

Polygons

POINTS TO REMEMBER

1. **Polygon** : A closed plane figure bounded by three or more line segments is called a polygon. These line segments are called its sides and by joining two non-consecutive vertices of a polygon is called its diagonal and the point of intersection of two consecutive sides of a polygon is called a vertex.
2. **Convex Polygon** : If each angle of a polygons is less than 180° , then it is called a convex polygon.
3. **Concave Polygon** : If at least one angle of a polygon is a reflex angle *i.e.* more than 180° , then it is called a concave polygon.
4. **Regular Polygon** : If all sides of a polygon are equal and all angles are equal, then it is called a regular polygon.

5. Theorems :

- (i) The sum of all the interior angles of a convex polygon of n sides is $(2n - 4)$ right angles.
- (ii) The sum of all the exterior angles of a convex polygon is 4 right angles.

6. Some more results :

(i) For All Convex Polygons :

- (i) Sum of all interior angles of a polygon of n sides = $(2n - 4)$ right angles.
- (ii) Sum of all exterior angles of a polygon of n sides = 4 right angles.
- (iii) At each vertex of a polygon, we have :

$$\text{Interior Angle} + \text{Exterior Angle} = 180^\circ.$$

(ii) For Regular Polygons :

- (i) Each interior angle of a regular polygon of n sides = $\frac{(2n-4)}{n} \text{ rt. } \angle s = \left[\frac{(2n-4) \times 90}{n} \right]^\circ$.
- (ii) Each exterior angle of a regular polygon of n sides = $\left(\frac{360}{n} \right)^\circ$.
- (iii) If each exterior angle of a regular polygon is x° , then number of its sides = $\left(\frac{360}{x} \right)$.

Note : Greater is the number of sides in a regular polygon, greater is the value of its interior angle and smaller is the value of its each exterior angle.

- (iii) **Number of diagonals in a polygon of n sides** = $\left[\frac{n(n-1)}{2} - n \right]$.

EXERCISE 14 (A)

Q. 1. Write in degrees the sum of all interior angles of a :

- (i) Hexagon (ii) Septagon
(iii) Nonagon (iv) 15-gon

Sol. (i) Sum of interior angles of a hexagon

$$= (2n - 4) \text{ right angles}$$

$$= (2 \times 6 - 4) \times 90^\circ$$

$$= (12 - 4) \times 90^\circ = 8 \times 90^\circ$$

$$= 720^\circ \text{ Ans.}$$

(ii) Sum of interior angles of a septagon

$$= (2n - 4) \text{ right angles}$$

$$= (2 \times 7 - 4) \times 90^\circ$$

$$= (14 - 4) \times 90^\circ = 10 \times 90^\circ$$

$$= 900^\circ \text{ Ans.}$$

(iii) Sum of interior angles of nonagon

$$= (2n - 4) \text{ right angles}$$

$$= (2 \times 9 - 4) \times 90^\circ = (18 - 4) \times 90^\circ$$

$$= 14 \times 90^\circ = 1260^\circ.$$

(iv) Sum of interior angles of a 15-gon

$$= (2n - 4) \text{ right angles}$$

$$= (2 \times 15 - 4) \times 90^\circ = (30 - 4) \times 90^\circ$$

$$= 26 \times 90^\circ = 2340^\circ \text{ Ans.}$$

Q. 2. Find the measure, in degrees, of each interior angle of a regular :

- (i) Pentagon (ii) Octagon
(iii) Decagon (iv) 16-gon

Sol. We know that each interior angle of a regular polygon of n sides

$$= \frac{(2n - 4)}{n} \text{ right angles.}$$

(i) Each interior angle of pentagon

$$= \frac{(2n - 4)}{n} \text{ right angles}$$

$$= \frac{2 \times 5 - 4}{5} \times 90^\circ$$

$$= \frac{10 - 4}{5} \times 90^\circ = \frac{6}{5} \times 90^\circ = 108^\circ \text{ Ans.}$$

(ii) Each interior angle of octagon

$$= \frac{2n - 4}{n} \text{ rt. angles.}$$

$$= \frac{2 \times 8 - 4}{8} \times 90^\circ$$

$$= \frac{16 - 4}{8} \times 90^\circ = \frac{12}{8} \times 90^\circ$$

$$= 135^\circ \text{ Ans.}$$

(iii) Each interior angle of decagon

$$= \frac{2n - 4}{n} \text{ rt. angles.}$$

$$= \frac{2 \times 10 - 4}{10} \times 90^\circ$$

$$= \frac{20 - 4}{10} \times 90^\circ = \frac{16}{10} \times 90^\circ$$

$$= 144^\circ \text{ Ans.}$$

(iv) Each interior angle of 16-gon

$$= \frac{2n - 4}{n} \text{ rt. angles.}$$

$$= \frac{2 \times 16 - 4}{16} \times 90^\circ = \frac{32 - 4}{16} \times 90^\circ$$

$$= \frac{28}{16} \times 90^\circ = \frac{315}{2} = 157.5^\circ \text{ Ans.}$$

Q. 3. Find the measure, in degrees, of each exterior angle of a regular polygon containing :

- (i) 6 sides (ii) 8 sides
(iii) 15 sides (iv) 20 sides

Sol. We know that each exterior angle of a

$$\text{polygon of } n \text{ sides} = \frac{360^\circ}{n}.$$

(i) Each exterior angle of 6 sided polygon

$$= \frac{360^\circ}{6} = 60^\circ$$

(ii) Each exterior angle of 8 sided polygon

$$= \frac{360^\circ}{8} = 45^\circ$$

(iii) Each exterior angle of 15 sided polygon

$$= \frac{360^\circ}{15} = 24^\circ$$

(iv) Each exterior angle of 20 sided polygon

$$= \frac{360^\circ}{20} = 18^\circ \text{ Ans.}$$

Q. 4. Find the number of sides of a polygon, the sum of whose interior angles is :

(i) 24 right angles (ii) 1620°

(iii) 2880°

Sol. We know that sum of interior angles of a regular polygon of n sides $= (2n - 4)$ right angles.

