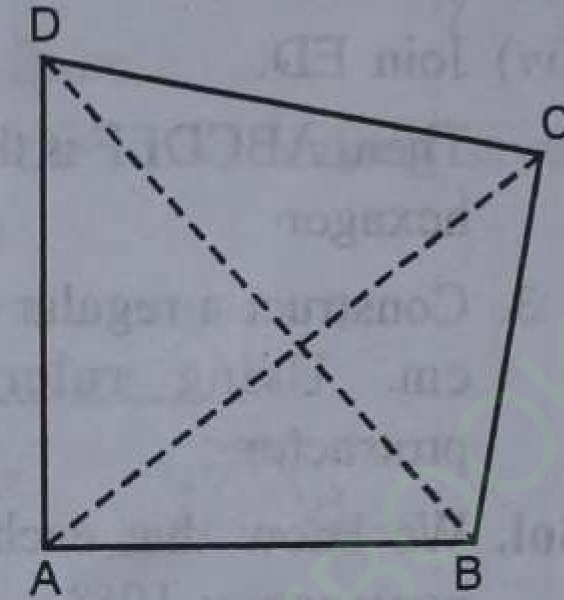
15

Quadrilaterals

POINTS TO REMEMBER

1. **Quadrilateral.** A closed four sided figure is called a quadrilateral.



(*i*) It has four sides, four vertices, four angles and two diagonals.

(*ii*) Sum of its four angles = 360° *i.e.* $\angle A + \angle B + \angle C + \angle D = 360°$.

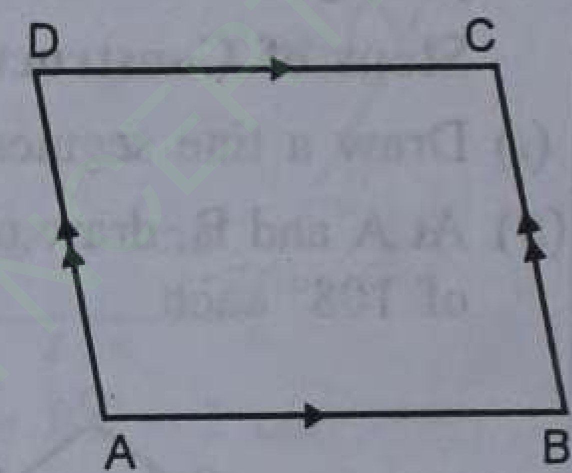
2. **Types of Quadrilaterals**

1. **Parallelogram.** *A quadrilateral in which opposite sides are parallel, is called a parallelogram.*

In the given figure, ABCD is a quadrilateral in which AB ∥ DC and AD ∥ BC.

∴ ABCD is a parallelogram.

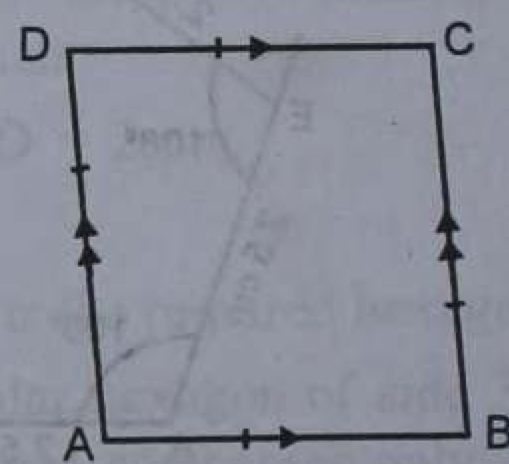
Its opposites sides are equal *i.e.* AB = CD and AD = BC.



2. **Rhombus.** *A parallelogram having all sides equal, is called a rhombus.*

In the given figure, ABCD is a parallelogram in which AB = BC = CD = DA.

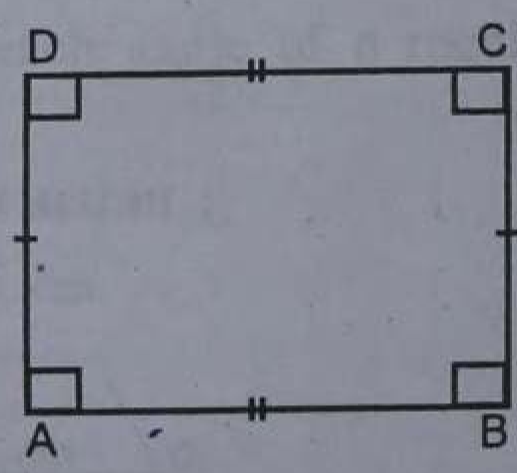
∴ ABCD is a rhombus.



3. **Rectangle.** *A parallelogram each of whose angles measures 90°, is called a rectangle.*

In the given figure, ABCD is a rectangle.

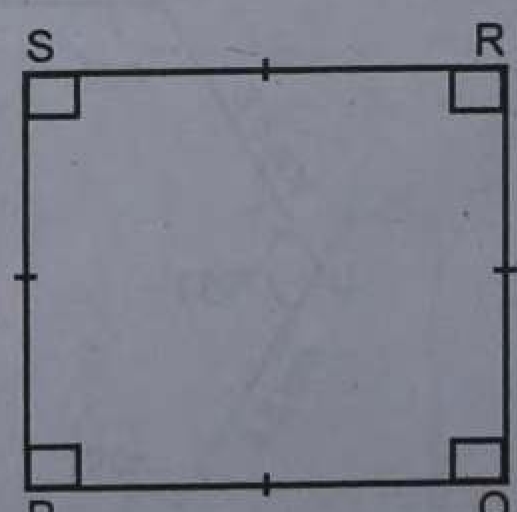
Its opposite sides are equal.



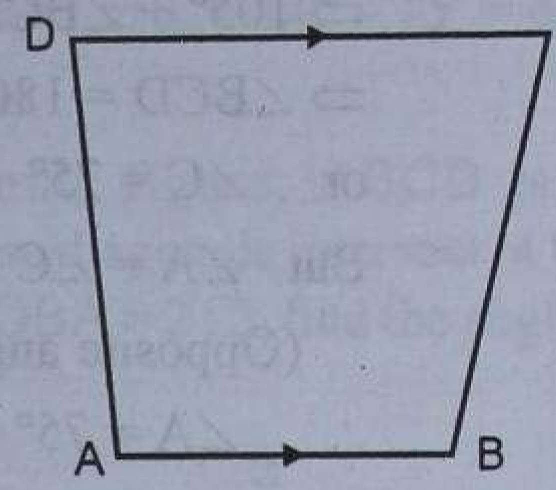
4. **Square.** *A rectangle having all sides equal, is called a square.*

In the given figure, PQRS is a square, in which PQ = QR = RS = SP.

Its each angle is of 90°.



5. **Trapezium.** *A quadrilateral having two parallel opposite sides and two non-parallel opposite sides is called a trapezium.*

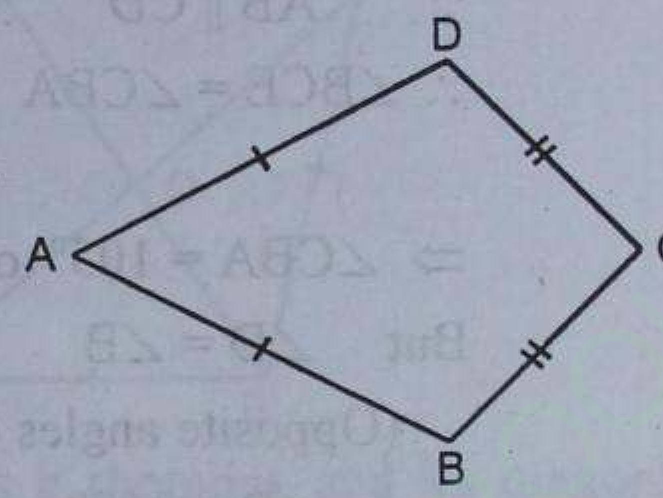


In the given figure, ABCD is a quadrilateral in which AB || DC and AD is not parallel to BC.

∴ ABCD is a trapezium.

If the non-parallel sides of a trapezium are equal, then it is called an **isosceles trapezium.**

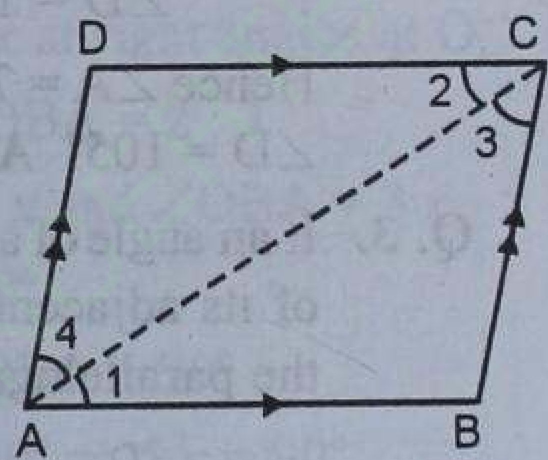
6. **Kite.** *A quadrilateral in which two pairs of adjacent sides are equal, is known as a kite.*



In the given figure, ABCD is a quadrilateral in which AB = AD and CB = CD.

∴ ABCD is a kite.

Results on Parallelogram



Theorem 1. *In a parallelogram :*

(*i*) *the opposite sides qre equal ;*

(*ii*) *the opposite angles are equal ;*

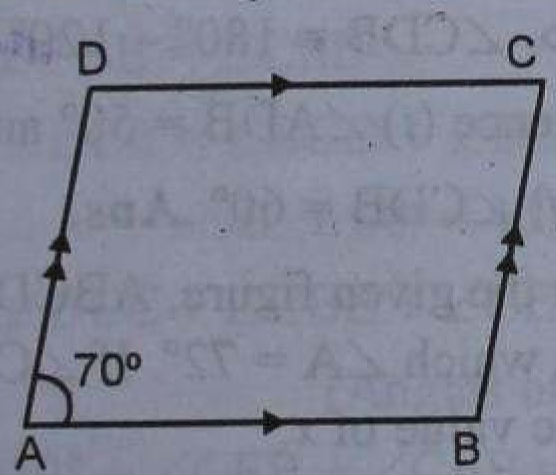
(*iii*) *each diagonal bisects the parallelogram.*

Theorem 2. *The diagonals of a parallelogram bisect each other.*

Theorem 3. *If a pair of opposite sides of a quadrilateral are parallel and equal, then it is a parallelogram*

EXERCISE 15 (A)

Q. 1. In the given figure, ABCD is a parallelogram in which $\angle A = 70^\circ$. Calculate $\angle B$, $\angle C$ and $\angle D$.



Sol. $\because$ ABCD is a parallelogram.

$\therefore \angle A = \angle C$ and $\angle B = \angle D$

$\therefore \angle C = \angle A = 70^\circ$

But $\angle A + \angle B = 180^\circ$

(Co. interior angles)

$\Rightarrow 70^\circ + \angle B = 180^\circ$

$\Rightarrow \angle B = 180^\circ - 70^\circ = 110^\circ$

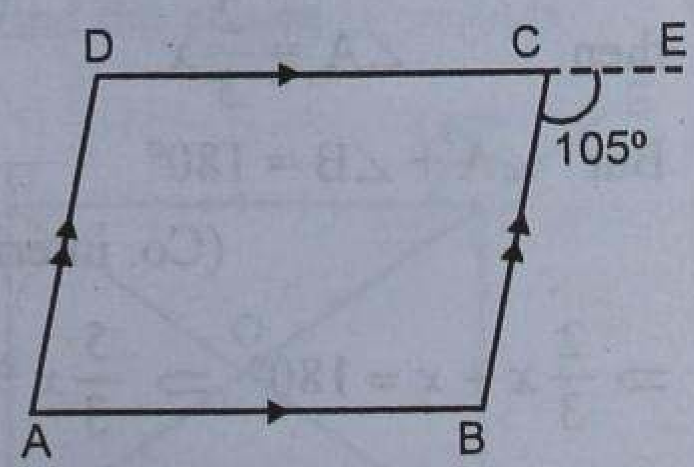
But $\angle D = \angle B$

$\therefore \angle D = 110^\circ$

Hence $\angle B = 110^\circ$, $\angle C = 70^\circ$

and $\angle D = 110^\circ$ **Ans.**

Q. 2. In the given figure, ABCD is a parallelogram. Side DC is produced to E and $\angle BCE = 105^\circ$.



Calculate $\angle A$, $\angle B$, $\angle C$ and $\angle D$.

Sol. ABCD is a parallelogram.

Side DC is produced to E

and $\angle BCE = 105^\circ$

But $\angle BCE + \angle BCD = 180^\circ$

(Linear pair)

$\Rightarrow 105° + \angle BCD = 180°$

$\Rightarrow \angle BCD = 180° - 105° = 75°$

or $\angle C = 75°$

But $\angle A = \angle C$

(Opposite angles of a parallelogram)

$\therefore \angle A = 75°$

$\because AB \parallel CD$

$\therefore \angle BCE = \angle CBA$

(Alternate angles)

$\Rightarrow \angle CBA = 105°$ or $\angle B = 105°$

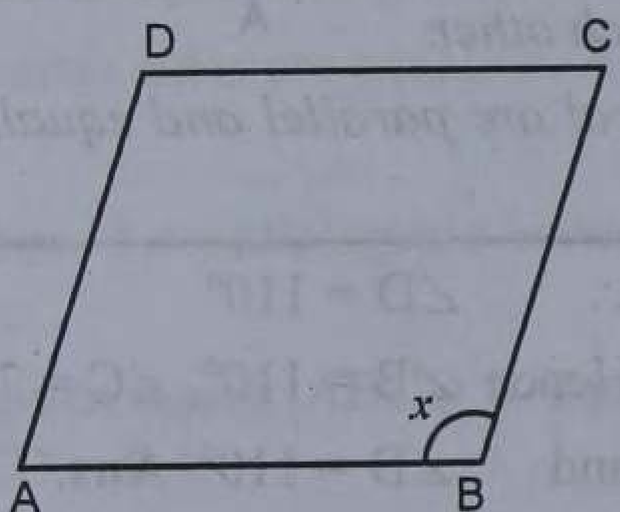
But $\angle D = \angle B$

(Opposite angles of a parallelogram)

$\therefore \angle D = 105°$

Hence $\angle A = 75°, \angle B = 105°, \angle C = 75°, \angle D = 105°$ **Ans.**

Q. 3. If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.



Sol. In parallelogram ABCD,

Let $\angle B = x$

then $\angle A = \frac{2}{3}x$

But $\angle A + \angle B = 180°$

(Co. interior angles)

$\Rightarrow \frac{2}{3}x + x = 180° \Rightarrow \frac{5}{3}x = 180°$

$\Rightarrow x = 180° \times \frac{3}{5} = 108°$

$\therefore \angle B = 108°$ and $\angle A = \frac{2}{3} \times 108° = 72°$

But $\angle C = \angle A$

(Opposite angles of a parallelogram)

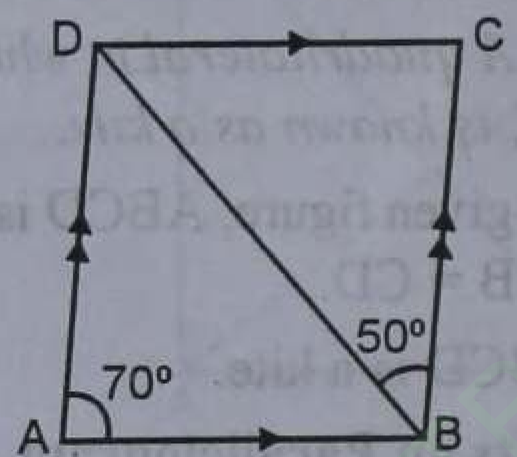
$\therefore \angle C = 72°$

Similarly $\angle D = \angle B = 108°$

Hence $\angle A = 72°, \angle B = 108°, \angle C = 72°$ and $\angle D = 108°$ **Ans.**

Q. 4. In the adjoining figure, ABCD is a parallelogram in which $\angle BAD = 70°$ and $\angle CBD = 50°$. Calculate :

(*i*) $\angle ADB$ (*ii*) $\angle CDB$.



