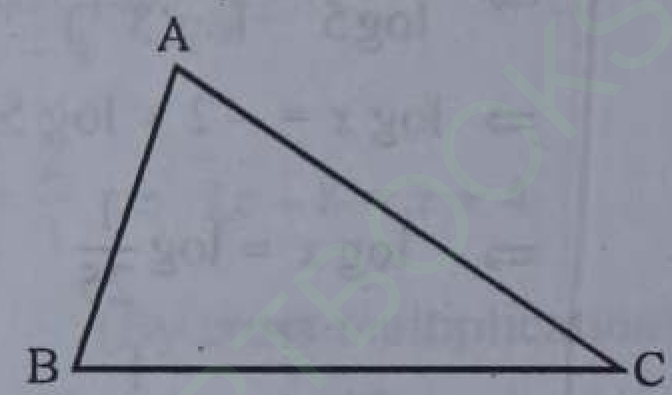
10

Triangles

POINTS TO REMEMBER :

1. **Triangle.** *A plane figure bounded by three line segments is called a triangle.* The line segments forming a triangle are called its **sides** and each point, where two sides intersect, is called its **vertex.**

We denote a triangle by the symbol Δ.



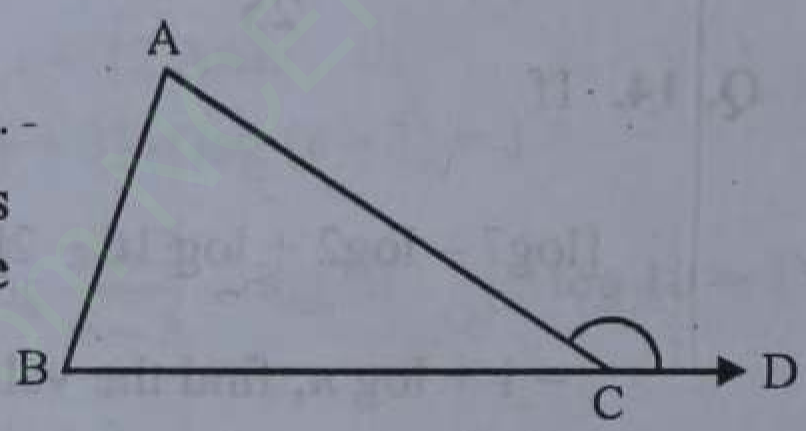
Thus, a ΔABC has :

(*i*) *three sides*, namely **AB, BC** and **CA** ;

(*ii*) *three vertices*, namely **A, B** and **C** ;

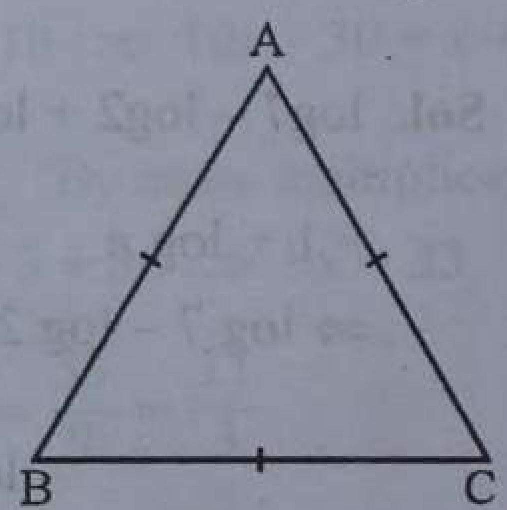
(*iii*) *three angles*, namely $\angle$**BAC**, $\angle$**ABC** and $\angle$**BCA**, to be denoted by $\angle$**A**, $\angle$**B** and $\angle$**C** respectively.

A triangle has six **elements**, namely three sides and three angles.