(i) Sum = 24 right angles

$$\therefore (2n - 4) = 24 \Rightarrow 2n = 24 + 4 = 28$$

$$\therefore n = \frac{28}{2} = 14$$

Hence polygon has 14 sides.

(ii) Sum = 1620°

$$\therefore (2n - 4) \text{ right angles} = 1620^\circ$$

$$\Rightarrow (2n - 4) \times 90^\circ = 1620^\circ$$

$$\Rightarrow 2n - 4 = \frac{1620^\circ}{90^\circ}$$

$$\Rightarrow 2n - 4 = 18^\circ \Rightarrow 2n = 18 + 4 = 22$$

$$\therefore n = \frac{22}{2} = 11$$

Hence polygon has 11 sides.

(iii) Sum = 2880°

$$\Rightarrow (2n - 4) \text{ right angles} = 2880^\circ$$

$$\Rightarrow 2n - 4 = \frac{2880^\circ}{90^\circ}$$

$$\Rightarrow 2n - 4 = 32$$

$$\Rightarrow 2n = 32 + 4 = 36$$

$$\therefore n = \frac{36}{2} = 18$$

Hence polygon has 18 sides. **Ans.**

Q. 5. Find the number of sides in a regular polygon, if each of its exterior angles is :

(i) 72°

(ii) 24°

(iii) $(22.5)^\circ$

(iv) 15°

Sol. We know that each exterior angle of a regular polygon of n sides $= \frac{360^\circ}{n}$

(i) Exterior angle = 72°

$$\therefore \frac{360^\circ}{n} = 72^\circ \Rightarrow n = \frac{360^\circ}{72^\circ} = 5$$

$\therefore$ No. of sides of polygon = 5 **Ans.**

(ii) Each exterior angle = 24°

$$\therefore \frac{360^\circ}{n} = 24^\circ \Rightarrow n = \frac{360^\circ}{24^\circ} = 15$$

$\therefore$ No. of sides of the regular polygon = 15 **Ans.**

(iii) Each exterior angle = $(22.5)^\circ$

$$\therefore \frac{360^\circ}{n} = 22.5^\circ$$

$$\Rightarrow n = \frac{360^\circ}{22.5^\circ} = \frac{360 \times 10}{225} = 16$$

$\therefore$ No. of sides of the regular polygon = 16. **Ans.**

(iv) Each exterior angle = 15°

$$\therefore \frac{360^\circ}{n} = 15^\circ \Rightarrow n = \frac{360^\circ}{15^\circ} = 24$$

Hence no. of sides of the regular polygon = 24 **Ans.**

Q. 6. Find the number of sides in a regular polygon, if each of its interior angles is :

(i) 120°

(ii) 150°

(iii) 160°

(iv) 165°

Sol. We known that each interior angle of a regular polygon of n sides $= \frac{2n - 4}{n}$ right angles.

(i) Each interior angle = 120°

$$\therefore = \frac{2n - 4}{n} \text{ rt. Angle} = 120^\circ$$

$$\Rightarrow \frac{2n - 4}{n} \times 90^\circ = 120^\circ$$

$$\Rightarrow \frac{2n-4}{n} = \frac{120^\circ}{90^\circ}$$

$$\Rightarrow \frac{2n-4}{n} = \frac{4}{3} \Rightarrow 6n-12=4n$$

$$\Rightarrow 6n-4n=12 \Rightarrow 2n=12$$

$$\therefore n=6$$

Hence number of sides = 6. **Ans.**

(ii) Each interior angle = 150°

$$\therefore \frac{2n-4}{n} \text{ right angle} = 150^\circ$$

$$\Rightarrow \frac{2n-4}{n} \times 90^\circ = 150^\circ$$

$$\Rightarrow \frac{2n-4}{n} = \frac{150^\circ}{90} = \frac{5}{3}$$

$$\Rightarrow 6n-12=5n \Rightarrow 6n-5n=12$$

$$\Rightarrow n=12$$

Hence no. of sides = 12 **Ans.**

(iii) Each interior angle = 160°

$$\therefore \frac{2n-4}{n} \text{ right angles} = 160^\circ$$

$$\Rightarrow \frac{2n-4}{n} \times 90^\circ = 160^\circ$$

$$\Rightarrow \frac{2n-4}{n} = \frac{160^\circ}{90} = \frac{16}{9}$$

$$18n-36=16n \Rightarrow 18n-16n=36$$

$$\Rightarrow 2n=36 \Rightarrow n=\frac{36}{2}=18$$

Hence no. of sides = 18 **Ans.**

(iv) Each interior angle = 165°

$$\therefore \frac{2n-4}{n} \text{ right angles} = 165^\circ$$

$$\Rightarrow \frac{2n-4}{n} \times 90^\circ = 165^\circ$$

$$\Rightarrow \frac{2n-4}{n} = \frac{165^\circ}{90^\circ} = \frac{11}{6}$$

$$\Rightarrow 12n-24=11n$$

$$\Rightarrow 12n-11n=24$$

$$\Rightarrow n=24$$

Hence no. of sides = 24 **Ans.**

Q. 7. Is it possible to describe a polygon, the sum of whose interior angles is :

(i) 320°

(ii) 540°

(iii) 11 right angles

(iv) 14 right angles ?