Sol. ABCD is a parallelogram. BD is joined.

$\angle BAD = 70°$ and $\angle CBD = 50°$

$\because AD \parallel BC$

$\therefore \angle ADB = \angle CBD$

(Alternate angles)

$= 50°$

But $\angle BAD + \angle ADC = 180°$

(Co. interior angles)

$\Rightarrow \angle BAD + \angle ADB + \angle CDB = 180°$

$\Rightarrow 70° + 50° + \angle CBD = 180°$

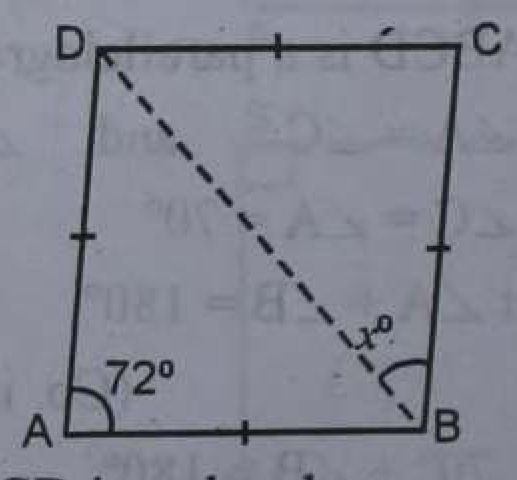
$\Rightarrow 120° + \angle CDB = 180°$

$\Rightarrow \angle CDB = 180° - 120° = 60°$

Hence (*i*) $\angle ADB = 50°$ and

(*ii*) $\angle CDB = 60°$ **Ans.**

Q. 5. In the given figure, ABCD is a rhombus in which $\angle A = 72°$. If $\angle CBD = x°$, find the value of x.



Sol. ABCD is a rhombus.

$\angle A = 72°$ and $\angle CBD = x°$

$\because$ ABCD is a rhombus

$\therefore$ Diagonal BD bisects $\angle B$ and $\angle D$

$\therefore\ \angle ABD = \angle CBD = x$

$\Rightarrow\ \angle ABC = x + x = 2x$

But $\angle A + \angle B = 180^\circ$

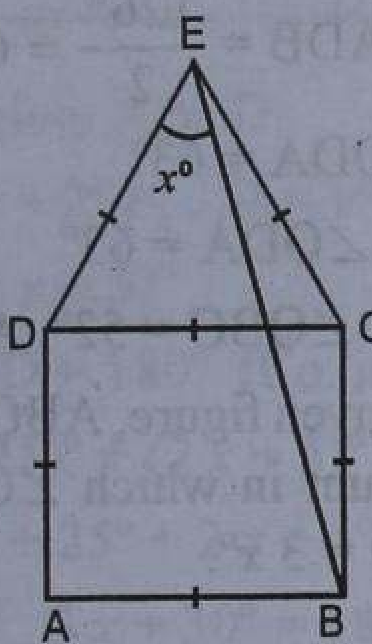
(Co. interior angles)

$\Rightarrow\ 72^\circ + 2x = 180^\circ$

$\Rightarrow\ 2x = 180^\circ - 72^\circ = 108^\circ$

$\therefore\ x = \dfrac{108}{2} = 54^\circ$ **Ans.**

Q. 6. In the adjoining figure, equilateral ΔEDC surmounts square ABCD. If $\angle DEB = x^\circ$, find the value of x.



Sol. In the figure, ABCD is a square and ΔCDE is an equilateral. BE is joined.

$\angle DEB = x^\circ$

In ΔBCE, BC = CE = CD

$\therefore\ \angle CBE = \angle CEB$

and $\angle BCE = \angle BCD + \angle DCE$

$= 90^\circ + 60^\circ = 150^\circ$

But $\angle BCE + \angle CBE + \angle CEB = 180^\circ$

(Angles of a triangle)

$\Rightarrow 150^\circ + \angle CEB + \angle CEB = 180^\circ$

$\Rightarrow 150^\circ + 2\angle CEB = 180^\circ$

$\Rightarrow 2\angle CEB = 180^\circ - 150^\circ = 30^\circ$

$\therefore\ \angle CEB = \dfrac{30^\circ}{2} = 15^\circ$

But $\angle CED = 60^\circ$

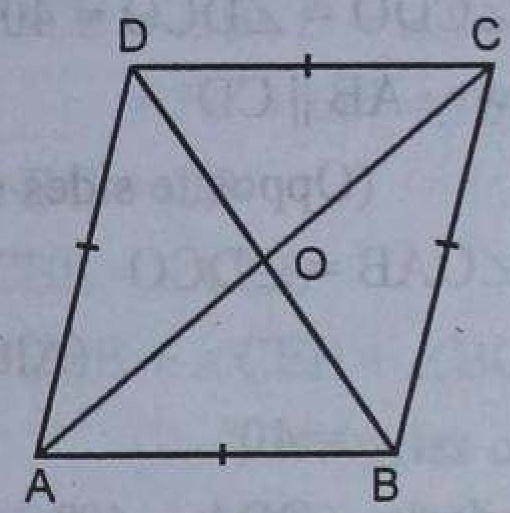
(Angle of an equilateral triangle)

$\Rightarrow x^\circ + \angle CEB = 60^\circ$

$\Rightarrow x^\circ + 15^\circ = 60^\circ \Rightarrow x^\circ = 60^\circ - 15^\circ = 45^\circ$

$\therefore\ x = 45$ **Ans.**

Q. 7. In the adjoining figure, ABCD is a rhombus whose diagonals intersect at O. If $\angle OAB : \angle OBA = 2 : 3$, find the angles of ΔOAB.



Sol. ABCD is a rhombus and its diagonal bisect each other at right angles at O.

$\angle OAB : \angle OBA = 2 : 3$

Let $\angle OAB = 2x$ and $\angle OBA = 3x$

But $\angle AOB = 90^\circ$

$\therefore \angle OAB + \angle OBA = 90^\circ$

$\Rightarrow 2x + 3x = 90^\circ \Rightarrow 5x = 90^\circ$

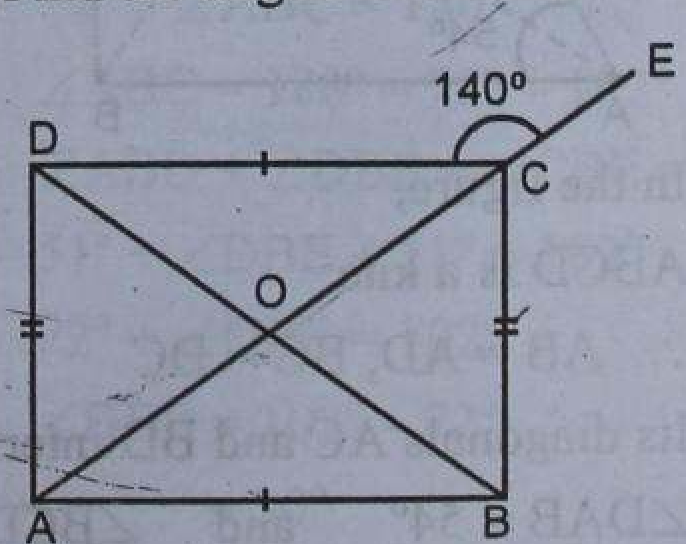
$\therefore\ x = \dfrac{90^\circ}{5} = 18^\circ$

$\therefore\ \angle OAB = 2x = 2 \times 18^\circ = 36^\circ$

$\angle OBA = 3x = 3 \times 18^\circ = 54^\circ$

and $\angle AOB = 90^\circ$ **Ans.**

Q. 8. In the given figure, ABCD is a rectangle whose diagonals intersect at O. Diagonal AC is produced to E and $\angle ECD = 140^\circ$. Find the angles of ΔOAB.



Sol. ABCD is a rectangle and its diagonals AC and BD bisect each other at O.

Diagonal AC is produced to E such that $\angle ECD = 140^\circ$

$\angle ECD + \angle DCO = 180^\circ$ (Linear pair)

$\Rightarrow 140^\circ + \angle DCO = 180^\circ$

$\Rightarrow \angle DCO = 180^\circ - 140^\circ = 40^\circ$

But $OC = OD$

(Half of equal diagonals)

$\therefore \angle CDO = \angle DCO = 40^\circ$

Now $\because AB \parallel CD$

(Opposite sides of a rectangle)

$\therefore \angle OAB = \angle DCO$

(Alternate angles)

$= 40^\circ$

Similarly $\angle OBA = 40^\circ$

Now in ΔAOB

$\angle OBA + \angle OAB + \angle AOB = 180^\circ$

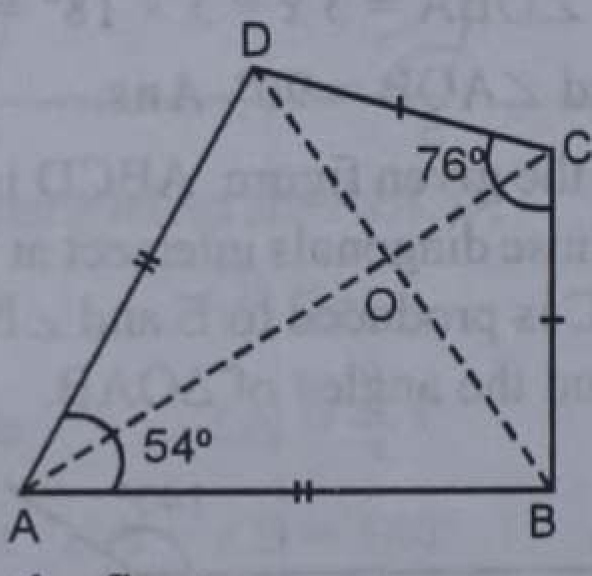
(Angles of a triangle)

$\Rightarrow 40^\circ + 40^\circ + \angle AOB = 180^\circ$

$\Rightarrow 80^\circ + \angle AOB = 180^\circ$

$\Rightarrow \angle AOB = 180^\circ - 80^\circ = 100^\circ$ **Ans.**

Q. 9. In the given figure, ABCD is a kite whose diagonals intersect at O. If $\angle DAB = 54^\circ$ and $\angle BCD = 76^\circ$, calculate : (*i*) $\angle ODA$ (*ii*) $\angle OBC$.



Sol. In the figure,

ABCD is a kite

$\therefore AB = AD, BC = DC$

Its diagonals AC and BD intersect at O.

$\angle DAB = 54^\circ$ and $\angle BCD = 76^\circ$

In ΔBCD,

$\angle CDB = \angle CBD$ ($\because BC = DC$)

But $\angle BCD + \angle CDB + \angle CBD = 180^\circ$

$\Rightarrow 76^\circ + \angle CBD + \angle CDB = 180^\circ$

$\Rightarrow 76^\circ + 2\angle CBD = 180^\circ$

$\Rightarrow 2\angle CBD = 180^\circ - 76^\circ = 104^\circ$

$\therefore \angle CBD = \frac{104^\circ}{2} = 52^\circ$

or $\angle OBC = 52^\circ$

Similarly in ΔABD,

$\angle DAB = 54^\circ$ and $\angle ABD = \angle ADB$

($\because AB = AD$)

But $\angle DAB + \angle ABD + \angle ADB = 180^\circ$

$\Rightarrow 54^\circ + \angle ADB + \angle ADB = 180^\circ$

$\Rightarrow 54^\circ + 2\angle ADB = 180^\circ$

$\Rightarrow 2\angle ADB = 180^\circ - 54^\circ = 126^\circ$

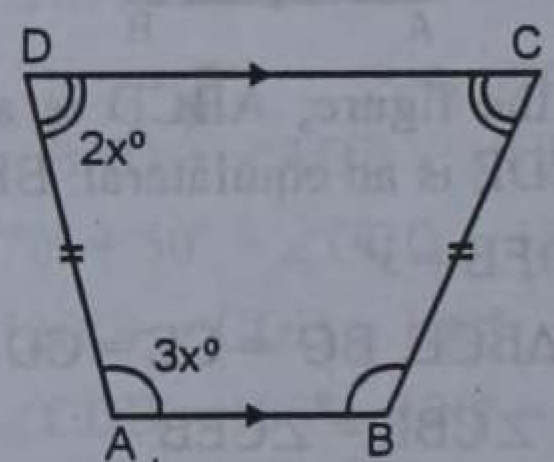
$\therefore \angle ADB = \frac{126^\circ}{2} = 63^\circ$

or $\angle ODA = 63^\circ$

Hence $\angle ODA = 63^\circ$

and $\angle OBC = 52^\circ$ **Ans.**

Q. 10. In the given figure, ABCD is an isosceles trapezium in which $\angle CDA = 2x^\circ$ and $\angle BAD = 3x^\circ$.



Find all the angles of the trapezium.

Sol. ABCD is an isosceles trapezium in which $AD = BC$ and $AB \parallel CD$.

$\angle BAD + \angle CDA = 180^\circ$

(Co. interior angles)

$\Rightarrow 3x + 2x = 180^\circ \Rightarrow 5x = 180^\circ$

$\therefore x = \frac{180^\circ}{5} = 36^\circ$

$\therefore \angle A = 3x = 3 \times 36^\circ = 108^\circ$

$\angle D = 2x = 2 + 36^\circ = 72^\circ$

$\because$ ABCD is an isosceles trapezium.

$\therefore \angle A = \angle B$ and $\angle C = \angle D$

$\therefore \angle B = 108^\circ$

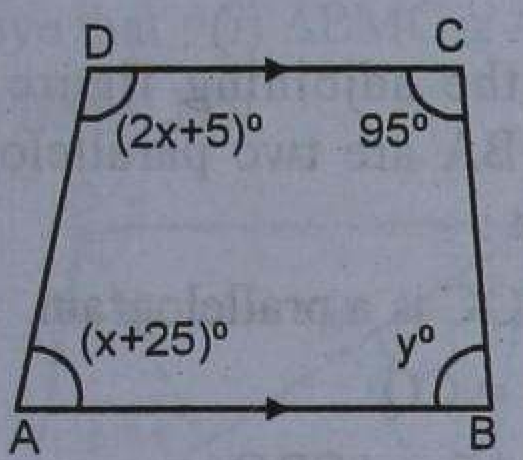
and $\angle C = 72^\circ$

Hence $\angle A = 108^\circ$, $\angle B = 108^\circ$,

$\angle C = 72^\circ$, $\angle D = 72^\circ$ **Ans.**

Q. 11. In the given figure, ABCD is a trapezium in which

$\angle A = (x + 25)^\circ$, $\angle B = y^\circ$, $\angle C = 95^\circ$ and $\angle D = (2x + 5)^\circ$. Find the values of x and y.



