11

Mid-Point and Intercept Theorems

POINTS TO REMEMBER

Theorems.

1. **Mid-point Theorem :** The line joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.
2. The straight line drawn through the mid-point of one side of a triangle and parallel to the other side, bisects the third side.
3. **Intercept Theorem :** If a transversal makes equal intercepts on three or more parallel lines, then any other line cutting them will also make equal intercepts.

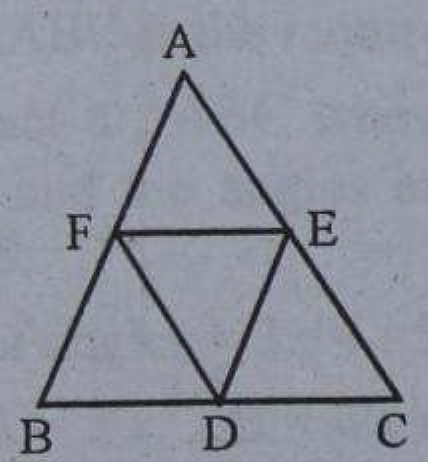
EXERCISE 11

Q. 1. In the given figure, D, E, F are the mid-points of the sides BC, CA and AB respectively.

(*i*) If AB = 6·2 cm, find DE.

(*ii*) If DF = 3·8 cm, find AC.

(*iii*) If perimeter of ΔABC is 21 cm, find FE.



Sol. D, E and F are the mid-points of the sides BC, CA and AB respectively.

$\therefore$ $EF \parallel BC$ and $EF = \frac{1}{2}BC$

Similarly $DE \parallel AB$ and $DE = \frac{1}{2}AB$

and $FD \parallel AC$ and $FD = \frac{1}{2}AC$

(*i*) Now if AB = 6·2 cm, then

$$DE = \frac{1}{2}AB = \frac{1}{2} \times 6{\cdot}2 = 3{\cdot}1\text{ cm}.$$

(*ii*) DF = 3·8 cm,

But $DF = \frac{1}{2}AC \Rightarrow AC = 2\,DF$

$\therefore$ $AC = 2 \times 3{\cdot}8 = 7{\cdot}6$ cm

(*iii*). If perimeter of ΔABC = 21 cm

$\Rightarrow AB + BC + CA = 21$ cm

$\Rightarrow 6{\cdot}2 + BC + 7{\cdot}6 = 21$ cm

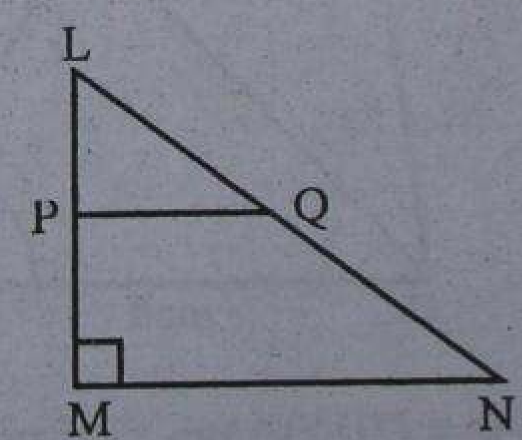
$\Rightarrow BC + 13{\cdot}8 = 21 \Rightarrow BC = 21 - 13{\cdot}8$

$\Rightarrow BC = 7{\cdot}2$ cm

But $FE = \frac{1}{2}BC$

$= \frac{1}{2} \times 7{\cdot}2 = 3{\cdot}6\text{ cm}$ **Ans.**

Q. 2. In the given figure, LMN is a right triangle in which $\angle M = 90°$, P and Q are mid-points of LM and LN respectively. If LM = 9 cm, MN = 12 cm and LN = 15 cm, find :



(*i*) the perimeter of trapezium MNQP,

(*ii*) the area of trapezium MNQP.

Sol. In right angled ΔLMN, $\angle M = 90^\circ$

P and Q are the mid-points of sides LM and LN respectively.

$\therefore PQ \parallel MN$ and $PQ = \frac{1}{2} MN$

Now LM = 9 cm, MN = 12 cm

and LN = 15 cm

$$\therefore \quad PQ = \frac{1}{2} MN = \frac{1}{2} \times 12 = 6\,cm$$

$$PM = \frac{1}{2} LM = \frac{1}{2} \times 9 = 4{\cdot}5\,cm$$

$$QN = \frac{1}{2} LN = \frac{1}{2} \times 15 = 7{\cdot}5\,cm.$$

(*i*) Perimeter of trapezium MNQP

$= PQ + QN + MN + PM$

$= 6 + 7{\cdot}5 + 12 + 4{\cdot}5 = 30$ cm.

