

Similarity

POINTS TO REMEMBER

1. **Similar figures** : Two figures are similar if they have the same shape. Similar figures may differ in size. The sign ' $\sim$ ' is used for similarity.

2. **Similar Triangles** : Two Δ s ABC and DEF are said to be similar, if their corresponding sides are proportional and we write,

$$\Delta ABC \sim \Delta DEF.$$

Thus, $\Delta ABC \sim \Delta DEF$

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}.$$

3. **Axioms of Similarity of Triangles**

(i) **(AA – axiom of Similarity)** :

If two triangles have two pairs of corresponding angles equal, then the triangles are similar.

In the given figure, ΔABC and ΔDEF are such that

$$\angle A = \angle D$$

and $\angle B = \angle E.$

$$\therefore \Delta ABC \sim \Delta DEF.$$

(ii) **(SAS – axiom of Similarity)** :

If two triangles have a pair of corresponding angles equal and the sides including them proportional, then the triangles are similar.

In the given figure, ΔABC and ΔDEF are such that

$$\angle A = \angle D$$

and $\frac{AB}{DE} = \frac{AC}{DF}.$

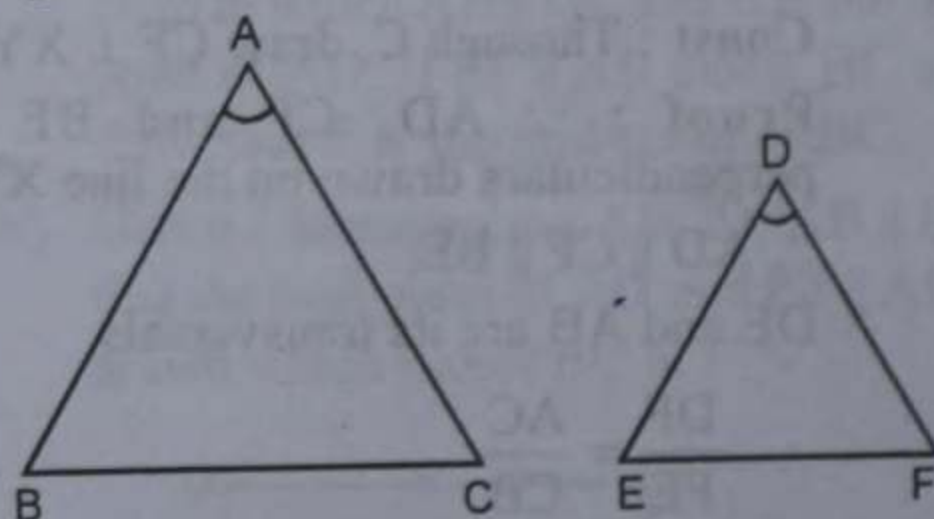
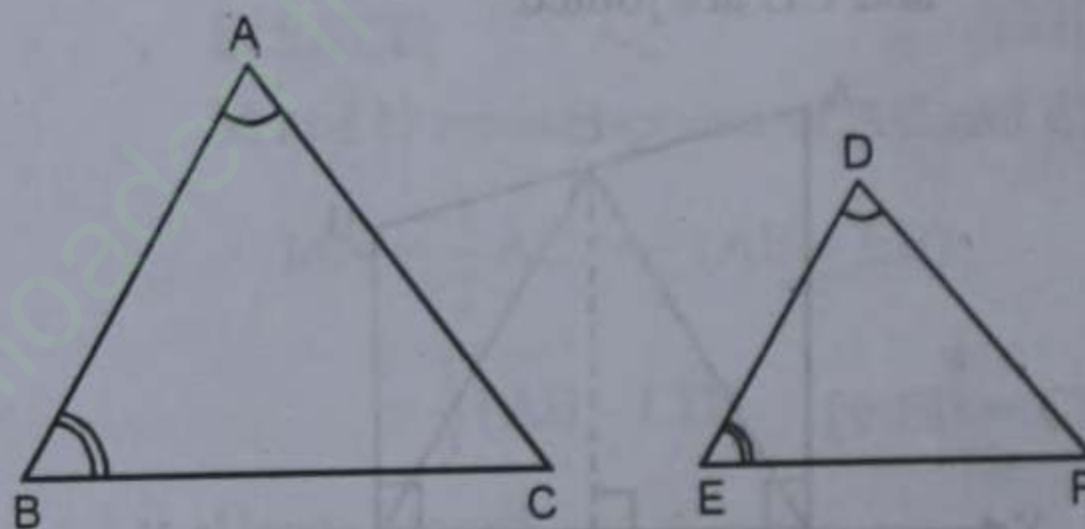
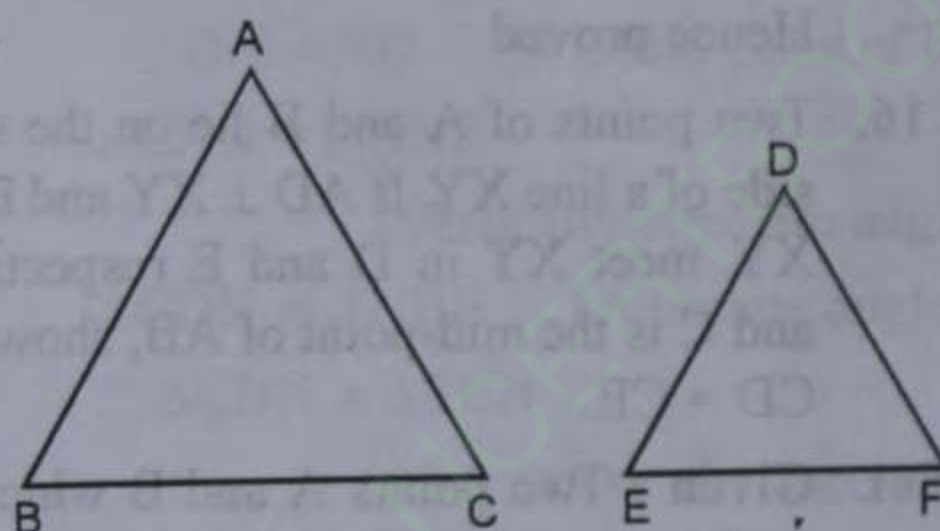
$$\therefore \Delta ABC \sim \Delta DEF.$$

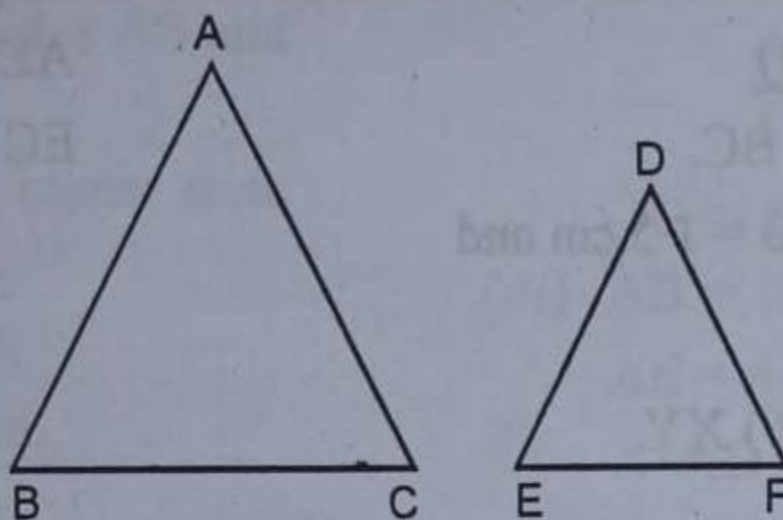
(iii) **(SSS – axiom of Similarity)** :

If two triangles have three pairs of corresponding sides proportional then the triangles are similar.

In the given figure, ΔABC and ΔDEF are such that

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}.$$





$$\therefore \triangle ABC \sim \triangle DEF.$$

4. **Size Transformation** : It is the process in which a given figure is enlarged or reduced by the scale factor ' k ' such that the resulting figure is similar to the given figure.

The given figure is called an object on the **Preimage** and the resulting figure is called its **image**.

5. **Properties of size-Transformation.**

(i) In the size transformation, the shape of the figure is preserved. Thus angle, perpendicular ship, parallelism etc. are preserved.

(ii) Let k be the scale factor of the given size transformation, then

(a) $k > 1 \Rightarrow$ The transformation is enlargement.

(b) $k < 1 \Rightarrow$ The transformation is reduction.

(c) $k = 1 \Rightarrow$ The transformation is an identity transformation.

(iii) Each side of resulting image = k times the corresponding side of the given object.

(iv) Area of resulting image = $k^2 \times$ (Area of given object).

(v) Volume of resulting image = $k^3 \times$ (Volume of given object).

6. **Model.** The model of a plane figure and the actual figure are similar to one another.

Let the model of a plane figure be drawn to the scale $1 : p$, then

$$\text{scale factor, } k = \frac{1}{p}$$

(a) Length of model = $k \times$ (Length of actual object).

(b) Area of model = $k^2 \times$ (Area of actual object).

(c) Volume of model = $k^3 \times$ (Volume of actual object).

7. **Map.** Let the map of a plane figure be drawn to the scale $1 : p$, then

$$\text{scale factor, } k = \frac{1}{p}$$

(a) Length in the map = $k \times$ (Actual length).