Sol. We know that sum of interior angles of a regular polygon of n sides = $(2n-4)$ right angles.

(i) Sum of interior angles = 320°

$$\therefore (2n-4) \text{ right angles} = 320^\circ$$

$$\Rightarrow (2n-4) \times 90^\circ = 320^\circ$$

$$\Rightarrow 2n-4 = \frac{320^\circ}{90^\circ} = \frac{32}{9}$$

$$\Rightarrow 2n = \frac{32}{9} + 4 = \frac{32+36}{9} = \frac{68}{9}$$

$$n = \frac{68}{9 \times 2} = \frac{34}{9}$$

Which is in fraction.

Hence it is not possible to describe a polygon.

(ii) Sum of interior angles = 540°

$$\therefore (2n-4) \text{ right angles} = 540^\circ$$

$$\Rightarrow (2n-4) \times 90^\circ = 540^\circ$$

$$\Rightarrow 2n-4 = \frac{540^\circ}{90^\circ} = 6$$

$$\Rightarrow 2n = 6 + 4 = 10 \Rightarrow n = \frac{10}{2} = 5$$

Yes, it is possible to describe a polygon.

(iii) Sum of interior angles = 11 right angles

$$\therefore (2n-4) \text{ right angles} = 11 \text{ right angles}$$

$$\Rightarrow 2n-4 = 11 \Rightarrow 2n = 11 + 4 = 15$$

$$\Rightarrow n = \frac{15}{2}$$

Which is in fraction.

Hence it is not possible to describe a polygon.

(iv) Sum of interior angles = 14 right angles

$$\therefore (2n-4) \text{ right angles} = 14 \text{ right angles}$$

$$\Rightarrow 2n-4 = 14$$

$$\Rightarrow 2n = 14 + 4 = 18$$

$$\Rightarrow n = \frac{18}{2} = 9$$

Hence it is possible to describe a polygon.

Ans.

Q. 8. Is it possible to have a regular polygon, each of whose exterior angle is :

(i) 32° (ii) 18°

(iii) $\frac{1}{8}$ of a right angle (iv) 80° ?

Sol. We know that exterior angle of a regular

polygon of n sides $= \frac{360^\circ}{n}$

(i) Exterior angle $= 32^\circ$

$$\therefore \frac{360^\circ}{n} = 32^\circ \Rightarrow n = \frac{360^\circ}{32^\circ} = \frac{45}{4}$$

which is in fraction.

$\therefore$ It is not possible to have a regular polygon.

(ii) Exterior angle $= 180^\circ$

$$\therefore \frac{360^\circ}{n} = 18^\circ \Rightarrow n = \frac{360^\circ}{18^\circ} = 20$$

Hence it is possible to have a regular polygon.

(iii) Exterior angle $= \frac{1}{8}$ of right angle
 $= \frac{1}{8} \times 90^\circ = \frac{45^\circ}{4}$

$$\therefore \frac{360^\circ}{n} = \frac{45^\circ}{4} \Rightarrow n = \frac{360^\circ \times 4}{45} = 32$$

$\therefore$ It is possible to have a regular polygon.

(iv) Exterior angle $= 80^\circ$

$$\therefore \frac{360^\circ}{n} = 80^\circ \Rightarrow \frac{360^\circ}{80^\circ} = \frac{9}{2}$$

which is in fraction.

$\therefore$ It is not possible to have a regular polygon. **Ans.**

Q. 9. Is it possible to have a regular polygon, each of whose interior angles is :

(i) 120° (ii) 105°

(iii) 175° (iv) 130° ?

Sol. We know that each interior angle of a regular polygon of n sides $= \frac{2n-4}{n}$ right angles.

(i) Interior angle $= 120^\circ$

$$\therefore \frac{2n-4}{n} \text{ right angles} = 120^\circ$$

$$\Rightarrow \frac{2n-4}{n} \times 90^\circ = 120^\circ$$

$$\Rightarrow \frac{2n-4}{n} = \frac{120^\circ}{90^\circ} = \frac{4}{3}$$

$$\Rightarrow 6n - 12 = 4n \Rightarrow 6n - 4n = 12$$

$$\Rightarrow 2n = 12 \Rightarrow n = 6$$

It is possible to have a regular polygon.

(ii) Interior angle $= 105^\circ$

$$\therefore \frac{2n-4}{n} \text{ right angle} = 105^\circ$$

$$\Rightarrow \frac{2n-4}{n} \times 90^\circ = 105^\circ$$

$$\Rightarrow \frac{2n-4}{n} = \frac{105^\circ}{90^\circ}$$

$$\Rightarrow \frac{2n-4}{n} = \frac{7}{6} \Rightarrow 12n - 24 = 7n$$

$$\Rightarrow 12n - 7n = 24 \Rightarrow 5n = 24$$

$$\Rightarrow n = \frac{24}{5}$$

which is in fraction.

Hence it is not possible to have a regular polygon.

(iii) Interior angle $= 175^\circ$

$$\therefore \frac{2n-4}{n} \text{ right angle} = 175^\circ$$

$$\Rightarrow \frac{2n-4}{n} \times 90^\circ = 175^\circ$$

$$\Rightarrow \frac{2n-4}{n} = \frac{175^\circ}{90^\circ}$$

$$\Rightarrow \frac{2n-4}{n} = \frac{35}{18} \Rightarrow 36n - 72 = 35n$$

$$\Rightarrow 36n - 35n = 72 \Rightarrow n = 72$$

$\therefore$ It is possible to have a regular polygon.

