= 378 \times 525 \text{ cm}^2$

Area of one tile $= 21 \times 21 \text{ cm}^2$

$\therefore$ No. of tiles

$$= \frac{\text{Total area of the courtyard}}{\text{area of one tile}}$$

$$= \frac{378 \times 525}{21 \times 21} = 18 \times 25$$

$= 450$

Q. 18. The cost of cultivating a square field at the rate of 160 per hectare is Rs. 1440. Find the cost of putting fence around it at the rate of 75 paise per metre.

Sol. Total cost = Rs. 1440

Rate of cultivation = Rs. 160 per hectare

$\therefore$ Area of the field $= \frac{1440}{160} = 9$ hectare

$= 90000 \text{ m}^2$

$\therefore$ Side of square field

$= \sqrt{90000} = 300$ m

and perimeter of the square field $= 4a$

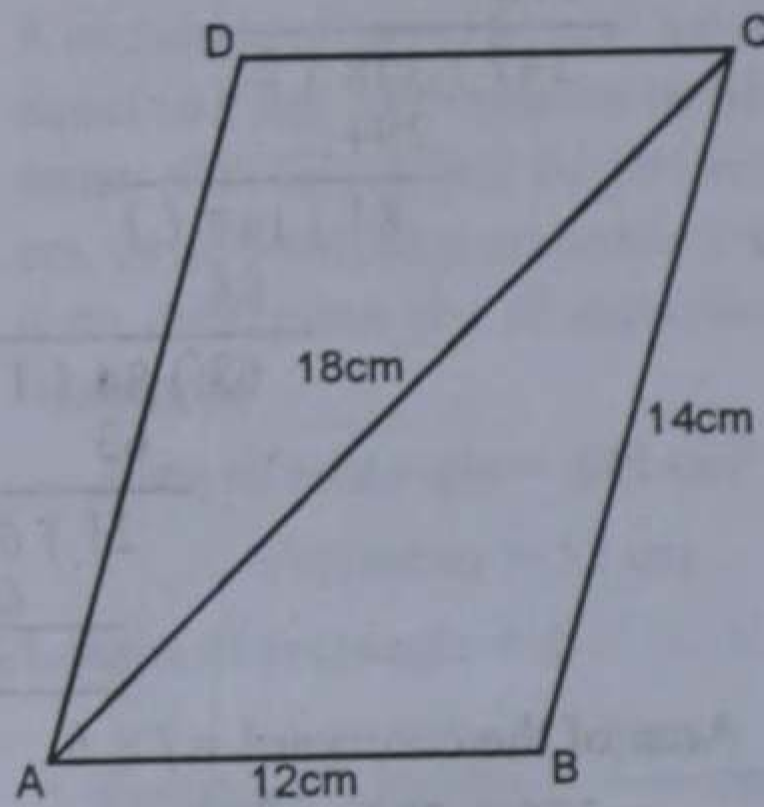
$= 4 \times 300 = 1200$ m

Rate of fencing = 75 paise per metre

$\therefore$ Total cost $= \text{Rs.} \frac{1200 \times 75}{100}$

= Rs. 900 **Ans.**

Q. 19. Find the area of a parallelogram, if its two adjacent sides are 12 cm and 14 cm and if the diagonal connecting their ends is 18 cm.



Sol. $\because$ Diagonals bisect the parallelogram into two triangle of equal area.

$\therefore$ Area of parallelogram ABCD = 2 area of ΔABC

Now sides of ΔABC are 12 cm, 14 cm, 18 cm

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{12+14+18}{2}$$

$$= \frac{44}{2} = 22$$

and Area of ΔABC

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{22(22-12)(22-14)(22-18)}$$

$$= \sqrt{22 \times 10 \times 8 \times 4}$$

$$= \sqrt{11 \times 2 \times 2 \times 5 \times 2 \times 2 \times 2 \times 2 \times 2}$$

$$= \sqrt{2^2 \times 2^2 \times 2^2 \times 110}$$

$$= 2 \times 2 \times 2\sqrt{110} = 8\sqrt{110} \text{ cm}^2$$

$\therefore$ Area of parallelogram ABCD

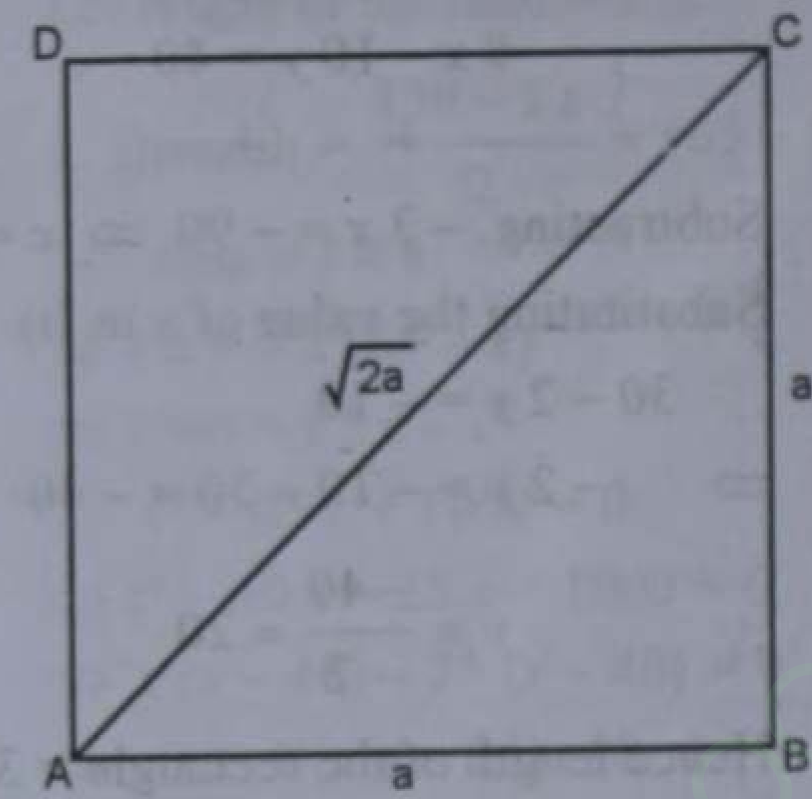
$= 2 \times$ area of ΔABC

$$= 2 \times 8\sqrt{110} = 16\sqrt{110} \text{ cm}^2$$

$$= 16 \times 10{\cdot}488 = 167{\cdot}81 \text{ cm}^2$$

$= 167{\cdot}8 \text{ cm}^2$ **Ans.**

Q. 20. Find the length of the diagonal of a square of area 200 cm^2.



Sol. Let side of square = a

$\therefore$ Area = a^2

and length of diagonal = $\sqrt{2}\,a$

$$= \sqrt{2a^2}$$

$$= \sqrt{2 \times \text{Area}}$$

$= \sqrt{2 \times 200} = \sqrt{400} = 20 \text{ cm}$ **Ans.**

Q. 21. The area of a square field is 8 hectares. How long would a man take to cross it diagonally by walking at the rate of 4 kmph ?

Sol. Area of square field = 8 hectare

$$= 80{,}000 \text{ m}^2$$

$\therefore$ Length of its diagonal = $\sqrt{2 \times \text{area}}$

$$= \sqrt{2 \times 80000} \text{ m} = \sqrt{160000} = 400 \text{ m}$$

Speed of a man = 4 kmph

$\therefore$ Time taken by him

$$= \frac{400 \times 1}{40 \times 1000} = \frac{1}{10} \text{ hour}$$

$= \frac{1}{10} \times 60 = 6$ minutes **Ans.**

Q. 22. Find the area and perimeter of a square plot of land whose diagonal is 15 m. Give your answer correct to two decimal places.