2. **Exterior Angle of a Triangle.** If a side BC of a ΔABC is produced to a point D, then $\angle ACD$ is called an **exterior angle** at C and $\angle B$ and $\angle A$ are called its **interior opposite angles.**

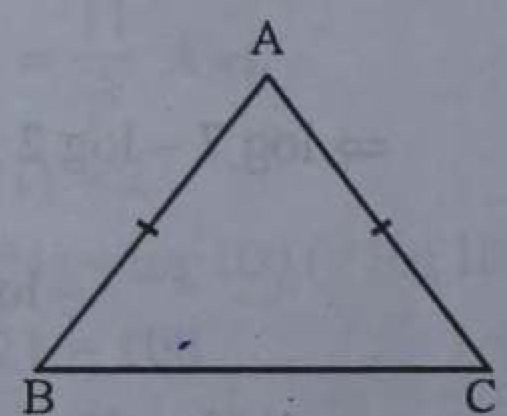
3. **Types of Triangles on the Basis of Sides**



(*i*) **Equilateral Triangle.** *A triangle having all sides equal, is called an equilateral triangle.*

In the given figure, in ΔABC, we have

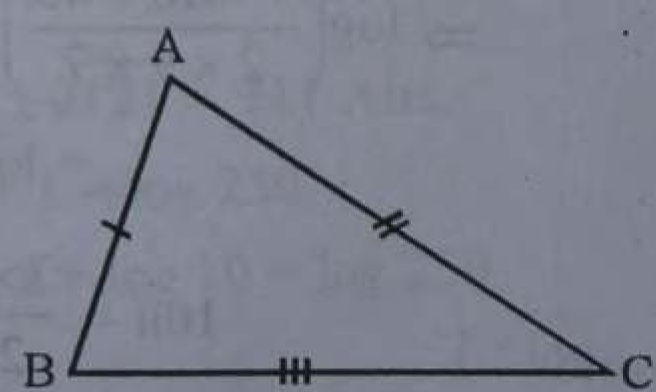
$$AB = BC = CA.$$



(*ii*) **Isosceles Triangle.** *A triangle having any two sides equal, is called an isosceles triangle.*

In the given figure, in ΔABC, we have

$$AB = AC.$$



(*iii*) **Scalene Triangle.** *A triangle in which all the sides are of different lengths is called a scalene triangle.*

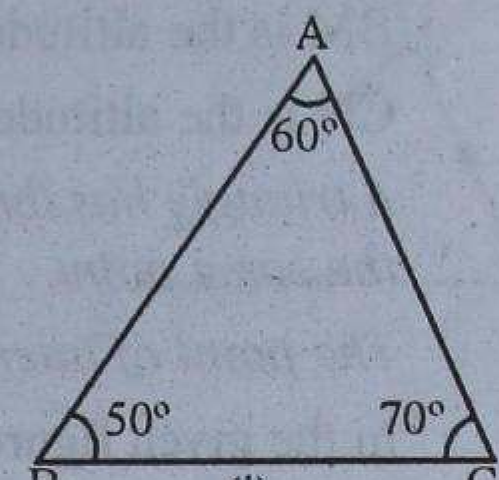
In the given figure, in ΔABC, we have

$$AB \neq AC \neq BC.$$

4. **Perimeter of a Triangle.** *The sum of the lengths of the sides of a triangle is called its perimeter.*

5. Types of Triangles on the Basis of Angles

(*i*) **Acute-Angled Triangle.** *A triangle in which every angle measures more than 0° but less than 90°, is called an acute-angled triangle.*



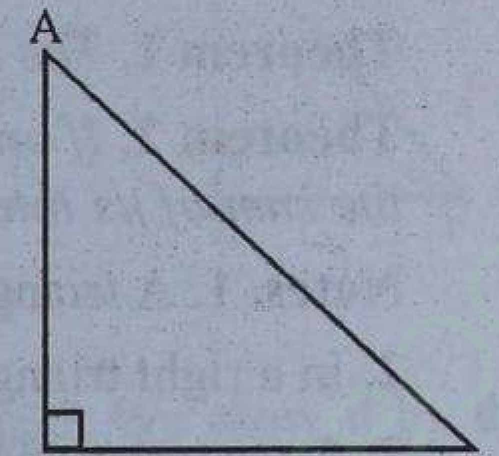
(i)

(*ii*) **Right-Angled Triangle.** *A triangle in which one of the angles measures 90°, is called a right-angled triangle or simply a right triangle.*

In a right triangle, the side opposite to the right angle is called its **hypotenuse** and the other two sides are called its **legs.**

In ΔABC, $\angle B = 90°$.