Sol. In trapezium ABCD

$\angle A = (x + 25)^\circ$, $\angle B = y^\circ$, $\angle C = 95^\circ$ and $\angle D = (2x + 5)^\circ$

$\angle A + \angle D = 180^\circ$ (Co. interior angles)

$\Rightarrow (x + 25)^\circ + (2x + 5)^\circ = 180^\circ$

$\Rightarrow \quad x + 25^\circ + 2x + 5^\circ = 180^\circ$

$\Rightarrow \quad 3x + 30^\circ = 180^\circ$

$\Rightarrow \quad 3x = 180^\circ - 30^\circ = 150^\circ$

$\therefore \quad x = \dfrac{150^\circ}{3} = 50^\circ$

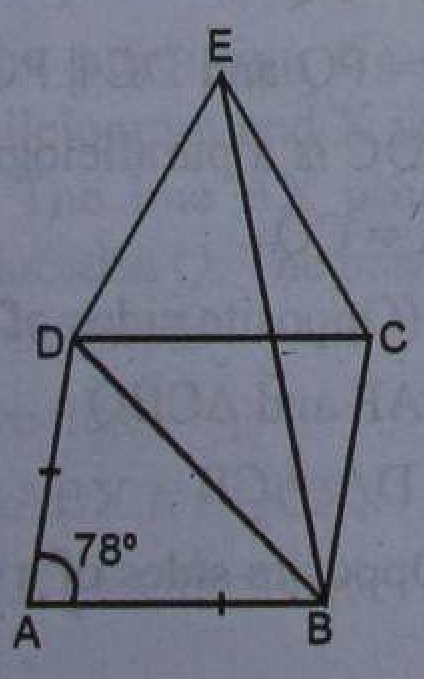
Similarly, $\angle B + \angle C = 180^\circ$

$\Rightarrow \quad y + 95^\circ = 180^\circ$

$\Rightarrow \quad y = 180^\circ - 95^\circ = 85^\circ$

Hence $x = 50^\circ$, $y = 85^\circ$ **Ans.**

Q. 12. In the given figure, ABCD is a rhombus and ΔEDC is equilateral. If $\angle BAD = 78^\circ$, calculate : (*i*) $\angle CBE$ (*ii*) $\angle DBE$.



Sol. (*i*) ABCD is a rhombus and ΔEDC is an equilateral triangle,

$\angle BAD = 78^\circ$

$\therefore \angle BCD = \angle A = 78^\circ$

(Opposite angles of a rhombus)

$\therefore \angle BCE = \angle BCD + \angle DCE$

$= 78^\circ + 60^\circ = 138^\circ$

$\because \quad BC = CE$

$\therefore \angle CBE = \angle CEB$

But $\angle CBE + \angle CEB + \angle BCE = 180^\circ$

(Sum of angles of a triangle)

$\Rightarrow \angle CBE + \angle CBE + 138^\circ = 138^\circ$

$\Rightarrow 2\angle CBE = 180^\circ - 138^\circ = 42^\circ$

$\therefore \quad \angle CBE = \dfrac{42^\circ}{2} = 21^\circ$

Now in ΔABD,

(*ii*) $AB = AD$ (Sides of a rhombus)

$\therefore \angle ABD = \angle ADB$

But $\angle ABD + \angle ADB + \angle BAD = 180^\circ$

$\Rightarrow \angle ABD + \angle ABD + 78^\circ = 180^\circ$

$\Rightarrow 2\angle ABD + 78^\circ = 180^\circ$

$\Rightarrow 2\angle ABD = 180^\circ - 78^\circ = 102^\circ$

$\therefore \angle ABD = \dfrac{102^\circ}{2} = 51^\circ$

But $\angle BAD + \angle ABC = 180^\circ$

(Co. interior angles)

$\Rightarrow \ 78^\circ + \angle ABC = 180^\circ$

$\Rightarrow \ \angle ABC = 180^\circ - 78^\circ = 102^\circ$

$\Rightarrow \ \angle ABD + \angle DBE + \angle CBE = 102^\circ$

$\Rightarrow \ 51^\circ + \angle DBE + 21^\circ = 102^\circ$

$\Rightarrow \ 72^\circ + \angle DBE = 102^\circ$

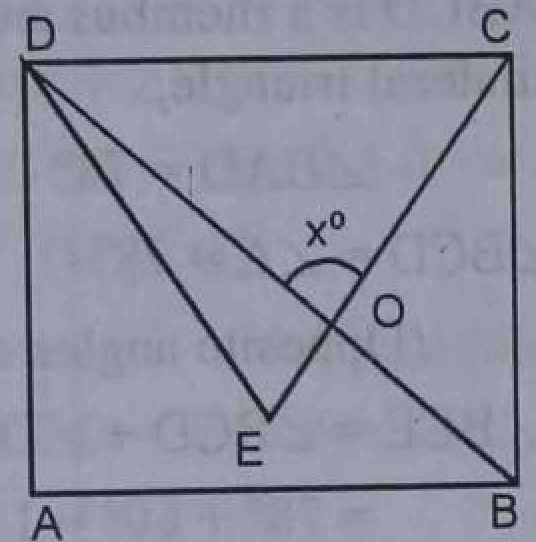
$\Rightarrow \ \angle DBE = 102^\circ - 72^\circ$

$= 30^\circ$

Hence $\angle CBE = 21^\circ$

and $\angle DBE = 30^\circ$ **Ans.**

Q. 13. DEC is an equilateral triangle in a square ABCD. If BD and CE intersect at O and $\angle COD = x^\circ$, find the value of x.



Sol. ABCD is a square and ΔECD is an equilateral triangle. Diagonal BD and CE intersect each other at O, $\angle COD = x^\circ$.

$\because$ BD is the diagonal of square ABCD

$\therefore \angle BDC = \frac{90^\circ}{2} = 45^\circ \Rightarrow \angle ODC = 45^\circ$

$\angle ECD = 60^\circ$

(Angle of equilateral triangle)

or $\angle OCD = 60^\circ$

Now in ΔOCD,

$\angle OCD + \angle ODC + \angle COD = 180^\circ$

(Angles of a triangle)

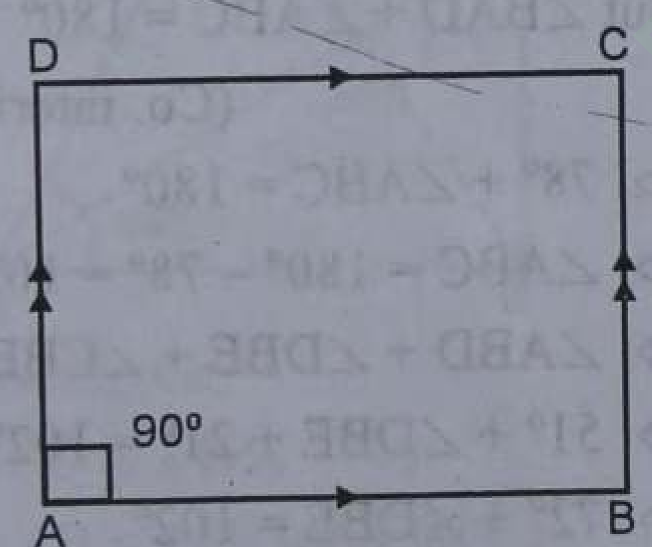
$\Rightarrow 45^\circ + 60^\circ + x^\circ = 180^\circ$

$\Rightarrow 105^\circ + x^\circ = 180^\circ$

$\therefore \quad x^\circ = 180^\circ - 105^\circ = 75^\circ$

Hence $x = 75$ **Ans.**

Q. 14. If one angle of a parallelogram is 90°, show that each of its angles measures 90°.



Sol. Given : ABCD is a parallelogram and $\angle A = 90^\circ$.

To prove : Each angle of the parallelogram ABCD is 90°.

Proof : $\because \angle A = \angle C$

(Opposite angles of a parallelogram)

$\therefore \angle C = 90^\circ \qquad (\because \angle A = 90^\circ)$

But $\angle A + \angle D = 180^\circ$

(Co. interior angles)

$\Rightarrow \quad \angle D = 180^\circ - 90^\circ = 90^\circ$

and $\angle B = \angle D$

(Opposite angles of a parallelogram)

$\therefore \quad \angle B = 90^\circ$

Hence $\angle B = \angle C = \angle D = 90^\circ$

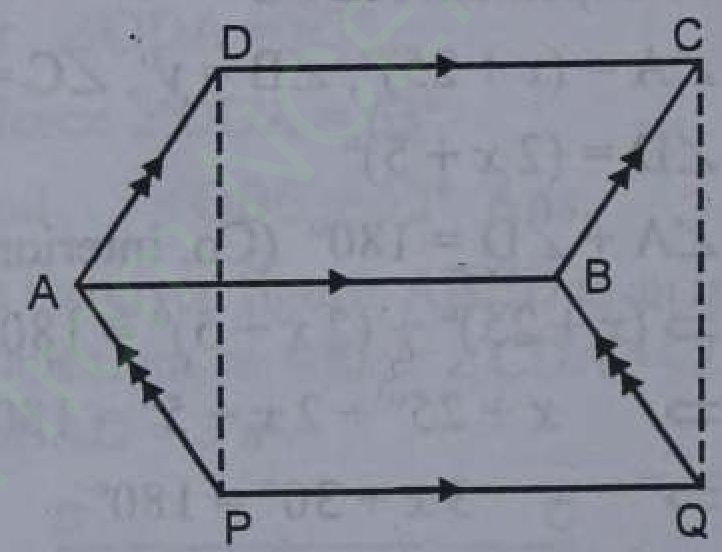
Q.E.D.

Q. 15. In the adjoining figure, ABCD and PQBA are two parallelograms. Prove that :

(*i*) DPQC is a prallelogram.

(*ii*) DP = CQ.

(*iii*) $\Delta DAP \cong \Delta CBQ$.



Sol. Given : ABCD and PQBA are two parallelogram PD and QC are joined.

To prove : (*i*) DPQC is a parallelogram.

(*ii*) DP = CQ (*iii*) $\Delta DAP \cong \Delta CBQ$.

Proof : (*i*) DC $\parallel$ AB and AB $\parallel$ PQ

(Given)

$\therefore$ DC $\parallel$ PQ

Again DC = AB and AB = PQ

(Opposite sides of parallelograms)

$\therefore$ DC = PQ

$\because$ DC = PQ and DC $\parallel$ PQ

$\therefore$ DPQC is a parallelogram.

(*ii*) $\therefore$ DA = CQ

(Opposite sides of parallelogram)

(*iii*) In ΔDAP and ΔCBQ

DA = CB

{Opposite sides of a parallelogram}

AP = BQ

(Opposite sides of parallelogram)

PD = CQ (Proved)

$\therefore \Delta DAP \cong \Delta CBQ$

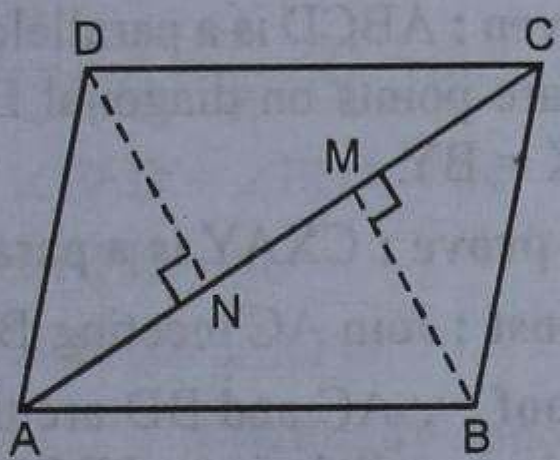
(S.S.S. axiom of congruency)

Hence proved.

Q. 16. In the adjoining figure, ABCD is a parallelogram. $BM \perp AC$ and $DN \perp AC$.

Prove that : (*i*) $\Delta BMC \cong \Delta DNA$.

(*ii*) BM = DN.



Sol. Given : ABCD is a parallelogram.

$BM \perp AC$ and $DN \perp AC$.

To prove :

(*i*) $\Delta BMC \cong \Delta DNA$

(*ii*) BM = DN.

Proof : In ΔBMC and ΔDNA

BC = AD

(Opposite sides of a parallelogram)

$\angle M = \angle N$ (Each 90°)

$\angle BCM = \angle DAN$

(Alternate angles)

(*i*) $\therefore \Delta BMC \cong \Delta DNA$

(AAS axiom of congruency)

(*ii*) $\therefore$ BM = DN (C.P.C.T.)

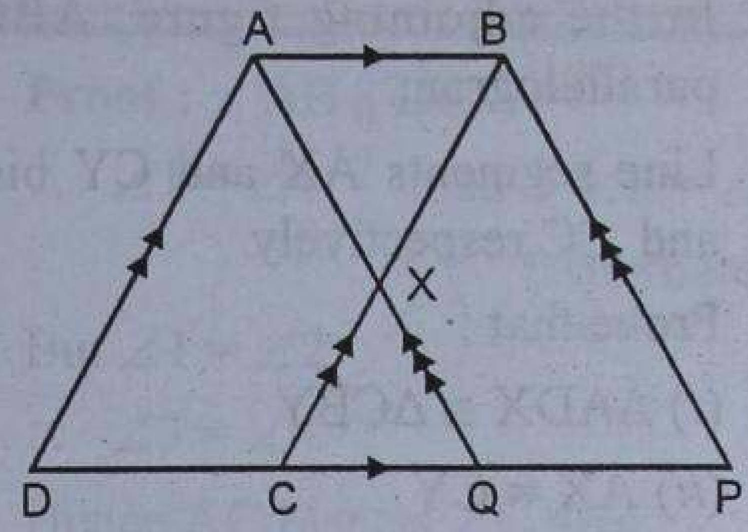
Hence proved.

Q. 17. In the adjoining figure, ABCD is a parallelogram and X is the mid-point of BC. The line AX produced meets DC produced at Q. The parallelogram AQPB is completed.

Prove that :

(*i*) $\Delta ABX \cong \Delta QCX$.

(*ii*) DC = CQ = QP.



Sol. Given : ABCD is a parallelogram. X is mid-point of BC.

AX is joined and produced to meet DC produced at Q. From B, BP is drawn parallel to AQ so that AQPB is a parallelogram.

To prove : (*i*) $\Delta ABX \cong \Delta QCX$.

(*ii*) DC = CQ = QP.

Proof : (*i*) In ΔABX and ΔQCX,

XB = XC ($\because$ X is mid-point of BC)

$\angle AXB = \angle CXQ$

(Vertically opposite angles)

$\angle BAX = \angle XQC$ (Alternate angles)

$\therefore \Delta ABX \cong \Delta QCX$

(AAS axiom of congruency)

Hence proved.

(*ii*) In parallelogram ABCD,

AB = DC ...(*i*)

(Opposite sides of a parallelogram)

Similarly, in parallelogram AQPB

AB = QP ...(*ii*)

$\therefore$ From (*i*) and (*ii*)

DC = QP ...(*iii*)

In ΔBCP,

X is mid-point of BC and AQ || BP.

$\therefore$ Q is mid-point of CP.

$\Rightarrow$ CQ = QP ...(*ii*)

From (*iii*) and (*iv*)

DC = QP = CQ

or DC = CQ = QP

Hence proved.

Q. 18. In the adjoining figure, ABCD is a parallelogram.

Line segments AX and CY bisect ∠A and ∠C respectively.

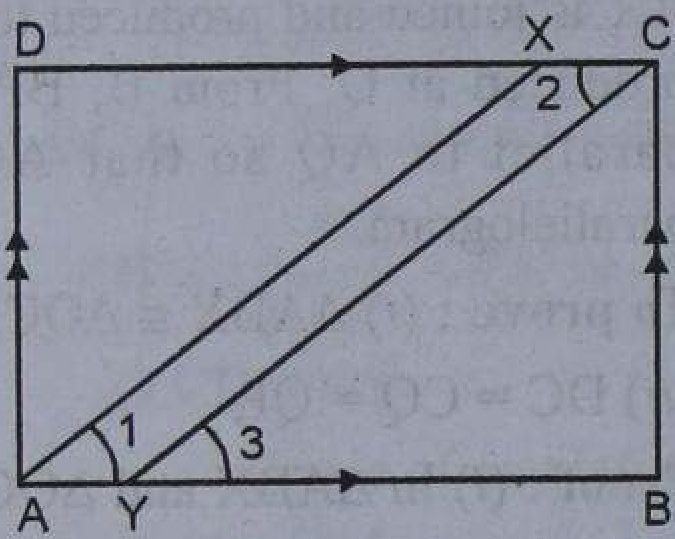
Prove that :

(*i*) ΔADX ≅ ΔCBY

(*ii*) AX = CY

(*iii*) AX ∥ CY

(*iv*) AYCX is a parallelogram.