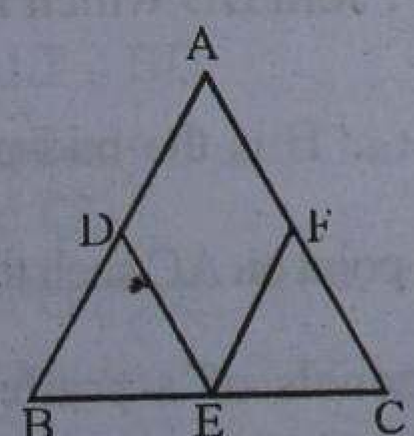
(*ii*) Area of trapezium MNQP

$= \frac{1}{2}(PQ + MN) \times PM \quad (\because \angle M = 90^\circ)$

$= \frac{1}{2}(6 + 12) \times 4{\cdot}5\,cm^2$

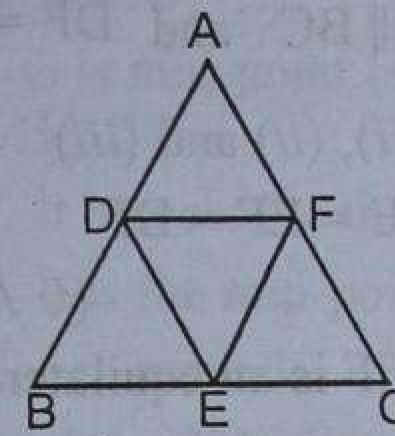
$= \frac{1}{2} \times 18 \times 4{\cdot}5\,cm^2 = 40{\cdot}5\,cm^2$. **Ans.**

Q. 3. In the given figure, D, E, F are respectively the mid-points of the sides AB, BC and CA of ΔABC. Prove that ADEF is a parallelogram.



Sol. Given : In the ΔABC, D, E and F are the mid-points of sides AB, BC and CA respectively.

To prove : ADEF is a parallelogram.



Const. Join DE, EF and FD.

Proof : $\because$ E and F are the mid-points of sides BC and CA respectively.

$\therefore \; EF \parallel AB$ ...(*i*)

Similarly D and E are the mid-points of sides AB and BC respectively.

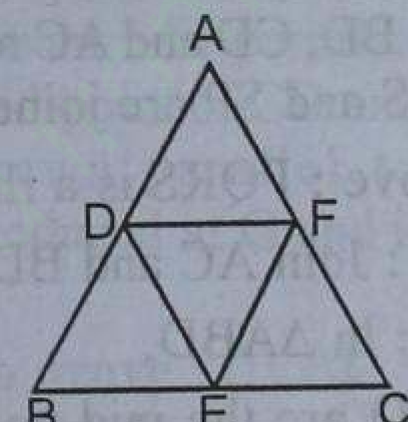
$\therefore \; DE \parallel AC$ ...(*ii*)

From (*i*) and (*ii*)

ADEF is a parallelogram.

Hence proved.

Q. 4. If D, E, F are respectively the mid-points of the sides AB, BC and CA of an equilateral triangle ABC, prove that ΔDEF is also an equilateral triangle.



Sol. Given : ΔABC is an equilateral triangle D, E and F are the mid-points of sides AB, BC and CA respectively. DE, EF and FD are joined.

To prove : ΔDEF is also an equilateral triangle.

Proof : $\because$ D and E are the mid-point of AB and BC respectively.

$\therefore DE \parallel AC$ and $DE = \frac{1}{2} AC$ ...(*i*)

Similarly E and F are the mid-points of BC and CA respectively.

$\therefore EF \parallel AB$ and $EF = \frac{1}{2} AB$ ...(*ii*)

and F and D are the mid-points of AC and AB respectively.

$\therefore$ DF || BC and $DF = \frac{1}{2} BC$ ...(*iii*)

From (*i*), (*ii*) and (*iii*)

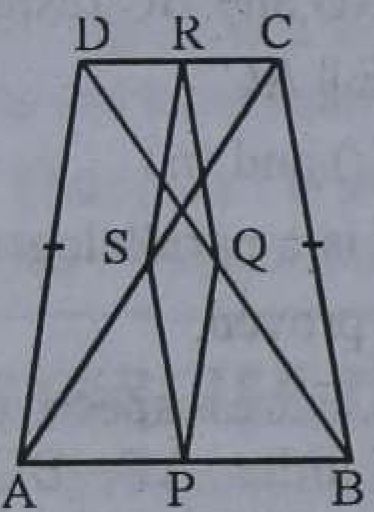
DE = EF = DF

($\because$ AB = BC = CA)

$\therefore$ ΔDEF is an equilateral triangle.