$\therefore$ It is a right angled triangle in which AC is the **hypotenuse** and AB, BC are its legs.



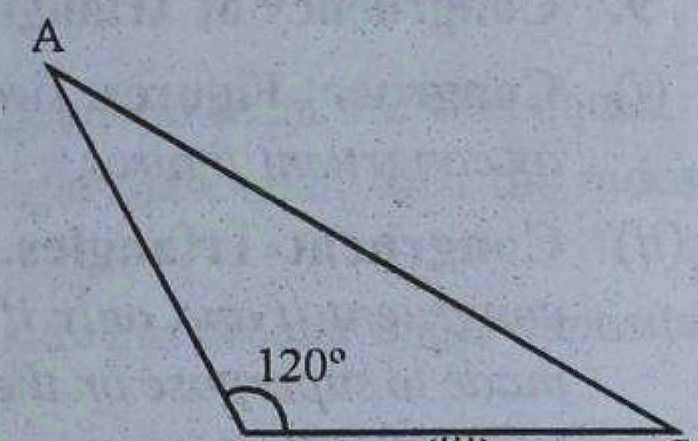
(ii)

(*iii*) **Obtuse-Angled Triangle.** *A triangle in which one of the angles measures more than 90° but less than 180°, is called an obtuse-angled triangle.*

Thus, in an obtuse-angled triangle, one of the angles is obtuse angle.

In ΔABC, we have $\angle ABC = 120°$, which is an obtuse-angle.

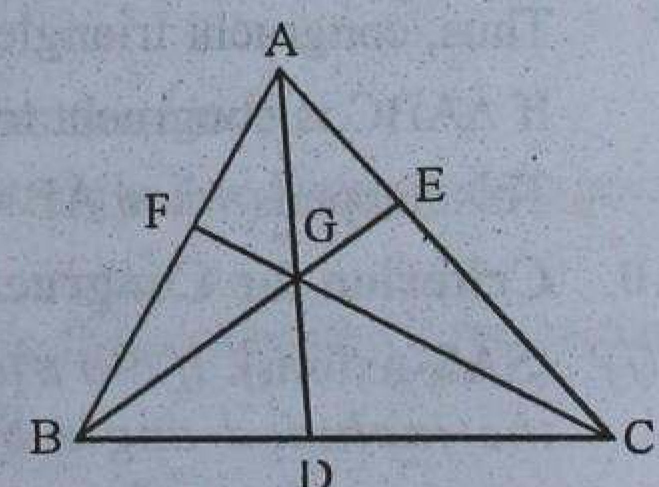
$\therefore$ ΔABC is an obtuse-angled triangle.



(iii)

6. Medians of a Triangle. *The median of a triangle corresponding to any side is the line segment joining the mid-point of that side with the opposite vertex.*

In the given figure D, E, F are the mid-points of the sides BC, CA and AB respectively of ΔABC.



Thus, AD is the median corresponding to side BC ;

BE is the median corresponding to side CA ;

CF is the median corresponding to side AB.

A triangle has three medians and the medians of a triangle are concurrent, i.e., they intersect at the same point.

The point of intersection of the medians of a triangle is called its **centroid.**

In the given figure, G is the centroid of ΔABC.

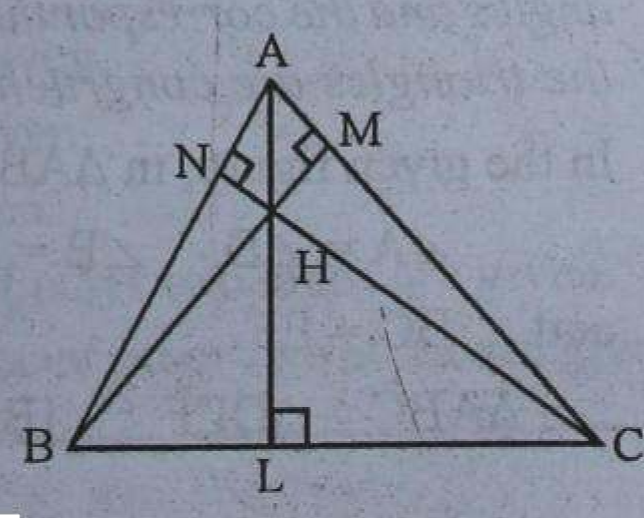
G divides AD in the ratio 2 : 1, *i.e.*, AG : GD = 2 : 1.

