16

Area

POINTS TO REMEMBER

1. **Equal figures :** Two plane figures having equal area are called equal figures.
2. **Congruent figures :** Two plane figures having the same shape and size are called congruent figures. But two plane figures having equal areas need not be congruent.
3. **Results on Area of polygon regions**

(*i*) Parallelograms on the same base and between the same parallels are equal in area.

(*ii*) The area of a parallelogram is equal to the area of the rectangle on the same base and of the same altitude *i.e.* between the same parallels.

(*iii*) Triangles are the same base and between the same parallels are equal in area.

4. **Some more results :**

(*i*) Area of a || gm = Base × height

(*ii*) Area of a triangle $= \frac{1}{2} \times$ Base × height

(*iii*) Area of trapezium $= \frac{1}{2}$ (sum of parallel sides) × height

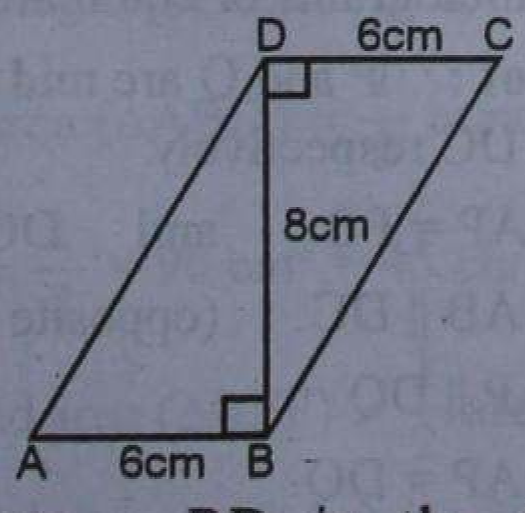
(*iv*) Area of rhombus $= \frac{1}{2} \times$ Product of diagonals

5. (*i*) If a triangle and a parallelogram are on the same base and between the same parallels, then the area of triangle is half of the area of the parallellogram.

(*ii*) Parallelograms on equal bases and between the same parallels are equal in area.

EXERCISE 16

Q. 1. In the adjoining figure, BD is a diagonal of quad. ABCD. Show that ABCD is a parallelogram and calculate the area of || gm ABCD.



Sol. Given : BD is the diagonal of quadrilateral ABCD

AB = 6 cm, CD = 6 cm and BD = 8 cm

$\angle ABD = \angle BDC = 90^\circ$

To prove : (*i*) ABCD is a parallelogram.

(*ii*) Find the area of ||gm ABCD.

Proof : $\because \angle ABD = \angle BDC$ (each $= 90^\circ$)

But these are alternate angles.

$\therefore$ AB || DC

But AB = DC = 6 cm

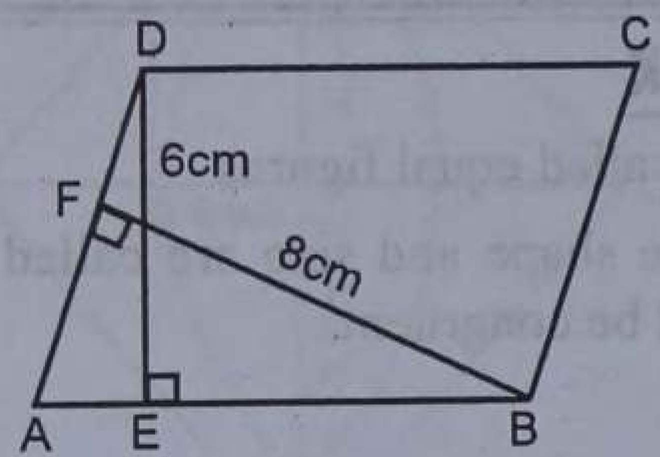
$\therefore$ ABCD is a || gm.

Now Area = Base × Altitude

$= 6 \times 8 \text{ cm}^2 = 48 \text{ cm}^2$ **Ans.**

Q. 2. In a ||gm ABCD, it is given that AB = 16 cm and the altitudes corresponding to the sides AB and AD are 6 cm and 8 cm respectively.

Find the length of AD.



Sol. In || gm ABCD, AB = 16 cm, altitudes on AB and AD are DE and BF are drawn and DE = 7 cm, BF = 8 cm

Area of || gm ABCD = Base × altitude

$= AB \times DE$

$= 16 \times 6 = 96 \text{ cm}^2$ ...(*i*)

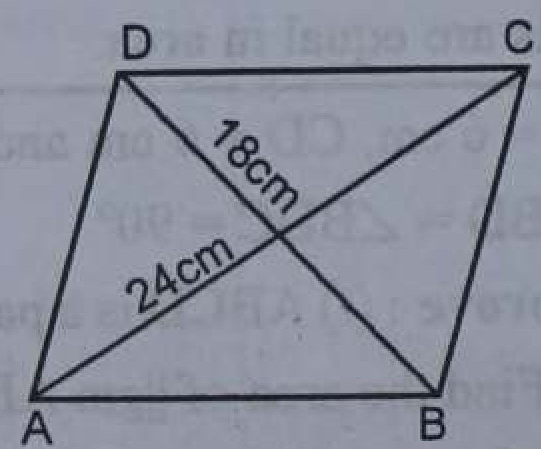
Again area of || gm = AD × BF

$= AD \times 8 \text{ cm}^2$ ...(*ii*)

From (*i*) and (*ii*)

$8\ AD = 96 \Rightarrow AD = \frac{96}{8} = 12$ cm **Ans.**

Q. 3. Find the area of a rhombus, the lengths of whose diagonals are 18 cm and 24 cm respectively.



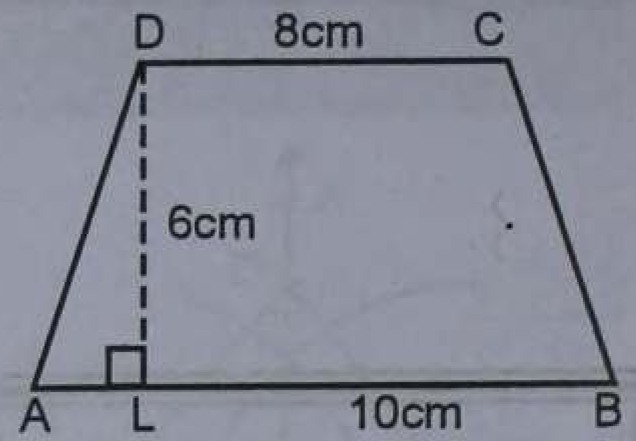
Sol. Let the first diagonal of rhombus $(d_1) = 18$ cm

and second diagonal $(d_2) = 24$ cm

$\therefore$ Area $= \frac{d_1 \times d_2}{2} = \frac{18 \times 24}{2} \text{cm}^2$

$= 216 \text{ cm}^2$ **Ans.**

Q. 4. Find the area of a trapezium whose parallel sides measure 10 cm and 8 cm respectively and the distance between these sides is 6 cm.



Sol. In trapezium ABCD

AB || DC and DL ⊥ AB

AB = 10 cm, DC = 8 cm

and DL = 6 cm

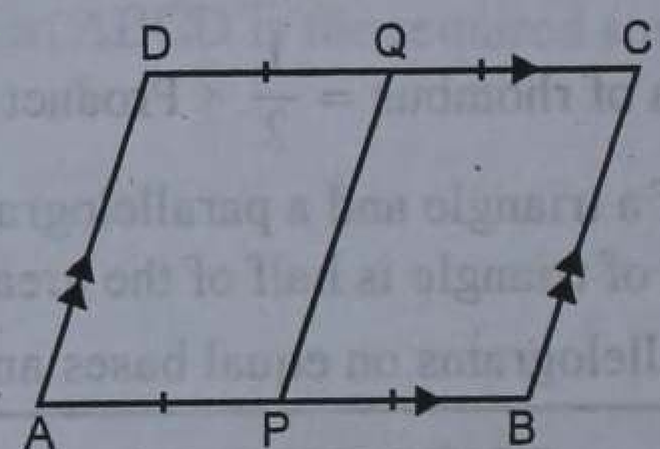
Area of trapezium ABCD

$= \frac{\text{Sum of parallel sides}}{2} \times \text{height}$

$= \frac{(10+8)}{2} \times 6 \text{ cm}^2$

$= \frac{18}{2} \times 6 = 54 \text{ cm}^2$ **Ans.**

Q. 5. Show that the line segment joining the mid-points of a pair of opposite sides of a parallelogram, divides it into two equal parallelograms.



Sol. **Given :** In || gm ABCD,

P and Q are the mid points of sides AB and DC respectively. PQ is joined.

To prove : APQD and PBCQ are parallelograms of equal areas.

Proof : ∵ P and Q are mid points of AB and DC respectively.

∴ AP = PB and DQ = QC

But AB || DC (opposite sides of ||gm)

∴ AP || DQ

and AP = DQ

∴ APQD is a || gm.

Similarly PBCQ is a || gm.

$\because$ || gms APQD and PBCQ are on the equal bases and between the same parallel lines.

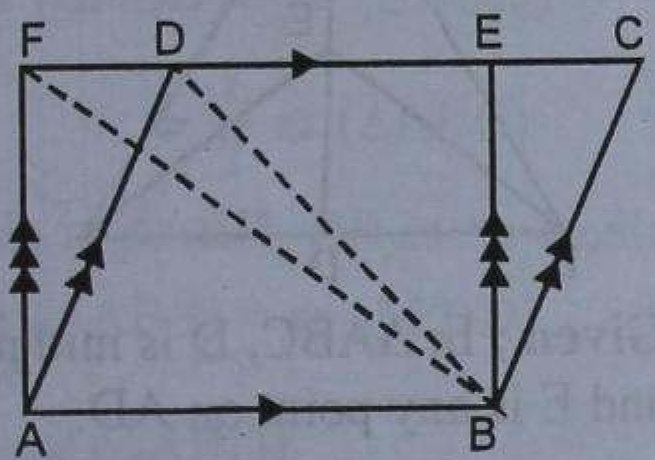
$\therefore$ area of || gm APQD = area of || gm PBCQ

Hence APQD and PBCQ are parallelograms of equal areas.