Sol. Given : ABCD is a parallelogram.

Line segments AX and CY bisect ∠A and ∠C respectively.

To prove : (*i*) ΔADX ≅ ΔCBY

(*ii*) AX = CY (*iii*) AX ∥ CY

(*iv*) AYCX is a parallelogram.

Proof : (*i*) In ΔADX and ΔCBY,

AD = BC

(Opposite sides of a parallelogram)

∠D = ∠B

(Opposite angles of the parallelogram)

∠DAX = ∠BCY

{half of equal angles A and C}

∴ ADX ≅ ΔCBY

(AAS axiom of congruency)

(*ii*) ∴ AX = CY (C.P.C.T.)

(*iii*) ∠1 = ∠2 (Half of equal angles)

But ∠2 = ∠3 (Alternate angles)

∴ ∠1 = ∠3

But these are corresponding angles.

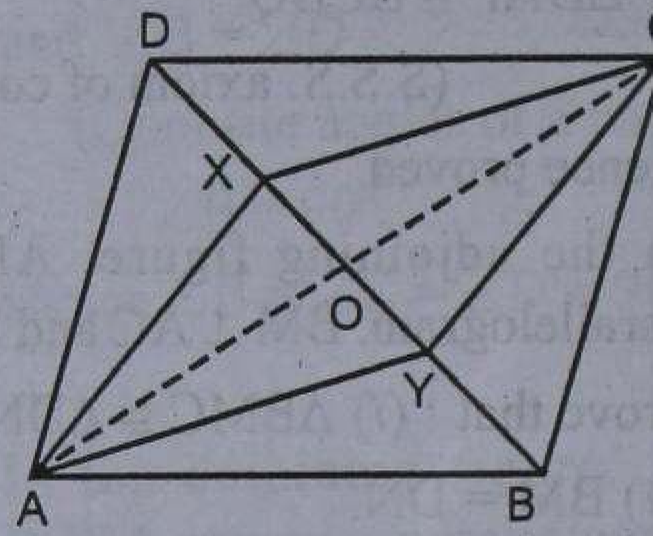
∴ AX ∥ CY

(*iv*) ∵ AX = CY and AX ∥ CY

∴ AYCX is a parallelogram.

Hence proved.

Q. 19. In the given figure, ABCD is a parallelogram and X, Y are points on diagonal BD such that DX = BY. Prove that CXAY is a parallelogram.



Sol. Given : ABCD is a parallelogram. X and Y are points on diagonal BD such that DX = BY.

To prove : CXAY is a parallelogram.

Const : Join AC meeting BD at O.

Proof : ∵ AC and BD are the diagonals of the parallelogram ABCD.

∴ AC and BD bisect each other at O.

∴ AO = OC and BO = OD

But DX = BY (Given)

∴ DO – DX = OB – BY

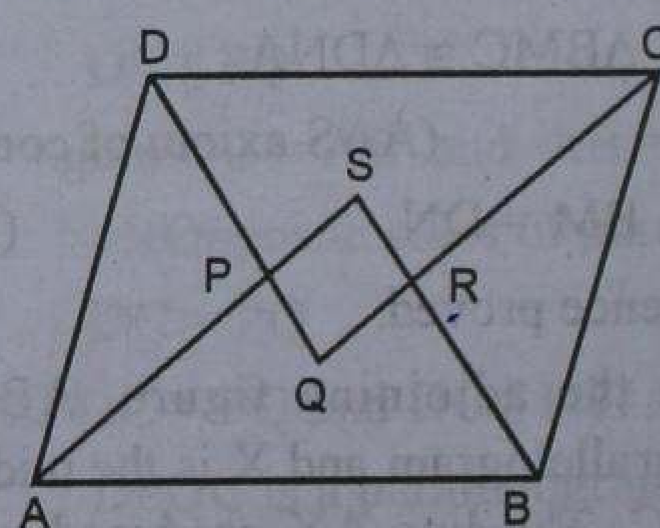
⇒ OX = OY

Now in quadrilateral CXAY, diagonals AC and XY bisect each other at O.

∴ CXAY is a parallelogram.

Hence proved.

Q. 20. Show that the bisectors of the angles of a parallelogram enclose a rectangle.



Sol. Given : ABCD is a parallelogram.

Bisectors of ∠A and ∠B meet at S and bisectors of ∠C and ∠D meet at Q.

To prove : PQRS is a rectangle.

Proof : ∵ ∠A + ∠B = 180°

$\therefore \quad \frac{1}{2}\angle A + \frac{1}{2}\angle B = 90^\circ$

$\Rightarrow \angle SAB = \angle SBA = 90^\circ$

$\therefore$ In ΔASB,

$\angle ASB = 90^\circ$

Similarly we can prove that

$\angle CQD = 90^\circ$

Again $\angle A + \angle D = 180^\circ$ (Co. interior angles)

$\therefore \quad \frac{1}{2}\angle A + \frac{1}{2}\angle D = 90^\circ$

$\Rightarrow \angle PAD = \angle PDA = 90^\circ$

$\therefore \quad \angle APD = 90^\circ$

But $\angle SPQ = \angle APD$ (Vertically opposite angles)

$\therefore \quad \angle SPQ = 90^\circ$

$\because$ Similarly we can prove that

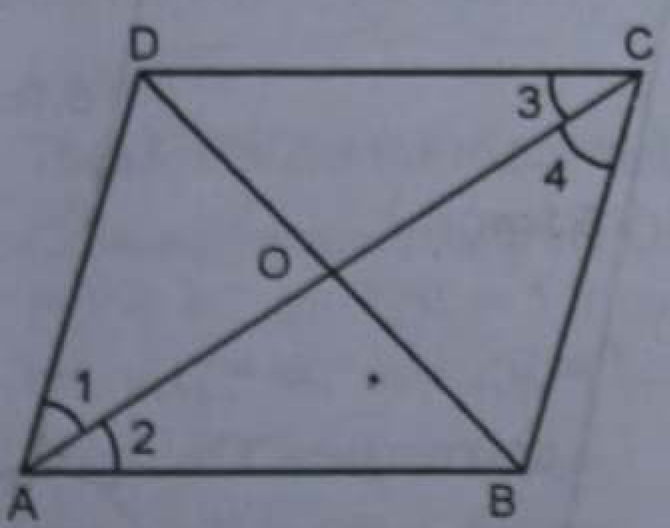
$\angle SRQ = 90^\circ$

$\because$ In quadrilateral PQRS, its each angle is of 90°

$\therefore$ PQRS is a rectangle.

Hence proved.

Q. 21. If a diagonal of a parallelogram bisects one of the angles of the parallelogram, prove that it also bisects the second angle and then the two diagonals are perpendicular to each other.



Sol. Given : In parallelogram ABCD, diagonal AC bisects $\angle A$. BD is joined meeting AC at O.

To prove : (*i*) AC bisects $\angle C$.

(*ii*) Diagonal AC and BD are perpendicular to each other.

Proof : $\because$ AB $\parallel$ DC

$\therefore \quad \angle 1 = \angle 4$ and $\angle 2 = \angle 3$ (Alternate angles)

But $\angle 1 = \angle 2$ (Given)

$\therefore \quad \angle 3 = \angle 4$

Hence AC bisects $\angle C$ also.

Similarly we can prove that diagonal BD will also bisect the $\angle B$ and $\angle D$.

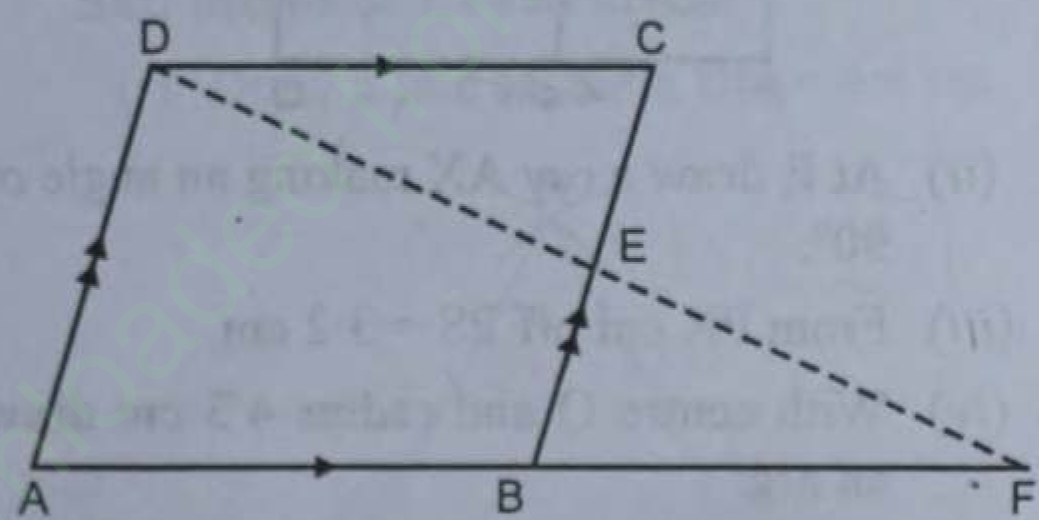
$\therefore$ ABCD is a rhombus.

But diagonals of a rhombus bisect each other at right angles.

$\therefore$ AC and BD are perpendicular to each other.

Hence proved.

Q. 22. In the given figure, ABCD is a parallelogram and E is the mid-point of BC. If DE and AB produced meet at F, prove that AF = 2 AB.



Sol. Given : ABCD is a parallelogram. E is mid-point of BC. DE and AB are produced to meet at F.

To prove : AF = 2 AB.

Proof : In ΔDEC and ΔFEB

CE = EB ($\because$ E is mid-point of BC)

$\angle DEC = \angle BEF$ (Vertically opposite angles)

$\angle DCE = \angle EBF$ (Alternate angles)

$\therefore \Delta DEC \cong \Delta FEB$ (AAS axiom of congruency)

$\therefore \quad$ CD = BF (C.P.C.T.)

But AB = CD (Opposite sides of a parallelogram)

$\therefore \quad AB = BF$

$AF = AB + BF$

$= AB + AB = 2\ AB$

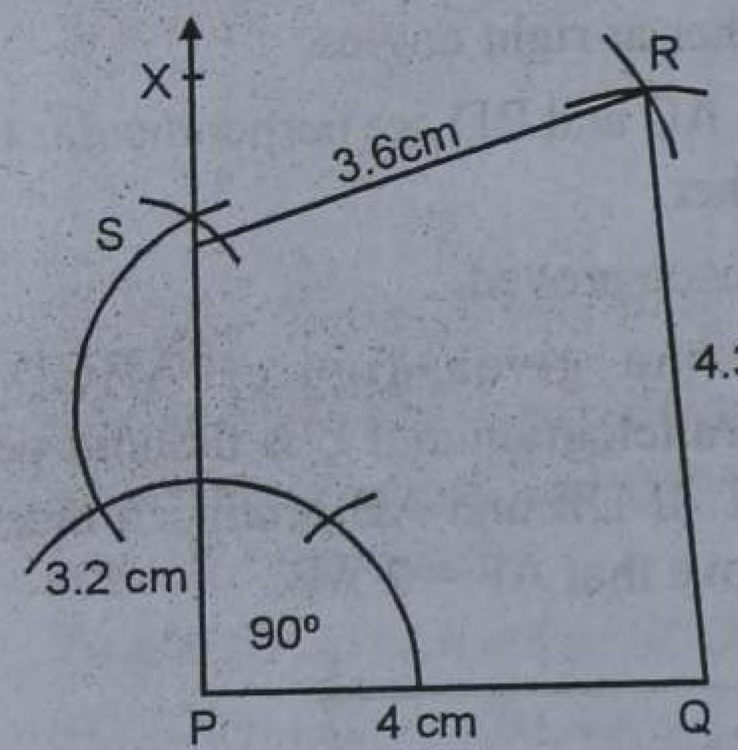
Hence proved.

EXERCISE 15 (B)

Q. 1. Construct a quadrilateral PQRS in which $PQ = 4$ cm, $\angle P = 90^\circ$, $QR = 4 \cdot 3$ cm, $RS = 3 \cdot 6$ cm and $SP = 3 \cdot 2$ cm.

Sol. Steps of Construction :

(*i*) Draw a line segment $PQ = 4$ cm.



(*ii*) At P, draw a ray AX making an angle of 90°.

(*iii*) From PX cut off $PS = 3 \cdot 2$ cm.

(*iv*) With centre Q and radius $4 \cdot 3$ cm draw an arc.

(*v*) With centre S and radius $3 \cdot 6$ cm draw another arc which intersects the first arc at R.

(*vi*) Join QR and SR.

Then PQRS is the required quadrilateral.

Q. 2. Construct a quadrilateral ABCD in which $AB = 4 \cdot 5$ cm, $BC = 5 \cdot 2$ cm, $CD = 5$ cm, $DA = 4 \cdot 7$ cm and $\angle ABC = 75^\circ$.

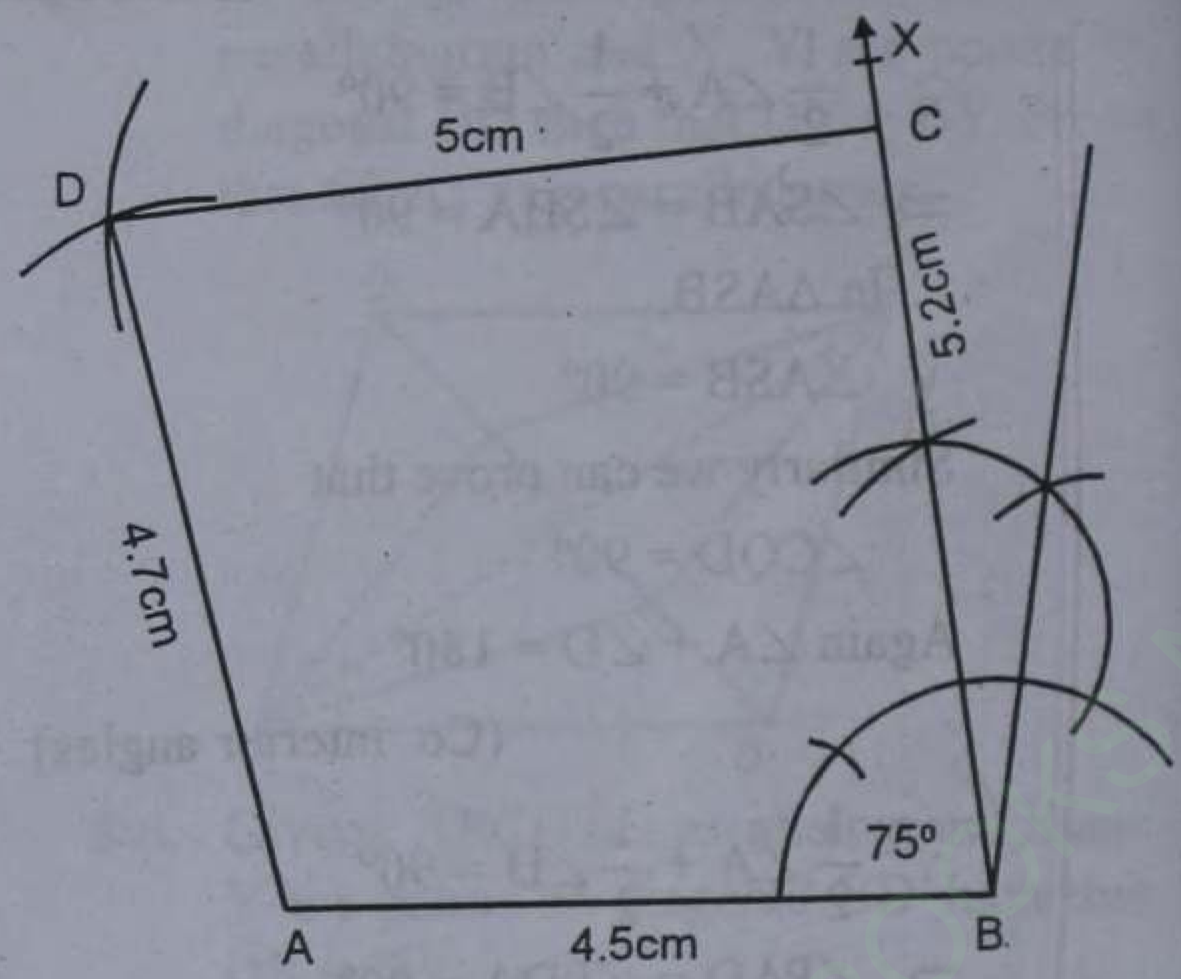
Sol. Steps of Construction :

(*i*) Draw a line segment $AB = 4 \cdot 5$ cm.