(b) Area in the map = $k^2 \times$ (Actual area).

8. **Theorems.** (i) (Basic Proportionality Theorem)

A line drawn parallel to one side of a triangle divides the other two sides proportionally.

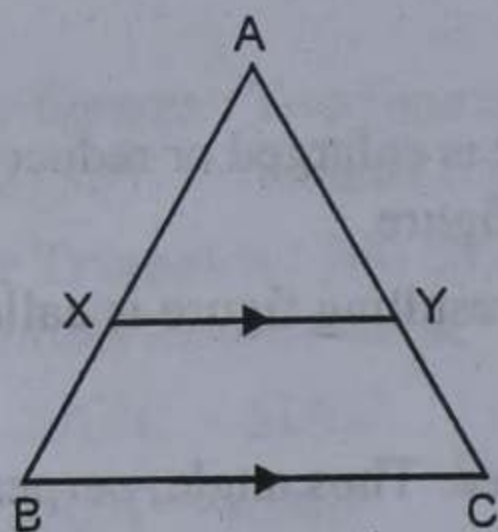
(ii) (Converse of Theorem) If a line divides any two sides of a triangle proportionally, the line is parallel to the third side.

EXERCISE 12 (A)

Q. 1. In the given figure, $XY \parallel BC$.

Given that $AX = 3$ cm, $XB = 1.5$ cm and $BC = 6$ cm.

Calculate : (i) $\frac{AY}{YC}$ (ii) XY .



Sol. In $\triangle ABC$, $XY \parallel BC$

$AX = 3$ cm, $XB = 1.5$ cm and $BC = 6$ cm

$$(i) \therefore \frac{AX}{XB} = \frac{AY}{YC}$$

$$\Rightarrow \frac{3}{1.5} = \frac{AY}{YC} \Rightarrow \frac{AY}{YC} = 2 \text{ Ans.}$$

(ii) In $\triangle AXY$ and $\triangle ABC$

$$\angle A = \angle A \quad (\text{Common})$$

$$\angle AXY = \angle ABC \quad (\text{Corresponding angles})$$

$$\angle AYX = \angle ACB \quad (\text{Corresponding angles})$$

$$\therefore \triangle AXY \sim \triangle ABC$$

(AAA axiom of similarity)

$$\therefore \frac{AX}{AB} = \frac{XY}{BC} \Rightarrow \frac{AX}{AX + XB} = \frac{XY}{BC}$$

$$\Rightarrow \frac{XY}{6} = \frac{3}{3 + 1.5} \Rightarrow \frac{XY}{6} = \frac{3}{4.5}$$

$$\Rightarrow XY = \frac{3 \times 6}{4.5} = 4 \text{ Ans.}$$

Q. 2. In the given figure, $DE \parallel BC$.

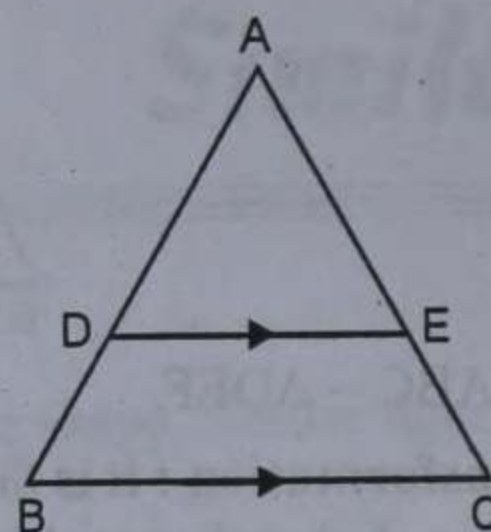
(i) If $AD = 3.6$ cm, $AB = 9$ cm and $AE = 2.4$ cm, find EC .

(ii) If $\frac{AD}{DB} = \frac{3}{5}$ and $AC = 5.6$ cm, find AE .

(iii) If $AD = x$ cm, $DB = (x - 2)$ cm,

$AE = (x + 2)$ cm and

$EC = (x - 1)$ cm, find the value of x .



Sol. (i) $AD = 3.6$ cm, $AB = 9$ cm,

$$AE = 2.4 \text{ cm}$$

$$DB = AB - AD = 9.0 - 3.6 = 5.4 \text{ cm}$$

Now in $\triangle ABC$, $DE \parallel BC$

$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{3.6}{5.4} = \frac{2.4}{EC}$$

$$\Rightarrow EC \times 3.6 = 2.4 \times 5.4$$

$$\Rightarrow EC = \frac{2.4 \times 5.4}{3.6} = 3.6 \text{ cm Ans.}$$

(ii) Let $AE = x$,

$$AC = 5.6 \text{ cm.}$$

$$\therefore EC = AC - AE = 5.6 - x$$

$$\therefore DE \parallel BC$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{3}{5} = \frac{x}{5.6 - x}$$

$$\Rightarrow (5.6 - x) \times 3 = 5x$$

$$\Rightarrow 16.8 - 3x = 5x \Rightarrow 16.8 = 5x + 3x$$

$$\Rightarrow 8x = 16.8 \Rightarrow x = \frac{16.8}{8} = 2.1$$

Hence $AE = 2.1$ cm Ans.

(iii) $AD = x$ cm $DB = (x - 2)$ cm

$$AE = (x + 2) \text{ cm and}$$

$$EC = (x - 1) \text{ cm}$$

$$\therefore DE \parallel BC$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{x}{x - 2} = \frac{x + 2}{x - 1}$$

$$\Rightarrow x(x - 1) = (x - 2)(x + 2)$$

$$\Rightarrow x^2 - x = x^2 - 4 \Rightarrow -x = -4$$

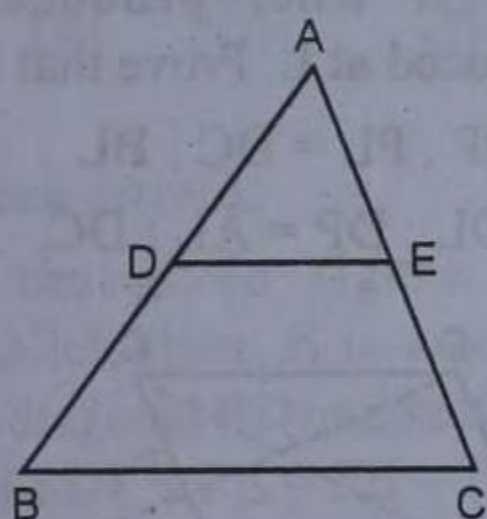
$$\therefore x = 4 \text{ Ans.}$$

Q. 3. D and E are points on the sides AB and AC respectively of $\triangle ABC$.

For each of the following cases, state whether $DE \parallel BC$:

- (i) $AD = 5.7$ cm, $BD = 9.5$ cm,
 $AE = 3.6$ cm and $EC = 6$ cm.
- (ii) $AB = 5.6$ cm, $AD = 1.4$ cm,
 $AC = 9.6$ cm and $EC = 2.4$ cm.
- (iii) $AB = 11.7$ cm, $BD = 5.2$ cm,
 $AE = 4.4$ cm and $AC = 9.9$ cm.
- (iv) $AB = 10.8$ cm, $BD = 4.5$ cm,
 $AC = 4.8$ cm and $AE = 2.8$ cm.

Sol. In $\triangle ABC$, D and E are points on the sides AB and AC respectively. D, E are joined.



- (i) Here $AD = 5.7$ cm, $BD = 9.5$ cm,
 $AE = 3.6$ cm and $EC = 6$ cm

$$\text{Now } \frac{AD}{DB} = \frac{5.7}{9.5} = \frac{3}{5}$$

$$\text{and } \frac{AE}{EC} = \frac{3.6}{6.0} = \frac{3}{5}$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

$\therefore DE \parallel BC$

- (ii) Here $AB = 5.6$ cm, $AD = 1.4$ cm,
 $AC = 9.6$ cm and $EC = 2.4$ cm.
 $DB = AB - AD = 5.6 - 1.4 = 4.2$ cm
 $AE = AC - EC = 9.6 - 2.4 = 7.2$ cm

$$\text{Now } \frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3}$$

$$\text{and } \frac{AE}{EC} = \frac{7.2}{2.4} = \frac{3}{1}$$

$$\therefore \frac{AD}{DB} \neq \frac{AE}{EC}$$

$\therefore DE$ is not parallel to BC .

- (iii) $AB = 11.7$ cm, $BD = 5.2$ cm,
 $AE = 4.4$ cm, $AC = 9.9$ cm
 $\therefore AD = AB - BD = 11.7 - 5.2 = 6.5$ cm
 $EB = AC - AE = 9.9 - 4.4 = 5.5$ cm.

$$\text{Now } \frac{AD}{DB} = \frac{6.5}{5.2} = \frac{5}{4}$$

$$\frac{AE}{EC} = \frac{4.4}{5.5} = \frac{4}{5}$$

$$\therefore \frac{AD}{DB} \neq \frac{AE}{EC}$$

$\therefore DE$ is not parallel to BC .