(iv) Interior angle = 130°

$$\begin{aligned}\therefore \frac{2n-4}{n} \text{ right angle} &= 130^\circ \\ \Rightarrow \frac{2n-4}{n} \times 90^\circ &= 130^\circ \Rightarrow \frac{2n-4}{n} = \frac{130^\circ}{90} \\ \Rightarrow \frac{2n-4}{n} &= \frac{13}{9} \Rightarrow 18n - 36 = 13n \\ \Rightarrow 18n - 13n &= 36 \Rightarrow 5n = 36 \\ n &= \frac{36}{5}\end{aligned}$$

which is in fraction.

$\therefore$ It is not possible to have a regular polygon. **Ans.**

Q. 10. The angles of a quadrilateral are in the ratio $6 : 3 : 2 : 4$. Find the angles.

Sol. Sum of four angles of a quadrilateral

$$ABCD = 360^\circ$$

$$\text{then } \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\text{and } \angle A : \angle B : \angle C : \angle D = 6 : 3 : 2 : 4$$

$$\text{Let } \angle A = 6x, \text{ then } \angle B = 3x,$$

$$\angle C = 2x \text{ and } \angle D = 4x$$

$$\text{then } 6x + 3x + 2x + 4x = 360^\circ$$

$$\Rightarrow 15x = 360^\circ \Rightarrow x = \frac{360^\circ}{15} = 24^\circ$$

$$\therefore \angle A = 6x = 6 \times 24^\circ = 144^\circ$$

$$\angle B = 3x = 3 \times 24^\circ = 72^\circ$$

$$\angle C = 2x = 2 \times 24^\circ = 48^\circ$$

$$\angle D = 4x = 4 \times 24^\circ = 96^\circ \text{ Ans.}$$

Q. 11. The angles of a pentagon are in the ratio $3 : 4 : 5 : 2 : 4$. Find the angles.

Sol. Sum of five angles of a pentagon ABCDE

$$= (2n - 4) \text{ right angles}$$

$$= (2 \times 5 - 4) \times 90^\circ = (10 - 4) \times 90^\circ$$

$$= 6 \times 90^\circ = 540^\circ$$

The ratio between the angles say $\angle A$, $\angle B$, $\angle C$, $\angle D$, $\angle E$

$$= 3 : 4 : 5 : 2 : 4$$

$$\text{Let } \angle A = 3x, \text{ then } \angle B = 4x, \angle C = 5x,$$

$$\angle D = 2x \text{ and } \angle E = 4x$$

$$\therefore 3x + 4x + 5x + 2x + 4x = 540^\circ$$

$$\Rightarrow 18x = 540^\circ \Rightarrow x = \frac{540^\circ}{18} = 30^\circ$$

$$\therefore \angle A = 3x = 3 \times 30^\circ = 90^\circ$$

$$\angle B = 4x = 4 \times 30^\circ = 120^\circ$$

$$\angle C = 5x = 5 \times 30^\circ = 150^\circ$$

$$\angle D = 2x = 2 \times 30^\circ = 60^\circ$$

$$\angle E = 4x = 4 \times 30^\circ = 120^\circ \text{ Ans.}$$

Q. 12. The angles of a pentagon are $(3x + 5)^\circ$, $(x + 16)^\circ$, $(2x + 9)^\circ$, $(3x - 8)^\circ$ and $(4x - 15)^\circ$ respectively. Find the value of x and hence find the measures of all the angles of the pentagon.

Sol. Let angles of pentagon ABCDE are $(3x + 5)^\circ$, $(x + 16)^\circ$, $(2x + 9)^\circ$, $(3x - 8)^\circ$ and $(4x - 15)^\circ$.

But the sum of these five angles

$$= (2n - 4) \text{ right angle}$$

$$= (2 \times 5 - 4) \times 90^\circ$$

$$= (10 - 4) \times 90^\circ = 6 \times 90^\circ = 540^\circ$$

$$\therefore 3x + 5 + x + 16 + 2x + 9 + 3x - 8 + 4x - 15 = 540^\circ$$

$$\Rightarrow 13x + 30 - 23 = 540^\circ$$

$$\Rightarrow 13x + 7 = 540^\circ$$

$$\Rightarrow 13x = 540^\circ - 7 = 533^\circ$$

$$x = \frac{533}{13} = 41$$

$$\therefore \text{First angle} = 3x + 5 = 3 \times 41 + 5 = 123 + 5 = 128^\circ$$

$$\text{Second angle} = x + 16 = 41 + 16 = 57^\circ$$

$$\text{Third angle} = 2x + 9 = 2 \times 41 + 9 = 82 + 9 = 91^\circ$$

$$\text{Fourth angle} = 3x - 8 = 3 \times 41 - 8 = 123 - 8 = 115^\circ$$

$$\text{Fifth angle} = 4x - 15 = 4 \times 41 - 15 = 164 - 15 = 149^\circ \text{ Ans.}$$

Q. 13. The angles of a hexagon are $2x^\circ$, $(2x + 25)^\circ$, $3(x - 15)^\circ$, $(3x - 20)^\circ$, $2(x + 5)^\circ$ and $3(x - 5)^\circ$ respectively. Find the value of x and hence find the measures of all the angles of the hexagon.

Sol. Angles a hexagon are $2x^\circ$, $(2x + 25)^\circ$, $3(x - 15)^\circ$, $(3x - 20)^\circ$, $2(x + 5)^\circ$ and $3(x - 5)^\circ$.