(*ii*) At B, draw an arc BX making an angle of 75°.

(*iii*) From BX, cut off $BC = 5 \cdot 2$ cm.

(*iv*) With centre C and radius 5 cm draw an arc.



(*v*) With centre A and radius $4 \cdot 7$ cm draw another arc which intersects the first arc at D.

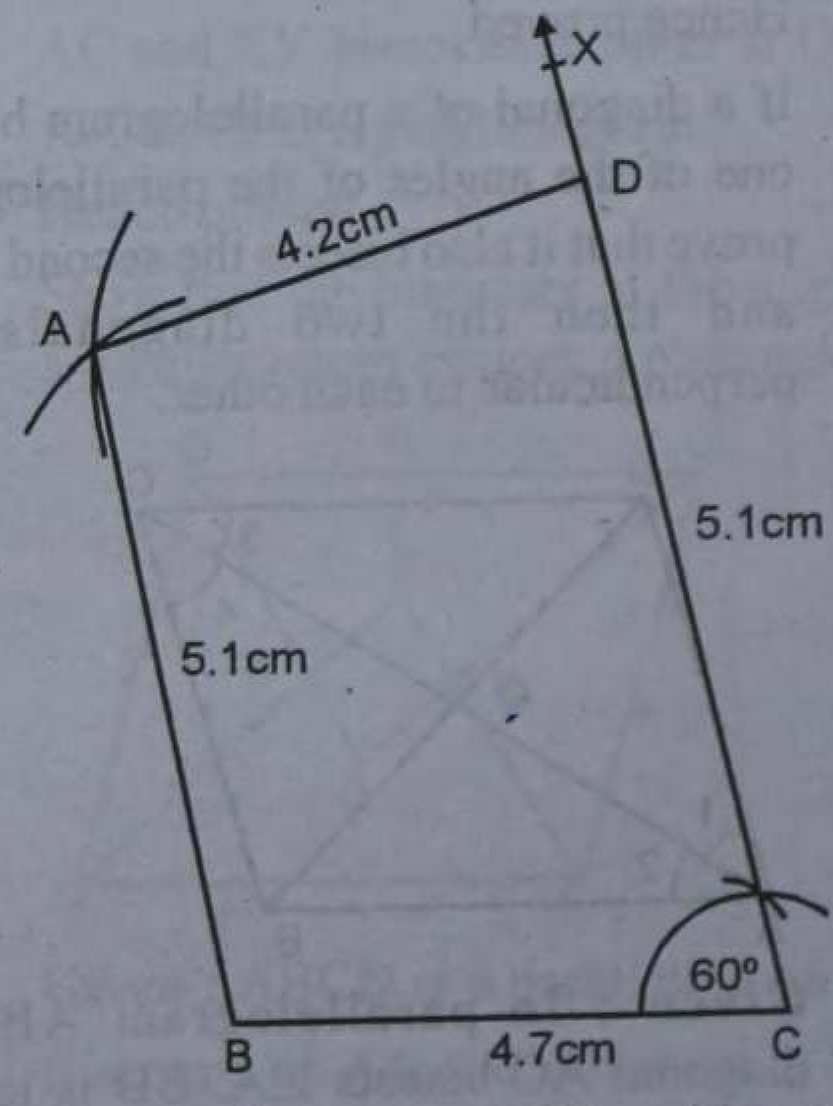
(*vi*) Join AD and CD.

Then ABCD is the required quadrilateral.

Q. 3. Construct a quadrilateral ABCD in which $AB = CD = 5 \cdot 1$ cm, $BC = 4 \cdot 7$ cm, $DA = 4 \cdot 2$ cm and $\angle BCD = 60^\circ$.

Sol. Steps of Construction :

(*i*) Draw a line segment $BC = 4 \cdot 7$ cm.



(*ii*) At C, draw an arc CX making an angle of 60° and cut off $CD = 5 \cdot 1$ cm.

(*iii*) With centre D and radius $4 \cdot 2$ cm, draw an arc.

(*iv*) With centre B and radius 5·1 cm, draw another arc intersecting the first arc at A.

(*v*) Join AB and AD.

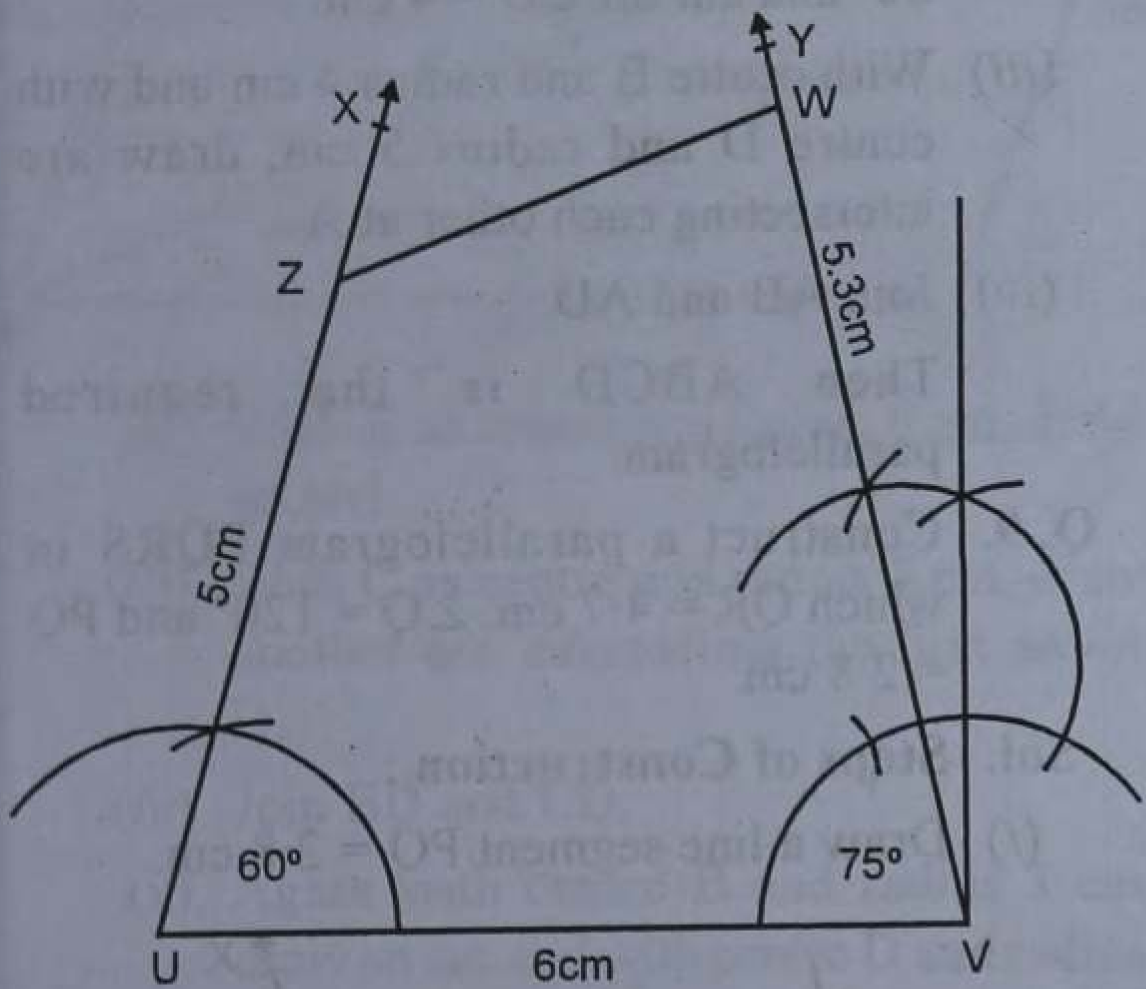
Then ABCD is the required quadrilateral.

Q. 4. Construct a quadrilateral UVWZ in which UV = 6 cm, VW = 5·3 cm, UZ = 5 cm, $\angle U = 60^\circ$ and $\angle V = 75^\circ$.

Sol. Steps of Construction :

(*i*) Draw a line segment UV = 6 cm.

(*ii*) At U, draw a ray UX making an angle of 60°.



(*iii*) At V, draw another ray VY making an angle of 75°.

(*iv*) From UX, cut UZ = 5 cm and from VY, cut off UW = 5·3 cm.

(*v*) Join WZ.

Then UVWZ is the required quadrilateral.

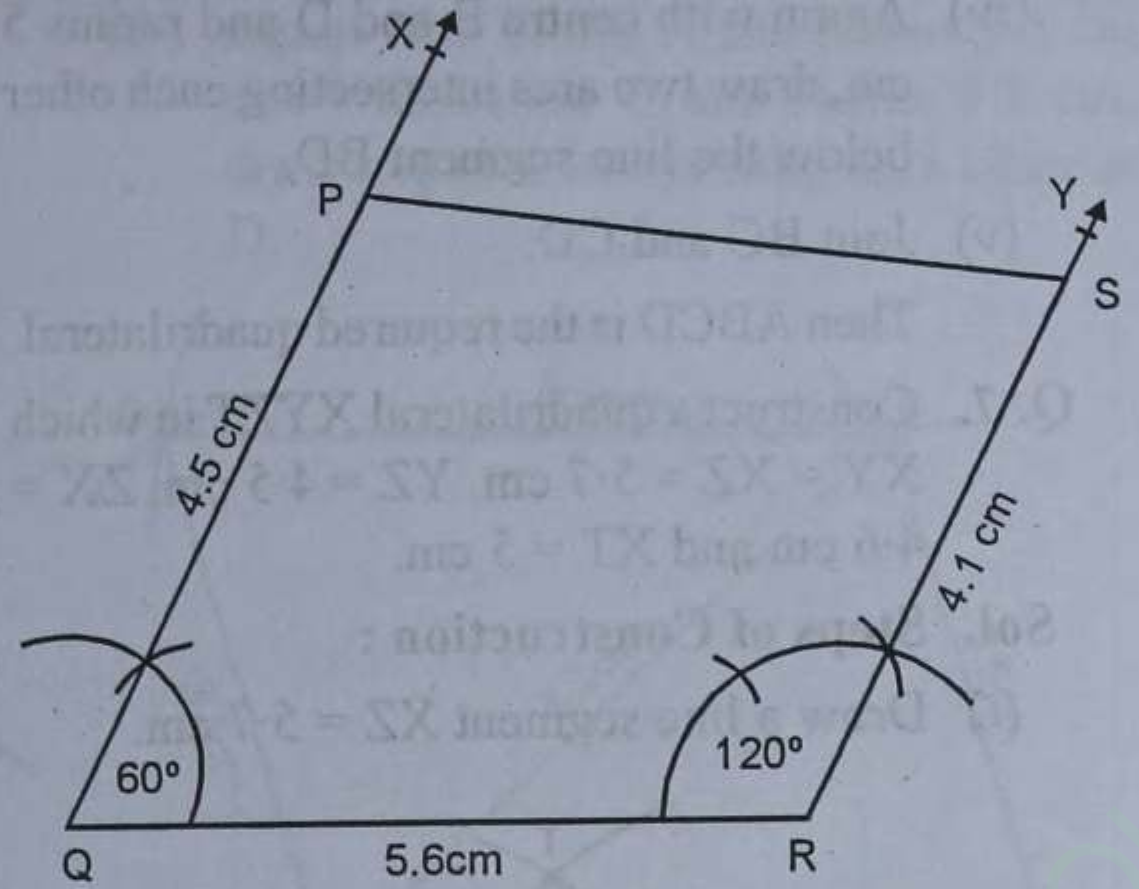
Q. 5. Construct a quadrilateral PQRS in which PQ = 4·5 cm, QR = 5·6 cm, RS = 4·1 cm, $\angle Q = 60^\circ$ and $\angle R = 120^\circ$

Sol. Steps of Construction :

(*i*) Draw a line QR = 5·6 cm.

(*ii*) At Q, draw a ray QX making an angle of 60°.

(*iii*) At R, draw another ray RY making an angle of 120°.



(*iv*) Cut off from QX, QP = 4·5 cm and from RY, RS = 4·1 cm.

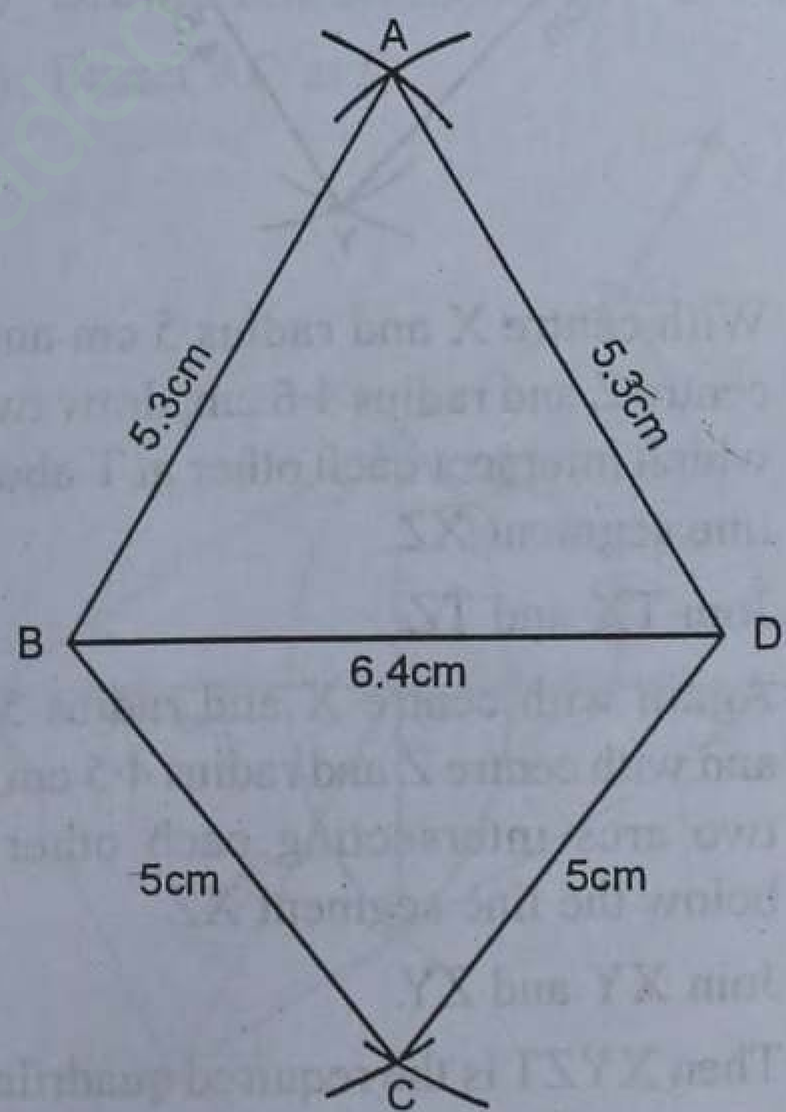
(*v*) Join PS.