Sol. Diagonal of a square plot = 15 m

$\therefore$ Its side $= \dfrac{\text{Diagonal}}{\sqrt{2}} = \dfrac{15}{\sqrt{2}}$ m

(*i*) Area $= (\text{side})^2 = \left(\dfrac{15}{\sqrt{2}}\right)^2$

$= \dfrac{225}{2} = 112{\cdot}5\,\text{m}^2$

(*ii*) and perimeter $= 4a = \dfrac{4 \times 15}{\sqrt{2}} = \dfrac{60}{\sqrt{2}}$ m

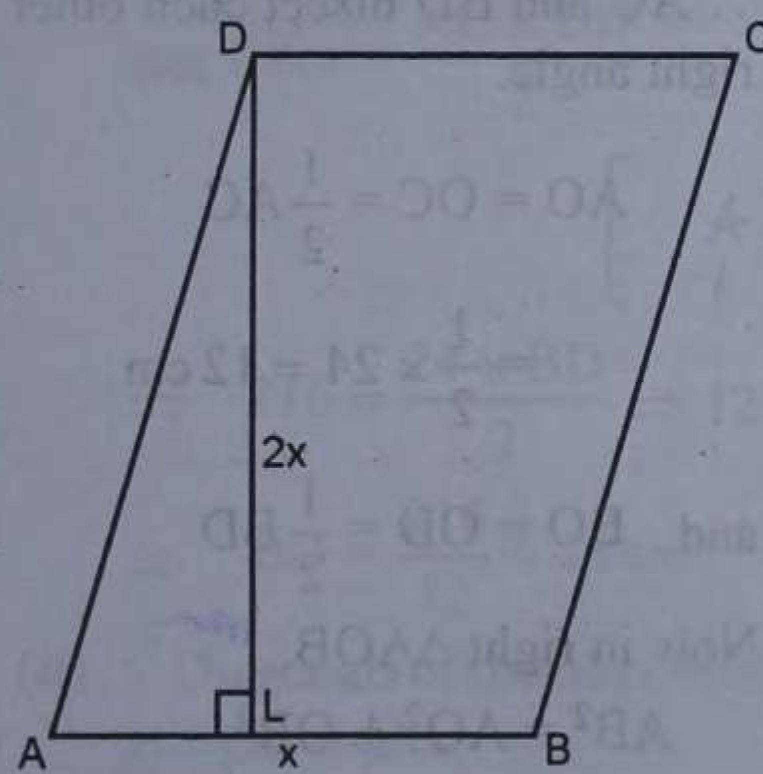
$= \dfrac{60 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \dfrac{60\sqrt{2}}{2} = 30\sqrt{2}$ m

$= 30\,(1{\cdot}414) = 42{\cdot}42$ m **Ans.**

Q. 23. The area of a parallelogram is 338 m². If its altitude is twice the corresponding base, determine the base and the altitude.

Sol. Area of parallelogram = 338 m²

Let base of the parallelogram = x m



$\therefore$ Altitude $= 2x$

and Area = Base × Altitude

$= x \times 2x = 2x^2$

$\therefore \quad 2x^2 = 338$

$\Rightarrow \quad x^2 = \dfrac{338}{2} = 169 = (13)^2$

$\therefore \quad x = 13$

$\therefore$ Base $= x = 13$ m and

altitude $= 2x = 2 \times 13$ m

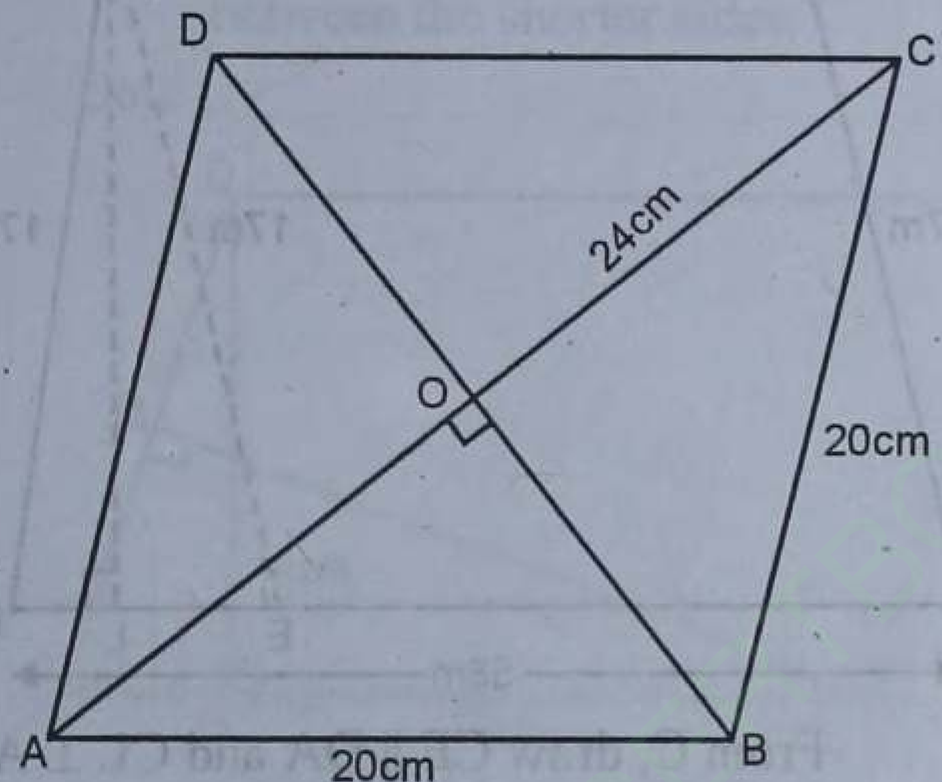
$= 26$ m **Ans.**

Q. 24. Find the area of a rhombus one side of which measures 20 cm and one of whose diagonals is 24 cm.

Sol. In rhombus ABCD,

each side = 20 cm and

one diagonal AC = 24 cm



$\because$ The diagonals of a rhombus bisect each other at right angles.

$\therefore$ AC and BD bisect each other at O at right angle.

Hence AOB is a right triangle in which AB = 20 cm,

$AO = \dfrac{1}{2}AC = \dfrac{1}{2} \times 24 = 12$ cm

But $AB^2 = AO^2 + OB^2$

$\Rightarrow (20)^2 = (12)^2 + OB^2$

$\Rightarrow \quad 400 = 144 + OB^2$

$\Rightarrow OB^2 = 400 - 144 = 256 = (16)^2$

$\therefore \quad OB = 16$ cm

$\therefore$ Diagonal $BD = 2 \times OB$

$= 2 \times 16 = 32$ cm

Now area of rhombus

$= \dfrac{\text{Product of diagonals}}{2}$

$= \dfrac{24 \times 32}{2} = 384\,\text{cm}^2$ **Ans.**

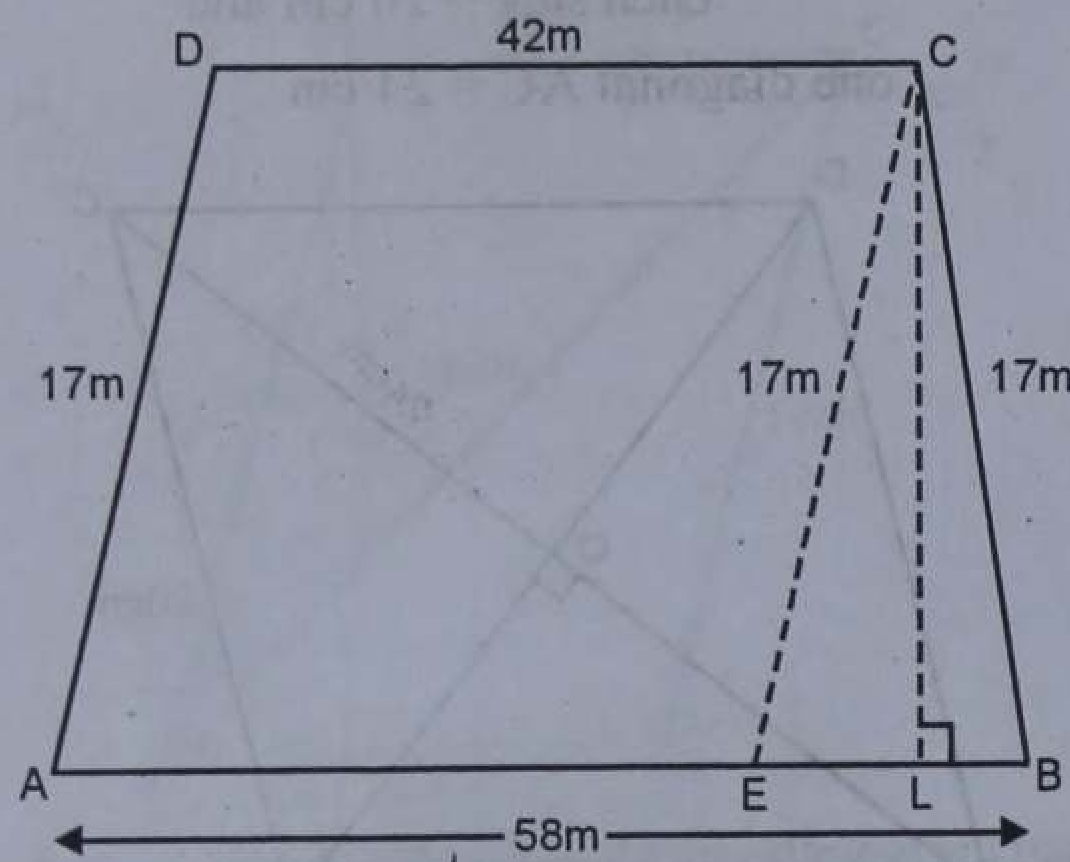
Q. 25. The two parallel sides of a trapezium are 58 m and 42 m long. The other two sides are equal, each being 17 m. Find its area.