Similarly, BG : GE = 2 : 1 and CG : GF = 2 : 1.

7. Altitudes of a Triangle. *The altitude of a triangle corresponding to any side is the length of perpendicular from the opposite vertex to that side.*

In the given figure, in ΔABC, we have $AL \perp BC$, $BM \perp CA$ and $CN \perp AB$.

$\therefore$ AL is the altitude corresponding to side BC ;



BM is the altitude corresponding to side CA ;

CN is the altitude corresponding to side AB.

A triangle has three altitudes and the altitudes of a triangle are concurrent, i.e., they intersect at the same point.

The point of intersection of the altitudes of a triangle is called its **orthocentre.**

In the given figure, H is the orthocentre of ΔABC.

8. Some Theorems and Their Applications

Theorem 1. *The sum of the angles of a triangle is equal to two right angles.*

Theorem 2. *If one side of a triangle is produced, then the exterior angle so formed is equal to the sum of its interior opposite angles.*

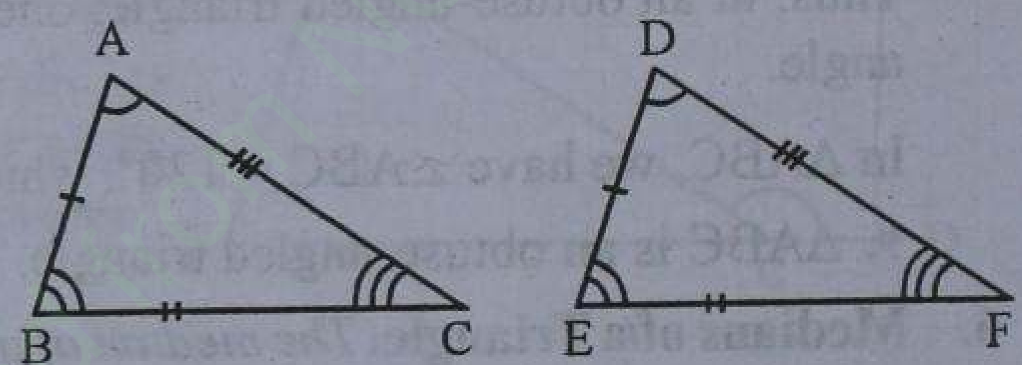
Notes. 1. A triangle cannot have more than one right angle or obtuse angle.

2. In a right triangle, sum of two acute angle is 90°.

3. An exterior angle of a triangle is greater than its interior opposite angle.

9. Congruency of Triangles

(*i*) **Congruent Figures.** *Two geometrical figures, having exactly the same shape and size are known as congruent figures.*



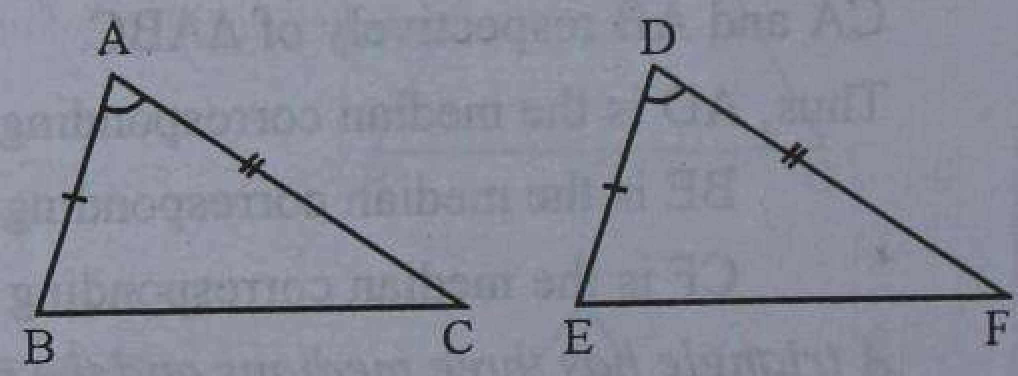
(*ii*) **Congruent Triangles.** *Two triangles are congruent if and only if one of them can be made to superpose on the other, so as to cover it exactly.*

Thus, congruent triangles are exactly identical.