Q. 6. In the given figure, the area of || gm ABCD is 90 cm^2. State giving reasons :

(*i*) ar (||gm ABEF) (*ii*) ar (ΔABD)

(*iii*) ar (ΔBEF).



Sol. Area of || gm ABCD = 90 cm^2

AF || BE are drawn and BD and BF are joined.

$\therefore$ ABEF is a parallelogram.

(*i*) Now || gm ABCD and || gm ABEF are on the same base and between the same parallel lines.

$\therefore$ area of || gm ABCD = area of || gm ABEF

But area of || gm ABCD = 90 cm^2

$\therefore$ Area of || gm ABEF = 90 cm^2

(*ii*) $\because$ BD and BF are the diagonals of || gm ABCD and || gm ABEF respectively and diagonals of a || gm bisect it into two triangles of equal area.

$\therefore$ Area (ΔABD) $= \frac{1}{2}$ area (|| gm ABCD)

$= \frac{1}{2} \times 90 \text{ cm}^2 = 45 \text{ cm}^2$

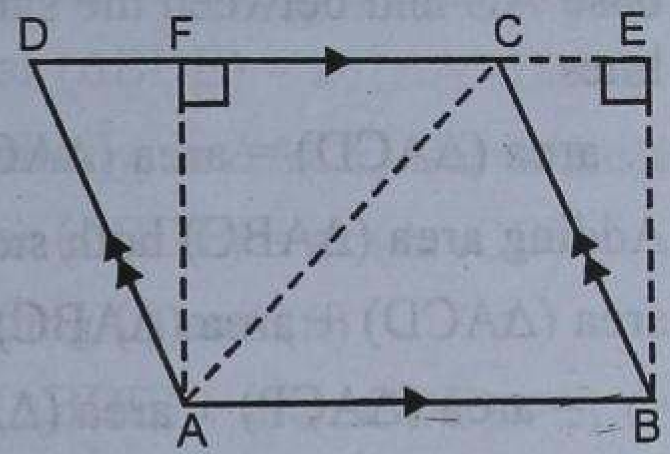
(*iii*) and area (ΔBEF) $= \frac{1}{2}$ area (|| gm ABEF)

$= \frac{1}{2} \times 90 \text{ cm}^2 = 45 \text{ cm}^2$ **Ans.**

Q. 7. In the given figure, the area of ΔABC is 64 cm^2. State giving reasons :

(*i*) ar (|| gm ABCD)

(*ii*) ar (rect. ABEF).



Sol. Area of ΔABC = 64 cm^2

|| gm ABCD and rectangle ABEF are drawn on the same base AB of ΔABC.

(*i*) In || gm ABCD, CA is its diagonal

$\therefore$ Area (ΔABC) $= \frac{1}{2}$ ar (|| gm ABCD)

$\Rightarrow$ Area || gm ABCD = 2 area (ΔABC)

$= 2 \times 64 \text{ cm}^2 = 128 \text{ cm}^2$

(*ii*) $\because$ || gm ABCD and rectangle are on the same base AB and between the same parallels.

$\therefore$ Area (|| gm ABCD)

= (rectangle ABEF)

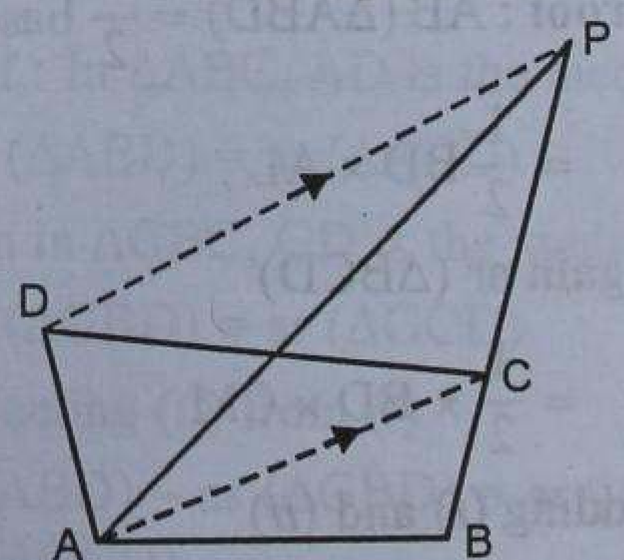
$\therefore$ Area (rectangle ABEF)

= 128 cm^2 **Ans.**

Q. 8. In the given figure, ABCD is a quadrilateral. A line through D, parallel to AC, meets BC produced in P.

Prove that : ar (ΔABP)

= ar (quad. ABCD).



Sol. Given : In quad. ABCD, a line through D is drawn parallel to AC and meets BC produced in P.

To prove : Area (ΔABP)

= area (quad. ABCD)

Proof : $\because$ AC || PD

and ΔACD and ΔACP are on the same base AC and between the same parallel lines.

$\therefore$ area (ΔACD) = area (ΔACP)

Adding area (ΔABC) both sides,

area (ΔACD) + area (ΔABC)

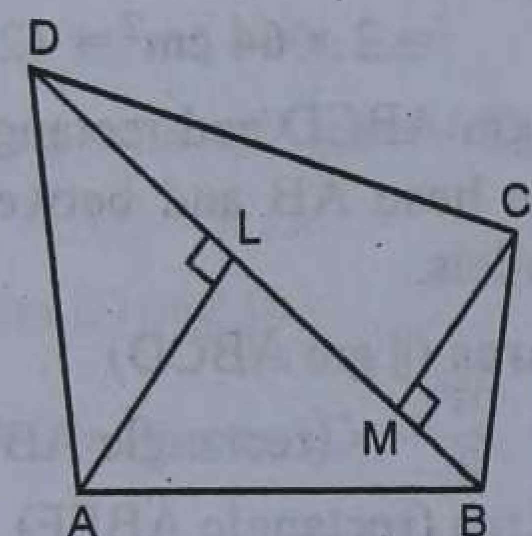
= area (ΔACP) + area (ΔABC)

$\Rightarrow$ area (quad ABCD) = area (ΔABP)

or ar (ΔABP) = ar (quad. ABCD)

Hence proved.

Q. 9. ABCD is a quadrilateral. If AL ⊥ BD and CM ⊥ BD, prove that : ar (quad. ABCD) $= \frac{1}{2} \times$ BD × (AL + CM).



Sol. Given : In quadrilateral ABCD, AL ⊥ BD and CM ⊥ BD.

To prove : ar (quad ABCD)

$= \frac{1}{2} \times BD \times (AL + CM)$

Proof : AB (ΔABD) $= \frac{1}{2}$ base × altitude

$= \frac{1}{2} BD \times AL$ ...(*i*)

Again ar (ΔBCD)

$= \frac{1}{2} \times BD \times CM$ ...(*ii*)

Adding (*i*) and (*ii*)

ar (ΔABD) + ar (ΔBCD)

$= \frac{1}{2} BD \times AL + \frac{1}{2} BD \times CM$

$\Rightarrow$ ar (quad ABCD)

$= \frac{1}{2} BD\ (AL + CM)$

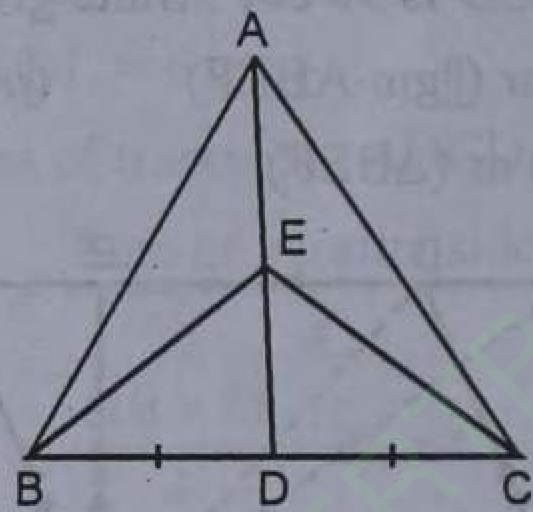
Hence proved

Q. 10. In the given figure, D is the mid-point of BC and E is any point on AD.

Prove that :

(*i*) ar (ΔEBD) = ar (ΔEDC).

(*ii*) ar (ΔABE) = ar (ΔACE).



Sol. Given : In ΔABC, D is mid point of BC and E is any point on AD.

To prove : (*i*) ar (ΔEBD) = ar (ΔEDC).

(*ii*) ar (ΔABE) = ar (ΔACE).

Proof : In ΔABC,

AD is the median of the triangle

$\therefore$ ar (ΔABD) = ar (ΔACD)

Again in ΔEBC,

ED is the median of ΔEBC

(*i*) $\therefore$ ar (ΔEBD) = ar (ΔEDC) ...(*ii*)

(*ii*) Subtracting (*ii*) from (*i*)

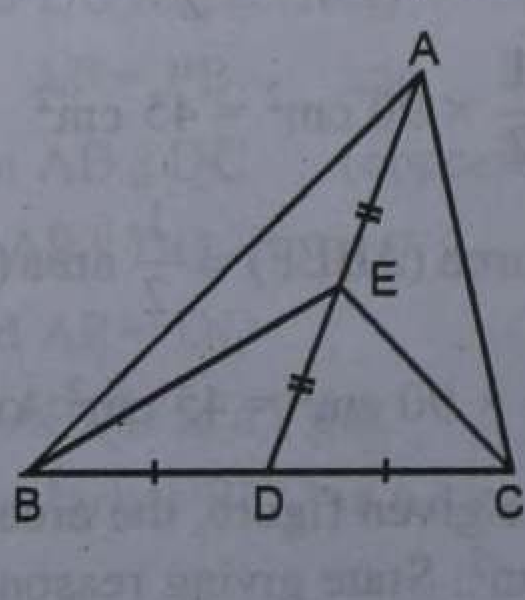
ar (ΔABD) – ar (ΔEBD)

= ar (ΔACE) – ar (ΔECB)

$\Rightarrow$ ar (ΔABE) = ar (ΔACE)

Hence proved.