Hence proved.

Q. 5. In the adjoining figure, ABCD is a quadrilateral in which AD = BC and P, Q, R, S are the mid-points of AB, BD, CD and AC respectively. Prove that PQRS is a rhombus.



Sol. Given : In quadrilateral ABCD, AD = BC and P, Q, R and S are the mid-points of AB, BD, CD and AC respectively. PQ, QR, RS and SP are joined.

To prove : PQRS is a rhombus.

Const : Join AC and BD.

Proof : In ΔABD,

P and Q are the mid-points of AB and AD

$\therefore$ PQ || AD and $PQ = \frac{1}{2} AD$ ...(*i*)

In ΔACD,

S and R are the mid-points of AC and CD

$\therefore$ SR || AD and $SR = \frac{1}{2} AD$ ...(*ii*)

In ΔBCD,

Q and R are the mid-points of BD and CD

$\therefore$ QR || BC and $QR = \frac{1}{2} BC$ ...(*iii*)

and in ΔABC,

P and S are the mid-points of AB and AC

PS || BC and $PS = \frac{1}{2} BC$ ...(*iv*)

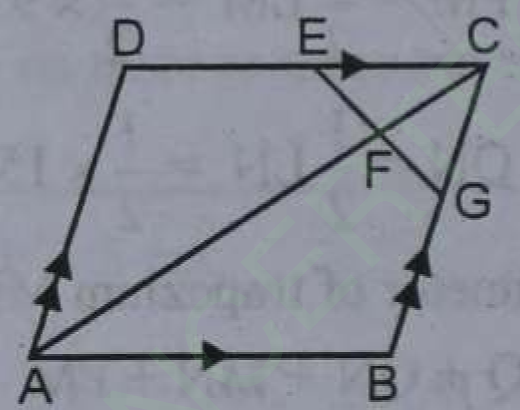
From (*i*), (*ii*) and (*iii*) and (*iv*)

PS = QR = RS = SP ($\because$ AD = BC)

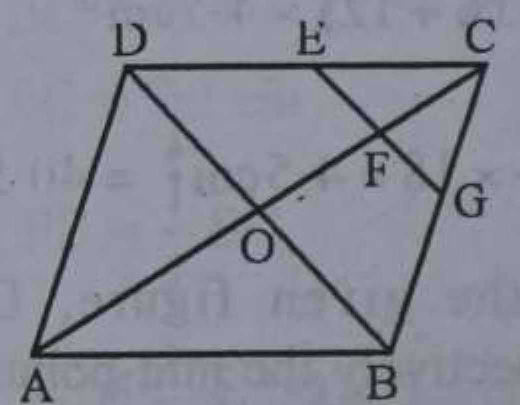
$\therefore$ PQRS is a rhombus.

Hence proved.

Q. 6. In the adjoining figure, ABCD is a parallelogram in which E is the mid-point of DC and F is a point on AC such that $CF = \frac{1}{4} AC$. If EF is produced to meet BC in G, prove that G is the mid-point of BC.



Sol. Given : In parallelogram ABCD, in which E is the mid-point of DC and F is a point on AC such that $CF = \frac{1}{4} AC$. EF is produced to meet BC at G.



To prove : G is the mid-point of BC.

Const : Join BD which intersects AC at O.

Proof : $\because$ E is the mid-point of DC and F is the point on AC such that $CF = \frac{1}{4} AC$.

$\because$ Diagonals of a prallelogram bisects each other.

$\therefore$ AO = OC

$CF = \frac{1}{2} OC$

$\Rightarrow$ F is the mid-point of OC.

Now in ΔDOC,

EF $\parallel$ DO or BD

We know that any line drawn from the mid-point of one side of a triangle, parallel to the other side, bisects the third side.

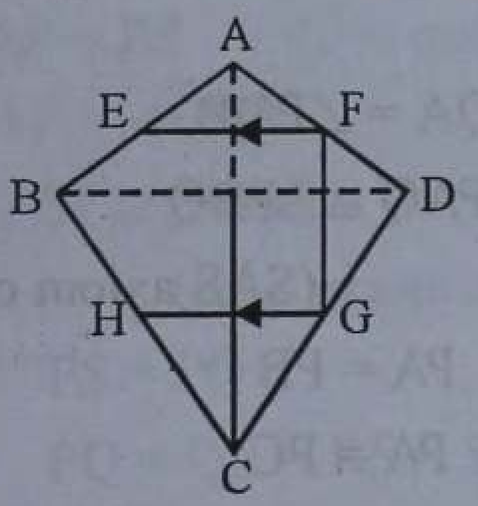
$\therefore$ G is mid-point of BC of ΔBCD.