If ΔABC is congruent to ΔDEF, we write **$\Delta ABC \cong \Delta DEF$.**

This happens when AB = DE, BC = EF, AC = DF and $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$.

10. Criterion for Congruence

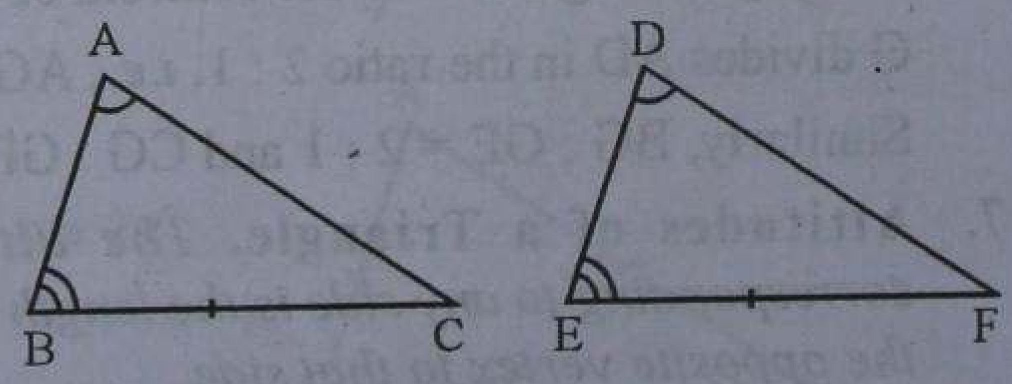


(*i*) **(SAS-axiom).** *If two triangles have two sides and the included angle of the one equal to the corresponding sides and the included angle of the other, then the triangles are congruent.*

In the given figure, in ΔABC and ΔDEF, we have :

$$AB = DE,\ AC = DF \text{ and } \angle A = \angle D.$$

$\therefore\ \Delta ABC \cong \Delta DEF$ [By SAS-axiom]



(*ii*) **(AAS-axiom).** *If two triangles have two angles and a side of the one equal to the corresponding two angles and the corresponding side of the other, then the triangles are congruent.*

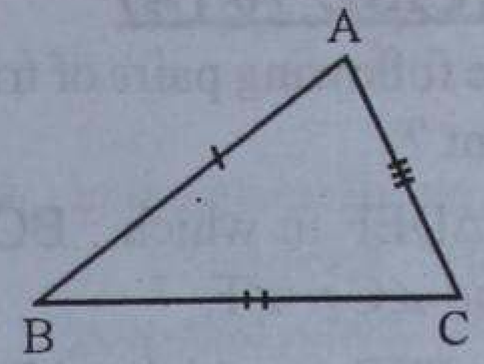
In the given figure, in ΔABC and ΔDEF, we have :

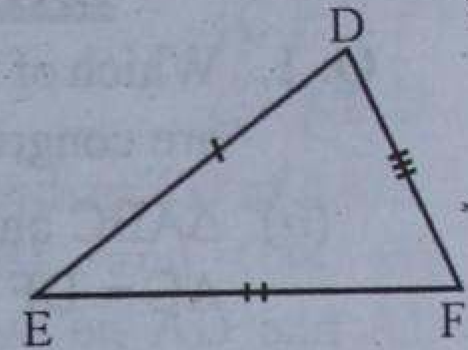
$$\angle A = \angle D,\quad \angle B = \angle E$$

and $\quad BC = EF$

$\therefore\ \Delta ABC \cong \Delta DEF$ [By AAS-axiom]

iii) **(SSS-axiom).** *If two triangles have three sides of the one equal to the corresponding three sides of the other, then the triangles are congruent.*