Q. 11. In the given figure, D is the mid-point of BC and E is the mid-point of AD.



Prove that : ar (ΔABE)

$$= \frac{1}{4} \text{ ar } (\Delta ABC).$$

Sol. Given : In ΔABC, D is mid point of BC and E is mid point on AD. CE and BE are joined.

To prove : ar (ΔABE)

$$= \frac{1}{4} \text{ ar } (\frac{1}{4} ABC).$$

Proof : In ΔABC, AD is the median

∴ ar (ΔABD) = ar (ΔACD)

$$= \frac{1}{2} \text{ ar } (\Delta ABC) \quad ...(i)$$

Again in ΔABD, BE is the median

∴ ar (ΔABE) = ar (ΔEBD)

$$= \frac{1}{2} \text{ ar } (\Delta ABD)$$

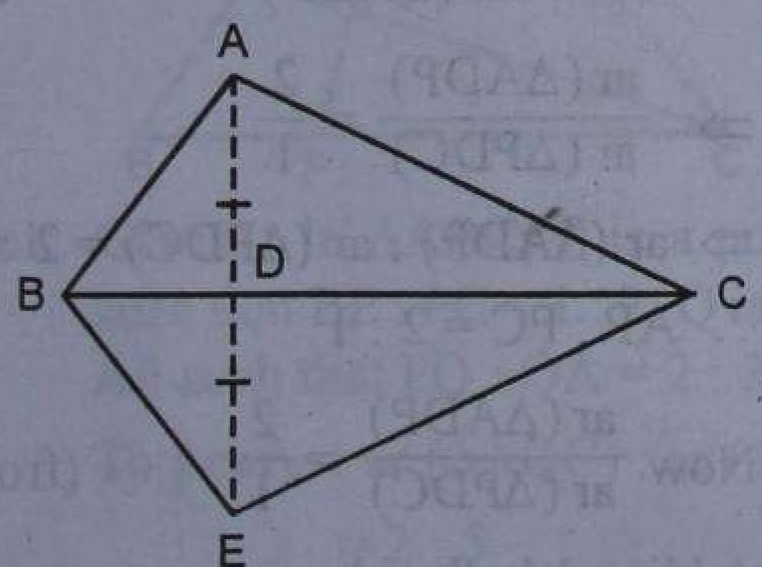
$$= \frac{1}{2} \times \frac{1}{2} \text{ ar } (\Delta ABC) \quad \text{[from } (i)]$$

$$= \frac{1}{4} \text{ ar } (\Delta ABC)$$

Hence proved.

Q. 12. In the given figure, a point D is taken on side BC of ΔABC and AD is produced to E, making DE = AD.

Show that : ar (ΔBEC) = ar (ΔABC).



Sol. Given : In ΔABC, D is any point on BC, AD is joined and produced to E such that DE = AD.

BE and CE are joined.

To prove : ar (BEC) = ar (ΔABC).

Proof : ∵ AD = DE (given)

∴ D is mid point of AE.

Now in ΔABE, BD is the median

∴ ar (ΔBDE) = ar (ΔABD) ...(*i*)

Similarly, in ΔACE, CD is the median

∴ ar (ΔCDE) = ar (ΔACD) ...(*ii*)

Adding (*i*) and (*ii*)

ar (ΔBDE) + ar (ΔCDE) = ar (ΔABD) + ar (ΔACD)

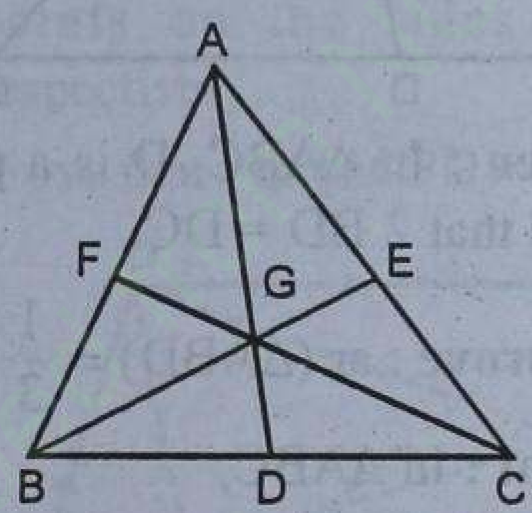
⇒ ar (ΔBEC) = ar (ΔABC)

Hence proved.

Q. 13. If the medians of a ΔABC intersect at G, show that :

ar (ΔAGB) = ar (ΔAGC) = ar (ΔBGC)

$$= \frac{1}{3} \text{ ar } (\Delta ABC)$$



Sol. Given : In ΔABC, AD, BE and CF are the medians of the sides BC, CA and AB respectively intersecting at the point G.

To prove : ar (ΔAGB) = ar (ΔAGC)

$$= \text{ar } (\Delta BGC) = \frac{1}{3} \text{ ar } (\Delta ABC)$$

Proof : In ΔABC, AD is the median

∴ ar (ΔABD) = ar (ΔACD) ...(*i*)

Again in ΔGBC, GD is the median

∴ ar (ΔGBD) = ar (ΔGCD) ...(*ii*)

Subtracting (*ii*) from (*i*)

ar (ΔABD) – ar (ΔGBD) = ar (ΔACD) – ar (ΔGCD)

⇒ ar (ΔAGB) = ar (ΔAGC) ...(*iii*)

Similarly we can prove that

ar (ΔAGC) = ar (ΔBGC) ...(*iv*)

From (*iii*) and (*iv*)

$\text{ar}(\Delta AGB) = \text{ar}(\Delta AGC) = \text{ar}(\Delta BGC)$

But $\text{ar}(\Delta AGB) + \text{ar}(\Delta AGC) + \text{ar}(\Delta BGC) = \text{ar}(\Delta ABC)$

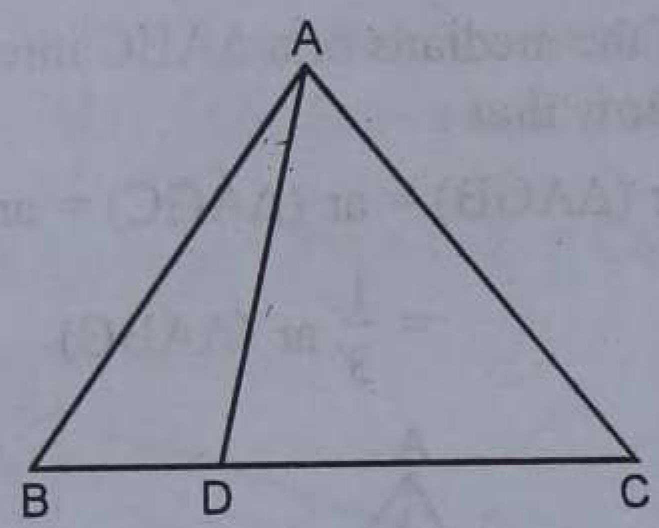
$= \text{ar}(\Delta AGB) = \text{ar}(\Delta AGC) = \text{ar}(\Delta BGC)$

$= \frac{1}{3}\ \text{ar}(\Delta ABC)$

Hence proved.

Q. 14. D is a point on base BC of a ΔABC such that $2\ BD = DC$.

Prove that : $\text{ar}(\Delta ABD) = \frac{1}{3}\ \text{ar}(\Delta ABC)$.



Sol. Given : In ΔABC, D is a point on BC such that $2\ BD = DC$.

To prove : $\text{ar}(\Delta ABD) = \frac{1}{3}\ \text{ar}(\Delta ABC)$.

Proof : In ΔABC,

$\because 2BD = DC \Rightarrow \frac{BD}{DC} = \frac{1}{2}$

$\Rightarrow BD : DC = 1 : 2$

$\therefore \text{ar}(\Delta ABD) : \text{ar}(\Delta ADC) = 1 : 2$

But $\text{ar}(\Delta ABD) + \text{ar}(\Delta ADC) = \text{ar}(\Delta ABC)$

$\Rightarrow \text{ar}(\Delta ABD) + 2\ \text{ar}(\Delta ABD) = \text{ar}(\Delta ABC)$

$\Rightarrow 3\ \text{ar}(\Delta ABD) = \text{ar}(\Delta ABC)$

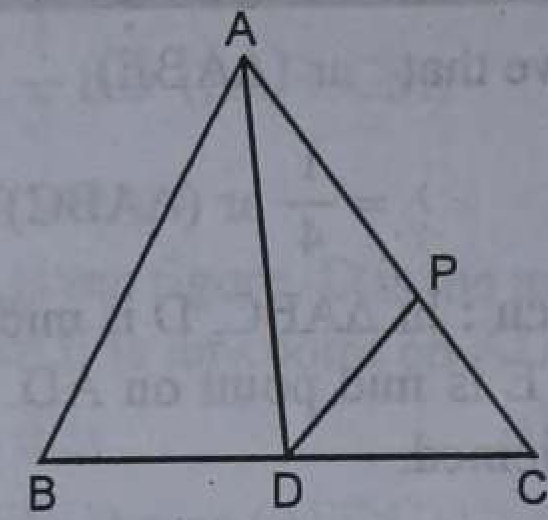
$\Rightarrow \text{ar}(\Delta ABD) = \frac{1}{3}\ \text{ar}(\Delta ABC)$

Hence proved.

Q. 15. In the given figure, AD is a median of ΔABC and P is a point on AC such that : $\text{ar}(\Delta ADP) : \text{ar}(\Delta ABD) = 2 : 3$.

Find : (*i*) AP : PC

(*ii*) $\text{ar}(\Delta PDC) : \text{ar}(\Delta ABC)$.



Sol. Given : In ΔABC, AD is median of the triangle, P is a point on AC such that : $\text{ar}(\Delta ADP) : \text{ar}(\Delta ABD) = 2 : 3$, now we have

To find : (*i*) AP : PC

(*ii*) $\text{ar}(\Delta PDC) : \text{ar}(\Delta ABC)$.