Q.E.D.

Q. 7. In the adjoining figure, ABCD is a kite in which AB = AD and CB = CD. If E, F, G are respectively the mid-points of AB, AD and CD, prove that :

(*i*) $\angle EFG = 90^\circ$,

(*ii*) If GH $\parallel$ FE, then H bisects CB.



Sol. In kite shaped quadrilateral ABCD, AB = AD and CB = CD. E, F and D are the mid-points of AB, AD and CD respectively.

To prove : (*i*) $\angle EFG = 90^\circ$

(*ii*) If GH $\parallel$ FE, then H bisects CB.

Const : Join AC and BD.

Proof : (*i*) In ΔABD,

E and F are mid-points of AB and AD.

$\therefore$ EF $\parallel$ BD.

Similarly F and G are mid-points of AD and CD.

$\therefore$ FG $\parallel$ AC.

$\because$ Diagonals of a kite intersect each other at right angles.

$\therefore$ AC and BD are perpendicular to each other.

$\therefore \angle EFG = 90^\circ$.

(*ii*) In ΔBCD,

$\because$ G is mid-point of DC and GH $\parallel$ EF or GH $\parallel$ BD.

$\therefore$ It bisects the third side.

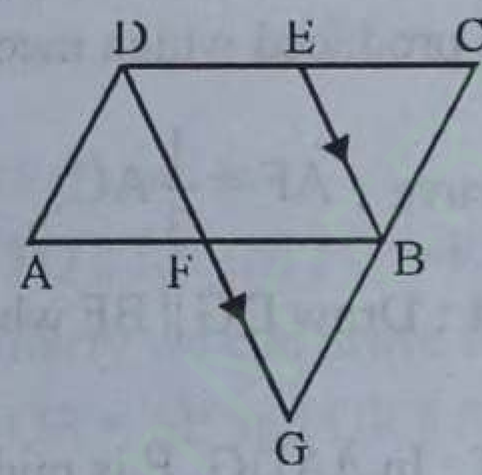
$\therefore$ It is the mid-point of BC.

Hence proved.

Q. 8. In the adjoining figure, ABCD is a parallelogram, E is the mid-point of CD and through D, a line is drawn parallel to EB to meet CB produced at G and intersecting AB at F.

Prove that : (*i*) $AD = \frac{1}{2} GC$

(*ii*) DG = 2 EB.



Sol. Given : In parallelogram ABCD, E is mid-point of CD and through D, a line is drawn parallel to EB to meet CB produced at G and intersects AB at F.

To prove : (*i*) $AD = \frac{1}{2} GC$

(*ii*) DG = 2 EB.

Proof : (*i*) In ΔCDG, E is mid-point of DC and EB $\parallel$ GD.

$\therefore$ B is mid-point of CG.

$$\Rightarrow \quad BC = \frac{1}{2} GC$$

$$\Rightarrow \quad AD = \frac{1}{2} GC$$

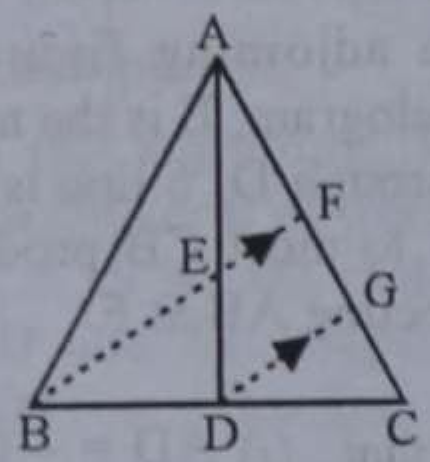
($\because$ AD = BC, Opposite sides of the parallelogram)

(*ii*) and $EB = \frac{1}{2} DG$

$\Rightarrow$ 2 EB = DG $\Rightarrow$ DG = 2 EB

Hence proved.

Q. 9. In the adjoining figure, in ΔABC, AD is the median through A and E is the mid-point of AD. If BE produced meets AC in F, prove that $AF = \frac{1}{3}AC$.



Sol. **Given :** In ΔABC, AD is the median through A and E is the mid-point of AD. BE is produced which meets AC in F.

To prove : $AF = \frac{1}{3}AC$.

Const : Draw DG || BF which meets AC at G.

Proof : In ΔADG, E is mid-point of AD and BEF || DG.