- (iv) $AB = 10.8$ cm, $BD = 4.5$ cm,
 $AC = 4.8$ cm and $AE = 2.8$ cm
 $AD = AB - BD = 10.8 - 4.5 = 6.3$ cm
 $EC = AC - AE = 4.8 - 2.8 = 2.0$ cm

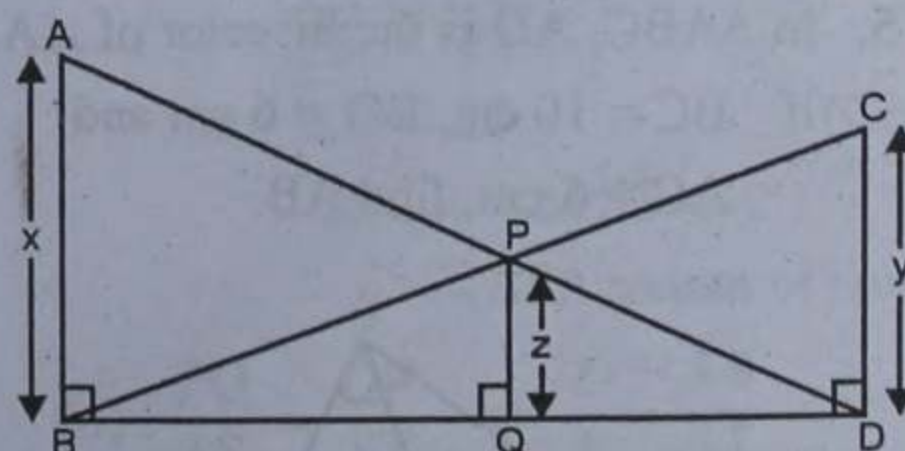
$$\text{Now } \frac{AD}{DB} = \frac{6.3}{4.5} = \frac{7}{5}$$

$$\text{and } \frac{AE}{EC} = \frac{2.8}{2.0} = \frac{7}{5}$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

$\therefore DE \parallel BC$

Q. 4. In the given figure, it is given that $\angle ABD = \angle CDB = \angle PQB = 90^\circ$. If $AB = x$ units, $CD = y$ units and $PQ = z$ units, prove that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$.



Sol. In $\triangle ABD$ and $\triangle PQD$

$$\angle D = \angle D \quad (\text{Common})$$

$$\angle B = \angle Q \quad (\text{Each } 90^\circ)$$

$\therefore \triangle ABD \sim \triangle PQD$ (AA Axiom)

$$\therefore \frac{BD}{QD} = \frac{AB}{PQ} = \frac{x}{z}$$

$$\Rightarrow \frac{BD}{QD} = \frac{x}{z}$$

$$\Rightarrow \frac{QD}{BD} = \frac{z}{x}$$

Similarly, we can prove that $\triangle BDC$ and $\triangle BQP$ are similar

$$\therefore \frac{BD}{BQ} = \frac{CD}{PQ} = \frac{y}{z}$$

$$\Rightarrow \frac{BD}{BQ} = \frac{y}{z}$$

$$\therefore \frac{BQ}{BD} = \frac{z}{y}$$

Adding the two results,

$$\frac{QD}{BD} + \frac{BQ}{BD} = \frac{z}{x} + \frac{z}{y}$$

$$\Rightarrow \frac{QD + BQ}{BD} = z \left(\frac{1}{x} + \frac{1}{y} \right)$$

$$\Rightarrow \frac{BD}{BD} = z \left(\frac{1}{x} + \frac{1}{y} \right)$$

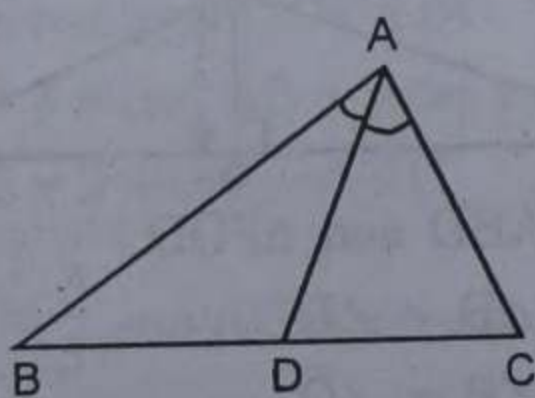
$$\Rightarrow z \left(\frac{1}{x} + \frac{1}{y} \right) = 1$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

Hence proved.

Q. 5. In $\triangle ABC$, AD is the bisector of $\angle A$.

If $BC = 10$ cm, $BD = 6$ cm and $AC = 6$ cm, find AB.



Sol. In $\triangle ABC$, AD is the bisector of $\angle A$

$$\therefore \frac{AB}{AC} = \frac{BD}{DC}$$

But $BC = 10$ cm, $BD = 6$ cm,
 $AC = 6$ cm

$$\therefore DC = BC - BD = 10 - 6 = 4 \text{ cm}$$

$$\therefore \frac{AB}{6} = \frac{6}{4}$$

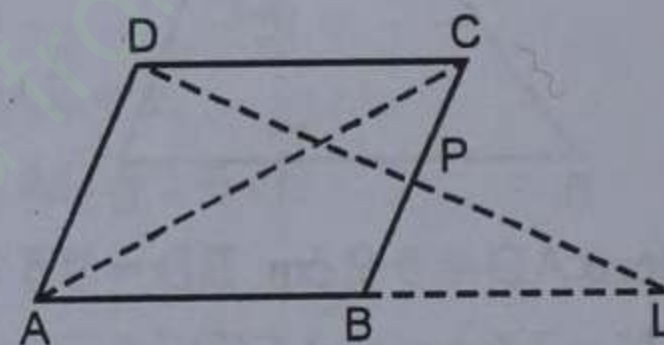
$$\Rightarrow AB = \frac{6 \times 6}{4} = 9$$

Hence $AB = 9$ cm.

Q. 6. In the adjoining figure, ABCD is a parallelogram, P is a point on side BC and DP when produced meets AB produced at L. Prove that :

(i) $DP : PL = DC : BL$

(ii) $DL : DP = AL : DC$.



Sol. Given : ABCD is a parallelogram. P is a point on side BC. DP is joined and produced to meet AB produced at L.

To prove :

(i) $DP : PL = DC : BL$

(ii) $DL : DP = AL : DC$

Proof : (i) In $\triangle DCP$ and $\triangle BPL$,

$$\angle CPD = \angle BPL$$

(Vertically opposite angles)

$$\angle CDP = \angle PLB \quad (\text{Alternate angles})$$

$$\angle DCP = \angle PBL \quad (\text{Alternate angles})$$

$$\therefore \triangle DCP \sim \triangle BPL$$

(AAA axioms of similarity)

$$\therefore \frac{DP}{PL} = \frac{DC}{BL}$$

$$\Rightarrow DP : PL = DC : BL.$$

(ii) Now in $\triangle ALD$

$BP \parallel AD$

(opposite sides of a parallelogram)

$$\therefore \frac{LP}{PD} = \frac{LB}{BA}$$

$$\Rightarrow \frac{DP}{PL} = \frac{AB}{BL}$$

$$\Rightarrow \frac{PL}{DP} = \frac{BL}{AB}$$

Adding 1 both sides

$$\frac{PL}{DP} + 1 = \frac{BL}{AB} + 1$$

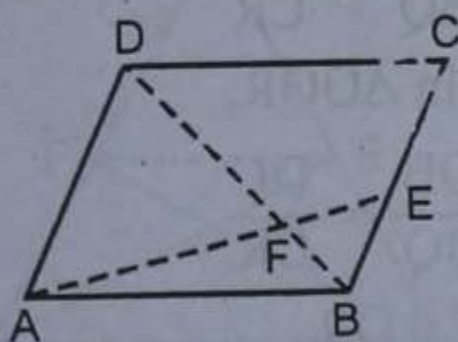
$$\frac{PL + DP}{DP} = \frac{BL + AB}{AB}$$

$$\Rightarrow \frac{DL}{DP} = \frac{AL}{AB} \Rightarrow \frac{DL}{DP} = \frac{AL}{DC}$$

($\because AB = DC$)

Hence proved.

Q. 7. In the given figure, ABCD is a parallelogram, E is a point on BC and the diagonal BD intersects AE at F. Prove that : $DF \times FE = FB \times FA$.



Sol. **Given :** ABCD is a parallelogram. E is a point on BC and diagonal BD intersects AE at F.

To prove : $DF \times FE = FB \times FA$.

Proof : In $\triangle AFD$ and $\triangle BFE$

$$\angle AFD = \angle BFE$$

(Vertically opposite angles)

$$\angle ADF = \angle FBE \quad (\text{Alternate angles})$$

$$\therefore \triangle AFD \sim \triangle BFE$$

(AA axiom of similarity)

$$\therefore \frac{FD}{FB} = \frac{FA}{FE}$$

$$\Rightarrow DF \times FE = FB \times FA$$

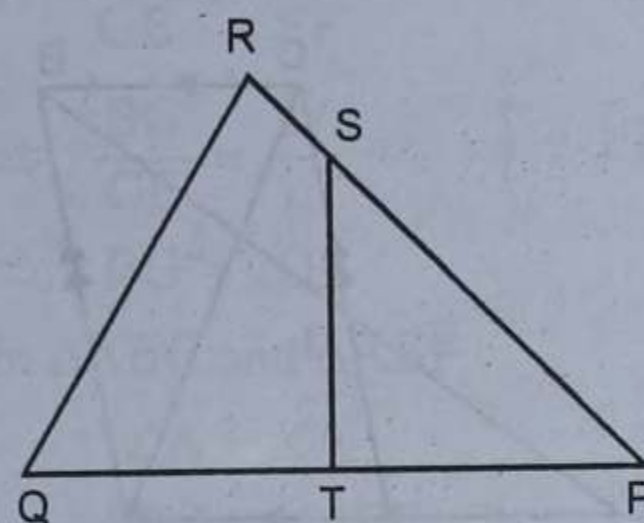
Hence proved.