Sol. In trapezium ABCD

AB || DC

and AB = 58 m, CD = 42 m

BC = AD = 17 m



From C, draw CE || DA and CL ⊥ AB meeting AB in E such that

AE = CD = 42 m and EB = AB – AE

$\Rightarrow$ EB = 58 – 42 = 16 m

CE = DA = 17 m

$\because$ ΔECB is an isosceles triangle and CL ⊥ EB

$\therefore$ CL bisects EB at L

$\therefore$ $EL = \frac{1}{2}EB = \frac{1}{2} \times 16 = 8$ m

Now in right ΔCEL,

$CE^2 = CL^2 + EL^2$

(Pythagoras Theorem)

$\Rightarrow (17)^2 = CL^2 + (8)^2$

$\Rightarrow 289 = CL^2 + 64$

$\Rightarrow CL^2 = 289 - 64 = 225 = (15)^2$

$\therefore$ CL = 15 m

Now area of trapezium ABCD

$= \frac{1}{2}(AB + CD) \times CL$

$= \frac{1}{2}(58 + 42) \times 15\,m^2$

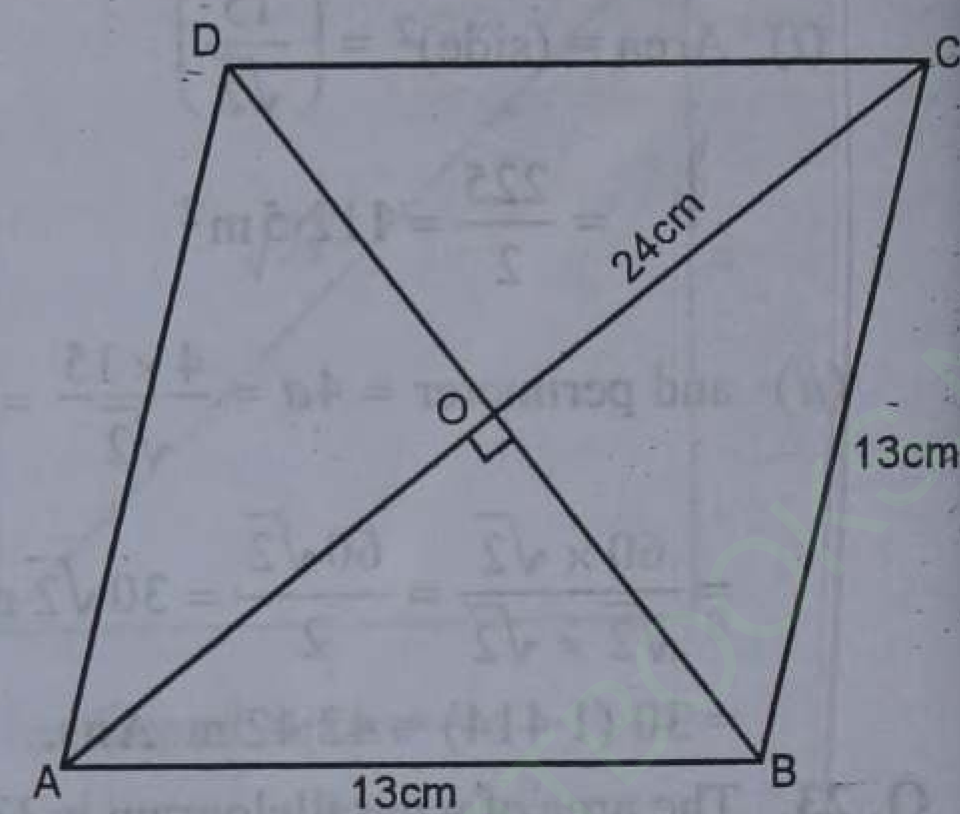
$= \frac{1}{2} \times 100 \times 15\ m^2 = 750\ m^2$ **Ans.**

Q. 26. The perimeter of a rhombus is 52 cm. If one of its diagonals is 24 cm long, find :

(*i*) The length of other diagonal.

(*ii*) The area of the rhombus.

Sol.



Side AB of the rhombus ABCD = 13 cm and diagonal AC = 24 cm

$\because$ Diagonals of a rhombus bisect each other at right angles.

$\therefore$ AC and BD bisect each other at O at right angle.

$\therefore$ $AO = OC = \frac{1}{2}AC$

$= \frac{1}{2} \times 24 = 12$ cm

and $BO = OD = \frac{1}{2}BD$

(*i*) Now in right ΔAOB,

$AB^2 = AO^2 + OB^2$

(Pythagoras Theorem)

$\Rightarrow (13)^2 = (12)^2 + OB^2$

$\Rightarrow 169 = 144 + OB^2$

$\Rightarrow OB^2 = 169 - 144 = 25 = (5)^2$

$\therefore$ OB = 5 cm

$\therefore$ BD = 2 OB = 2 × 5 = 10 cm

(*ii*) Now area of rhombus ABCD

$= \frac{\text{Product of diagonals}}{2} = \frac{AC \times BD}{2}$

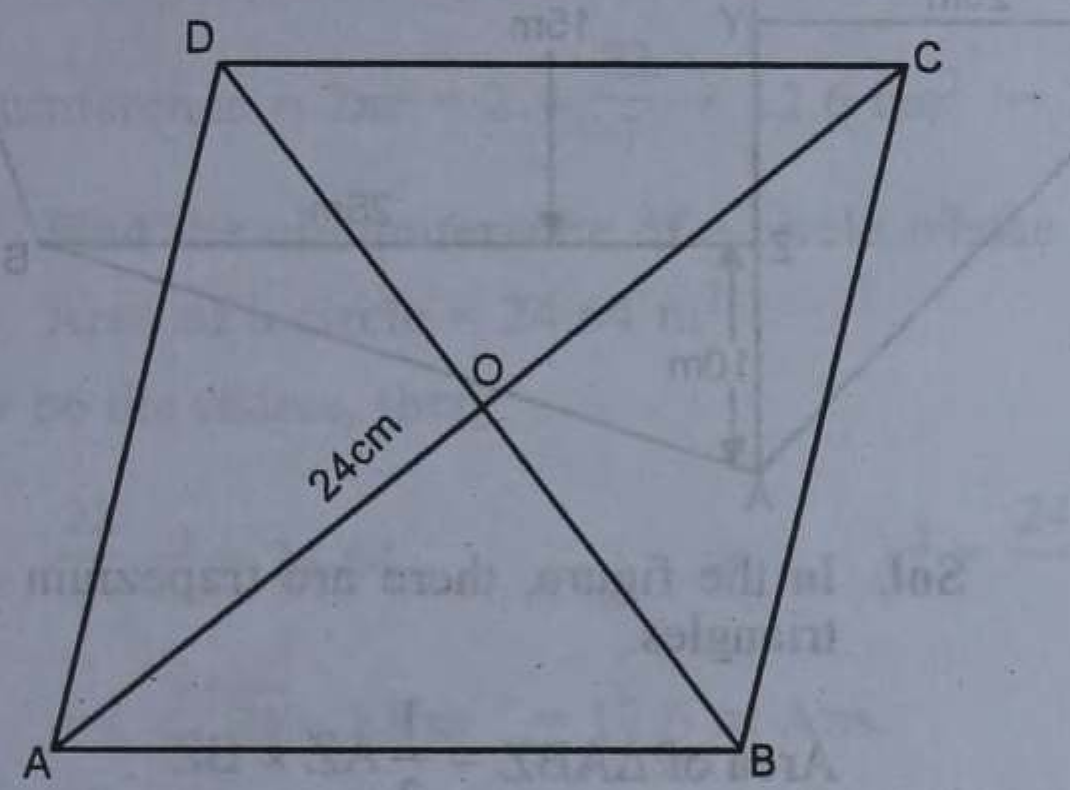
$= \frac{24 \times 10}{2} = 120\,cm^2$ **Ans.**

Q. 27. The area of a rhombus is 216 cm^2 and one of its diagonals measures 24 cm. Find :

(*i*) The length of other diagonal.