$\therefore$ F is mid-point of AG

or $\quad AF = FG \quad ...(i)$

Again in ΔBCF,

D is mid-point of BC and DG || BEF

$\therefore$ G is mid-point of FC

or $\quad FG = GC \quad ...(ii)$

From (*i*) and (*ii*)

$$AF = FG = GC$$

But $AF + FG + GC = AC$

$\Rightarrow AF + AF + AF = AC$

$\Rightarrow 3\,AF = AC$

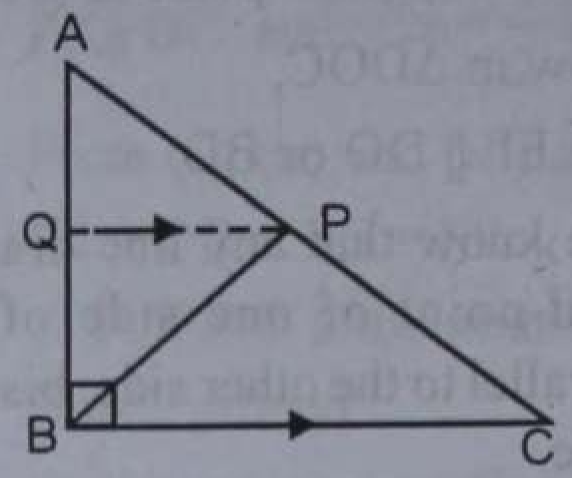
$\therefore \quad AF = \frac{1}{3}AC$

Hence proved.

Q. 10. In the adjoining figure, ΔABC is a right-angled at B and P is the mid-point of AC. Show that PA = PB = PC.

Sol. **Given :** In ΔABC, $\angle B = 90^\circ$

P is mid-point of AC. PB is joined.



To prove : PA = PB = PC.

Const : Through P, draw PQ || BC.

Proof : $\because$ P is mid-point of AC and PQ || BC.

$\therefore$ Q is mid-point of AB.

In ΔPAQ and ΔPBQ,

$PQ = PQ$ (common)

$AQ = BQ$ ($\because$ Q is mid-point of AB)

$\angle PQA = \angle PQB$ (each $= 90^\circ$)

$\therefore \Delta PAQ \cong \Delta PBQ$

(SAS axiom of congruency)

$\therefore \quad PA = PB$

But $\quad PA = PC$

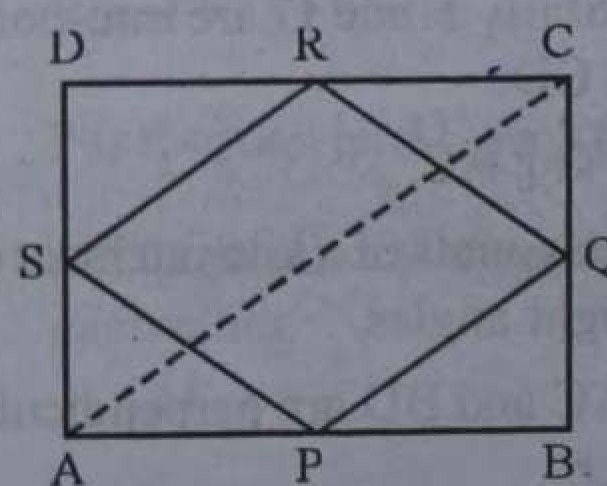
($\because$ P is mid-point of AC)

$\therefore \quad PA = PB = PC$

Hence proved.

Q. 11. Show that the quadrilateral formed by joining the mid-points of the pairs of adjacent sides of a rectangle is a rhombus.

Sol. **Given :** In rectangle ABCD, P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.



To prove : PQRS is a rhombus.

Const : Join AC.

Proof : In ΔABC,

P and Q are the mid-points of sides AB and BC respectively.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ ...(*i*)

Similarly S and R are the mid-points of sides AD and DC respectively.

$\therefore SR \parallel AC$ and $SR = \frac{1}{2}AC$ ...(*ii*)

From (*i*) and (*ii*)

$PQ \parallel SR$ and $PQ = SR$

$\therefore$ PQRS is a parallelogram.

Now in ΔASP and ΔBQP,

$AS = BQ$ (Half of equal sides)

$AP = BP$ (P is mid-point of AB)

$\angle SAP = \angle QBP$ (Each = 90°)

$\therefore \Delta ASP \cong \Delta BQP$

(SAS axiom of congruency)

$\therefore$ $PS = PQ$ (c.p.c.t.)