Q. 8. In the adjoining figure (not drawn to scale),

$PS = 4$ cm, $SR = 2$ cm, $PT = 3$ cm and $QT = 5$ cm.

(i) Show that $\triangle PQR \sim \triangle PST$

(ii) Calculate ST, if $QR = 5.8$ cm.



Sol. **Given :** $PS = 4$ cm, $SR = 2$ cm,
 $PT = 3$ cm, $QT = 5$ cm

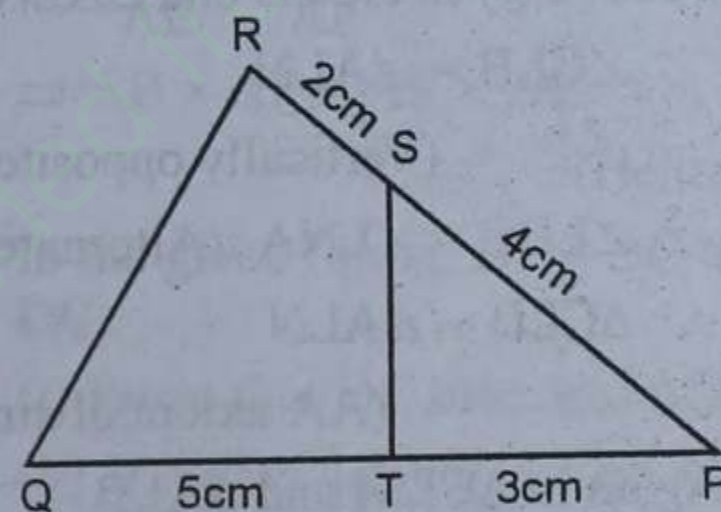
To prove : (i) Show that $\triangle PQR \sim \triangle PST$

(ii) Calculate ST, if $QR = 5.8$ cm.

Proof : (i) In $\triangle PQR$ and $\triangle PST$

$$PQ = 5 + 3 = 8 \text{ cm}$$

$$PR = 4 + 2 = 6 \text{ cm}$$



$$\therefore \frac{PQ}{PS} = \frac{8}{4} = \frac{2}{1}$$

$$\text{and } \frac{PR}{PT} = \frac{6}{3} = \frac{2}{1}$$

$$\therefore \frac{PQ}{PS} = \frac{PR}{PT}$$

$$\text{and } \angle P = \angle P \quad (\text{Common})$$

$$\therefore \triangle PQR \sim \triangle PST$$

(ASA axiom of similarity)

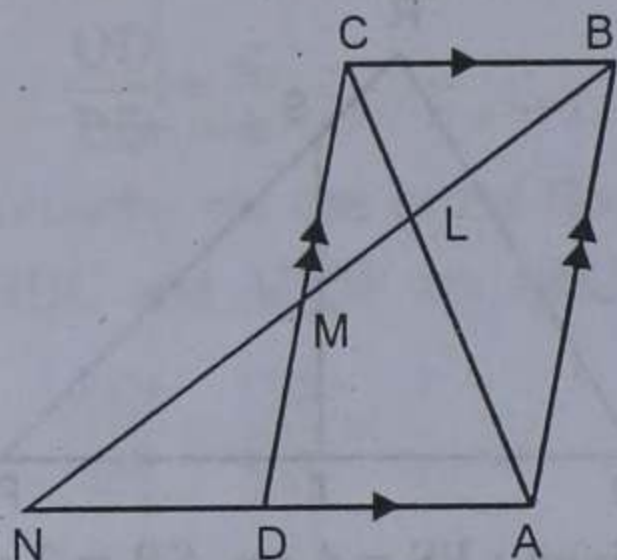
$$(ii) \frac{PQ}{PS} = \frac{QR}{ST} \Rightarrow \frac{2}{1} = \frac{5.8}{ST}$$

$$\Rightarrow 2 \times ST = 5.8 \times 1$$

$$\Rightarrow ST = \frac{5.8}{2} = 2.9 \text{ cm Ans.}$$

Q. 9. In the adjoining figure, ABCD is a parallelogram in which $AB = 16$ cm, $BC = 10$ cm and L is a point on AC such that $CL : LA = 2 : 3$. If BL produced meets CD at M and AD produced at N, prove that :

(i) $\triangle CLB \sim \triangle ALN$ (ii) $\triangle CLM \sim \triangle ALB$.



Sol. Given : In $\parallel gm$ ABCD, $AB = 16$ cm, $BC = 10$ cm and L is a point on AC such that $CL : LA = 2 : 3$. BL is produced to meet CD at M and AD produced at N.

To prove : (i) $\triangle CLB \sim \triangle ALN$

(ii) $\triangle CLM \sim \triangle ALB$

Proof : (i) In $\triangle CLB$ and $\triangle ALN$,

$$\angle CLB = \angle ALN$$

(Vertically opposite angles)

$$\angle LBC = \angle LNA \text{ (Alternate angles)}$$

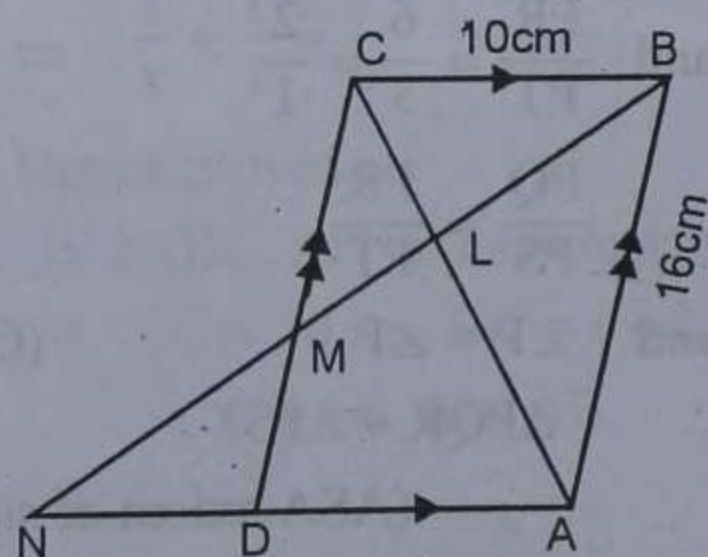
$$\therefore \triangle CLB \sim \triangle ALN$$

(AA axiom of similarity)

Again in $\triangle CLM$ and $\triangle ALB$,

$$\angle CLM = \angle ALB$$

(Vertically opposite angles)



$$\angle LCM = \angle LAB \text{ (Alternate angles)}$$

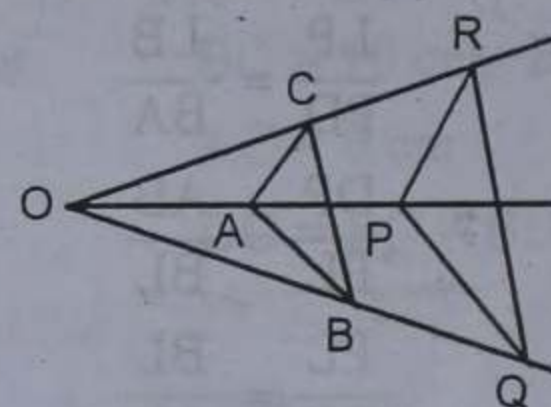
$$\therefore \triangle CLM \sim \triangle ALB$$

(AA axiom of similarity)

Hence proved.

Q. 10. In the given figure, $AB \parallel PQ$ and $AC \parallel PR$.

Prove that $BC \parallel QR$.



Sol. Given : In figure, $AB \parallel PQ$ and $AC \parallel PR$

To prove : $BC \parallel QR$

Proof : In $\triangle OPQ$,

$$AB \parallel PQ \quad (\text{given})$$

$$\therefore \frac{OB}{BQ} = \frac{OA}{AP} \quad \dots(i)$$

Similarly in $\triangle OPR$,

$$AC \parallel PR \quad (\text{given})$$

$$\therefore \frac{OA}{AP} = \frac{OC}{CR} \quad \dots(ii)$$

from (i) and (ii)

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

Now in $\triangle OQR$,

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

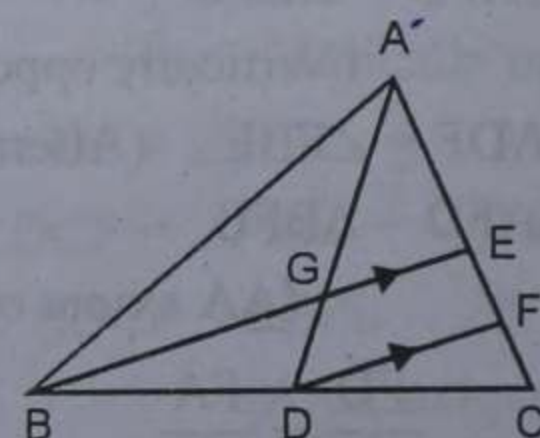
$$\therefore BC \parallel QR$$

Hence proved.