(*ii*) The length of each of its sides.

(*iii*) Its perimeter.



Sol. (*i*) Area of rhombus ABCD = 216 cm^2

one diagonal AC = 24 cm

But Area $= \frac{AC \times BD}{2}$

$$\left[\because A = \frac{d_1 \times d_2}{2}\right]$$

$\Rightarrow \quad 216 = \frac{24 \times BD}{2} \Rightarrow 12\,BD = 216$

$\Rightarrow \quad BD = \frac{216}{12} = 18\,cm$

(*ii*) ∵ Diagonals of rhombus bisect each other at right angles.

∴ AC and BD bisect at O at right angles.

$\therefore AO = OC = \frac{1}{2} AC$

$= \frac{1}{2} \times 24 = 12\,cm$

$BO = OD = \frac{1}{2} BD = \frac{1}{2} \times 18 = 9\,cm$

Now in right ΔAOB,

$AB^2 = AO^2 + OB^2 = (12)^2 + (9)^2$

$= 144 + 81 = 225 = (15)^2$

∴ AB = 15

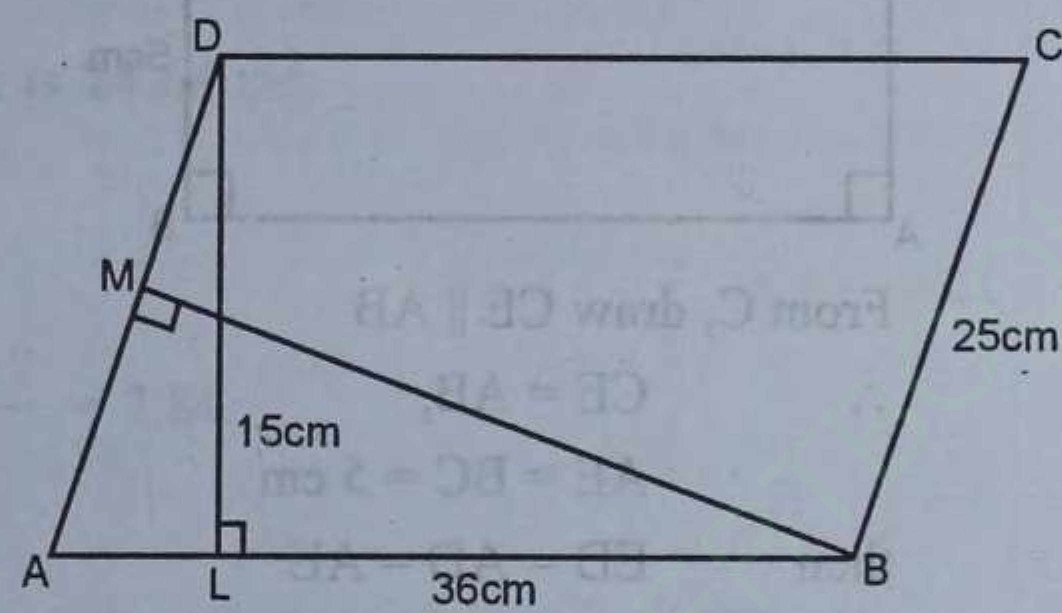
Hence each side of the rhombus = 15 cm.

(*iii*) Perimeter of rhombus = 4 × side

= 4 × 15 = 60 cm **Ans.**

Q. 28. Two adjacent sides of a parallelogram are 36 cm and 25 cm. If the distance between longer sides is 15 cm, find the distance between the shorter sides.

Sol.



In parallelogram ABCD,

AB = 36 cm, BC = 25 cm

DL ⊥ AB, BM ⊥ AD

DL = 15 cm

Now area of parallelogram ABCD

= Base × Altitude = AB × DL

= 36 × 15 = 540 cm^2

Again area of parallelogram ABCD

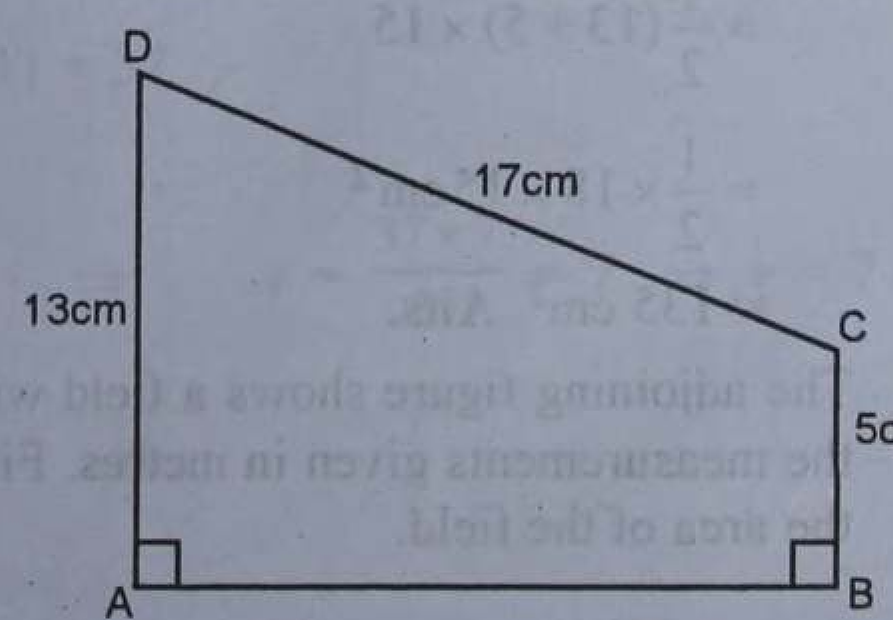
= AD × BM

$\Rightarrow \quad 540 = 25 \times BM$

$\Rightarrow \quad BM = \frac{540}{25}$

$\therefore \quad BM = 21{\cdot}6$ cm **Ans.**

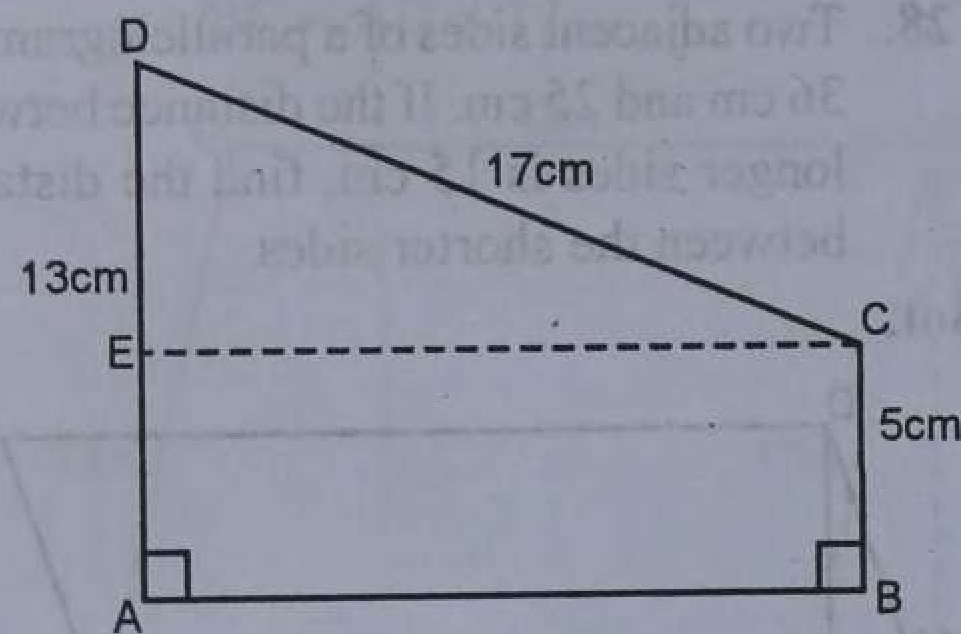
Q. 29. In the given figure, ABCD is a trapezium in which AD = 13 cm, BC = 5 cm, CD = 17 cm and ∠A = ∠B = 90°. Calculate : (*i*) AB (*ii*) Area of trap. ABCD.