But sum of angles of a hexagon

$$= (2n - 4) \text{ right angles}$$

$$= (2 \times 6 - 4) \times 90^\circ$$

$$= (12 - 4) \times 90^\circ = 8 \times 90^\circ$$

$$= 720^\circ$$

$$\therefore 2x + 2x + 25 + 3(x - 15) + 3x - 20 + 2(x + 5) + 3(x - 5) = 720^\circ$$

$$\Rightarrow 2x + 2x + 25 + 3x - 45 + 3x - 20$$

$$+ 2x + 10 + 3x - 15 = 720^\circ$$

$$\Rightarrow 15x + 35^\circ - 80^\circ = 720^\circ$$

$$\Rightarrow 15x - 45^\circ = 720^\circ$$

$$\Rightarrow 15x = 720^\circ + 45^\circ \Rightarrow 15x = 765^\circ$$

$$\Rightarrow x = \frac{765}{15} = 51^\circ$$

$$\therefore \text{First angle} = 2x = 2 \times 51^\circ = 102^\circ$$

$$\text{Second angle} = 2x + 25 = 2 \times 51^\circ + 25^\circ = 102 + 25 = 127^\circ$$

$$\text{Third angle} = 3(x - 15) = 3(51^\circ - 15^\circ) = 3 \times 36^\circ = 108^\circ$$

$$\text{Fourth angle} = 3x - 20 = 3 \times 51^\circ - 20 = 153 - 20 = 133^\circ$$

$$\text{Fifth angle} = 2(x + 5) = 2(51 + 5) = 2 \times 56 = 112^\circ$$

$$\text{Sixth angle} = 3(x - 5) = 3(51 - 5) = 3 \times 46 = 138^\circ$$

Hence angles are 102° , 127° , 108° , 133° , 112° and 138° . **Ans.**

Q. 14. Three of the exterior angles of a hexagon as 40° , 52° and 85° respectively and each of the remaining exterior angles is x° . Calculate the value of x .

Sol. Sum of exterior angles of a hexagon $= 360^\circ$

Three angles are 40° , 52° and 85° and three angles are x° each.

$$\therefore 40^\circ + 52^\circ + 85^\circ + x^\circ + x^\circ + x^\circ = 360^\circ$$

$$\Rightarrow 177^\circ + 3x^\circ = 360^\circ$$

$$\Rightarrow 3x^\circ = 360^\circ - 177^\circ = 183^\circ$$

$$\therefore x = \frac{183}{3} = 61^\circ$$

Hence $x = 61^\circ$ **Ans.**

Q. 15. One angle of an octagon is 100° and the other angles are all equal. Find the measure of each of the equal angles.

Sol. One angles of an octagon $= 100^\circ$

Let each of the other 3 angles $= x^\circ$

But sum of interior angles of an octagon

$$= (2x - 4) \text{ right angles}$$

$$= (2 \times 8 - 4) \times 90^\circ = (16 - 4) \times 90^\circ$$

$$= 12 \times 90^\circ = 1080^\circ$$

$$\therefore 100 + 7x = 1080 \Rightarrow 7x = 1080 - 100$$

$$\Rightarrow 7x = 980^\circ \Rightarrow x = \frac{980}{7} = 140^\circ$$

Hence each angle of the remaining angles $= 140^\circ$ **Ans.**

Q. 16. The interior angle of a regular polygon is double the exterior angle. Find the number of sides in the polygon.

Sol. Let no. of sides of a regular polygon $= x$

But sum of interior angle and exterior angle $= 180^\circ$

Let each exterior angle $= x^\circ$

then interior angle $= 2x$

$$\therefore x + 2x = 180^\circ \Rightarrow 3x = 180^\circ$$

$$x = \frac{180^\circ}{3} = 60^\circ$$

Now $x \times \text{exterior angle} = 360^\circ$

$$x \times 60^\circ = 360^\circ \Rightarrow x = \frac{360^\circ}{60^\circ} = 6$$

$\therefore$ No. of sides of the regular polygon $= 6$.

Q. 17. The ratio of each interior angle to each exterior angle of a regular polygon is $7 : 2$. Find the number of sides in the polygon.

Sol. Let number of sides of regular polygon = 3

and ratio of interior angle with exterior angle = 7 : 2

Let each interior angle = $7x$

and each exterior angle = $2x$

$$\therefore 7x + 2x = 180^\circ$$

$$\Rightarrow 9x = 180^\circ$$

$$x = \frac{180^\circ}{9} = 20^\circ$$

$$\therefore \text{Each exterior angles} = 20x^\circ = 2 \times 20^\circ = 40^\circ$$

But sum of exterior angles of a regular polygon of x sides = 360°

$$\Rightarrow x \times 40^\circ = 360^\circ$$

$$\Rightarrow x = \frac{360^\circ}{40^\circ} = 9$$

$\therefore$ No. of sides of a regular polygon = 9.

Q. 18. The sum of the interior angles of a polygon is 6 times the sum of its exterior angles. Find the number of sides in the polygon.

Sol. Sum of the exterior angles of a regular polygon of x sides = 360°

$$\therefore \text{Sum of its interior angles} = 360^\circ \times 6 = 2160^\circ$$

But sum of interior angles of the polygon

$$= (2x - 4) \text{ right angles}$$

$$\therefore (2x - 4) \times 90^\circ = 2160^\circ$$

$$\Rightarrow 2x - 4 = \frac{2160^\circ}{90^\circ}$$

$$\Rightarrow 2x - 4 = 24$$

$$\Rightarrow 2x = 24 + 4 = 28$$

$$\therefore x = \frac{28}{2} = 14$$

Hence number of sides = 14 **Ans.**

Q. 19. Two angles of a convex polygon are right angles and each of the other angles is 120° . Find the number of sides of the polygon.