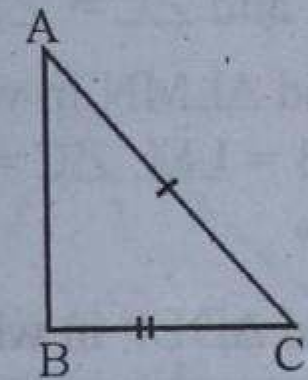


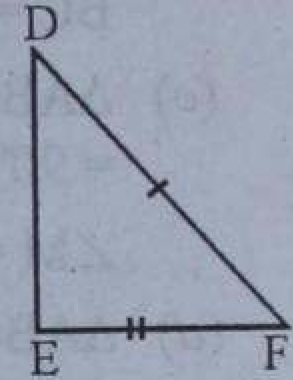
In the given figure, in ΔABC and ΔDEF, we have :

$AB = DE$, $BC = EF$ and $AC = DF$.

$\therefore$ $\Delta ABC \cong DEF$ [By SSS-axiom]

(iv) **(RHS-axiom).** *In two right-angled triangles if the hypotenuse and one side of the one are equal to the hypotenuse and the corresponding side of the other, then the triangles are congruent.*





In the given figure, ΔABC and ΔDEF are right-angled triangles in which Hyp. AC = Hyp. DF and $BC = EF$.

$\therefore$ $ABC \cong \Delta DEF$ [By RHS-axiom]

Note. The corresponding parts of two congruent triangles are always equal. We show it by the abbreviation 'c.p.c.t.' which means 'corresponding parts of congruent triangles.'

11. Isosceles Triangles.

Theorem 1. If two sides of a triangle are equal, then the angles opposite to them are also equal.

Theorem 2. (Converse of Theorem 1). If two angles of a triangle are equal, then the sides opposite to them are also equal.

Theorem 3. If two sides of a triangle are unequal, then the greater side has greater angle opposite to it.

Theorem 4. (Converse of Theorem 3). If two angles of a triangle are unequal, then the greater angle has greater side opposite to it.

Theorem 5. The sum of any two sides of a triangle is greater than its third side.

Theorem 6. Of all the line segments that can be drawn to a given straight line from a given point outside it, the perpendicular is the shortest.

12. Construction of Triangles. We know that a triangle has six elements, three sides and three angles. Therefore to construct a triangle, atleast three elements are required.

(i) Three sides.

(ii) Two sides and included angle.

(iii) Two angles and included sides. Beside these, we can also construct a triangle with given data as given below.

(iv) Two sides and an altitude to the third side.

(v) To construct an isosceles triangle whose base and height are given.

(vi) To construct an isosceles triangle whose one altitude and vertical angle are given.

(vii) To construct an equilateral triangle whose height is given.

(viii) To construct a right triangle whose one side and hypotenuse is given.

(ix) To construct a triangle whose perimeter and ratio of sides are given.

(x) To construct a triangle whose perimeter and base angles are given.

(xi) To construct a triangle in which base, one base angle and sum of other two sides are given.

(xii) To construct a triangle whose base, one base angle and difference of other two sides are given.

EXERCISE 10 (A)

Q. 1. Which of the following pairs of triangles are congruent ?

(*a*) ΔABC and ΔDEF in which : $BC = EF$, $AC = DF$ and $\angle C = \angle F$.

(*b*) ΔABC and ΔPQR in which : $AB = PQ$, $BC = QR$ and $\angle C = \angle R$.

(*c*) ΔABC and ΔLMN in which : $\angle A = \angle L = 90^o$, $AB = LM$, $\angle C = 40^o$ and $\angle M = 50^o$.

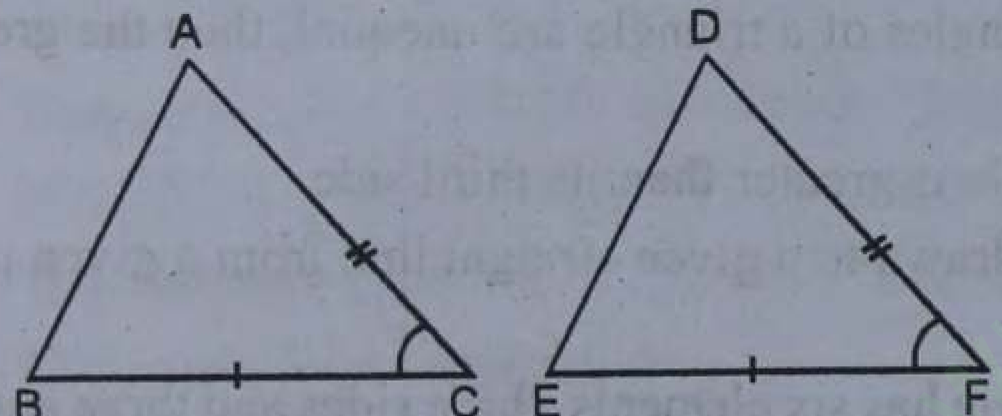
(*d*) ΔABC and ΔDEF in which : $\angle B = \angle E = 90^o$ and $AC = DF$.