Sol. In trapezium ABCD,

AD = 13 cm, BC = 5 cm, CD = 17 cm

$\angle A = \angle B = 90^\circ$



From C, draw CE || AB

$\therefore$ CE = AB,

AE = BC = 5 cm

then ED = AD – AE

= 13 – 5 = 8 cm

$\because$ $\angle A = 90^\circ$

$\therefore \angle DEC = 90^\circ$

(*i*) Now in right ΔDEC,

$CD^2 = DE^2 + EC^2$ (Pythagoras Theorem)

$\Rightarrow (17)^2 = (8)^2 + EC^2$

$\Rightarrow 289 = 64 + EC^2$

$\Rightarrow EC^2 = 289 - 64$

$\Rightarrow EC^2 = 225 = (15)^2$

$\therefore$ EC = 15 cm

But AB = EC $\therefore$ AB = 15 cm

(*ii*) Now area of trapezium ABCD

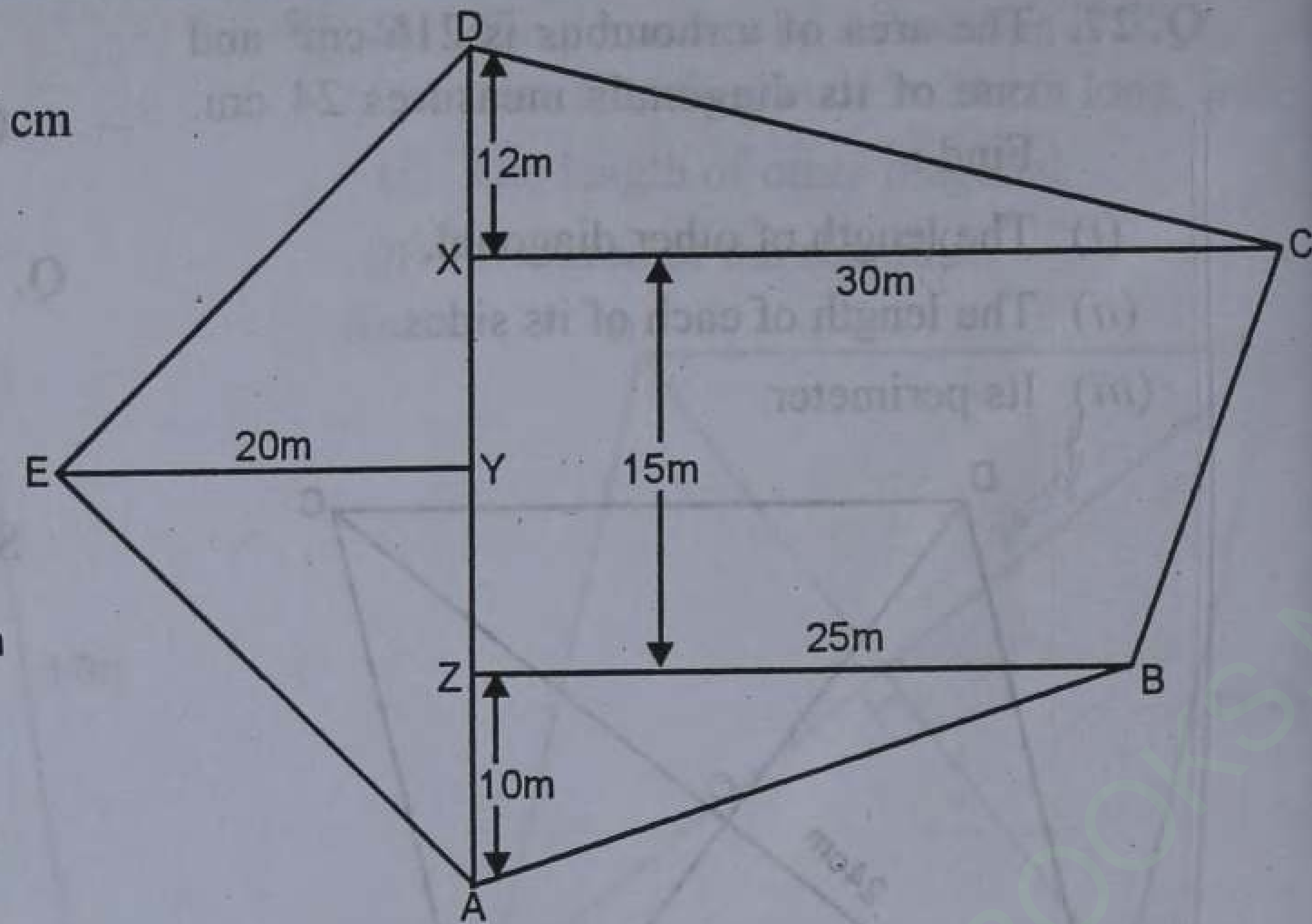
$= \frac{1}{2}(AD + BC) \times AB$

$= \frac{1}{2}(13 + 5) \times 15$

$= \frac{1}{2} \times 18 \times 15 \text{ cm}^2$

$= 135 \text{ cm}^2$ **Ans.**

Q. 30. The adjoining figure shows a field with the measurements given in metres. Find the area of the field.



Sol. In the figure, there are trapezium and triangles.

Area of $\Delta ABZ = \frac{1}{2} AZ \times BZ$

$= \frac{1}{2} \times 10 \times 25 = 125 \text{ m}^2$

Area of trapezium BZXC

$= \frac{1}{2}(BZ + XC) \times ZX$

$= \frac{1}{2}(25 + 30) \times 15 \text{ m}^2$

$= \frac{1}{2} \times 55 \times 15 \text{ m}^2 = \frac{825}{2} = 412{\cdot}5 \text{ m}^2$

Area of $\Delta XCD = \frac{1}{2} CX \times DX$

$= \frac{1}{2} \times 30 \times 12 = 180 \text{ m}^2$

and area of $\Delta AED = \frac{1}{2} \times AD \times EY$

$= \frac{1}{2}(10 + 15 + 12) \times 20 \text{ m}^2$

$= \frac{1}{2} \times 37 \times 20 = 370 \text{ m}^2$

$\therefore$ Area of ABCDE = area of ΔABZ + area of BZXC + area of ΔXCD + area of ΔAED

$= 125 + 412{\cdot}5 + 180 + 370 \text{ m}^2$

$= 1087{\cdot}5 \text{ m}^2$ **Ans.**

EXERCISE 20 (C)

1. Find the area and circumference of the circle whose radius is 12.6 cm.

Sol. Radius (r) = 12.6 cm

$$\therefore \text{ Area} = \pi r^2 = \frac{22}{7} \times (12.6)^2 \text{ cm}^2 = \frac{22}{7} \times 12.6 \times 12.6 \text{ cm}^2 = 498.96 \text{ cm}^2$$

$$\text{Circumference} = 2\pi r = 2 \times \frac{22}{7} \times 12.6 \text{ cm}^2 = 79.2 \text{ cm } \textbf{Ans.}$$

2. Find the circumference of a circle whose area is 24.64 m^2.

Sol. Area of a circle = 24.64 m^2

Let r be the radius, then

$$\Rightarrow \frac{22}{7} r^2 = 24.64 \qquad \Rightarrow \qquad r^2 = \frac{24.64 \times 7}{22} = 7.84$$

$$\therefore \quad r = \sqrt{7.84} = 2.8 \text{ m} \quad = 17.6 \text{ m } \textbf{Ans.}$$

3. The circumference of a circle exceeds its diameter by 42 cm. Find the area of the circle.

Sol. Let r be the radius of the circle

$\therefore$ Diameter = $2r$

and C = $2r + 42$

$$\Rightarrow 2\pi r = 2r + 42 \qquad \Rightarrow \qquad \frac{2 \times 22}{7} r - 2r = 42$$

$$\Rightarrow \frac{44r - 14r}{7} = 42 \qquad \Rightarrow \qquad \frac{30r}{7} = 42 \qquad \Rightarrow \qquad r = \frac{42 \times 7}{30} = \frac{49}{5} \text{ cm}$$

$$\text{Area} = \pi r^2 = \frac{22}{7} \times \frac{49}{3} \times \frac{49}{3} = \frac{7446}{25} = 301.84 \text{ cm}^2 \textbf{ Ans.}$$

4. The difference between the circumference and the radius of a circle is 37 cm. Find the area of the circle.

Sol. Let r be the radius

$\therefore$ $C - r = 37$

$$\Rightarrow 2\pi r - r = 37 \qquad \Rightarrow \qquad r\left(2 \times \frac{22}{7} - 1\right) = 37$$

$$\Rightarrow r\left(\frac{44-7}{7}\right) = 37 \qquad \Rightarrow \qquad r \times \frac{37}{7} = 37 \qquad \Rightarrow \qquad r = \frac{37 \times 7}{37} = 7 \Rightarrow r = 7 \text{ cm}$$

$$\text{Now area} = \pi r^2 = \frac{22}{7} \times 7 \times 7$$

$$= 154 \text{ cm}^2 \textbf{ Ans.}$$

5. Between a square of perimeter 44 cm a circle of circumference 44 cm, which figure has larger area and by how much ?

Sol. In first case,

Perimeter of a square = 44 cm

$\therefore$ Side $= \frac{44}{4} = 11$ cm

and area = (side)$^2 = 11 \times 11 = 121$ cm^2

In second case,

Circumference of a circle = 44 cm

$\therefore 2\pi r = 44 \Rightarrow \frac{2 \times 22}{7} r = 44 \Rightarrow r = \frac{44 \times 7}{44} = 7$ cm

$\therefore$ Area $= \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154$ cm^2

It is clear that area of circle is greater

$\therefore$ Difference $= 154 - 121 = 33$ cm^2 Ans.