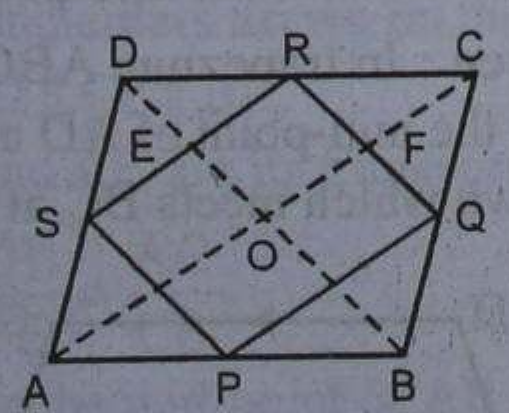
$\therefore$ $PQ = QR = RS = SP$

Hence PQRS is a rhombus.

Hence proved.

Q. 12. Show that the quadrilateral formed by joining the mid-points of the pairs of adjacent sides of a rhombus is a rectangle.

Sol. Given : In rhombus ABCD, P, Q, R and S are the mid-points of its sides AB, BC, CD and DA respectively and PQ, QR, RS and SP are joined.



To prove : PQRS is a rectangle.

Const : Join AC and BD which intersect each other at O.

Proof : $\because$ The diagonals of a rhombus bisect each other at right angles.

$\therefore AO = OC, BO = OD$ and AC and BD bisect each other at right angles.

Now in ΔABC,

P and Q are the mid-points of AB and BC respectively.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ ...(*i*)

Similarly, In ΔADC,

S and R are the mid-points of CD and DA respectively.

$\therefore RS \parallel AC$ and $RS = \frac{1}{2}AC$ ...(*ii*)

From (*i*) and (*ii*)

$PQ \parallel RS$ and $PQ = RS$

Similarly we can prove that

$QR = SP$ and $QR \parallel SP$

$\therefore$ PQRS is a parallelogram.

$\because$ $PQ \parallel AR$ and $QR \parallel BD$

and $AC \perp BD$

$\therefore$ PQRS is a rectangle.

Hence proved.

Q. 13. Show that the quadrilateral formed by joining the mid-points of the pairs of adjacent sides of a square is a square.

Sol. Given : In square ABCD, P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.

To prove : PQRS is a square.

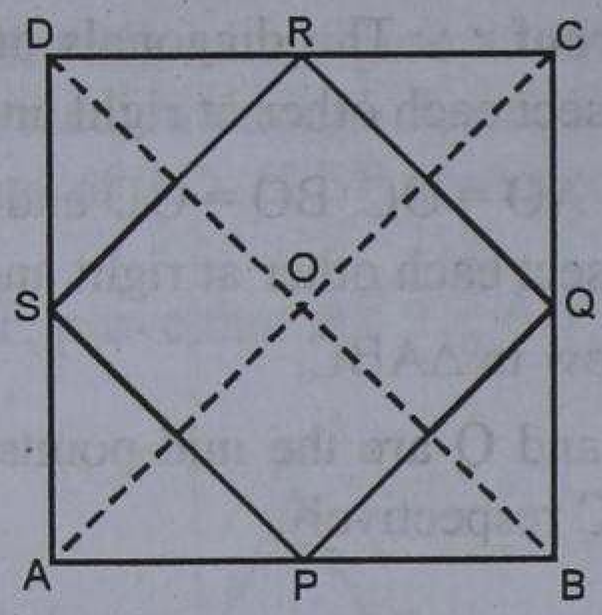
Construction : Join AC and BD.

Proof : In ΔABC,

P and Q are the mid-points of sides AB and BC respectively.

$\therefore$ $PQ \parallel AC$

and $PQ = \frac{1}{2}AC$ ...(*i*)



Similarly in ΔADC,

S and R are the mid-points of AD and CD respectively

$\therefore$ $SR \parallel AC$ and $SR = \frac{1}{2}AC$ ...(*ii*)

From (*i*) and (*ii*),

$PQ = SR$ and $PQ \parallel SR$

$\therefore$ PQRS is a parallelogram.

Now, in ΔAPS and ΔBPQ

$AP = PB$

($\because$ P, Q, S are the mid-points of the sides)

$AS = BQ$

and $\angle A = \angle B$ (Each = 90°)

$\therefore$ $\Delta APS \cong \Delta BPQ$ (SAS axioms)

$\therefore$ $PS = PQ$ (c.p.c.t.)

$\therefore$ $PQ = QR = RS = SP$

$\therefore$ PQRS is a rhombus or a square.

$\because$ $PQ \parallel AC : QR$ and $PS \parallel BD \parallel QR$

(Proved)

But, diagonals AC and BD of a square bisect each other at right angles.