Q. 11. In the given figure, medians AD and BE, of $\triangle ABC$ meet at G and $DF \parallel BE$.

Prove that : (i) $EF = FC$

(ii) $AG : GD = 2 : 1$.



Sol. Given : In $\triangle ABC$, AD and BE are the medians which intersect each other at G and $DF \parallel BE$.

To prove : (i) $EF = FG$

(ii) $AG : GD = 2 : 1$

Proof : (i) In $\triangle BCE$,

D is mid point of BC and $DF \parallel BE$

$\therefore$ F is mid point of EC

$\therefore EF = FC$

(ii) $\therefore$ E is mid point of AC

$\therefore AE = EC = 2EF$

Now in $\triangle ADF$,

$DF \parallel BGE$

$$\therefore \frac{AG}{GD} = \frac{AE}{EF} = \frac{2EF}{EF} = \frac{2}{1}$$

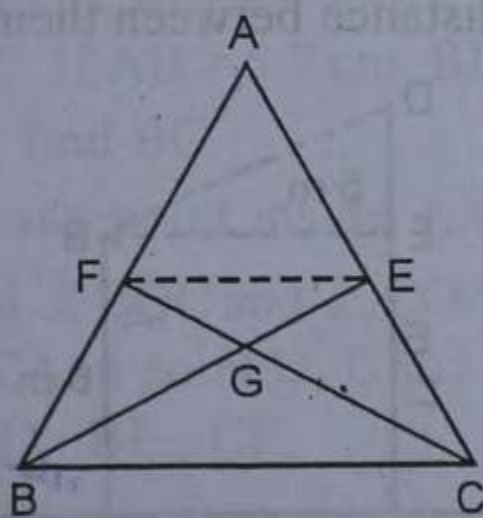
$\therefore AG : GD = 2 : 1$

Hence proved.

Q. 12. In the given figure, the medians BE and CF of $\triangle ABC$ meet at G.

Prove that : (i) $\triangle GEF \sim \triangle GBC$ and therefore, $BG = 2GE$

(ii) $AB \times AF = AE \times AC$.



Sol. Given : In $\triangle ABC$, BE and CF are the medians which intersect each other at G.

To prove : (i) $\triangle GEF \sim \triangle GBC$ and therefore, $BG = 2GE$

(ii) $AB \times AF = AE \times AC$

Construction. Join EF

Proof : $\therefore$ E and F are the mid points of AC and AB respectively.

$$\therefore EF \parallel BC \text{ and } EF = \frac{1}{2} BC$$

(i) In $\triangle GEF$ and $\triangle GBC$,

$\angle GEF = \angle GBC$ (Alternate angles)

$\angle EFG = \angle BCF$ (Alternate angles)

$$\angle EGF = \angle BGC$$

(Vertically opposite angles)

$$\therefore \triangle GEF \sim \triangle GBC$$

(AAA axiom of similarity)

$$\therefore \frac{BG}{GE} = \frac{BC}{EF}$$

$$\Rightarrow \frac{BG}{GE} = \frac{2}{1} \quad \left(\because EF = \frac{1}{2} BC \right)$$

$$\Rightarrow BG = 2GE$$

(ii) In $\triangle ABC$ and $\triangle AEF$,

$$\angle A = \angle A \quad (\text{common})$$

$$\angle ABC = \angle AFE$$

(Corresponding angles)

$$\angle ACB = \angle AEF$$

(Corresponding angles)

$$\therefore \triangle ABC \sim \triangle AEF$$

(AAA axiom of similarity)

$$\therefore \frac{AB}{AE} = \frac{AC}{AF}$$

$$\Rightarrow AB \times AF = AE \times AC$$

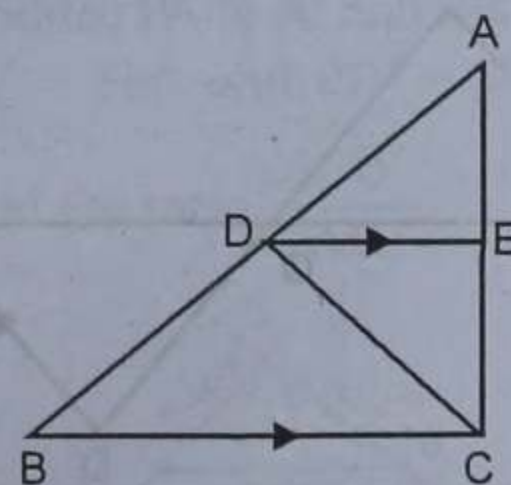
(Hence proved)

Q. 13. In the given figure, $DE \parallel BC$ and $BD = DC$.

(i) Prove that DE bisects $\angle ADC$

(ii) If $AD = 4.5$ cm, $AE = 3.9$ cm and $DC = 7.5$ cm, find CE

(iii) Find the ratio $AD : DB$.



Sol. Given : In $\triangle ABC$, $DE \parallel BC$

$$BD = DC$$

To prove : (i) DE bisects $\angle ADC$

(ii) If $AD = 4.5$ cm, $AE = 3.9$ cm and $DC = 7.5$ cm, find CE

(iii) Find the ratio $AD : DB$.

Proof : (i) In $\triangle ABC$,

$$\therefore DE \parallel BC$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

(Basic proportionality Theorem)

$$\Rightarrow \frac{AD}{DC} = \frac{AE}{EC} \quad [\because DB = DC \text{ (given)}]$$

$\therefore$ In $\triangle ADC$

DE bisects $\angle ADC$

Hence proved

(ii) $AD = 4.5$ cm, $AE = 3.9$ cm,
 $DC = 7.5$ cm

$$\text{Now } \frac{AD}{DC} = \frac{AE}{CE} \Rightarrow \frac{4.5}{7.5} = \frac{3.9}{CE}$$

$$\Rightarrow 4.5 \times CE = 3.9 \times 7.5$$

$$\Rightarrow CE = \frac{3.9 \times 7.5}{4.5} = 6.5 \text{ cm}$$

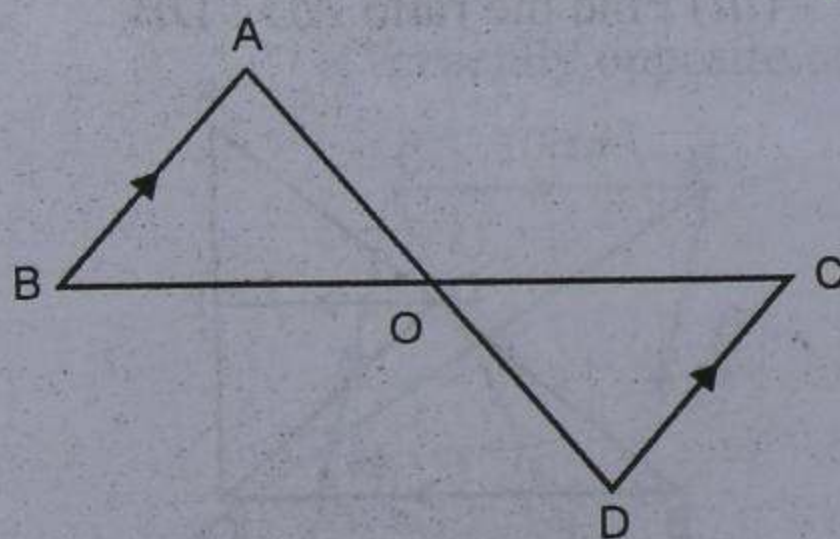
$$(iii) \therefore \frac{AD}{DB} = \frac{AE}{EC}$$

$$\therefore \frac{AD}{DB} = \frac{3.9}{6.5} = \frac{3}{5}$$

$$\Rightarrow AD : DB = 3 : 5 \text{ Ans.}$$

Q. 14. In the given figure, $BA \parallel DC$. Show that $\triangle OAB \sim \triangle ODC$.

If $AB = 4$ cm, $CD = 3$ cm, $OC = 5.7$ cm and $OD = 3.6$ cm, find OA and OB .