Q. 6. The circumference and the area of a circle are numerically equal. Find the diameter of the circle drawn inside a rectangle with sides 18 cm and 14 cm ?

Sol. Let radius of the circle = r

$\therefore$ circumference = $2\pi r$ and area = πr^2

$\because$ The circumference and area of a circle a equal

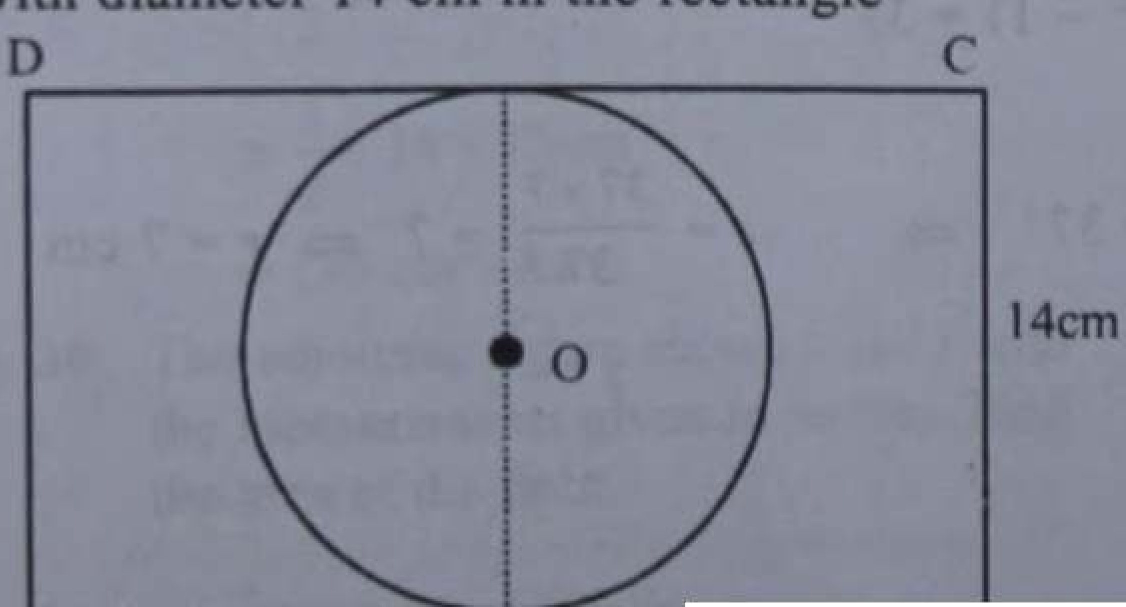
$\therefore 2\pi r = \pi r^2 \Rightarrow 2 = r$

$\therefore$ Diameter $= 2r = 2 \times 2 = 4$ units Ans.

7. Find the area of the largest circle that can be drawn inside a rectangle with sides 18 cm and 14 cm.

Sol. ABCD is rectangle whose length AB = 18 cm and breadth BC = 14 cm

A largest circle can be drawn with diameter 14 cm in the rectangle



$\therefore$ Radius $(r) = \frac{14}{2} = 7$ cm

Now area $= \pi r^2 = \frac{22}{7} \times 7 \times 7$ cm^2

$= 154$ cm^2 Ans.

8. A circular wire of radius 56 cm is bent in the form of a square. Find the area of the square formed.

Sol. Radius of the circular wire $(r) = 56$ cm

$\therefore$ Circumference $= 2\pi r$

$= 2 \times \frac{22}{7} \times 56 = 352$ cm

Now perimeter of square wire formed by the circular wire = 352 cm

$\therefore$ Side $= \frac{352}{4} = 88$ cm

Area of the square $= a^2 = (85)^2$ cm^2

$= 7744$ cm^2 Ans.

9. A wire can be bent in form of a circle of radius 42 cm. If it is bent in the form of a rectangle whose sides are in the ratio 6 : 5, find the area of the rectangle formed.

Sol. Radius of the circular wire = 42 cm

$\therefore$ Circumference $= 2\pi r$

$= 2 \times \frac{22}{7} \times 42 = 264$ cm

Now perimeter of the rectangle formed by the circular wire = 264 cm

$\Rightarrow 2(l + b) = 264$

$\Rightarrow l + b = \frac{264}{2} = 132$

But ratio in $l : b = 6 : 5$

$\therefore$ Length $(l) = \frac{132 \times 6}{6+5} = \frac{132 \times 6}{11} = 72$ cm

and width $(b) = \frac{132 \times 5}{11} = 60$ cm

$\therefore$ Area of the rectangle $= l \times b$

$= 72 \times 60 = 4320$ cm^2 Ans.

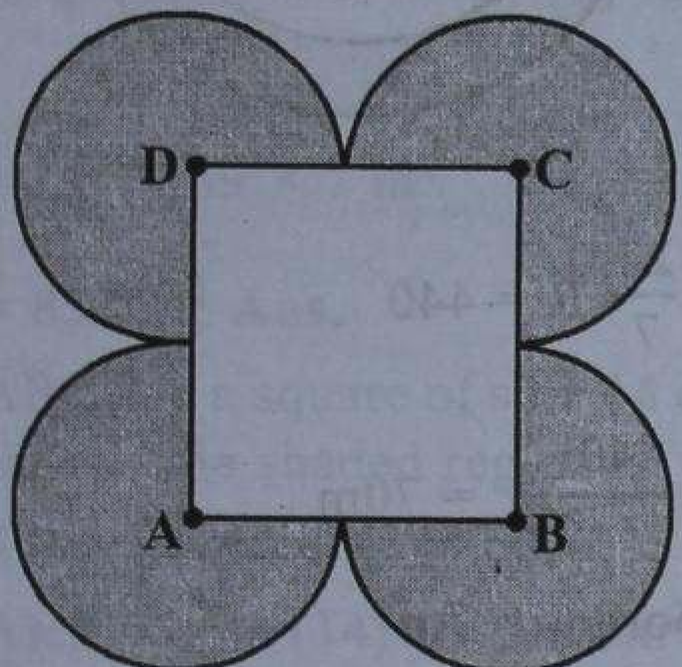
10. A square tank has an area of 1764 m^2. There are four semicircular plots arround it. Find the cost of turfing the plots at Rs. 15 per m^2.

Sol. Area of the square tank = 1764 m^2

$\therefore$ Side = $\sqrt{\text{Area}} = \sqrt{1764}$ m = 42 m

Now diameter of each semicircular lower plots on its sides = 42 m

$\therefore$ Radius $(r) = \dfrac{42}{2} = 21$ m



$\therefore$ Area of four semicular plots $= 4 \times \dfrac{1}{2}\pi r^2$

$= 2 \times \dfrac{22}{7} \times 21 \times 21 \text{ m}^2 = 2772 \text{ m}^2$

Rate of turfing = Rs. 15 per m^2

$\therefore$ Total cost = Rs. 15 × 2772

= Rs. 41580 **Ans.**

11. The cost of fencing a circular field at Rs. 11.50 per metre is Rs. 2530. The field is to be ploughed at the rate of Rs. 6.50 per m^2. Find the cost of ploughing the field.