Sol. $\therefore$ Two angles of a convex polygon = 90° each

$\therefore$ Exterior angles will be $180^\circ - 90^\circ = 90^\circ$ each

Each of other interior angles is 120°

$\therefore$ Each of exterior angles will be $180^\circ - 120^\circ = 60^\circ$

But, the sum of its exterior angles = 360°

Let no. of sides = n

$$\text{Then } 90^\circ + 90^\circ + (n - 2) \times 60^\circ = 360^\circ$$

$$180^\circ + (n - 2) 60^\circ = 360^\circ$$

$$60^\circ (n - 2) = 360^\circ - 180^\circ = 180^\circ$$

$$n - 2 = \frac{180^\circ}{60^\circ} = 3$$

$$\therefore n = 3 + 2 = 5$$

Hence, number of sides = 5 **Ans.**

Q. 20. The ratio between the number of sides of two regular polygons is 3 : 4 and the ratio between the sum of their interior angles is 2 : 3. Find the number of sides in each polygon.

Sol. The ratio between the sides of two regular polygon = 3 : 4

Let number of sides in the first polygon = $3x$

and number of sides in the second = $4x$

Sum of interior angles of the first polygon

$$= (2 \times 3x - 4) \text{ right angles}$$

$$= (6x - 4) \text{ right angles}$$

and sum of interior angles of the second polygon

$$= (2 \times 4x - 4) \text{ right angles}$$

$$= (8x - 4) \text{ right angles.}$$

$$\therefore (6x - 4) \text{ right angles}$$

$$: (8x - 4) \text{ right angles} = 2 : 3$$

$$\Rightarrow (6x - 4) : (8x - 4) = 2 : 3$$

$$\Rightarrow \frac{6x - 4}{8x - 4} = \frac{2}{3}$$

$$\begin{aligned} \Rightarrow 18x - 12 &= 16x - 8 \\ \Rightarrow 18x - 16x &= -8 + 12 \\ \Rightarrow 2x &= 4 \\ \therefore x &= 2 \end{aligned}$$

Hence number of sides in the first polygon

$$= 3x = 3 \times 2 = 6$$

and no. of sides of the second polygon

$$= 4x = 4 \times 2 = 8 \text{ Ans.}$$

Q. 21. The number of sides of two regular polygons are in the ratio 4 : 5 and their interior angles are in the ratio 15 : 16. Find the number of sides in each polygon.

Sol. Ratio between the sides of two regular polygon = 4 : 5

Let no. of sides of first polygon = $4x$

and no. of sides the second polygon = $5x$

$\therefore$ Interior angle of the first polygon

$$= \frac{2 \times 4x - 4}{4x} \text{ right angles}$$

$$= \frac{8x - 4}{4x} = \frac{2x - 1}{x} \text{ right angles}$$

and interior angle of second polygn

$$= \frac{2 \times 5x - 4}{5x} \text{ right angle}$$

$$= \frac{10x - 4}{5x} \text{ right angle}$$

$$\therefore \frac{2x - 1}{x} : \frac{10x - 4}{5x} = 15 : 16$$

$$\Rightarrow \frac{2x - 1}{x} \times \frac{5x}{10x - 4} = \frac{15}{16}$$

$$\Rightarrow \frac{5(2x - 1)}{10x - 4} = \frac{15}{16}$$

$$\Rightarrow \frac{10x - 5}{10x - 4} = \frac{15}{16}$$

$$\Rightarrow 160x - 80 = 150x - 60$$

$$\Rightarrow 160x - 150x = -60 + 80$$

$$\Rightarrow 10x = 20$$

$$x = \frac{20}{10} = 2$$

$\therefore$ No. of sides of the first polygon = $4x$

$$= 4 \times 2 = 8$$

and no. of sides of the second polygon

$$= 5 \times 2 = 10 \text{ Ans.}$$

Q. 22. How many diagonals are there in a

(i) Pentagon (ii) Hexagon

(iii) Octagon ?

Sol. No. of diagonals of a polygon of n sides

$$= \frac{1}{2}n(n - 1) - n$$

(i) $\therefore$ No. of diagonals in a pentagon

$$= \frac{1}{2}n(n - 1) - n$$

$$= \frac{1}{2} \times 5(5 - 1) - 5 \quad (\text{Here } n = 5)$$

$$= \frac{1}{2} \times 5 \times 4 - 5$$

$$= 10 - 5 = 5$$

(ii) No. of diagonals in a hexagon

$$= \frac{1}{2} \times 6(6 - 1) - 6 \quad (\text{Here } n = 6)$$

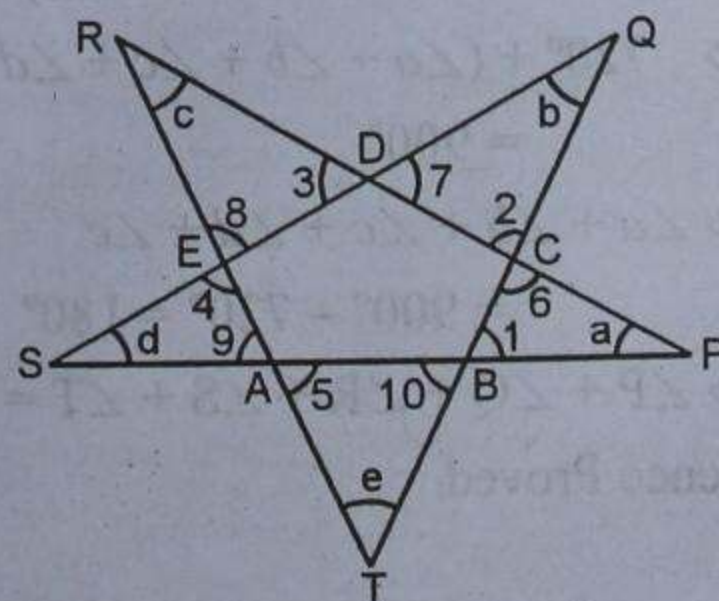
$$= \frac{1}{2} \times 6 \times 5 - 6 = 15 - 6 = 9$$

(iii) No. of diagonals in an octagon

$$= \frac{1}{2} \times 8(8 - 1) - 8 \quad (\text{Here } x = 8)$$

$$= \frac{1}{2} \times 8 \times 7 - 8 = 28 - 8 = 20 \text{ Ans.}$$

Q. 23. The alternate sides of any pentagon are produced to meet, so as to form a star-shaped figure, shown in the figure. Prove that the sum of measures of the angles at the vertices of the star is 180° .