Sol. In $\triangle OAB$ and $\triangle ODC$

$$\angle AOB = \angle COD$$

(Vertically opposite angles)

$$\angle BAO = \angle ODC \quad (\text{Alternate angles})$$

$$\angle ABO = \angle OCD \quad (\text{Alternate angles})$$

$$\therefore \triangle OAB \sim \triangle ODC$$

(AAA axiom of similarity)

$$\therefore \frac{OA}{OD} = \frac{OB}{OC} = \frac{AB}{CD}$$

Now $AB = 4$ cm, $CD = 3$ cm,

$OC = 5.7$ cm and $OD = 3.6$ cm

$$\therefore \frac{OA}{3.6} = \frac{OB}{5.7} = \frac{4}{3} \Rightarrow \frac{OA}{3.6} = \frac{4}{3}$$

$$\Rightarrow OA = \frac{3 \times 3.6}{3} = 4.8 \text{ cm}$$

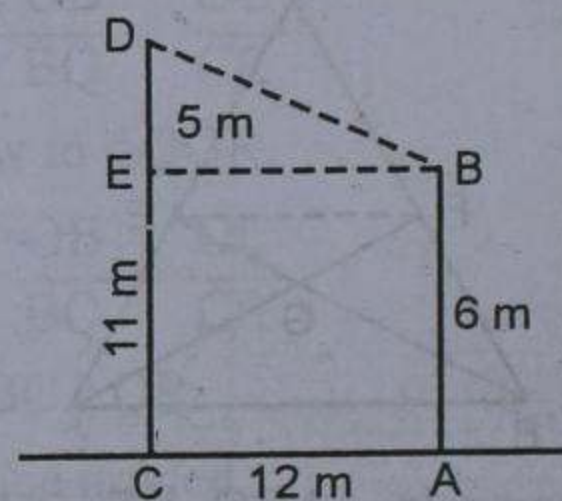
$$\text{and } \frac{OB}{5.7} = \frac{4}{3}$$

$$\Rightarrow OB = \frac{4 \times 5.7}{3} = 7.6 \text{ cm}$$

Hence $OA = 4.8$ cm and

$OB = 7.6$ cm **Ans.**

Q. 15. AB and CD are two vertical poles of height 6 m and 11 m respectively. If the distance between their feet is 12 m, find the distance between their tops.



Sol. AB and CD are two vertical poles such that $AB = 6$ m and $CD = 11$ m and distance between them *i.e.* $CA = 12$ m.

Draw $BE \perp CD$.

$$\therefore EC = BA = 6 \text{ m}$$

$$\text{and } DE = CD - EC \\ = 11 - 6 = 5 \text{ m}$$

$$\text{and } EB = CA = 12 \text{ m}$$

Now in right $\triangle BDE$,

$$BD^2 = EB^2 + DE^2 = (12)^2 + (5)^2 \\ = 144 + 25 = 169 = (13)^2$$

$$\therefore BD = 13 \text{ m.}$$

Hence distance between their tops

Q. 16. In the given figure $\angle ABC = 90^\circ$ and $BD \perp AC$. If $AB = 5.7$ cm, $BD = 3.8$ cm and $CD = 5.4$ cm, find BC .

Sol. In $\triangle ABC$, $\angle ABC = 90^\circ$ and $BD \perp AC$.

$AB = 5.7$ cm, $BD = 3.8$ cm and $CD = 5.4$ cm

Now in $\triangle ABC$ and $\triangle BDC$,

$$\angle ABC = \angle BDC \quad (\text{each} = 90^\circ)$$

$$\angle C = \angle C \quad (\text{Common})$$

$$\therefore \triangle ABC \sim \triangle BDC$$

(AA axiom of similarity)

$$\therefore \frac{AB}{BD} = \frac{BC}{DC} \Rightarrow \frac{5.7}{3.8} = \frac{BC}{5.4} \Rightarrow BC = \frac{5.7 \times 5.4}{3.8}$$

Hence $BC = 8.1$ cm. **Ans.**

Q. 17. O is any point inside a

$\triangle ABC$. The bisectors of

$\angle AOB$, $\angle BOC$ and $\angle COA$

meet the sides AB , BC and CA

in points D , E and F

respectively. Prove that $AD \cdot BE \cdot CF = DB \cdot EC \cdot FA$.

Sol. O is any point inside a $\triangle ABC$ and $BD \perp AC$. If $AB = 5.7$ cm, $BD = 3.8$ cm and $CD = 5.4$ cm, find BC .

Given : O is any point inside $\triangle ABC$. Bisectors of $\angle AOB$ and $\angle BOC$ and $\angle COA$ meet the sides AB , BC and CA in point D , E and F respectively.

To prove : $AD \cdot BE \cdot CF$

$= DB \cdot EC \cdot FA$

Proof : In $\triangle AOB$,

$\therefore OD$ is the bisector of $\angle AOB$

$$\therefore \frac{OA}{OB} = \frac{AD}{DB} \quad \dots(i)$$

(Angle bisector Theorem)

Similarly in $\triangle BOC$,

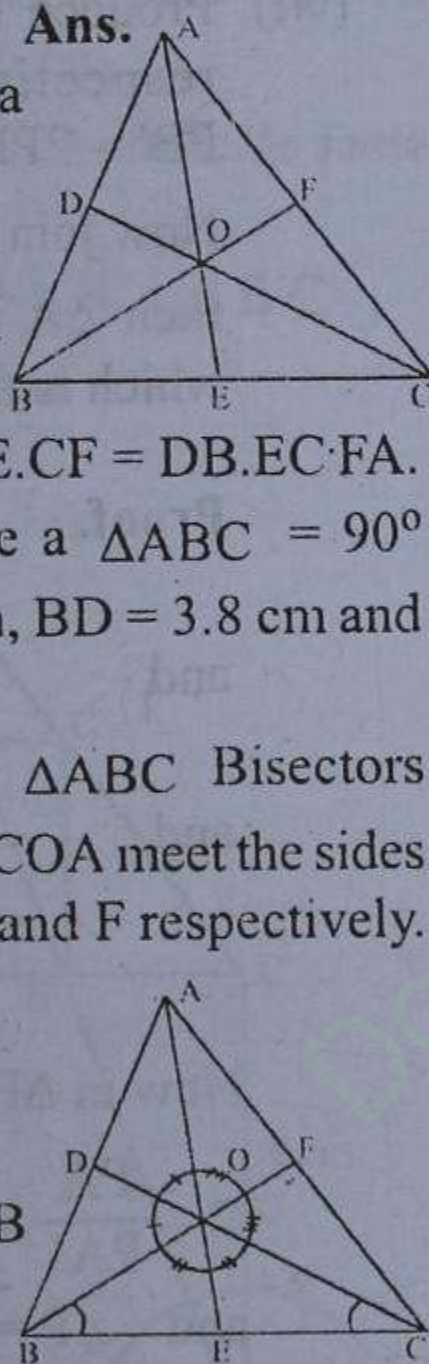
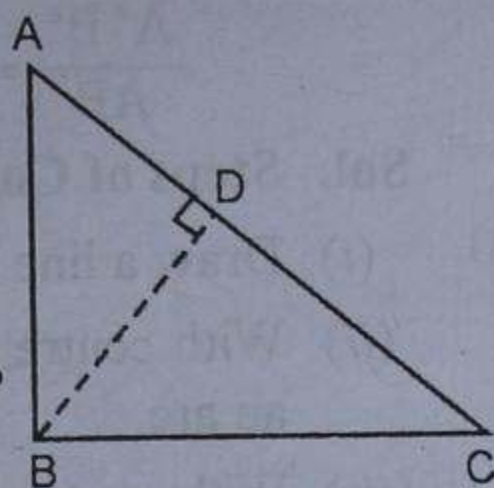
$$\frac{OB}{OC} = \frac{BE}{EC} \quad \dots(ii)$$

and in $\triangle COA$,

$$\frac{OC}{OA} = \frac{CF}{FA} \quad \dots(iii)$$

Multiplying (i), (ii) and (iii), we set

$$\frac{OA}{OB} \times \frac{OB}{OC} \times \frac{OC}{OA} = \frac{AD}{DB} \times \frac{BE}{EC} \times \frac{CF}{FA}$$



$$\Rightarrow 1 = \frac{AD \cdot BE \cdot CF}{DB \cdot EC \cdot FA} \Rightarrow AD \cdot BE \cdot CF = DB \cdot EC \cdot FA$$

Hence proved.

EXERCISE 12 (B)

Q. 1. A $\triangle ABC$ has been reduced by scale factor 0.8 to $\triangle A'B'C'$. Calculate :

(i) the length $A'B'$, when $AB = 10$ cm.

(ii) the length BC , when $B'C' = 8.4$ cm.

Sol. Scale factor $= k = 0.8 = \frac{8}{10} = \frac{4}{5}$

(i) $AB = 10$ cm.

$$\therefore A'B' = k \times AB = \frac{4}{5} \times 10 = 8 \text{ cm}$$

(ii) $B'C' = 8.4$ cm,

$$\text{then } BC = \frac{1}{k} B'C' = \frac{1}{\frac{4}{5}} \times 8.4 = \frac{8.4 \times 5}{4} = 10.5 \text{ cm}$$

Q. 2. A $\triangle PQR$ has been enlarged by scale factor 2.5 to $\triangle P'Q'R'$. Calculate :

(i) the length $Q'R'$, when $QR = 6$ cm.

(ii) the length PQ , when $P'Q' = 10$ cm.