Sol. Cost of fencing the circular field

= Rs. 2530

Rate of fencing = Rs. 11.50 per metre

$\therefore$ Circumference of the field $= \dfrac{2530}{11.50}$

$= \dfrac{2530}{11.50} \times 100\text{m} = \dfrac{2530 \times 100}{1150}$ m

= 220 m

Let r be the radius of the field

$\therefore \quad 2\pi r = 220$

$\Rightarrow \quad \dfrac{2 \times 22}{7} r = 220$

$\Rightarrow \quad r = \dfrac{220 \times 7}{2 \times 22} = 35$ m

and area of the field = πr^2

Area of the field = πr^2

$= \dfrac{22}{7} \times 35 \times 35 \text{ m}^2$

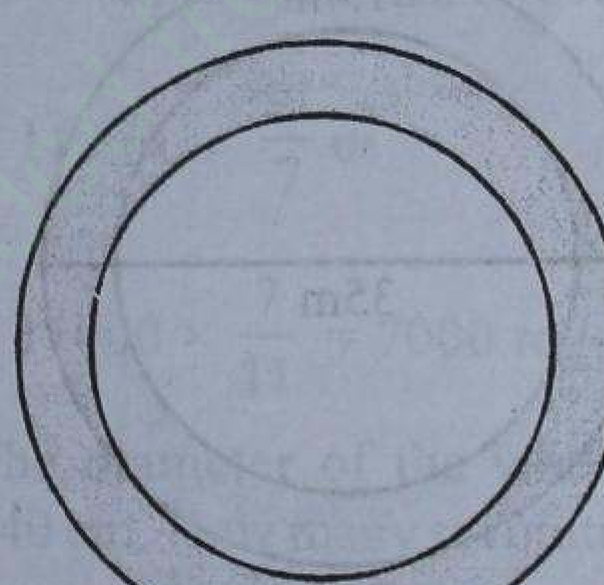
$= 3850 \text{ m}^2$

Rate of ploughing the field = Rs. 6.50 per m

$\therefore$ Total cost = Rs. 3850 × 6.50

= Rs. 25025 Ans.

12. The areas of two concentric circles forming or ring are 154 cm^2 and 616 cm^2. Find the width of the ring.



Sol. Area of outer circle = 616 cm^2

$\therefore$ Outer radius $= \sqrt{\dfrac{\text{Area}}{\pi}}$

$= \sqrt{\dfrac{616 \times 7}{22}}$ cm

$= \sqrt{196} = 14$ cm

Area of inner circle = 154 cm^2

$\therefore$ Inner radius $= \sqrt{\dfrac{\text{Area}}{\pi}}$

$= \sqrt{\frac{154 \times 7}{22}}$ cm

$= \sqrt{49} = 7$ cm

$\therefore$ Width of the ring so formed

$= 14 - 7 = 7$ cm Ans.

13. A circular ground whose diameter is 35 metres has a 1.4 m broad garden around it. What is the area of the garden is square meters.

Sol. Diameter of the circular ground = 35 m

$\therefore$ Radius (R) $= \frac{35}{2}$ m

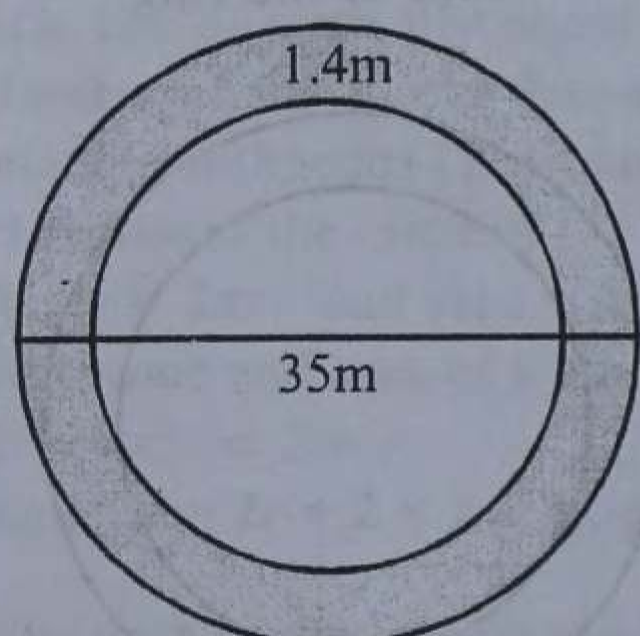
Width of garden around it = 1.4 cm

$\therefore$ Inner radius = (r)

$= (17.5 - 1.4)$ m

$= 16.1$ m

Now, area of the garden



$= \pi R^2 - \pi r^2 = \pi (R^2 - r^2)$

$= \pi (R + r)(R - r)$

$= \frac{22}{7}$ [17.5 + 16.1] [17.5 − 16.1]

$= \frac{22}{7} \times 33.6 \times 1.4$ m^2

$= 147.84$ m^2 Ans.

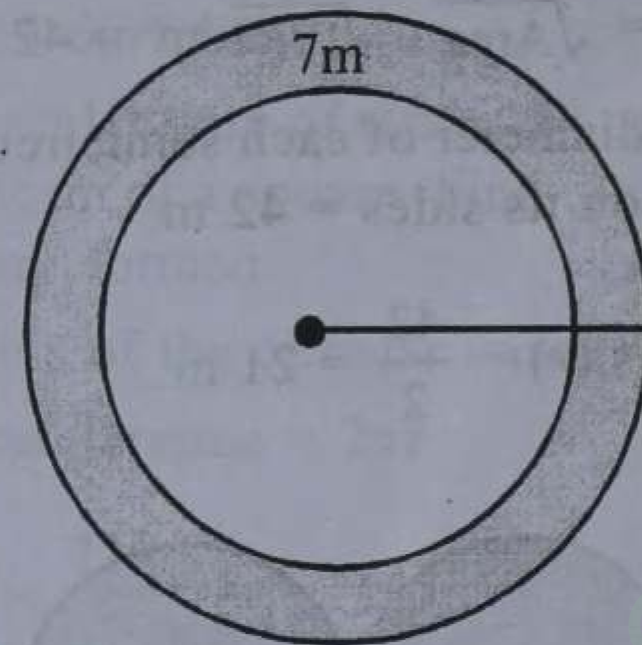
14. A circular garden has circumference of 440 m. There is a 7 m wide border inside the garden along its periphery. Find the area of the border.

Sol. Width of border = 7cm

Circumference of the garden = 440 m

Let R be the radius of the garden

Then $2\pi R = 440$



$\Rightarrow 2 \times \frac{22}{7} R = 440$

$\Rightarrow R = \frac{440 \times 7}{2 \times 22} = 70$m

Width of border = 7m

$\therefore$ Inner radius $(r) = 70 - 7 = 63$ m

Now area of border $= \pi [R^2 - r^2]$

$= \pi (R + r)(R - r)$

$= \frac{22}{7} (70 + 63)(70 - 63)$

$= \frac{22}{7} \times 133 \times 7$ m^2

$= 2926$ m^2 Ans.

15. A rope by which a cow is tethered is increased from 16m to 23 m. How much additional ground does it have now to graze ?

Sol. Let length of rope in first case (r)