Then PQRS is the required quadrilateral.

Q. 6. Draw a quadrilateral ABCD in which AB = AD = 5·3 cm, BC = CD = 5 cm and diagonal BD = 6·4 cm.

Sol. Steps of Construction :

(*i*) Draw a line segment BD = 6·4 cm.



(*ii*) With B and D as centres and radius 5·3 cm each draw two arcs intersecting each other at A above BD.

(*iii*) Join AB and AD.

(iv) Again with centre B and D and radius 5 cm, draw two arcs intersecting each other below the line segment BD.

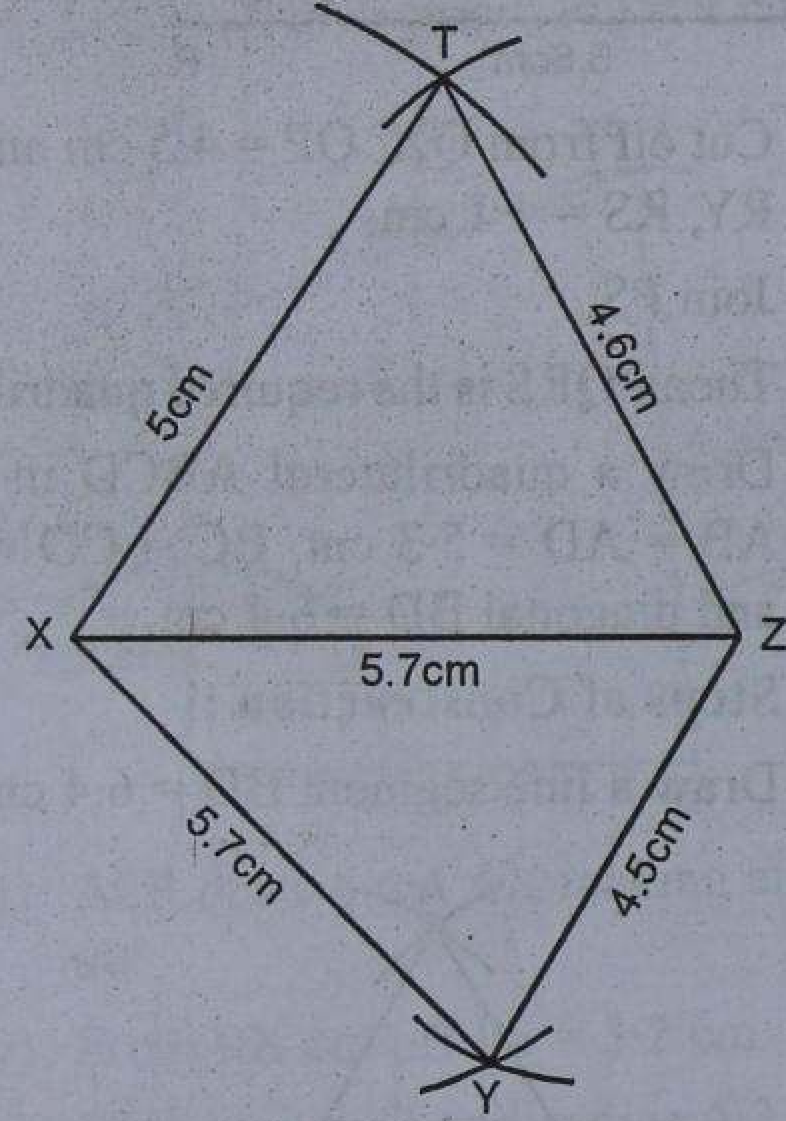
(v) Join BC and CD.

Then ABCD is the required quadrilateral.

Q. 7. Construct a quadrilateral XYZT in which XY = XZ = 5·7 cm, YZ = 4·5 cm, ZX = 4·6 cm and XT = 5 cm.

Sol. Steps of Construction :

(i) Draw a line segment XZ = 5·7 cm.



(ii) With centre X and radius 5 cm and with centre Z and radius 4·6 cm, draw two arcs which intersect each other at T above the line segment XZ.

(iii) Join TX and TZ.

(iv) Again with centre X and radius 5·7 cm and with centre Z and radius 4·5 cm, draw two arcs intersecting each other at Y below the line segment XZ.

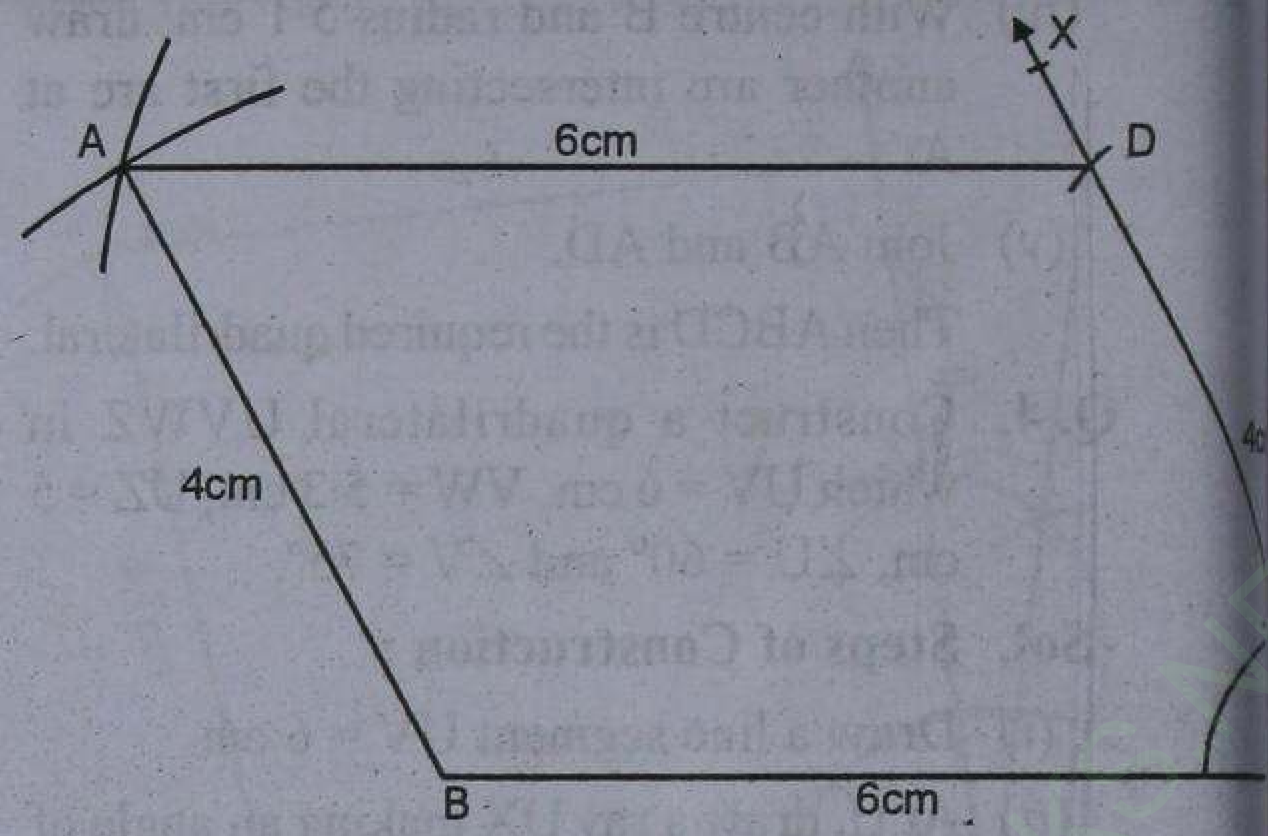
(v) Join XY and ZY.

Then XYZT is the required quadrilateral.

Q. 8. Construct a parallelogram ABCD in which BC = 6 cm, CD = 4 cm and ∠C = 60°.

Sol. Steps of Construction :

(i) Draw a line segment BC = 6 cm.



(ii) At C, draw a ray CX making an angle of 60° and cut off CD = 4 cm.

(iii) With centre B and radius 4 cm and with centre D and radius 5 cm, draw arc intersecting each other at A.

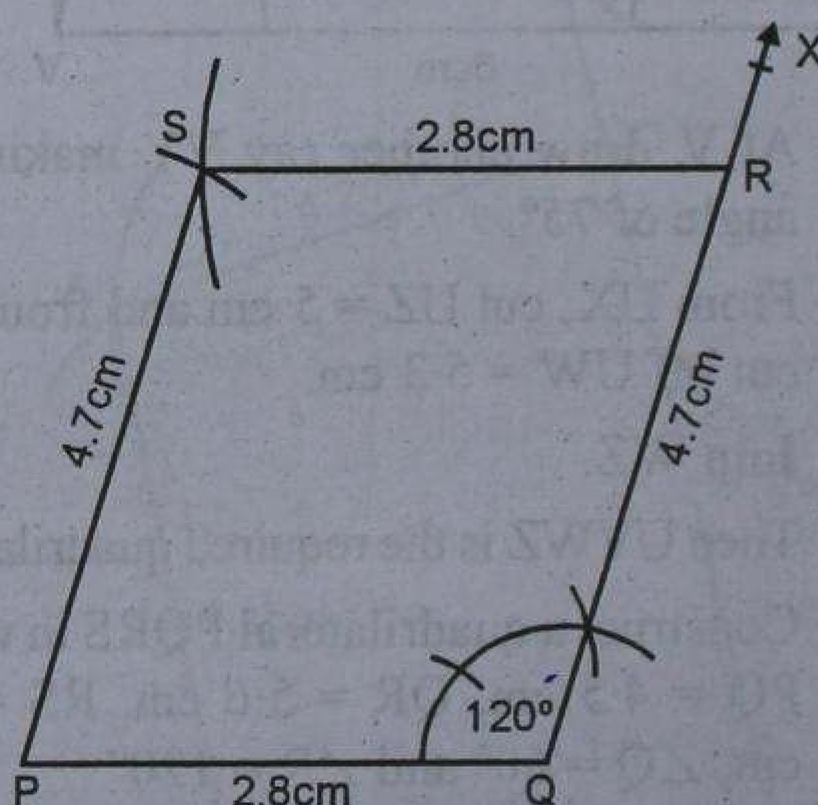
(iv) Join AB and AD.

Then ABCD is the required parallelogram.

Q. 9. Construct a parallelogram PQRS in which QR = 4·7 cm, ∠Q = 120° and PQ = 2·8 cm.

Sol. Steps of Construction :

(i) Draw a line segment PQ = 2·8 cm.



(ii) At Q, draw a ray QX making an angle of 120° and cut off QR = 4·7 cm.

(iii) With centre P and radius 4·7 cm and with centre R and radius 2·8 cm draw arcs intersecting each other at S.

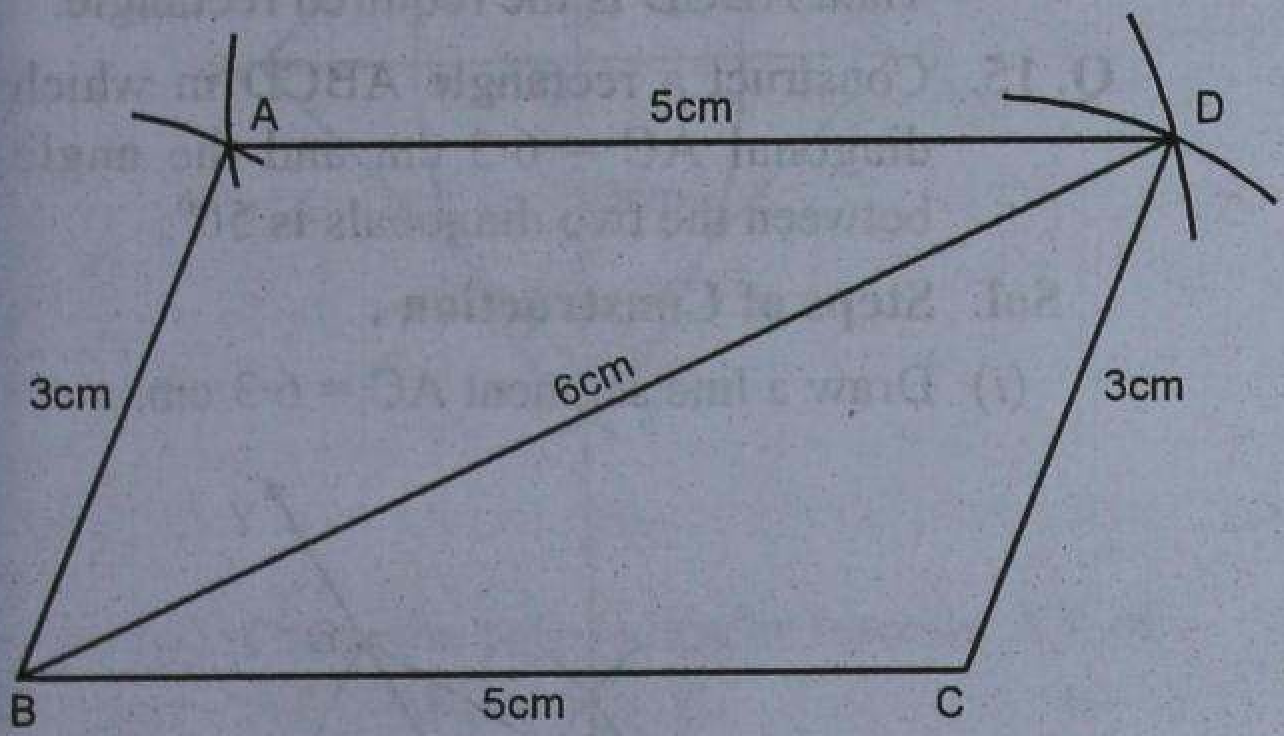
(iv) Join SP and SR.

Then PQRS is the required parallelogram.

Q. 10. Construct a parallelogram ABCD in which BC = 5 cm, CD = 3 cm and diagonal BD = 6 cm.

Sol. Steps of Construction :

(*i*) Draw a line segment BC = 5 cm.



(*ii*) With B as centre and radius 6 cm, draw an arc.

(*iii*) With C as centre and radius 3 cm, draw another arc intersecting the first arc at D.

(*iv*) Join BD and CD.

(*v*) Again with centre B and radius 3 cm draw an arc and with centre D and radius 5 cm draw another arc intersecting each other at A.

(*vi*) Join AB and AD.

Then ABCD is the required parallelogram.

Q. 11. Construct a parallelogram ABCD in which AB = 4·8 cm, BC = 3·5 cm and diagonal AC = 5·4 cm.

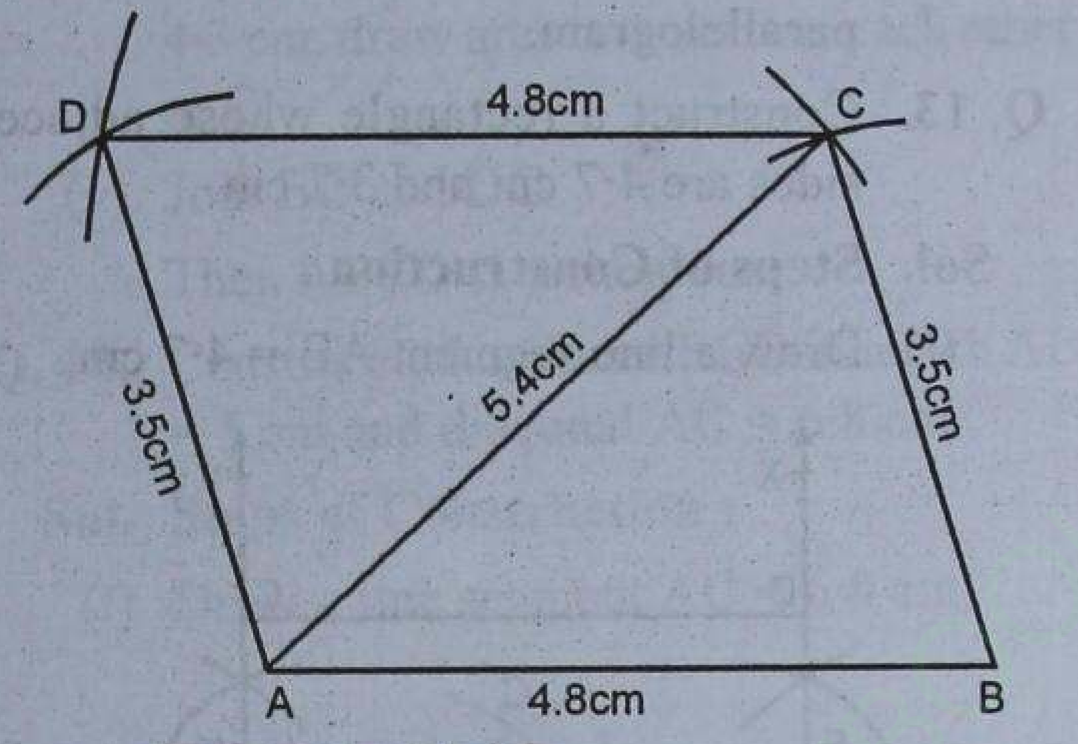
Sol. Steps of Construction :

(*i*) Draw a line segment AB = 4·5 cm.