(*i*) In ΔABC, AD is the median

$\therefore \text{ar}(\Delta ABD) = \text{ar}(\Delta ADC)$ ...(*i*)

$\because \text{ar}(\Delta ADP) : \text{ar}(\Delta ABD) = 2 : 3$

$\Rightarrow \text{ar}(\Delta ADP) : \text{ar}(\Delta ADC) = 2 : 3$ [from (*i*)]

$\Rightarrow \text{ar}(\Delta ADC) : \text{ar}(\Delta ADP) = 3 : 2$

$\Rightarrow \frac{\text{ar}(\Delta ADC)}{\text{ar}(\Delta ADP)} = \frac{3}{2}$

$\Rightarrow \frac{\text{ar}(\Delta ADC)}{\text{ar}(\Delta ADP)} - 1 = \frac{3}{2} - 1$

(Substracting 1 from both sides)

$\Rightarrow \frac{\text{ar}(\Delta ADC) - \text{ar}(\Delta ADP)}{\text{ar}(\Delta ADP)} = \frac{1}{2}$

$\Rightarrow \frac{\text{ar}(\Delta ADP)}{\text{ar}(\Delta PDC)} = \frac{2}{1}$ ...(*ii*)

$\Rightarrow \text{ar}(\Delta ADP) : \text{ar}(\Delta PDC) = 2 : 1$

$\therefore AP : PC = 2 : 1$

(*ii*) Now $\frac{\text{ar}(\Delta ADP)}{\text{ar}(\Delta PDC)} = \frac{2}{1}$ (from (*ii*))

Adding 1 both sides, we get

$\frac{\text{ar}(\Delta ADP)}{\text{ar}(\Delta PDC)} + 1 = \frac{2}{1} + 1$

$\frac{\text{ar}(\Delta ADP) + \text{ar}(\Delta PDC)}{\text{ar}(\Delta PDC)} = \frac{2}{1} + 1$

$\frac{\text{ar}(\Delta ADC)}{\text{ar}(\Delta PDC)} = \frac{3}{1}$

But ar (ΔADC) = ar (ΔABD) [from (*i*)]

$$\therefore \frac{ar(\Delta ADB)}{ar(\Delta PDC)} = \frac{3}{1}$$

$$\Rightarrow \frac{ar(\Delta PDC)}{ar(\Delta ABD)} = \frac{1}{3}$$

But $ar(\Delta ABD) = \frac{1}{2} ar(\Delta ABC)$

$$\therefore \frac{ar(\Delta PDC)}{\frac{1}{2} ar(\Delta ABC)} = \frac{1}{3}$$

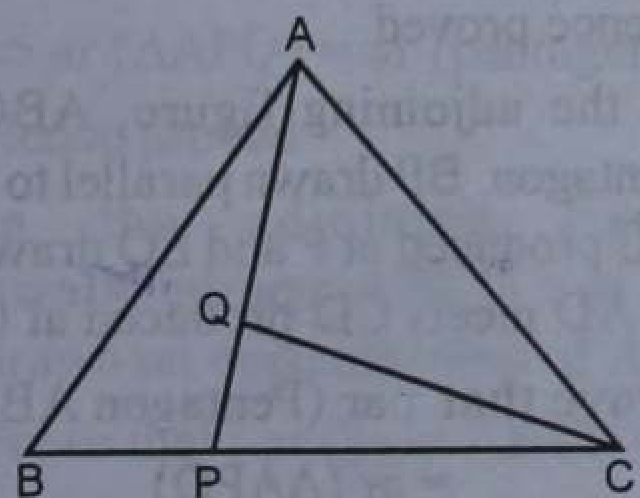
$$\Rightarrow \frac{2\, ar(\Delta PDC)}{ar(\Delta ABC)} = \frac{1}{3}$$

$$\Rightarrow \frac{ar(\Delta PDC)}{ar(\Delta ABC)} = \frac{1}{3 \times 2} = \frac{1}{6}$$

Hence ar (ΔPDC) : ar (ΔABC) = 1 : 6

Q. 16. In the given figure, P is a point on side BC of ΔABC such that BP : PC = 1 : 2 and Q is a point on AP such that PQ : QA = 2 : 3.

Show that : ar (ΔAQC) : ar (ΔABC) = 2 : 5.



Sol. Given : In ΔABC, P is a point on BC such that BP : PC = 1 : 2. Q is a point on AP such that PQ : QA = 2 : 3.

To prove : ar (ΔAQC) : ar (ΔABC) = 2 : 5

Proof : In ΔABC, P is a point on BC such that

BP : PC = 1 : 2

∴ ar (ΔAPB) : ar (ΔAPC) = 1 : 2

$$ar(\Delta APC) = \frac{2}{3} ar(\Delta ABC)$$

Again In ΔAPC,

Q is a point on AP such that PQ : QA = 2 : 3

⇒ ar (ΔAQC) : ar (ΔPQC) = 3 : 2

$$\text{or } ar(\Delta AQC) = \frac{3}{5} ar(\Delta APC)$$

$$= \frac{3}{5} \times \frac{2}{3} \times ar(\Delta ABC)$$

$$= \frac{2}{5} ar(\Delta PBC)$$

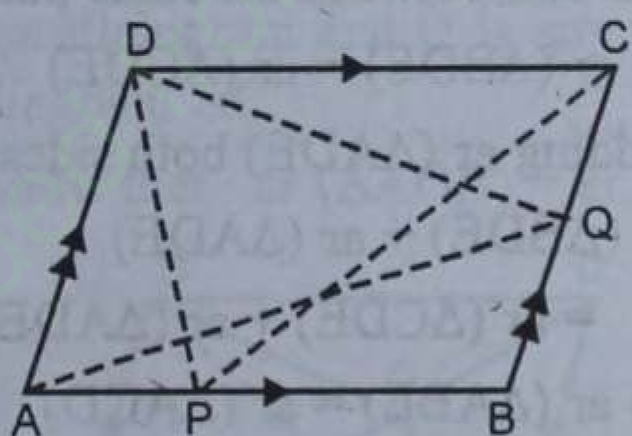
$$\Rightarrow \frac{ar(\Delta AQC)}{ar(\Delta ABC)} = \frac{2}{5}$$

∴ ar (ΔAQC) : ar (ΔABC) = 2 : 5

Hence proved.

Q. 17. In the adjoining figure, ABCD is a parallelogram. P and Q are any two points on the sides AB and BC respectively.

Prove that : ar (ΔCPD) = ar (ΔAQD).



Sol. Given : In ∥gm ABCD, P and Q are any two points on the sides AB and BC respectively.

AQ, DQ, CP and DP are joined.

To prove : ar (ΔCPD) = ar (ΔAQD).

Proof : ΔCPD and ∥ gm ABCD are on the same base CD and between the same parallel lines.

$$\therefore ar(\Delta CPD) = \frac{1}{2} ar(\| gm\ ABCD) \ ...(i)$$

Similarly, ΔAQD and ∥ gm ABCD are on the same base AD and between the same parallel lines.

$$\therefore ar(\Delta AQD) = \frac{1}{2} ar(\| gm\ ABCD) \ ...(ii)$$

From (*i*) and (*ii*)

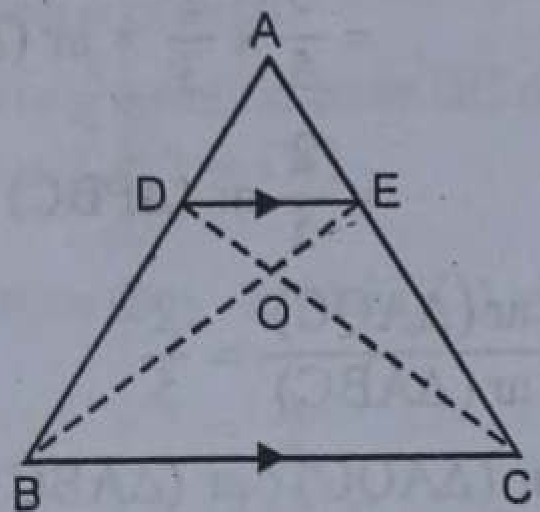
ar (ΔCPD) = ar (ΔAQD)

Hence proved.

Q. 18. In the adjoining figure, DE ∥ BC.

Prove that : (*i*) ar (ΔABE) = ar (ΔACD)

(*ii*) ar (ΔOBD) = ar (ΔOCE)



Sol. Given : In ΔABC, DE ∥ BC

To prove : (*i*) ar (ΔABE) = ar (ΔACD)

(*ii*) ar (ΔOBD) = ar (ΔOCE)

Proof : (*i*) In ΔABC, DE ∥ BC

ΔBDE and ΔCDE are on the same base DE and between the same parallels.

∴ ar (ΔBDE) = ar (ΔCDE) ...(*i*)

Adding ar (ΔADE) both sides of (*i*)

ar (ΔBDE) + ar (ΔADE)

= ar (ΔCDE) + ar (ΔADE)

⇒ ar (ΔABE) = ar (ΔACD)

(*ii*) Subtracting ar (ΔDOE) from both sides of (*i*)

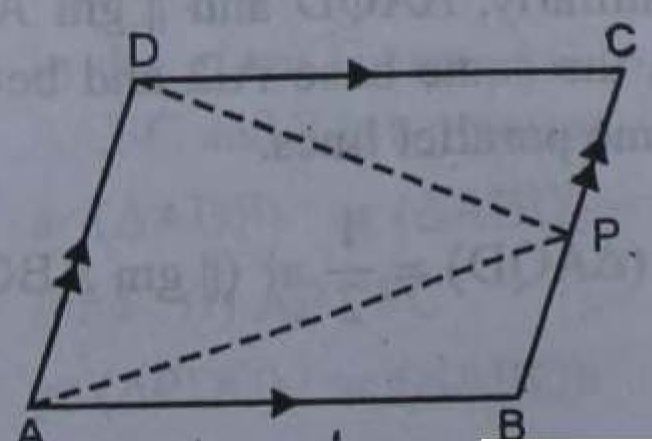
ar (ΔBDE) − ar (ΔDOE) = ar (ΔCDE) − ar (ΔDOE)

⇒ ar (ΔOBD) = ar (ΔOCE)

Hence proved.