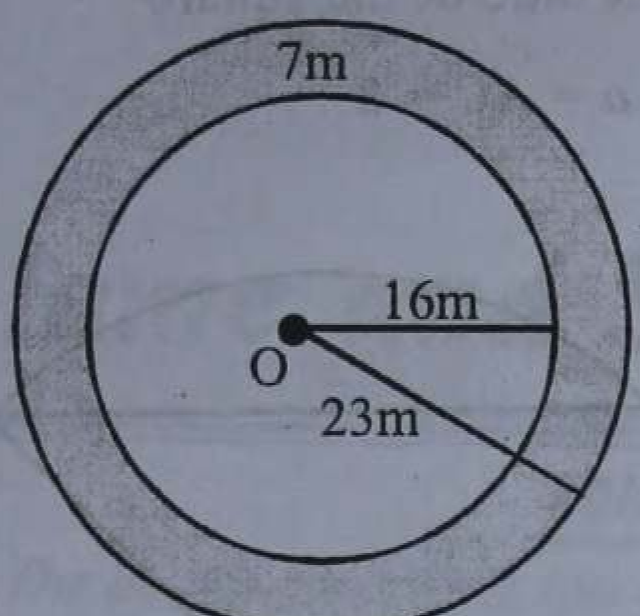
$\therefore r = 16$ m

and by increasing it R = 23 m

$\therefore$ Area of additional ground to be grazed

$= \pi(R^2 - r^2)$

$= \frac{22}{7} (23^2 - 16^2)$ m^2



$= \frac{22}{7}\ (23 + 16)\ (23 - 16)\ m^2$

$= \frac{22}{7} \times 39 \times 7\ m^2$

$= 858\ m^2$ **Ans.**

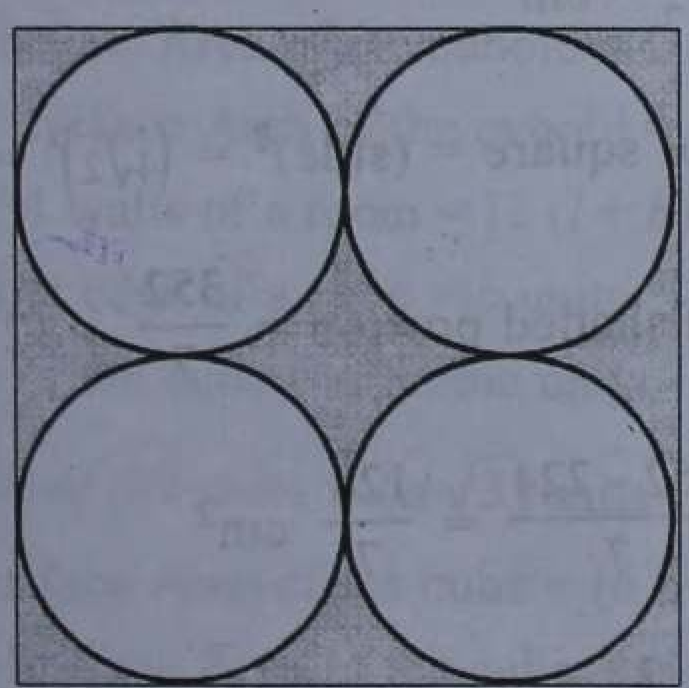
16. ABCD is a square of side 14 cm. Find the area of the shaded region.

Sol. Side of square = 14 cm

$\therefore$ Area = $a^2 = (14)^2 = 196\ cm^2$

Diameter of each circle drawn in it = $\frac{14}{2}$

= 7 cm



14cm

$\therefore$ Radius $(r) = \frac{7}{2}$ cm

Now area of each circle = πr^2

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\ cm^2$

$= \frac{77}{2}\ cm^2$

and area of 4 circle = $\frac{77}{2} \times 4 = 154\ cm^2$

$\therefore$ Area of shaded portion = $(196 - 154)\ cm^2$

$= 42\ cm^2$ Ans.

17. The radius of a wheel is 0.25 m. Find the number of revolutions it will make travel a distance of 11 km.

Sol. Radius of wheel $(r) = 0.25$ m

$\therefore$ Circumference = $2\pi r$

$= 2 \times \frac{22}{7} \times 0.25$

$= \frac{11}{7}$ m

$\therefore$ Wheel will travel in one revolution = $\frac{11}{7}$ m

$\therefore$ In 11 km, it will make revolutions

$= 11\ km \div \frac{11}{7}\ m$

$= 11000 \times \frac{7}{11} = 7000$ revolutions

18. The diameter of the wheels of a bus is 140 cm. How many revolutions per minute must a wheel make in order to move at a speed of 66 km per hour.

Sol. Diameter of wheel = 140 cm.

$\therefore$ Radius $(r) = \frac{140}{2} = 70$ cm

Circumference = $2\pi\ r^2 = 2 \times \frac{22}{7} \times 70$

$= 440\ cm = \frac{440}{100}\ m$

Speed of bus = 66 km per hour.

$\therefore$ Number of revolutions in 1 hour

$$= \frac{66 \times 1000 \times 100}{440}$$

$$= 15000$$

$\therefore$ Revolution in 1 minute $= \dfrac{15000}{60}$

$= 250$ **Ans.**

19. A bicycle wheel makes 5000 revolutions in moving 11 km. Find the diameter of the wheel.

Sol. Number of revolutions = 5000

Distance = 11 km = 11000 m

$\therefore$ Distance covered in one revolution

$$= \frac{11000}{5000} = \frac{11}{5} \text{ m}$$

$\therefore$ Circumference of wheel of the cycle

$$= \frac{11}{5} \text{ m} = \frac{11}{5} \times 100 = 220 \text{ cm}$$

Let d be the diameter of the wheel

$\because \quad \pi d = 220$

$\Rightarrow \quad \dfrac{22}{7} d = 220$

$\Rightarrow \quad d = \dfrac{220 \times 7}{22} = 70\text{cm}$ **Ans.**

20. A square is inscribed in a circle whose radius is 4 cm. Find the area of the portion between the circle and the square.

Sol. A square ABCD is inscribed in a circle with centre O.

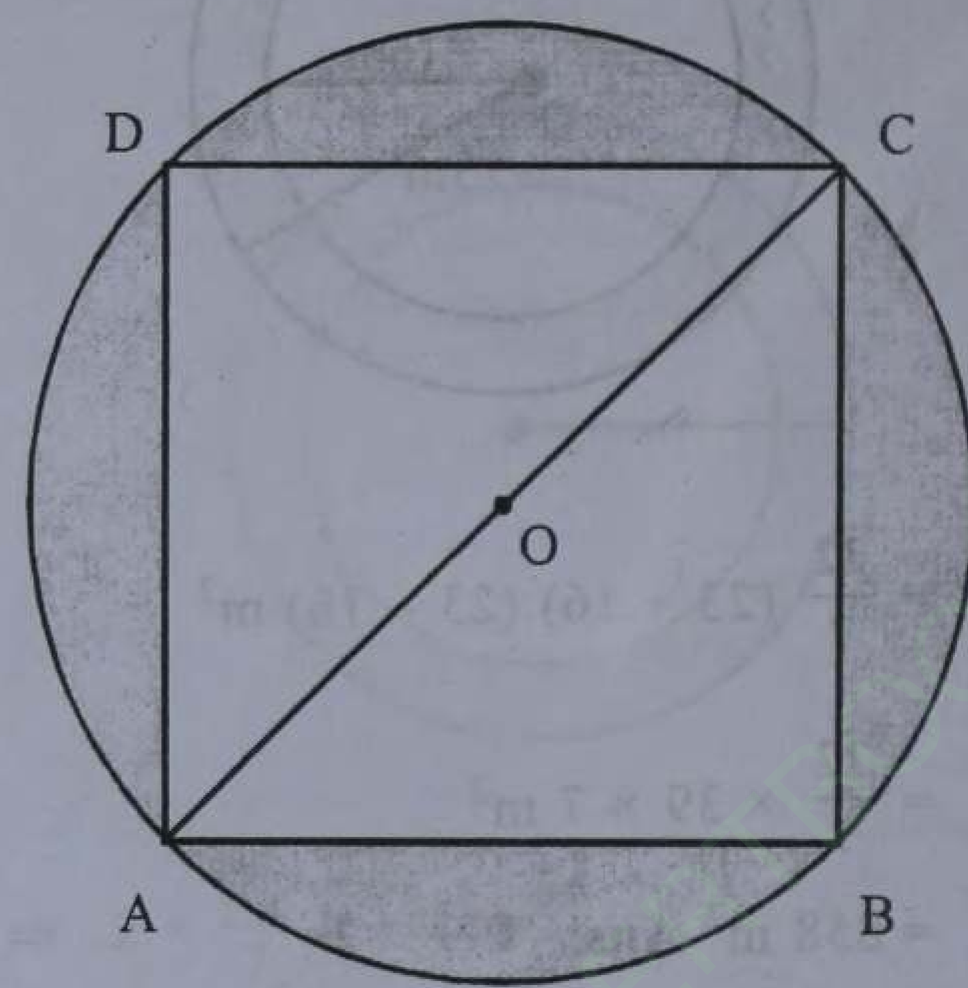
Radius of circle = 4 cm

$\therefore$ Diameter $= 4 \times 2 = 8$ cm

$\therefore$ Diagonal of square = 8 cm

Let a be the side of the square

$\therefore \quad \sqrt{2}\, a = AC = 8$



$$a = \frac{8}{\sqrt{2}} = \frac{8 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{8 \times \sqrt{2}}{2} = 4\sqrt{2} \text{ cm}$$

Now area of circle $= \pi r^2 = \dfrac{22}{7} \times 4 \times 4 \text{ cm}^2$

$$= \frac{352}{7} \text{ cm}^2$$

and area of square $= (\text{side})^2 = (4\sqrt{2})^2 = 32 \text{ cm}^2$

$\therefore$ Area of shaded portion $= \dfrac{352}{7} - 32$

$$= \frac{352 - 224}{7} = \frac{128}{7} \text{ cm}^2$$

$$= 18\frac{2}{7} = 18.28 \text{ cm}^2 \textbf{ Ans.}$$