Sol. (*a*) In ΔABC and ΔDEF,

$$BC = EF$$
$$AC = DF$$
$$\angle C = \angle F$$

$\therefore \Delta ABC \cong \Delta DEF$

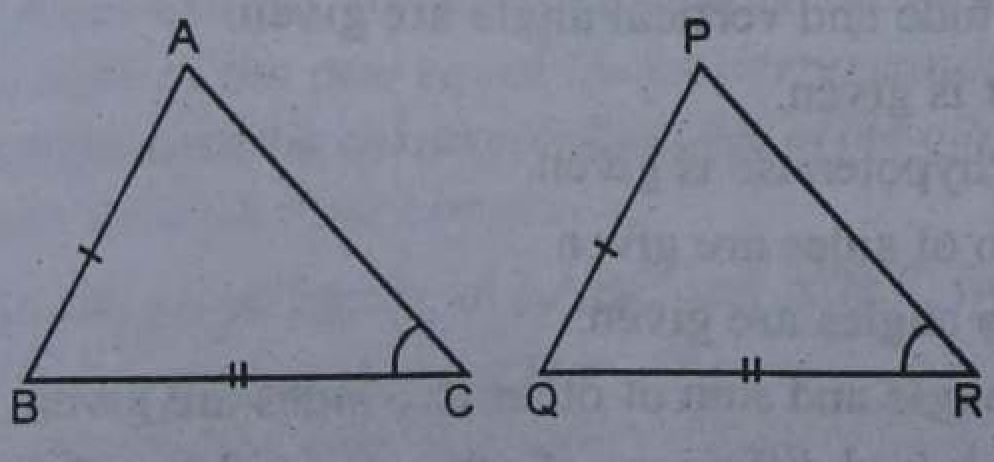
(SAS axiom of congruency)



(*b*) In ΔABC and ΔPQR

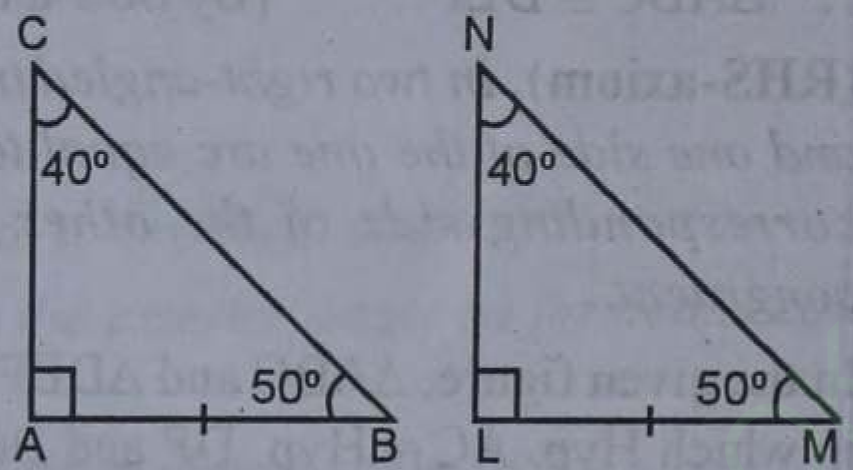
$$AB = PQ$$
$$BC = QR$$
$$\angle C = \angle R$$

$\therefore \Delta ABC$ and ΔPQR are not congruent.