Q. 19. In the given figure, ABCD is a parallelogram and P is a point on BC.

Prove that : ar (ΔABP) + ar (ΔDPC) = ar (ΔAPD)



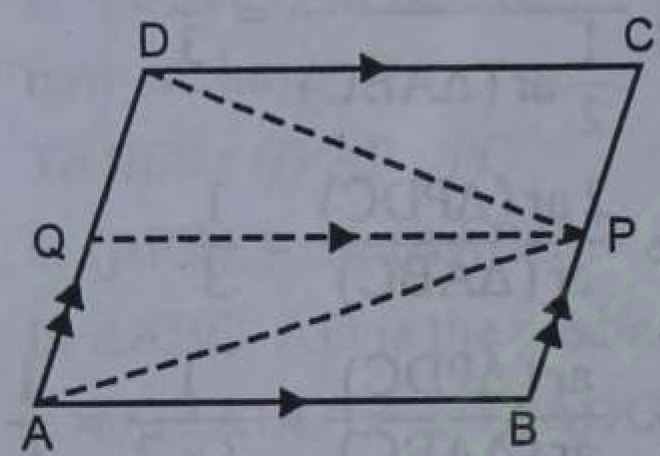
Sol. Given : In ∥ gm ABCD, P is a point on BC

To prove : ar (ΔABP) + ar (ΔDPC) = ar (ΔAPD)

Construction : From P, draw PQ ∥ AB or DC

Proof : ∵ QPCD is a ∥ gm and PD is the diagonal

∴ ar (ΔDPC) = ar (ΔQPD) ...(*i*)



Similarly ABPQ is a ∥ gm and AP is the diagonal

∴ ar (ΔABP) = ar (ΔAPQ) ...(*ii*)

Adding (*i*) and (*ii*)

ar (ΔDPC) + ar (ΔABP)

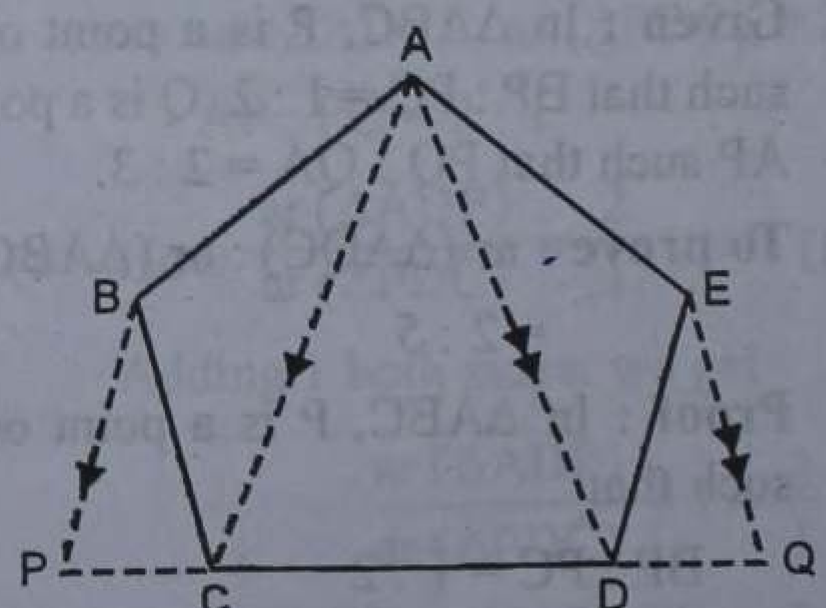
= ar (ΔQPD) + ar (ΔAPQ)

⇒ ar (ΔABP) + ar (ΔDPC) = ar (ΔAPD)

Hence proved.

Q. 20. In the adjoining figure, ABCDE is a pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q.

Prove that : ar (Pentagon ABCDE) = ar (ΔAPQ)

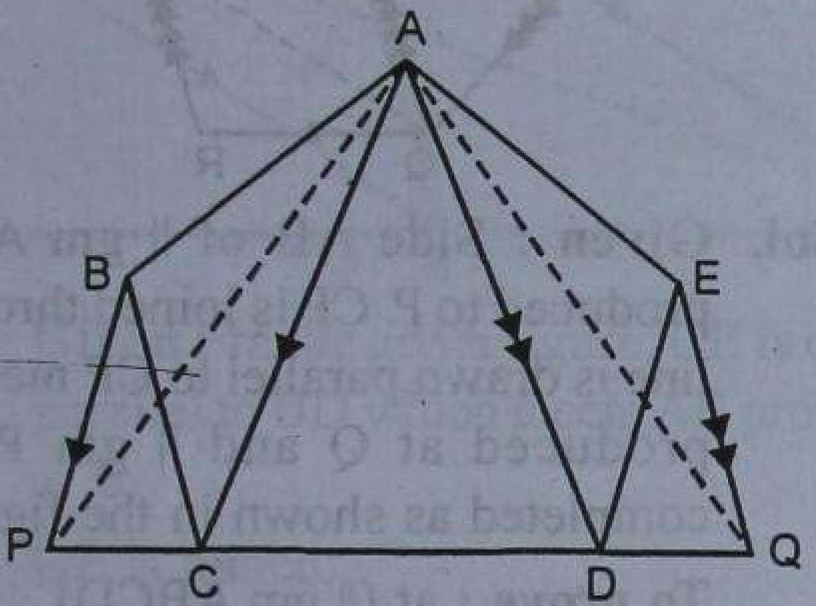


Sol. Given : In a pentagon ABCDE, AC and AD are joined. From B, BP ∥ AC and from E, EQ ∥ AD are drawn to meet CD produced on both sides at P and Q respectively.

To prove : ar (Pentagon ABCDE) = ar (ΔAPQ)

Construction : Given AP and AQ

Proof : $\because$ BP $\parallel$ AC and ΔABC and ΔAPC are on the same base AC and between the same parallel lines.



$\therefore$ ar (ΔABC) = ar (ΔAPC) ...(*i*)

Similarly ar (ΔADE) = ar (ΔAPQ) ...(*ii*)

and ar (ΔACD) = ar (ΔACD) ...(*iii*)

Adding (*i*), (*ii*) and (*iii*)

ar (ΔABC) + ar (ΔADE) + ar (ΔACD)

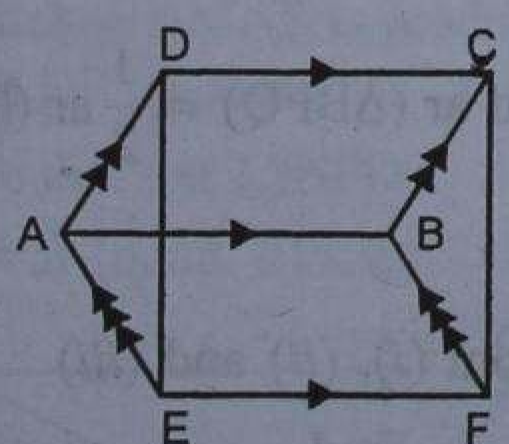
= ar (ΔAPC) + ar (ΔADQ) + ar (ΔACD)

$\Rightarrow$ ar (ΔAPQ) = ar (pentagon ABCDE)

Hence proved.

Q. 21. In the adjoining figure, two parallelograms ABCD and AEFB are drawn on opposite sides of AB.

Prove that : ar ($\parallel$ gm ABCD) + ar ($\parallel$ gm AEFB) = ar ($\parallel$ gm EFCD).



Sol. Given : $\parallel$ gm ABCD and $\parallel$ gm AEFB are drawn on the opposite sides of AB. DE and FC are joined.

To prove : ar ($\parallel$ gm ABCD) + ar ($\parallel$ gm AEFB) = ar ($\parallel$ gm EFCD)

Proof : In ΔADE and ΔBFC

AD = BC

{$\because$ Opposite sides of a parallelogram are equal}

AE = BF

DE = CF

$\therefore$ ΔADE $\cong$ ΔBFC

$\Rightarrow$ ar (ΔADE) = ar (ΔBFC) ...(*i*)

($\because$ Congruent triangles are equal in area)

Now ar ($\parallel$ gm ABCD) + ar ($\parallel$ gm AEFB)

= ar ($\parallel$ gm EFCD) – ar (ΔADE) + ar (ΔBFC)

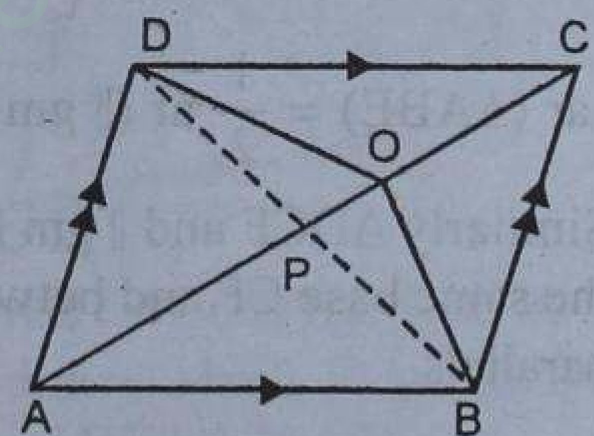
= ar ($\parallel$ gm EFCD) – ar (ΔADE) + ar (ΔADE) [from (*i*)]

= ar ($\parallel$ gm EFCD)

Hence proved.

Q. 22. In the adjoining figure, ABCD is a parallelogram and O is any point on its diagonal AC.

Show that : ar (ΔAOB) = ar (ΔAOD).



Sol. In $\parallel$ gm ABCD, O is any point on its diagonal. OB and OD are joined.

To prove : ar (ΔAOB) = ar (ΔAOD)

Construction : Join BD which intersects AC at P.