Given : The alternate sides of a pentagon ABCDE are produce to meet at P, Q, R, S and T so as to form a star shaped figure.

To Prove : $\angle P + \angle Q + \angle R + \angle S + \angle T = 180^\circ$... (i)

or $\angle a + \angle b + \angle c + \angle d + \angle e = 180^\circ$

Proof : $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 360^\circ$... (i)

(Sum of ext. angles of a polygon)

Similarly $\angle 6 + \angle 7 + \angle 8 + \angle 9 + \angle 10 = 360^\circ$... (ii)

But in $\triangle BCP$, $\angle 1 + \angle b + \angle a = 180^\circ$... (iii)

(Sum of angles of a triangle)

Similarly in $\triangle CDQ$, $\angle 2 + \angle 7 + \angle b = 180^\circ$... (iv)

In $\triangle DER$,
 $= \angle 3 + \angle 8 + \angle c = 180^\circ$... (v)

In $\triangle EAS$,
 $= \angle 4 + \angle 9 + \angle d = 180^\circ$... (vi)

and in $\triangle ABT$, $\angle 5 + \angle 10 + \angle e = 180^\circ$... (vii)

Adding (iii) to (vii)

$$\angle 1 + \angle 6 + \angle a + \angle 2 + \angle 7 + \angle b + \angle 3 + \angle 8 + \angle c + \angle 4 + \angle 9 + \angle d + \angle 5 + \angle 10 + \angle e$$

$$= 180^\circ + 180^\circ + 180^\circ + 180^\circ + 180^\circ$$

$$\Rightarrow (\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5) + (\angle 6 + \angle 7 + \angle 8 + \angle 9 + \angle 10)$$

$$+ (\angle a + \angle b + \angle c + \angle d + \angle e) = 900^\circ$$

$$\Rightarrow 360^\circ + 360^\circ + (\angle a + \angle b + \angle c + \angle d + \angle e) = 900^\circ$$

$$\Rightarrow 720^\circ + (\angle a + \angle b + \angle c + \angle d + \angle e) = 900^\circ$$

$$\Rightarrow \angle a + \angle b + \angle c + \angle d + \angle e = 900^\circ - 720^\circ = 180^\circ$$

$$\Rightarrow \angle P + \angle Q + \angle R + \angle S + \angle T = 180^\circ$$

Hence Proved.

EXERCISE 14 (B)

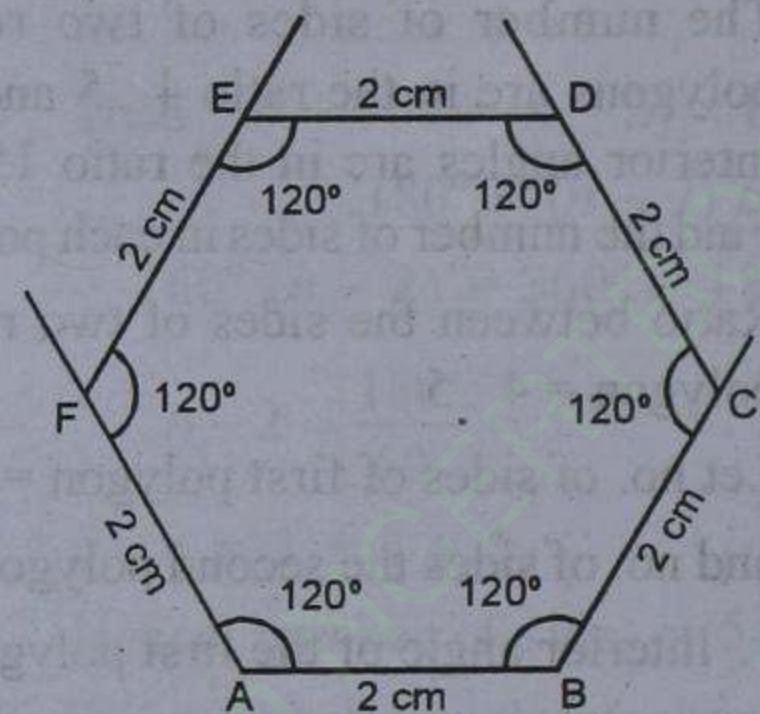
Q. 1. Construct a regular hexagon of side 2 cm, using ruler and compasses only.

Sol. We know that each angle of a regular hexagon $= 120^\circ$.

Steps of Construction :

(i) Draw $AB = 2$ cm.

(ii) At A and B, draw rays making an angle of 120° each.



(iii) Cut off $AF = BC = 2$ cm.

(iv) Again at F and C, draw rays making an angle of 120° each.

(v) Cut off $FE = CD = 2$ cm.

(vi) Join ED.