(*ii*) With centre A and radius 5·4 cm draw an arc and with centre B and radius 3·5 cm, draw another arc intersecting the first arc at C.

(*iii*) Join AC and BC.

(*iv*) Again with centre A and radius 3·5 cm and with centre C and radius 4·8 cm, draw two arcs intersecting each other at D.



(*v*) Join CD and AD.

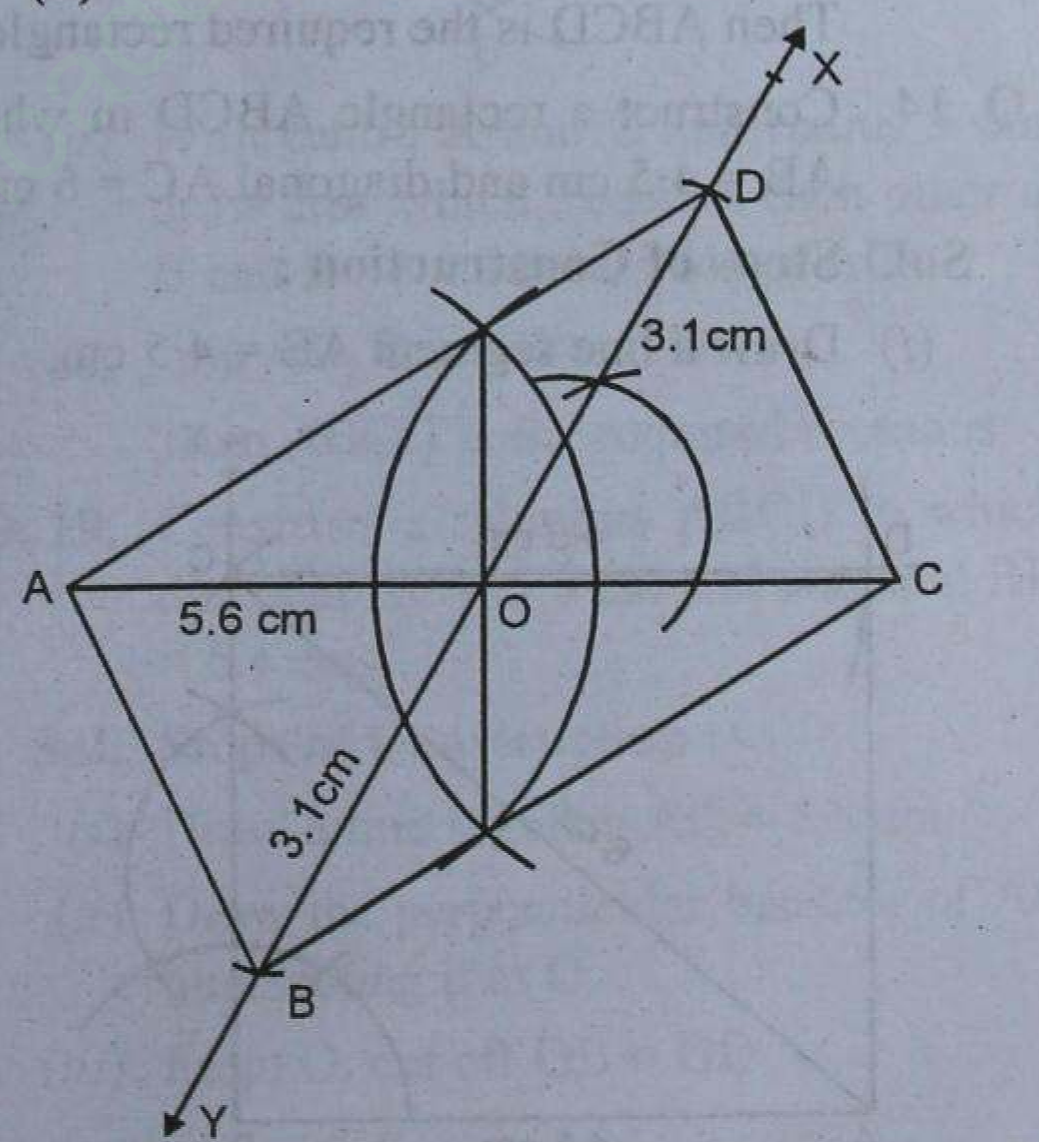
Then ABCD is the required parallelogram.

Q. 12. Construct a parallelogram ABCD in which diagonal AC = 5·6 cm, diagonal BD = 6·2 cm and angle between them is 60°.

Sol. Steps of Construction :

(*i*) Draw a line segment AC = 5·6 cm.

(*ii*) Bisect AC at O.



(*iii*) At O, draw a line XY making an angle of 60° and cut off OB = OD

$$= \frac{6\cdot2}{2} = 3\cdot1\,\text{cm}.$$

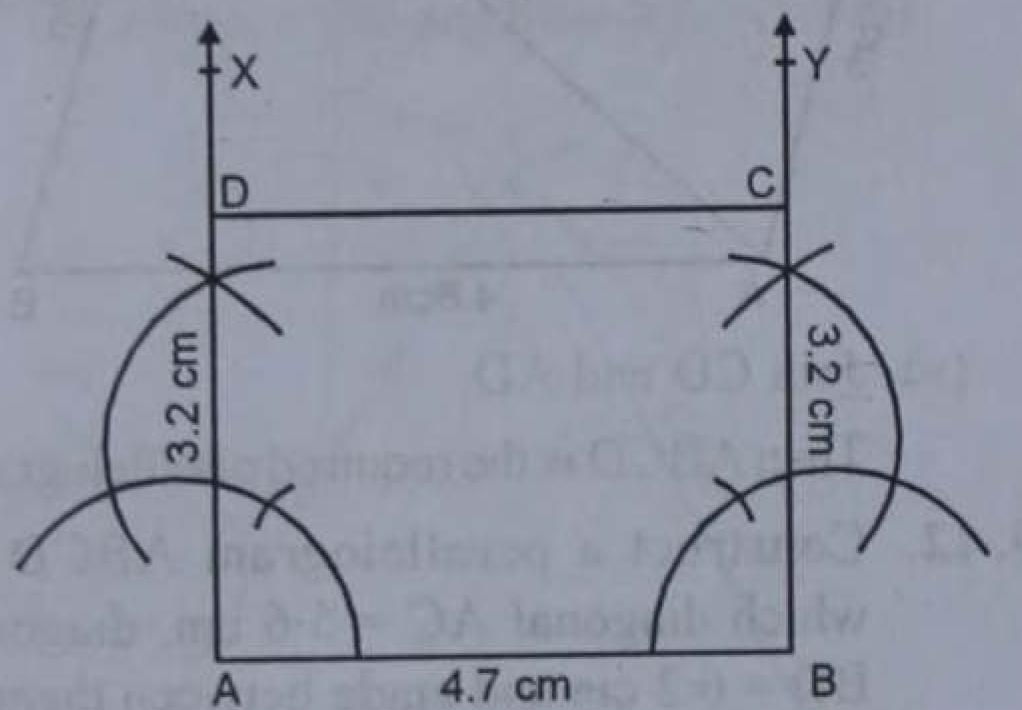
(*iv*) Join AB, BC, CD and DA.

Then ABCD is the required parallelogram.

Q. 13. Construct a rectangle whose adjacent sides are 4·7 cm and 3·2 cm.

Sol. Steps of Construction :

(*i*) Draw a line segment AB = 4·7 cm.



(*ii*) At A and B, draw rays AX and BY making an angle of 90° each and cut off AD = BC = 3·2 cm.

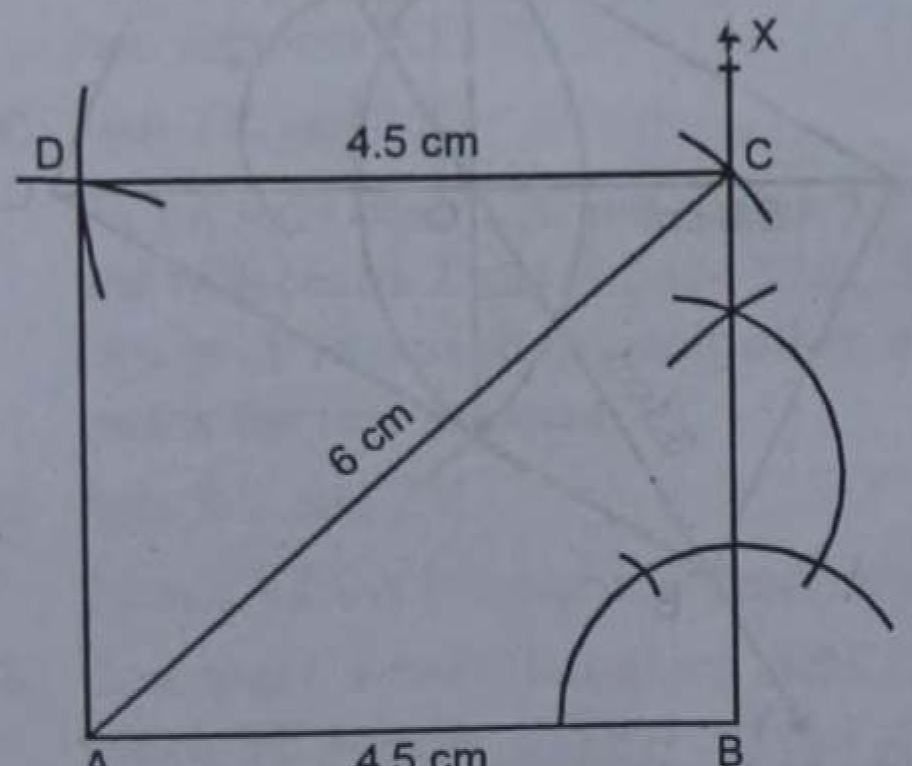
(*iii*) Join CD.

Then ABCD is the required rectangle.

Q. 14. Construct a rectangle ABCD in which AB = 4·5 cm and diagonal AC = 6 cm.

Sol. Steps of Construction :

(*i*) Draw a line segment AB = 4·5 cm.



(*ii*) At B, draw a ray BX making an angle of 90°.

(*iii*) With centre A and radius 6 cm draw an arc which intersects the ray BX at C.

(*iv*) Join AC.

(*v*) With centre A and radius BC and with centre C and radius 4·5 cm, draw arcs intersecting each other at D.

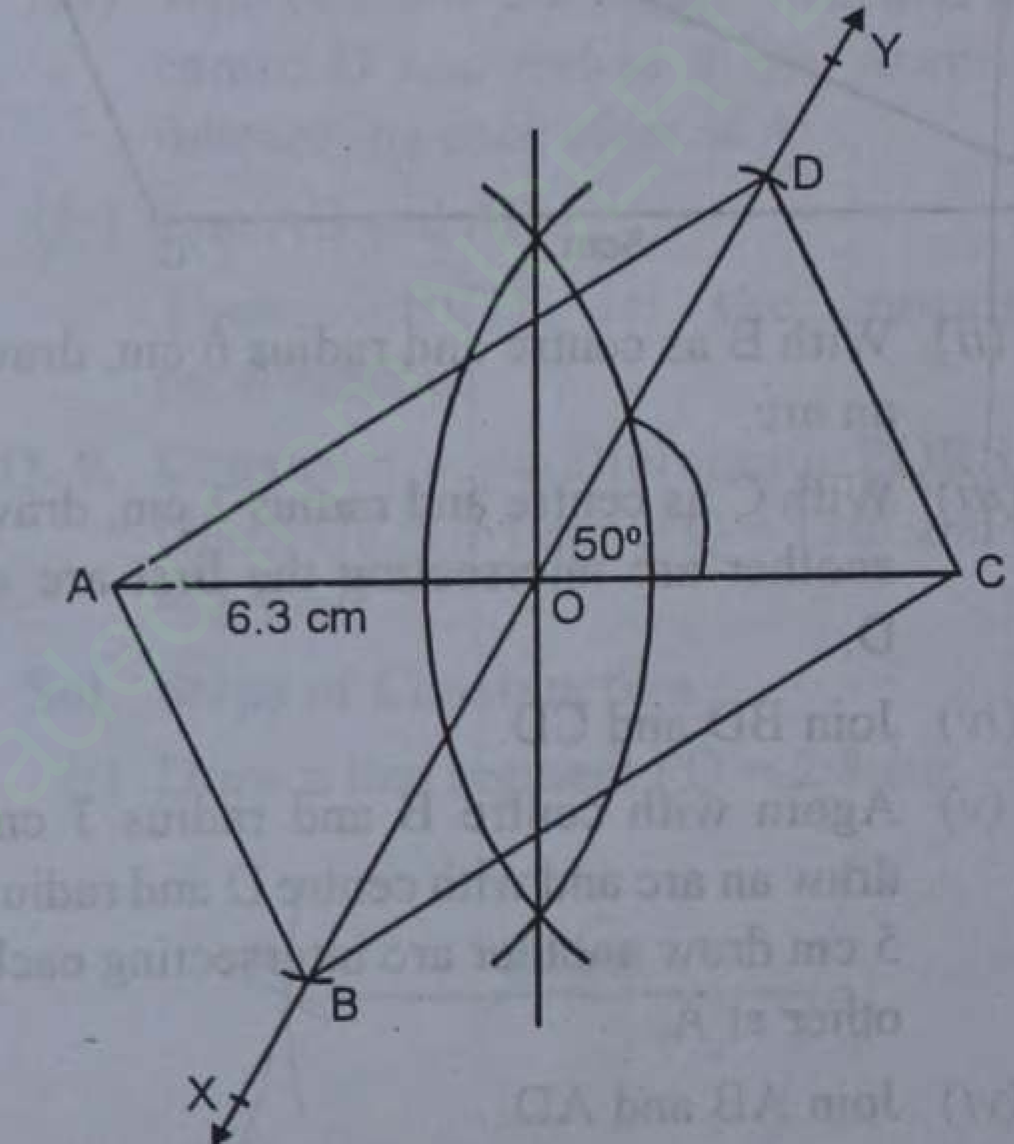
(*vi*) Join AD and CD.

Then ABCD is the required rectangle.

Q. 15. Construct a rectangle ABCD in which diagonal AC = 6·3 cm and the angle between the two diagonals is 50°.

Sol. Steps of Construction :

(*i*) Draw a line segment AC = 6·3 cm.



(*ii*) Bisect AC at O.

(*iii*) At O, draw a line XY making an angle of 50° and produce it both sides.