Proof : $\because$ Diagonals of a $\parallel$ gm bisect each other

$\therefore$ AP = PC and BP = PD

Now in ΔABD, AP is its median

$\therefore$ ar (ΔABP) = ar (ΔADP) ...(*i*)

Similarly in ΔOBD, OP is the median

$\therefore$ ar (ΔOBP) = ar (ΔODP) ...(*ii*)

Adding (*i*) and (*ii*)

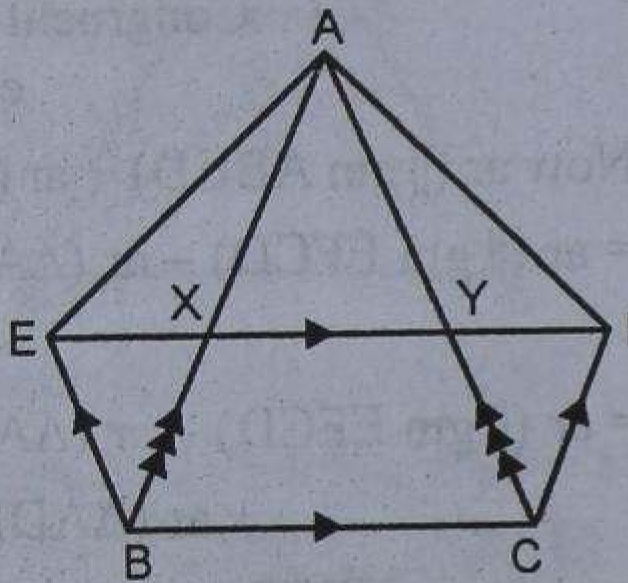
ar (ΔAPB) + ar (ΔOBP) = ar (ΔADP) + ar (ΔODP)

$\Rightarrow$ ar (ΔAOB) = ar (ΔAOD)

Hence proved

Q. 23. In the given figure, XY $\parallel$ BC, BE $\parallel$ CA and FC $\parallel$ AB.

Prove that : ar (ΔABE) = ar (ΔACF)



Sol. **Given :** In the figure, XY $\parallel$ BC, BE $\parallel$ CA and FC $\parallel$ AB.

To prove : ar (ΔABE) = ar (ΔACF)

Proof : ΔABE and $\parallel$ gm BCYE are on the same base BE and between the same parallels

$\therefore$ ar (ΔABE) $= \frac{1}{2}$ ar ($\parallel$ gm BCYE) ...(*i*)

Similarly ΔDCF and $\parallel$ gm BCFX are on the same base CF and between the same parallels.

$\therefore$ ar (ΔACF) $= \frac{1}{2}$ ar ($\parallel$ gm BCFX) ...(*ii*)

But $\parallel$ gm BCFX and $\parallel$ gm BCYE are on the same base BC and between the same parallels.

$\therefore$ ar ($\parallel$ gm BCFX) = ar ($\parallel$ gm BCYE) ...(*iii*)

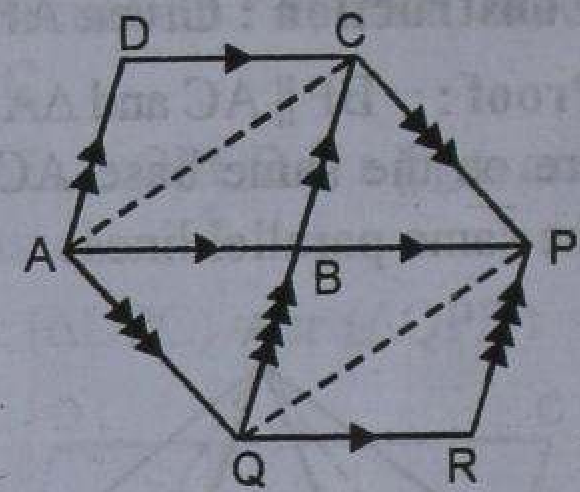
From (*i*), (*ii*) and (*iii*)

ar (ΔABE) = ar (ΔACF)

Hence proved.

Q. 24. In the given figure, the side AB of $\parallel$ gm ABCD is produced to a point P. A line through A drawn parallel to CP meets CB produced in Q and the parallelogram PBQR is completed.

Prove that : ar ($\parallel$ gm ABCD) = ar ($\parallel$ gm BPRQ).



Sol. **Given :** Side AB of $\parallel$ gm ABCD is produced to P. CP is joined through A, a line is drawn parallel to CP meeting CB produced at Q and $\parallel$ gm PBQR is completed as shown in the figure.

To prove : ar ($\parallel$ gm ABCD) = ar ($\parallel$ gm BPRQ)

Construction : Join AC and PQ.

Proof : ΔAQC and ΔAQP are on the same base AQ and between the same parallels

$\therefore$ ar (ΔAQC) = ar (ΔAQP)

Subtracting ar (ΔAQB) from both sides,

ar (ΔAQC) – ar (ΔAQB) = ar (ΔAQP) – ar (ΔAQB)

$\Rightarrow$ ar (ΔABC) = ar (ΔBPQ) ...(*i*)

But ar (ΔABC) $= \frac{1}{2}$ ar ($\parallel$ gm ABCD) ...(*ii*)

and ar (ΔBPQ) $= \frac{1}{2}$ ar ($\parallel$ gm BPRQ) ...(*iii*)

From (*i*), (*ii*) and (*iii*)

$= \frac{1}{2}$ ar ($\parallel$ gm ABCD)

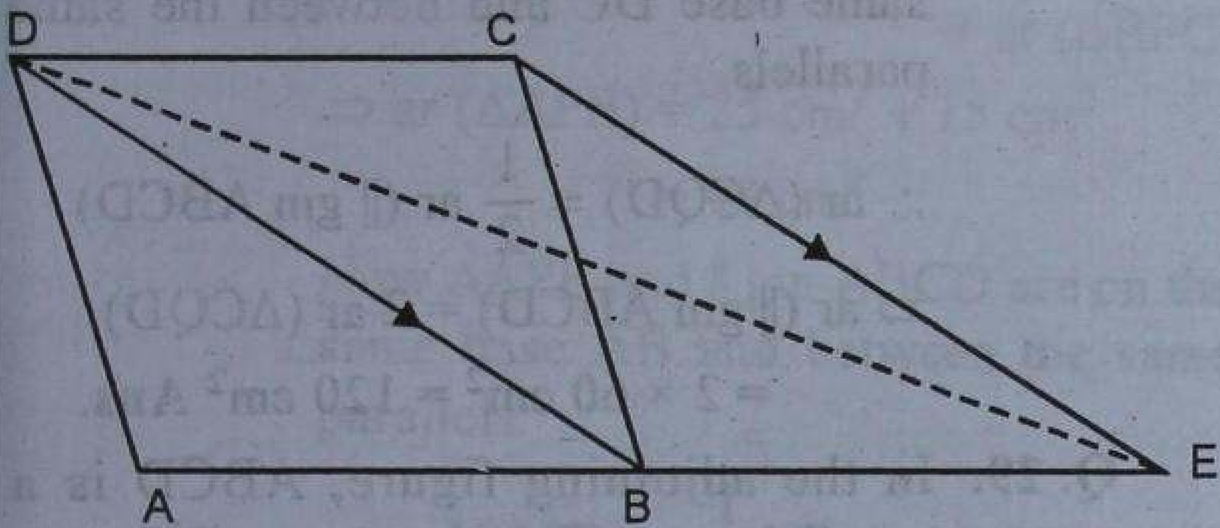
$= \frac{1}{2}$ ar ($\parallel$ gm BPRQ)

$\Rightarrow$ ar ($\parallel$ gm ABCD) = ar ($\parallel$ gm BPRQ)

Hence proved.

Q. 25. In the adjoining figure, CE is drawn parallel to DB to meet AB produced at E.

Prove that : ar (quad. ABCD)
= ar (ΔDAE).



Sol. Given : In the given figure, CE is drawn parallel to BD which meets AB produced at E.

DE is joined.

To prove : ar (quad. ABCD)
= ar (ΔDAE)

Proof : ΔDBE and ΔDBC are on the same base BD and between the same parallels.

∴ ar (ΔDBE) = ar (ΔDBC)

Adding ar (ΔABD) both sides,

ar (ΔDBE) + ar (ΔABD) = ar (ΔDBC)
+ ar (ΔABD)

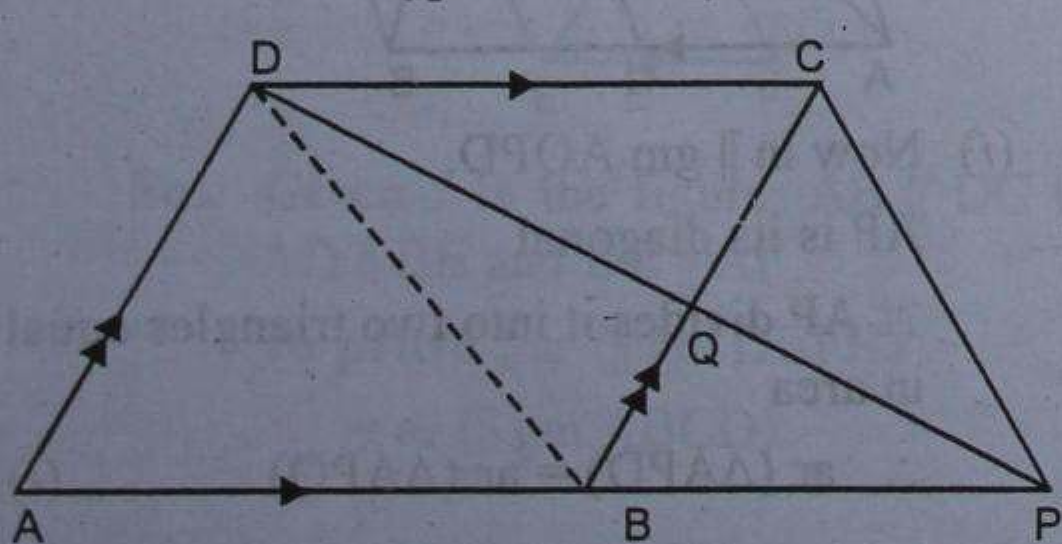
⇒ ar (ΔADE) = ar (quad ABCD)

⇒ ar (quad ABCD) = ar (ΔDAE)

Hence proved.

Q. 26. In the adjoining figure, ABCD is a parallelogram. AB is produced to a point P and DP intersects BC at Q.

Prove that : ar (ΔAPD)
= ar (quad. BPCD).



Sol. Given : In || gm ABCD, AB is produced point P and DP intersects BC at Q.

To prove : ar (ΔAPD) = ar (quad BPCD)

Construction : Join BD.

Proof : ΔBPD and ΔBPC are on the same base BP and between the same parallels.