Sol. Scale factor $= k = 2.5$

(i) $QR = 6$ cm.

$$\text{then } Q'R' = kQR = 2.5 \times 6 = 15 \text{ cm.}$$

(ii) $P'Q' = 10$ cm

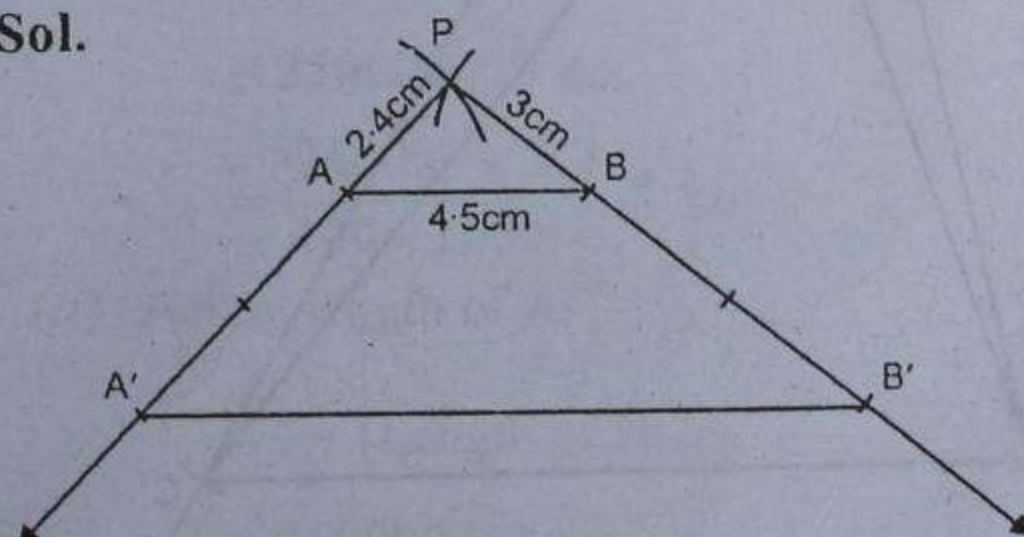
$$\text{then } PQ = \frac{1}{k} P'Q' = \frac{1}{2.5} \times 10 = 10 \times \frac{10}{25} = 4 \text{ cm. Ans.}$$

Q. 3. Draw a line segment $AB = 4.5$ cm. Locate a point P outside AB such that $PQ = 2.4$ cm and $PB = 3$ cm.

Produce PA to A' and PB to B' such that $PA' = 3PA$ and $PB' = 3PB$. Join $A'B'$.

Find the ratio $\frac{A'B'}{AB}$.

Sol.



Steps of Construction :

(i) Draw a line segment $AB = 4.5$ cm.

- (ii) With centre A and radius 2.4 cm, draw an arc.
- (iii) With centre B and radius 3 cm, draw another arc intersecting the first arc at P.
- (iv) Join PA and PB.
- (v) Now produce PA to A' and PB to B' such that $PA' = 3PA$ and $PB' = 3PB$.
- (vi) Join A'B'.

$$\text{Now } \frac{PA'}{PA} = \frac{3PA}{PA} = \frac{3}{1}$$

$$\text{and } \frac{PB'}{PB} = \frac{3PB}{PB} = \frac{3}{1}$$

$$\therefore \frac{PA'}{PA} = \frac{PB'}{PB}$$

$$\text{and } \angle P = \angle P \quad (\text{common})$$

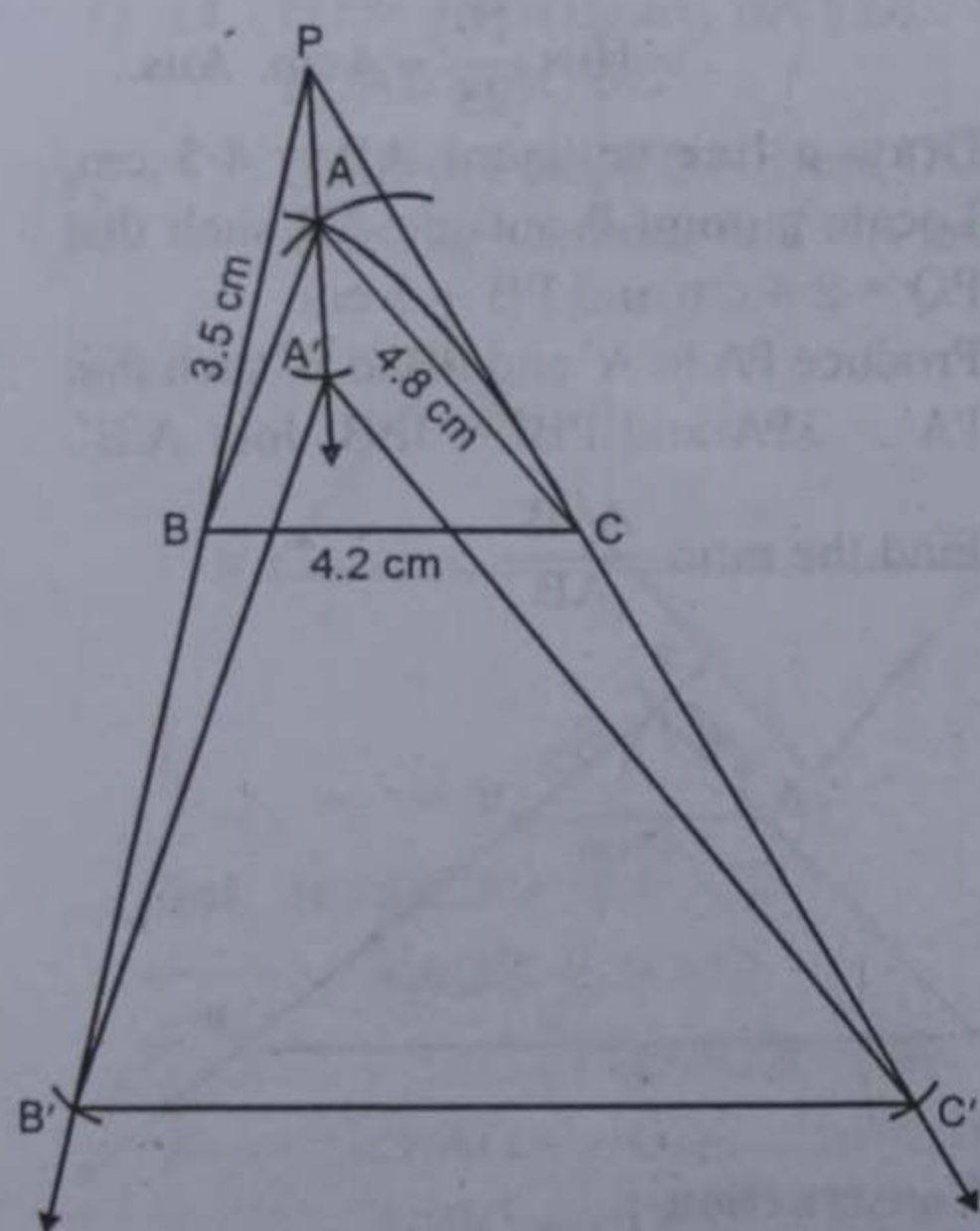
$$\therefore \Delta PA'B' \sim \Delta PAB$$

(SAS axiom of similarity)

$$\therefore \frac{A'B'}{AB} = \frac{PA'}{PA} = \frac{3}{1}$$

$$\text{or } A'B' : AB = 3 : 1$$

Q. 4. Construct a ΔABC with $AB = 3.5$ cm, $BC = 4.2$ cm and $AC = 4.8$ cm. Enlarge ΔABC with scale factor $k = 2$ to $\Delta A'B'C'$. Show that



$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC} = 2$$

Sol. Steps of Construction :

- (i) Draw a line segment $BC = 4.2$ cm.
 - (ii) With centre B and radius 3.5 cm draw an arc.
 - (iii) With centre C and radius 4.8 cm, draw another arc intersecting the first arc at A.
 - (iv) Join AB and AC.
- Then ΔABC is the triangle.
- Now, the scale factor $k = 2$.
- (v) Take a point P outside ΔABC .
 - (vi) Join PA, PB and PC.
 - (vii) Produce PA, PB and PC to A', B', C' respectively such that $PA' = 2PA$, $PB' = 2PB$ and $PC' = 2PC$.

Now join A'B', B'C' and A'C'

then $\Delta A'B'C'$ is the required triangle which is the image of ΔABC .