$\therefore$ Each angle of PQRS is a right angle

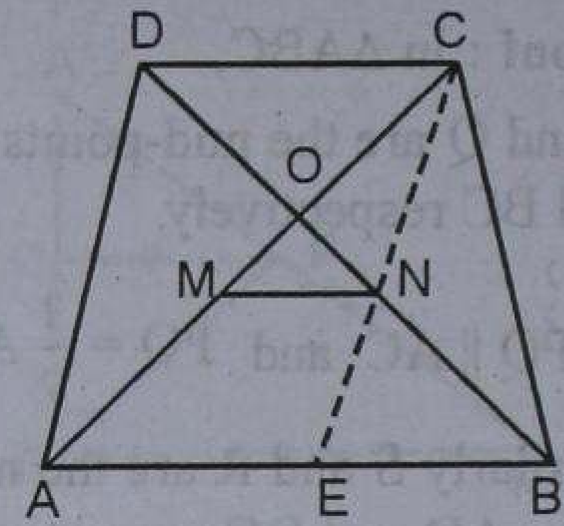
Hence, PQRS is square.

Hence proved.

Q. 14. In the adjoining figure, ABCD is a trapezium in which $AB \parallel DC$. If M and N are the mid-points of AC and BD respectively, prove that

$$MN = \frac{1}{2}(AB - CD).$$

Sol. **Given :** In trapezium ABCD, $AB \parallel DC$. M and N are the mid-points of AC and BD respectively and MN are joined.



To prove : $MN = \frac{1}{2}(AB - CD)$.

Const : Join CN and produce it to meet AB at E.

Proof : In ΔCDN and ΔEBN

$DN = NB$ (N is mid-point of BD)

$\angle CND = \angle BNE$

(Vertically opposite angles)

$\angle CDN = \angle NBE$ (Alternate angles)

$\therefore$ $\Delta CDN \cong \Delta EBN$

(ASA axiom of congruency)

$\therefore$ $CD = EB$ (C.P.C.T.)

and $CN = NE$ (C.P.C.T.)

In ΔACE,

M and N are mid-points of AC and CE

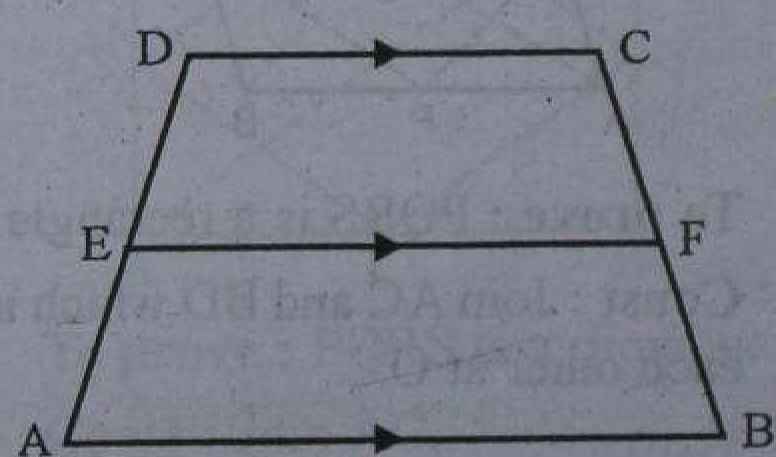
$\therefore$ $MN = \frac{1}{2}AE = \frac{1}{2}(AB - EB)$

$= \frac{1}{2}(AB - CD)$ [$\because EB = CD$]

Hence proved.

Q. 15. In the adjoining figure, ABCD is a trapezium in which $AB \parallel DC$ and E is the mid-point of AD. If $EF \parallel AB$ meets BC at F, show that F is the mid-point of BC.

Sol. **Given :** In trapezium ABCD, $AB \parallel DC$. E is the mid-point of AD and $EF \parallel AB$ is drawn which meets BC at F.



To prove : F is the mid-point of BC.

Proof : $\because$ AB || EF || DC, CB and AD are its transversals.