∴ ar (ΔBPD) = ar (ΔBPC) ...(*i*)

In || gm ABCD, BD is its diagonal

∴ ar (ΔABD) = ar (ΔDBC) ...(*ii*)

Adding (*i*) and (*ii*)

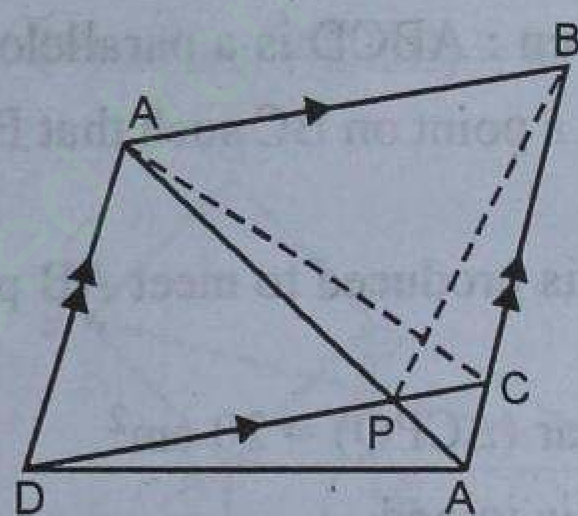
ar (ΔBPD) + ar (ΔABD) = ar (ΔBPC)
+ ar (ΔDBC)

⇒ ar (ΔAPD) = ar (quad BPCD)

Hence proved.

Q. 27. In the adjoining figure, ABCD is a parallelogram. Any line through A cuts DC at a point P and BC produced at Q.

Prove that : ar (ΔBPC) = ar (ΔDPQ).



Sol. Given : ABCD is a || gm. A line through A, drawn which intersects DC at a point P and BC produced at Q.

To prove : ar (ΔBPC) = ar (ΔDPQ)

Construction : Join AC and BP.

Proof : ΔBPC and ΔAPC are on the same base BC and between the same parallels.

∴ ar (ΔBPC) = ar (ΔAPC) ...(*i*)

Again ΔAQC and ΔDQC and on the same base QC and between the same parallels.

∴ ar (ΔAQC) = ar (ΔDQC) ...(*ii*)

Now ar (ΔBPC) = ar (ΔAPC)
[from (*i*)]

$= ar(\Delta AQC) - ar(\Delta PQC)$

$= ar(\Delta DQC) - ar(\Delta PQC)$ [from (*ii*)]

$= ar(\Delta DPQ)$

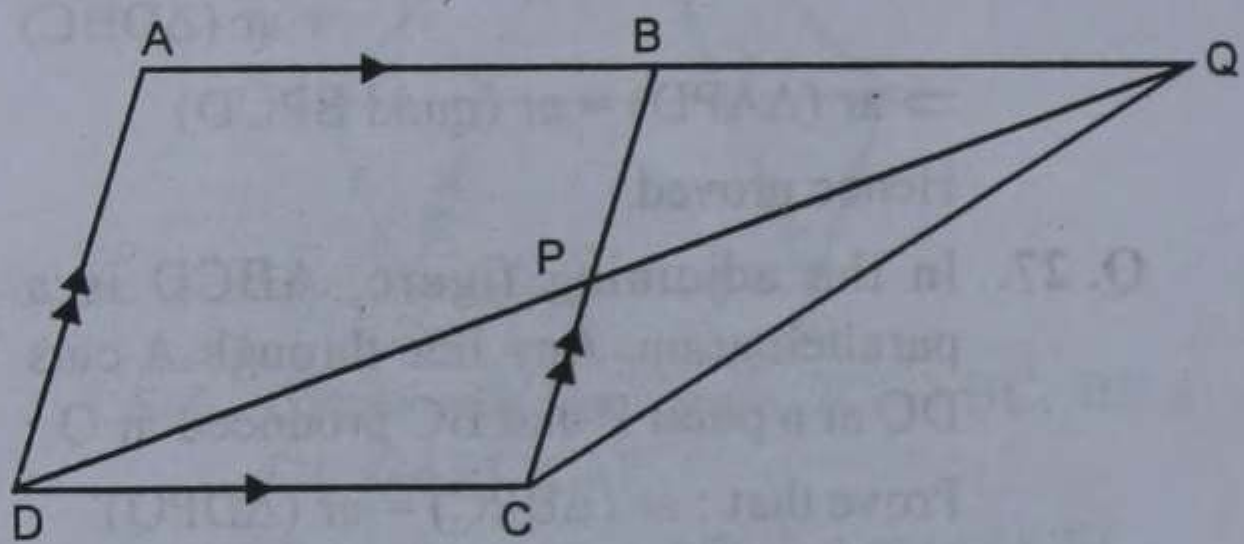
Hence proved.

Q. 28. In the adjoining figure, ABCD is a parallelogram. P is a point on BC such that BP : PC = 1 : 2. DP produced meets AB produced at Q.

Given ar $(\Delta CPQ) = 20$ cm^2.

Calculate : (*i*) ar (ΔCDP)

(*ii*) ar (|| gm ABCD).



Sol. Given : ABCD is a parallelogram.

P is a point on BC such that BP : PC = 1 : 2

DP is produced to meet AB produced at Q

ar $(\Delta CPQ) = 20$ cm^2

CQ is joined.

(*i*) $\because$ BP : PC = 1 : 2

$\therefore$ ar (ΔBPQ) : ar $(\Delta CPQ) = 1 : 2$

$\Rightarrow$ ar $(\Delta BPQ) = \frac{1}{2}$ ar (ΔCPQ)

$= \frac{1}{2} \times 20 \text{ cm}^2 = 10 \text{ cm}^2$

Now in ΔBPQ and ΔCPD,

$\angle BPQ = \angle DPC$

(Vertically opposite angles)

$\angle BQP = \angle PDC$ (alternate angles)

$\therefore \angle BPQ \sim \Delta CPD$

$$\therefore \frac{ar(\Delta CPD)}{ar(\Delta BPQ)} = \frac{(PC)^2}{BP^2} = \frac{(2)^2}{(1)^2} = \frac{4}{1}$$

$\therefore$ ar $(\Delta CPD) = 4$ ar (ΔBPQ)

$= 4 \times 10 \text{ cm}^2 = 40 \text{ cm}^2$ **Ans.**

(*ii*) ar (ΔCQD) = ar (ΔCPD) + ar (ΔCPQ)

$= (40 + 20) \text{ cm}^2 = 60 \text{ cm}^2$

But ΔCQD and || gm ABCD are on the same base DC and between the same parallels.

$\therefore$ ar $(\Delta CQD) = \frac{1}{2}$ ar (|| gm ABCD)

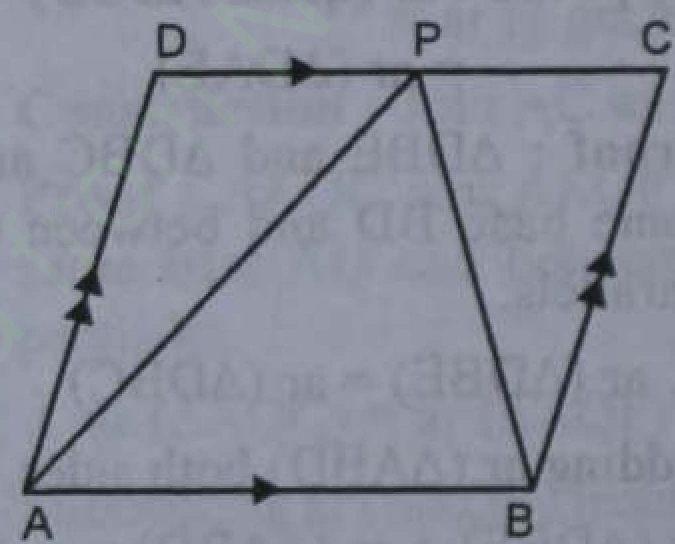
$\Rightarrow$ ar (|| gm ABCD) = 2 ar (ΔCQD)

$= 2 \times 60 \text{ cm}^2 = 120 \text{ cm}^2$ **Ans.**

Q. 29. In the adjoining figure, ABCD is a parallelogram. P is a point on DC such that ar $(\Delta APD) = 25$ cm^2 and ar $(\Delta BPC) = 15$ cm^2.

Calculate : (*i*) ar (|| gm ABCD)

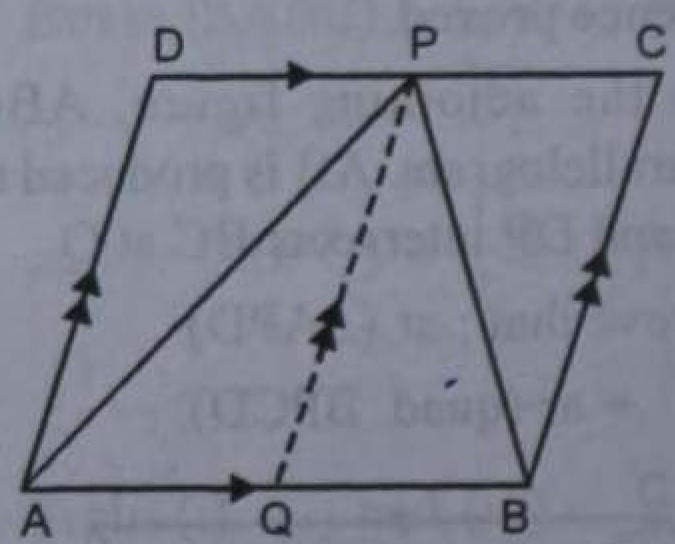
(*ii*) DP : PC.



Sol. ABCD is a || gm. P is a point on DC such that ar $(\Delta APD) = 25$ cm^2

and ar $(\Delta BPC) = 15$ cm^2

Through P, draw PQ || AD or BC



(*i*) Now in || gm AQPD,

AP is its diagonal

$\therefore$ AP divides it into two triangles equal in area

$\therefore$ ar (ΔAPD) = ar (ΔAPQ) ...(*i*)

Similarly in || gm QBCP,
PB is its diagonal

$\therefore$ ar (ΔBPC) = ar (ΔPQB) ...(*ii*)

Adding (*i*) and (*ii*)

ar (ΔAPQ) + ar (ΔPQB) = ar (ΔAPD) + ar (ΔBPC)

$\Rightarrow$ ar (ΔAPB) = 25 cm^2 + 15 cm^2 = 40 cm^2

Now ΔAPB and || gm ABCD are on the same base AB and between the same parallels.