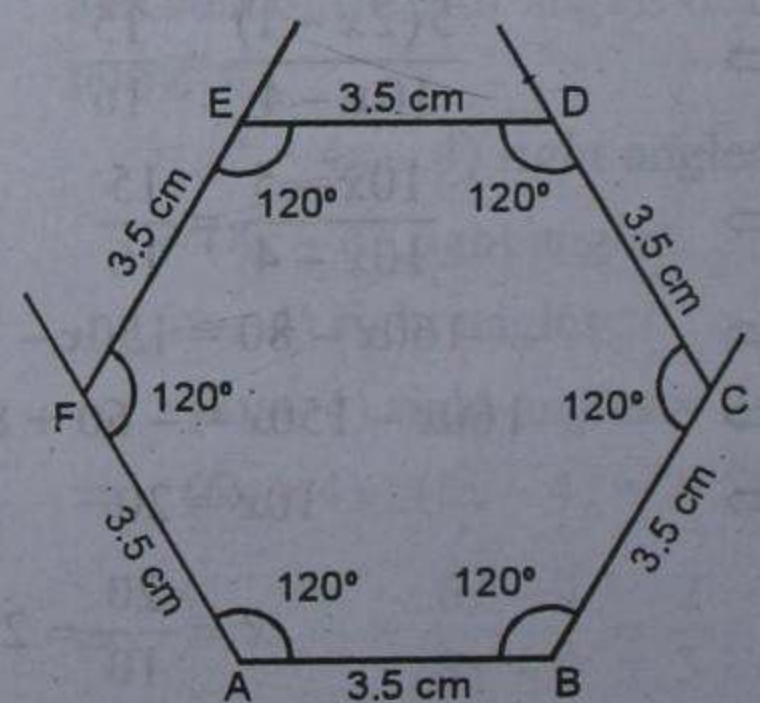
Then, ABCDEF is the required hexagon.

Q. 2. Construct a regular hexagon of side 3.5 cm, using ruler and compasses only.

Sol. We know that each angle of a regular hexagon $= 120^\circ$

Steps of Construction :

(i) Draw $AB = 3.5$ cm.



- (ii) At A and B, draw rays making an angle of 120° each.
- (iii) Cut off $AF = BC = 3.5$ cm
- (iv) At F and C, draw rays making an angle of 120° each.
- (v) Cut off $FE = CD = 3.5$ cm.
- (vi) Join ED.

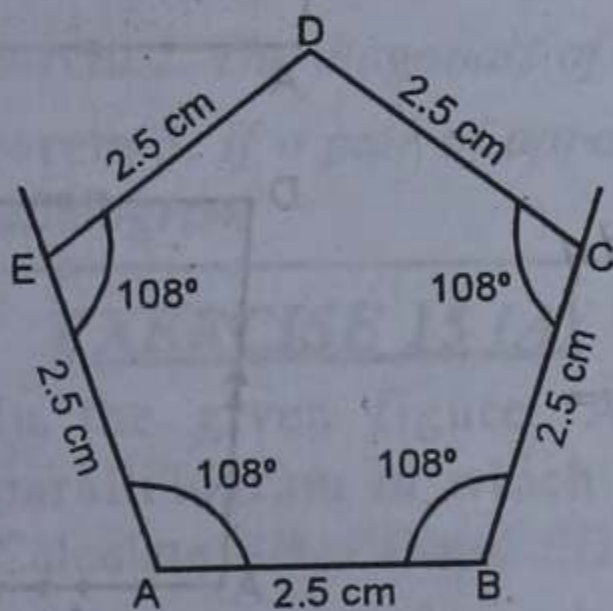
Then, ABCDEF is the required regular hexagon.

Q. 3. Construct a regular pentagon of side 2.5 cm. Using ruler, compasses and protractor.

Sol. We know that each angle of a regular pentagon = 108° .

Steps of Construction :

- (i) Draw a line segment $AB = 2.5$ cm.
- (ii) At A and B, draw rays making an angle of 108° each.



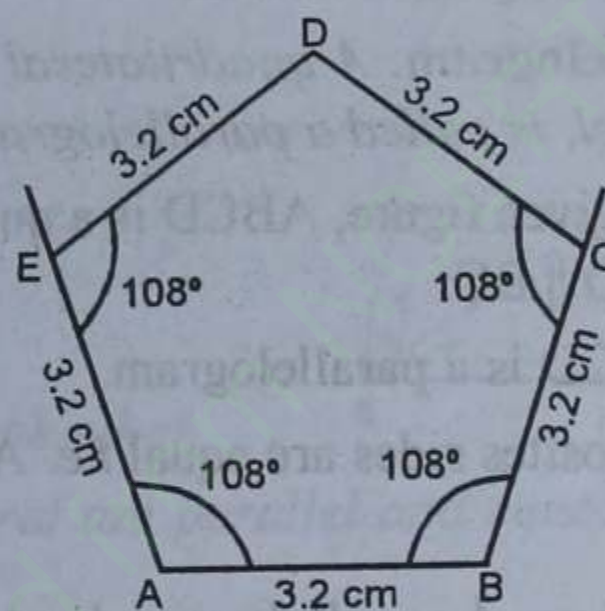
- (iii) Cut off $AE = BC = 2.5$ cm.
 - (iv) At E and C, draw rays making an angle of 108° each meeting each other at D.
- Then, ABCDE is the required regular pentagon.

Q. 4. Construct a regular pentagon of side 3.2 cm, using ruler, compasses and protractor.

Sol. We know that each angle of a regular pentagon = 108°

Steps of Construction :

- (i) Draw a line segment $AB = 3.2$ cm.
- (ii) At A and B, draw rays making an angle of 108° .



- (iii) Cut off $AE = BC = 3.2$ cm.
 - (iv) At E and C, draw rays making an angle of 108° each meeting each other at D.
- Then, ABCDE is the required pentagon.