$$\text{Proof. } \therefore \frac{PB'}{PB} = \frac{2PB}{PB} = \frac{2}{1}$$

$$\text{and } \frac{PA'}{PA} = \frac{2PA}{PA} = \frac{2}{1}$$

$$\text{and } \frac{PC'}{PC} = \frac{2PC}{PC} = \frac{2}{1}$$

$$\therefore \frac{PA'}{PA} = \frac{PB'}{PB} = \frac{PC'}{PC}$$

Now in ΔPAB and $\Delta PA'B'$

$$\therefore \frac{PA}{PA'} = \frac{PB}{PB'} \quad (\text{proved})$$

$$\text{and } \angle P = \angle P \quad (\text{Common})$$

$$\therefore \Delta PAB \sim \Delta PA'B'$$

(SAS axiom of similarity)

$$\therefore \frac{AB}{A'B'} = \frac{PA}{PA'} = \frac{1}{2}$$

$$\Rightarrow \frac{A'B'}{AB} = \frac{2}{1} \quad \dots(i)$$

Similarly we can prove that

$$\Delta PAC \sim \Delta PA'C'$$

$$\therefore \frac{AC}{A'C'} = \frac{1}{2}$$

$$\Rightarrow \frac{A'C'}{AC} = \frac{2}{1} \quad \dots(ii)$$

and $\triangle PBC \sim \triangle PB'C'$

$$\therefore \frac{BC}{B'C'} = \frac{1}{2}$$

$$\Rightarrow \frac{B'C'}{BC} = \frac{2}{1} \quad \dots(iii)$$

$\therefore$ From (i), (ii) and (iii)

$$\frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC} = \frac{2}{1}$$

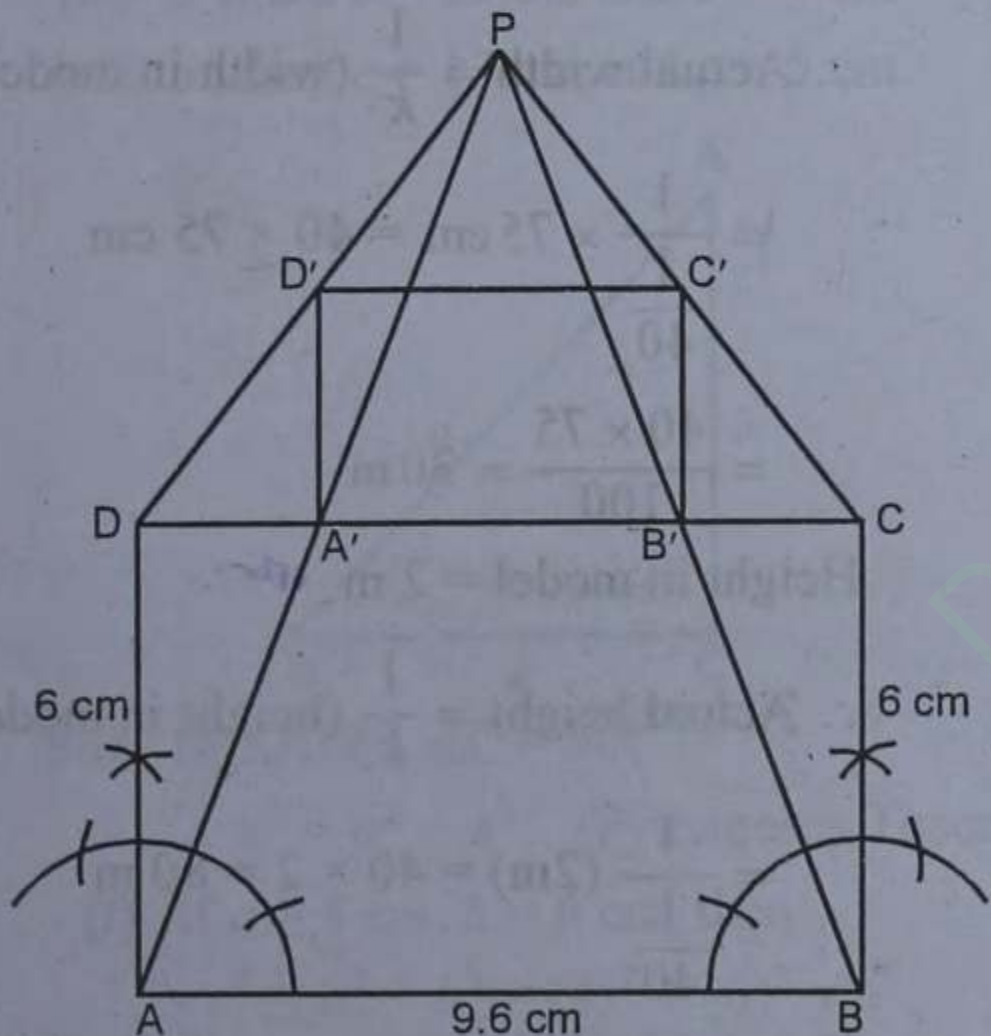
Hence proved.

Q. 5. Construct a rectangle ABCD with AB = 9.6 cm and BC = 6 cm.

(i) Reduce it to $A'B'C'D'$ with scale factor 0.5.

(ii) Measure the lengths $A'B'$ and $B'C'$.

Steps of Construction :



(i) Draw a line segment AB = 9.6 cm.

(ii) At A and B, draw perpendiculars and cut off AD = BC = 6 cm.

(iii) Join CD.

ABCD is a rectangle.

(iv) Take a point P outside the rectangle.

(v) Join PA, PB, PC and PD.

(vi) As $k = 0.5$, then take mid points of line segments PA, PB, PC and PD as A' , B' , C' and D' .

(vii) Join $A'B'$, $B'C'$, $C'D'$, $D'A'$.

Then $A'B'C'D'$ is the required rectangle.

Measuring the lengths of $A'B' = 4.8$ cm and $B'C' = 3$ cm.

Q. 6. On a map drawn to a scale 1 : 250000, a triangular plot of land has the following measurements :

AB = 3 cm, BC = 4 cm, $\angle ABC = 90^\circ$

Calculate : (i) the actual length of AB in km.

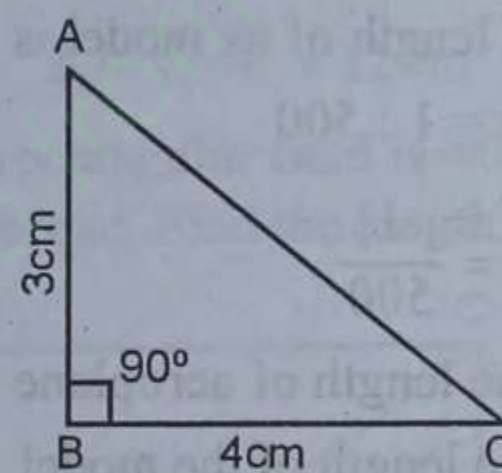
(ii) the actual length of AC in km.

(iii) the actual area of the plot in km^2 .

Sol. Scale = 1 : 250000 $\Rightarrow k = \frac{1}{250000}$

On map of a plot, measurements are

AB = 3 cm, BC = 4 cm, $\angle ABC = 90^\circ$



$$\therefore \text{Lengths of } AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{(3)^2 + (4)^2}$$

$$= \sqrt{9 + 16} = \sqrt{25} = 5 \text{ cm}$$

(i) The actual length AB

$$= \frac{1}{k} \text{ (length of AB on the map)}$$

$$= 250000 \times 3 \text{ cm}$$

$$= \frac{250000 \times 3}{100 \times 1000} = 7.5 \text{ km}$$

(ii) Actual length of AC

$$= \frac{1}{k} \text{ (Length of map)}$$

$$= 250000 \times 5 \text{ cm}$$

$$= \frac{250000 \times 5}{100 \times 1000} = 12.5 \text{ km}$$

(iii) Actual area of the plot

$$= \frac{1}{k^2} (\text{Area of plot on the map})$$

$$= \frac{1}{k^2} \left(\frac{3 \times 4}{2} \right) = \frac{6}{k^2}$$

$$= (250000)^2 \times 6 \text{ cm}^2$$

$$= \frac{250000 \times 250000 \times 6}{100 \times 1000 \times 100 \times 1000}$$

$$= \frac{75}{2} = 37.5 \text{ km}^2 \text{ Ans.}$$

Q. 7. A model of an aeroplane is made to a scale 1 : 500

Calculate : (i) the length in cm of the model, if the length of the aeroplane is 50 m.

(ii) the length in meters of the aeroplane, if the length of its model is 18 cm.

Sol. Scale = 1 : 500

$$\therefore k = \frac{1}{500}$$

(i) The length of aeroplane = 50 m.

$\therefore$ The length of the model

$$= \frac{1}{k} (\text{actual length})$$

$$= \frac{1}{500} \times 50 = \frac{1}{10} \text{ m} = 10 \text{ cm}$$

(ii) The length of model = 18 cm

$\therefore$ The length of actual aeroplane

$$= \frac{1}{k} (\text{length in model})$$

$$= \frac{1}{\frac{1}{500}} (18 \text{ cm})$$

$$= 500 \times 18 \text{ cm}$$

$$= \frac{500 \times 18}{100} = 90 \text{ m Ans.}$$

Q. 8. The dimensions of the model of a multistorey building are 1.2 m $\times$ 75 cm $\times$ 2 m. If scale factor is 1 : 40, find the actual dimensions of the building.

Sol. Scale factor = 1 : 40

$$\therefore k = \frac{1}{40}$$

Length in model = 1.2 m

$$\therefore \text{Actual length} = \frac{1}{k} (\text{length in model})$$

$$= \frac{1}{\frac{1}{40}} (1.2 \text{ m}) = 40 \times 1.2 \text{ m}$$

$$= 48 \text{ m}$$

Width in model = 75 cm.

$$\therefore \text{Actual width} = \frac{1}{k} (\text{width in model})$$

$$= \frac{1}{\frac{1}{40}} \times 75 \text{ cm} = 40 \times 75 \text{ cm}$$

$$= \frac{40 \times 75}{100} = 30 \text{ m}$$

Height in model = 2 m

$$\therefore \text{Actual height} = \frac{1}{k} (\text{height in model})$$

$$= \frac{1}{\frac{1}{40}} (2 \text{ m}) = 40 \times 2 = 80 \text{ m}$$

Hence actual dimensions of the building are 48 m $\times$ 30 m $\times$ 80 m **Ans.**