$\therefore$ ar (ΔAPB) = $\frac{1}{2}$ (|| gm ABCD)

$\Rightarrow$ ar (|| gm ABCD) = 2 ar (ΔAPB)
= 2 × 40 cm^2 = 80 cm^2

(*ii*) Now ar (ΔAPQ) : ar (ΔPQB) = AQ : QB

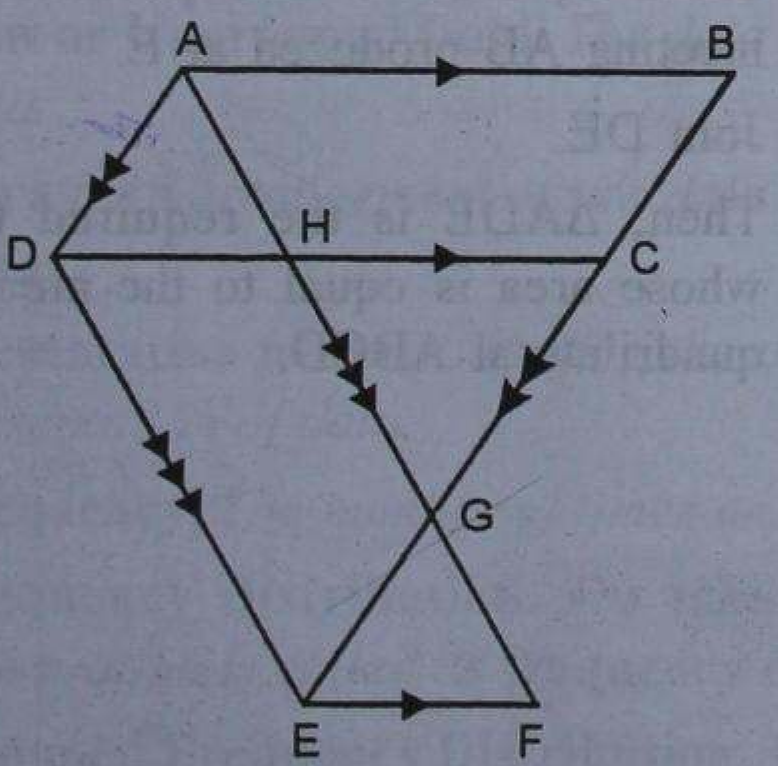
$\Rightarrow$ ar (ΔAPD) : ar (ΔBPC) = DP : PC

$\Rightarrow$ 25 cm^2 = 15 cm^2 = DP : PC

$\Rightarrow$ DP : PC = 25 : 15 (Dividing by 5)

$\Rightarrow$ DP : PC = 5 : 3 **Ans.**

Q. 30. In the given figure, AB || DC || EF, AD || BE and DE || AF.
Prove that : ar (|| gm DEFH) = ar (|| gm ABCD).



Sol. Given : In the figure, AB || DC || EF, AD || BE and DE || AF.

To prove : ar (|| gm DEFH) = ar (|| gm ABCD)

Proof : || gm ABCD and || gm ADGE are on the same base AD and between the same parallels.

$\therefore$ ar (|| gm ABCD) = ar (|| gm ADGE) ...(*i*)

Similarly || gm DEFH and || gm ADEG are on the same base DE and between the same parallels.

$\therefore$ ar (|| gm DEFH) = ar (|| gm ADGE) ...(*ii*)

From (*i*) and (*ii*)

ar (|| gm ABCD) = ar (|| DEFH)

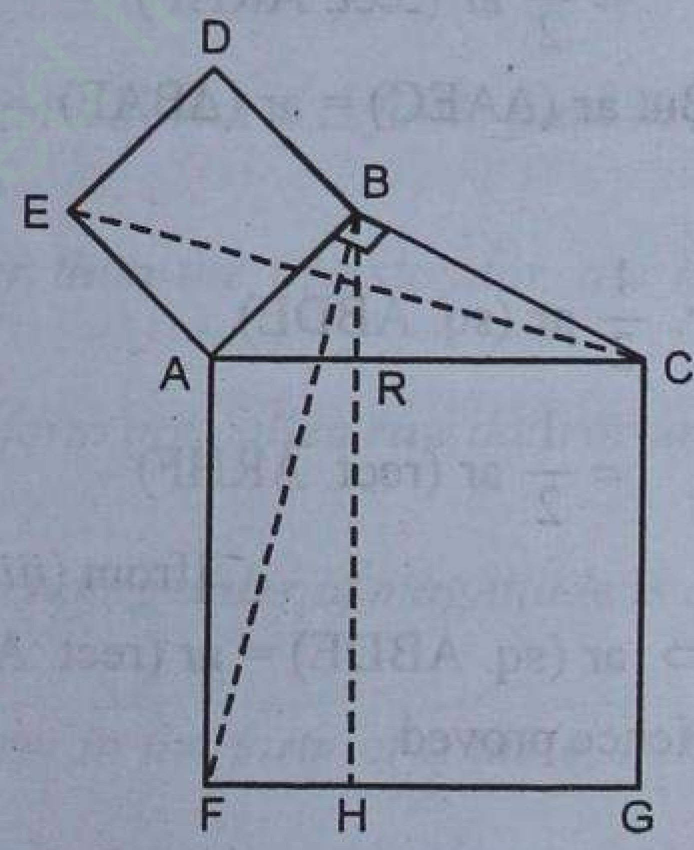
Hence proved.

Q. 31. In the given figure, squares ABDE and AFGC are drawn on the side AB and hypotenuse AC of right triangle ABC and BH $\perp$ FG.

Prove that :

(*i*) ΔEAC $\cong$ ΔBAF.

(*ii*) ar (sq. ABDE) = ar (rect. ARHF).



Sol. Given : Square ABDE and AFGC are drawn on side AB and hypotenuse AC of right triangle ABC. BH $\perp$ FG intersecting AC at R.

To prove : (*i*) ΔEAC $\cong$ ΔBAF.

(*ii*) ar (sq. ABCD) = as (rect. ARHF).

Construction : Join BF and CE.

Proof : $\angle$CAE = $\angle$CAB + $\angle$BAE
= $\angle$CAB + 90° ...(*i*)

Similarly $\angle BAF = \angle CAB + \angle CAF$
$= \angle CAB + 90^\circ$ ...(*ii*)

From (*i*) and (*ii*)
$\angle CAE = \angle BAF$

(*i*) Now in ΔEAC and ΔBAF
$\angle CAE = \angle BAF$ (proved)
$AE = AB$ (sides of a square)
$AC = AF$ (sides of a square)
$\therefore\ \Delta EAC \cong \Delta BAF$
(SAS criterion of congruency)
and ar (ΔEAC) = ar (ΔBAF)

(*ii*) Square ABDE and ΔEAC are on the same base AE and between the same parallels.

$\therefore$ ar $(\Delta AEC) = \frac{1}{2}$ ar (square ABDE) ...(*iii*)

Similarly ar (ΔBAF)
$= \frac{1}{2}$ ar (rect. ARHF) ...(*iv*)

But ar (ΔAEC) = ar (ΔBAF) (proved)

$\therefore\ \frac{1}{2}$ ar (sq. ABDE)
$= \frac{1}{2}$ ar (rect. ARHF)
[from (*iii* and *iv*)]

$\Rightarrow$ ar (sq. ABDE) = ar (rect. ARHF)

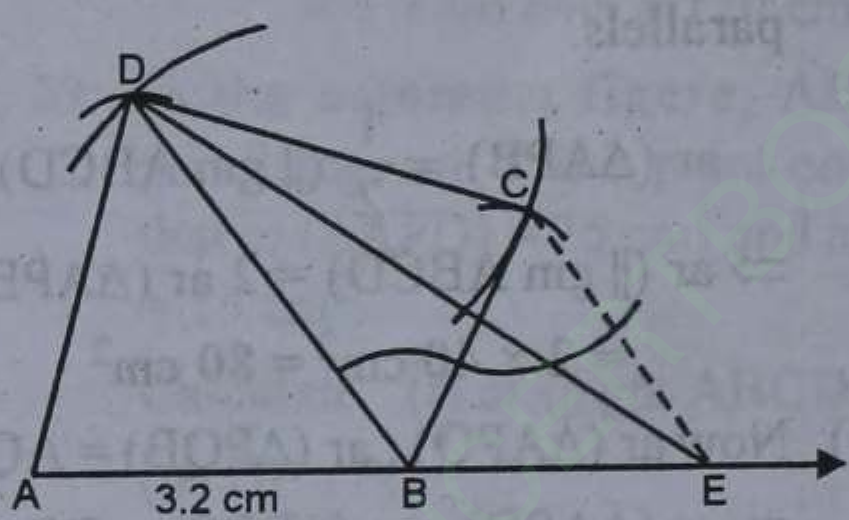
Hence proved.

Q. 32. Construct a quadrilateral ABCD in which AB = 3·2 cm, BC = 2·8 cm, CD = 4 cm, DA = 4·5 cm and BD = 5·2 cm. Also construct a triangle equal in area to this quadrilateral.

Sol. Steps of construction :

(*i*) Draw a line segment AB = 3·2 cm.

(*ii*) With centre A and radius 4·5 cm and with centre B and radius 5·2 cm, draw arcs which intersect each other at D.



(*iii*) Join AD and BD.

(*iv*) Again, with centre B and radius 2·8 cm, and with centre D and radius 4 cm, draw two arcs intersecting each other at C.

(*v*) Join BC and CD.
ABCD is the given quadrilateral.

(*vi*) Produce AB.

(*vii*) From C, draw a line parallel to BD meeting AB produced at E.

(*viii*) Join DE.
Then, ΔADE is the required triangle whose area is equal to the area of the quadrilateral ABCD.