$\therefore \quad \frac{DE}{EA} = \frac{CF}{FB}$ (Intercept Theorem)

But $DE = EA \quad \Rightarrow \frac{DE}{EA} = 1$

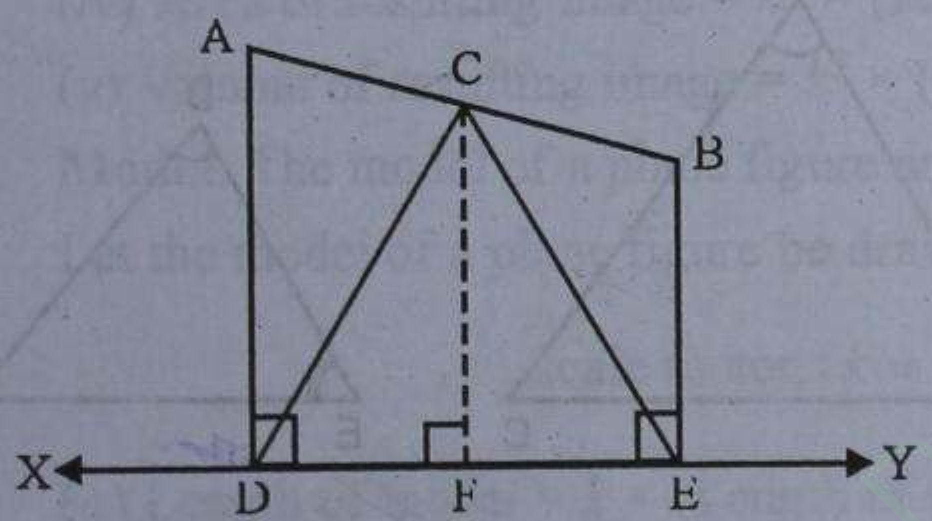
$\therefore \quad \frac{CF}{FB} = 1 \quad \Rightarrow \quad CF = FB$

Hence F is the mid-point of BC.

Hence proved.

Q. 16. Two points of A and B lie on the same side of a line XY. If $AD \perp XY$ and $BE \perp XY$, meet XY in D and E respectively and C is the mid-point of AB, show that CD = CE.

Sol. Given : Two points A and B which lie on the same side of a line XY. $AD \perp XY$ and $BE \perp XY$. C is mid-point of AB. CD and CE are joined.



To prove : CD = CE.

Const : Through C, draw $CF \perp XY$.

Proof : $\because$ AD, CF and BE are perpendiculars drawn on the line XY.

$\therefore$ AD || CF || BE,

DE and AB are its transversals.

$\therefore \quad \frac{DF}{FE} = \frac{AC}{CB}$

$\because$ C is the mid-point of AB.

$\therefore \quad AC = CB \Rightarrow \frac{AC}{CB} = 1$

$\Rightarrow \frac{DF}{FE} = \quad \Rightarrow \quad DF = FE$

Now in ΔCDF and ΔCEF,

$CF = CF$ (Common)

$\angle CFD = \angle CFE$ (Each 90°)

$DF = FE$ (Proved)

$\therefore \Delta CDF \cong \Delta CEF$

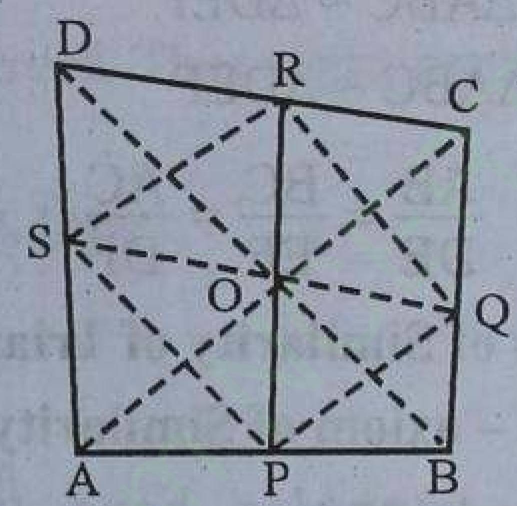
(SAS axiom of congruency)

$\therefore \quad CD = CE$ (C.P.C.T.)

Hence proved.

Q. 17. Prove that the straight lines joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Sol. Given : In quadrilateral ABCD, P, Q, R and S are the mid-points of its sides AB, BC, CD and DA respectively.



PR and QS are joined to intersect each other at O.

To prove : QO = OS and PO = OR.

Const : Join PQ, QR, RS and SP, AC and BD.

Proof : In ΔABC,

P and Q are the mid-points of AB and BC respectively.

$\therefore \quad PQ \parallel AC$ and $PQ = \frac{1}{2}AC \quad ...(i)$

Similarly in ΔADC,

S and R are the mid-points of AD and CD respectively.

$\therefore SR \parallel AC$ and $SR = \frac{1}{2}AC \quad ...(ii)$

From (*i*) and (*ii*)

PQ || SR and PQ = SR

$\therefore$ PQRS is a parallelogram.

But the diagonals of a parallelogram bisect each other.

$\therefore$ PR and QS bisect each other at O.

Hence PO = OR and QO = OS.

Hence proved.