(*iv*) Cut off OB = OD = $\frac{6\cdot3}{2}$ cm = 3·15 cm

(∵ Diagonal of a rectangle are equal)

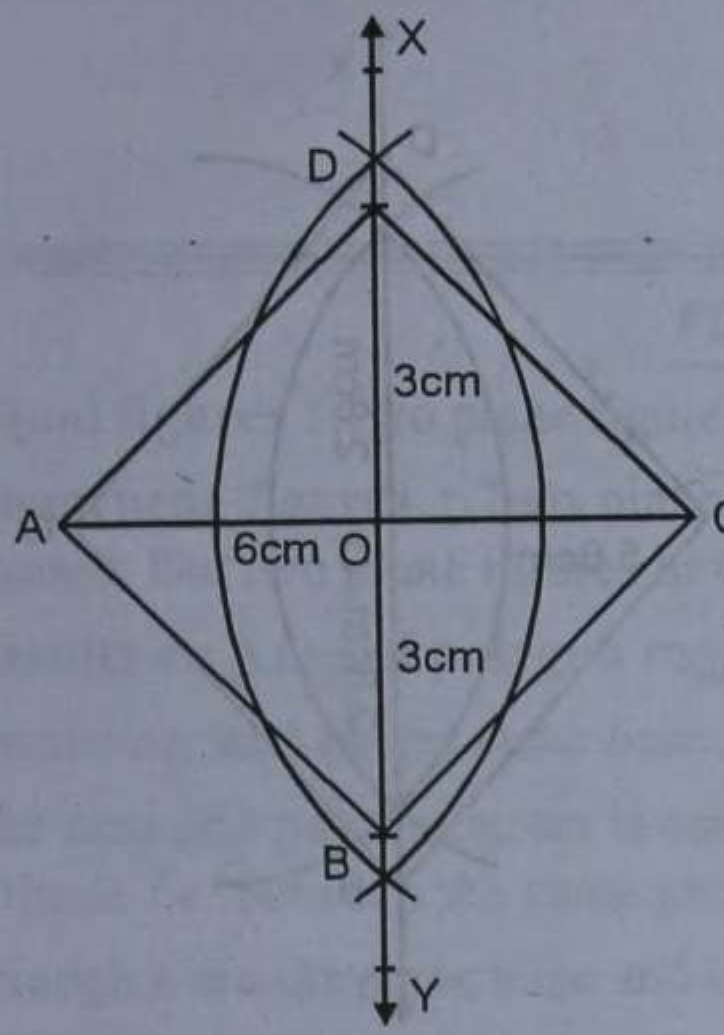
(*v*) Join AB, BC, CD and DA.

Then ABCD is the required rectangle.

Q. 16. Construct a square one of whose diagonals measures 6 cm.

Sol. Steps of Construction :

(*i*) Draw a line segment AC = 6 cm.



(*ii*) Draw the perpendicular bisector XY of AC cutting AC at O.

(*iii*) From O, cut off $OB = OD = \frac{6}{2} = 3\,\text{cm}$

($\because$ Diagonals of a square are equal and bisect each other at right angles)

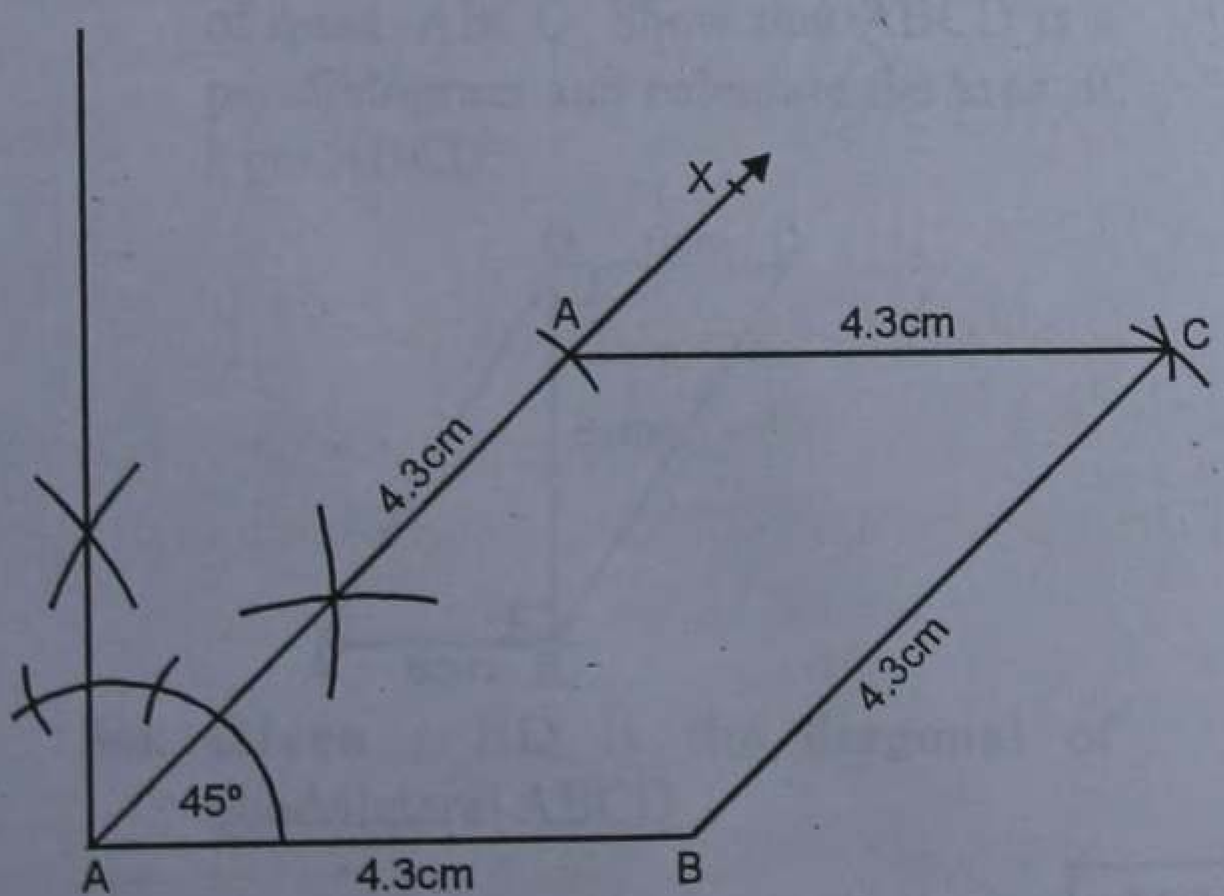
(*iv*) Join AB, BC, CD and DA.

Then ABCD is the required square.

Q. 17. Construct a rhombus ABCD in which AB = 4·3 cm and $\angle A = 45°$.

Sol. Steps of Construction :

(*i*) Draw a line segment AB = 4·3 cm.



(*ii*) At A, draw a ray AX making an angle of 45°.

(*iii*) From AX cut off AD = 4·3 cm.

(*iv*) With centre B and D and radius equal to 4·3 cm, draw arcs intersecting each other at C.

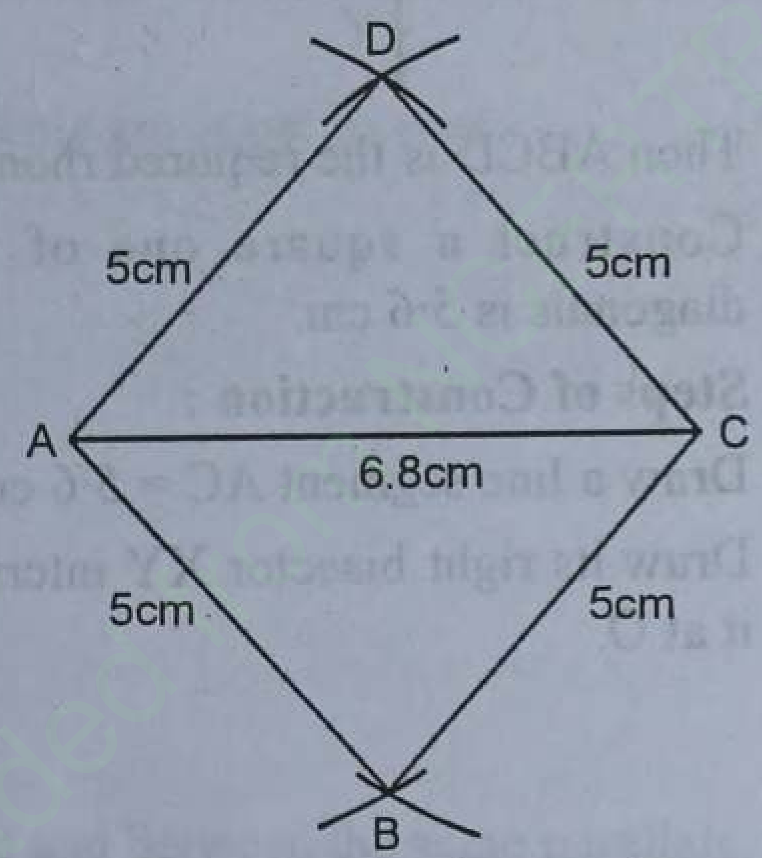
(*v*) Join BC and DC.

Then ABCD is a rhombus.

Q. 18. Construct a rhombus ABCD in which AB = 5 cm and diagonal AC = 6·8 cm.

Sol. Steps of Construction :

(*i*) Draw a line segment AC = 6·8 cm.



(*ii*) With centre A and C and radius 5 cm, draw arcs which intersect each other at B and D *i.e.* on both sides of AC.

(*iii*) Join AB, BC, CD and DA.

Then ABCD is the required rhombus.

Q. 19. Construct a rhombus ABCD in which diagonal AC = 5·8 cm and diagonal BD = 6·4 cm.

Sol. Steps of Construction :

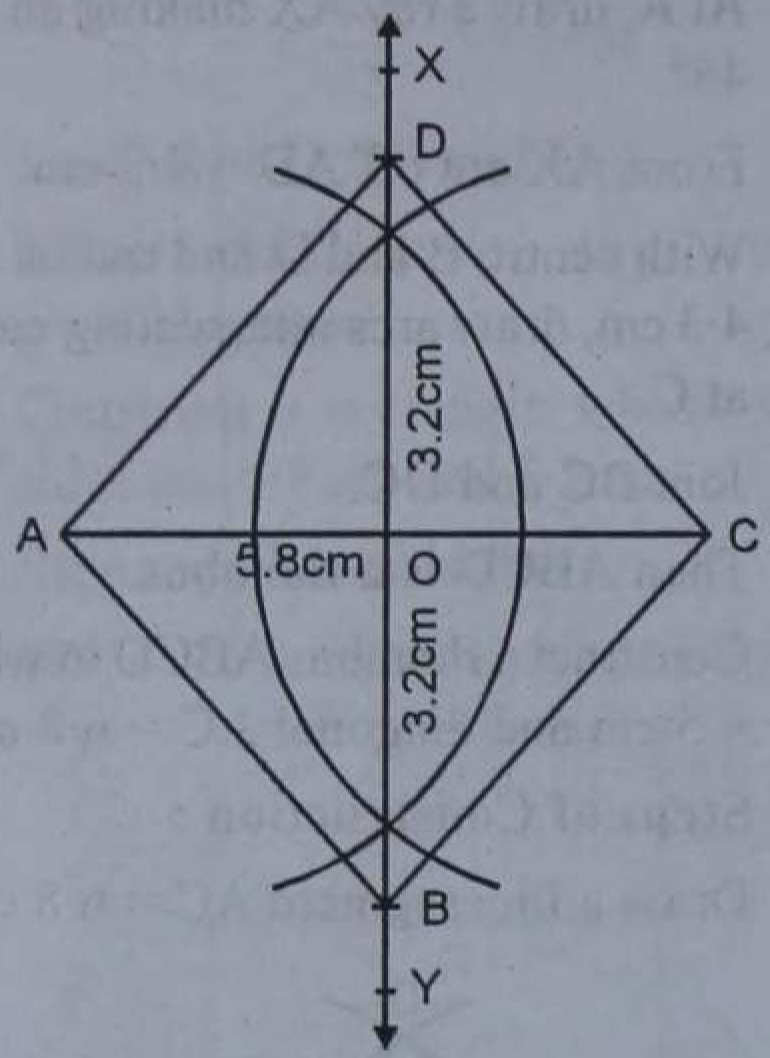
(*i*) Draw a line segment AC = 5·8 cm.

(*ii*) Draw the perpendicular bisector of AC intersecting it at O.

(*iii*) From O, cut off OB = OD

$$= \frac{6 \cdot 4}{2} = 3 \cdot 2\,\text{cm}.$$

(*iv*) Join AB, BC, CD and DA.



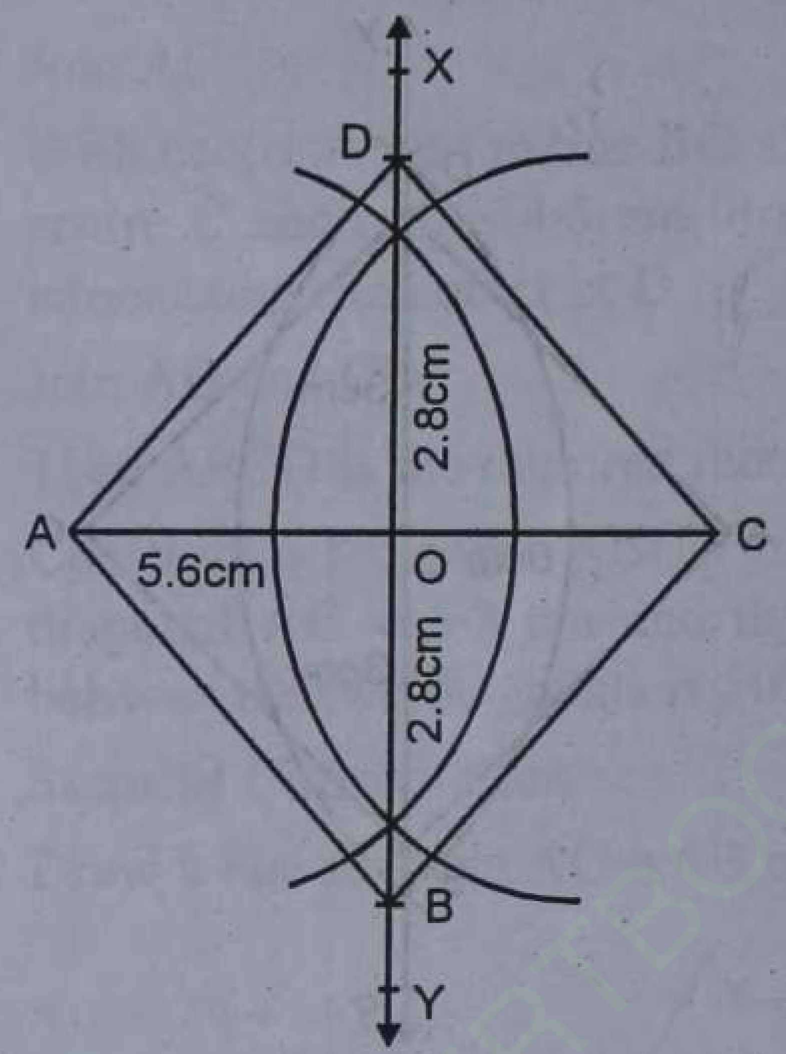
Then ABCD is the required rhombus.

Q. 20. Construct a square one of whose diagonals is 5·6 cm.

Sol. Steps of Construction :

(*i*) Draw a line segment AC = 5·6 cm.

(*ii*) Draw its right bisector XY intersecting it at O.



(*iii*) From O, cut off OB = OD

$$= \frac{5 \cdot 6}{2} = 2 \cdot 8 \text{ cm}.$$

(*iv*) Join AB, BC, CD and DA.

Then ABCD is the required square.