(*c*) In ΔABC and ΔLMN,

$$\angle A = \angle L = 90^o$$
$$AB = LM$$
$$\angle C = 40^o$$
$$\text{and } \angle M = 50^o$$



In ΔABC,

$$\angle A = 90^o \text{ and } \angle C = 40^o$$
$$\therefore \angle B = 180^o - (\angle A + \angle C)$$
$$= 180^o - (90^o + 40^o)$$
$$= 180^o - 130^o$$
$$= 50^o$$

$\therefore \angle B = \angle M$ (each $= 50^o$)

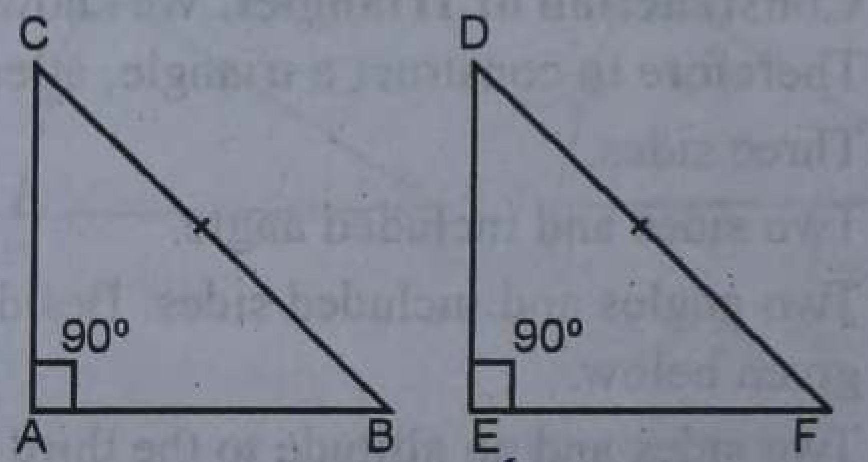
Hence $\Delta ABC \cong \Delta LMN$

(AAS axiom of congruency)

(*d*) In ΔABC and ΔDEF

$\angle B = \angle E$ (each 90^o)

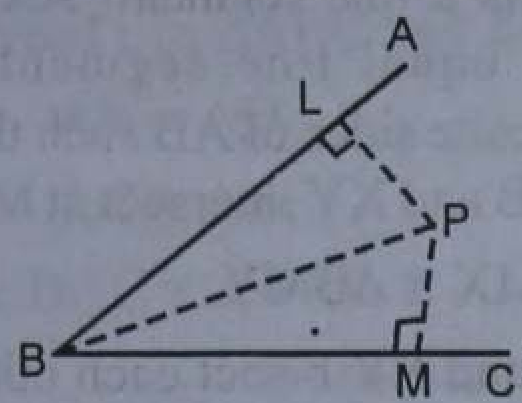
and $AC = DF$



But this is not sufficient to prove the congruency of triangle.

Hence ΔABC and ΔDEF are not congruent.

Q. 2. In the given figure, P is a point in the interior of $\angle ABC$. If $PL \perp BA$ and $PM \perp BC$ such that $PL = PM$, prove that BP is the bisector of $\angle ABC$.



Sol. Given : P is a point in the interior of $\angle ABC$

$PL \perp BA$ and $PM \perp BC$ and $PL = PM$.

To prove : BP is the bisector of $\angle ABC$

i.e. $\angle LBP = \angle MBP$

Proof : In right Δs BLP and BMP,

Hypotenuse $BP = BP$ (Common)

Side $PL = PM$ (Given)

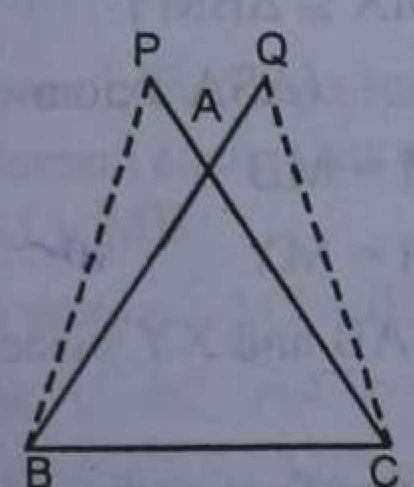
$\therefore \Delta BLP \cong \Delta BMP$

(R.H.S. axiom of congruency)

$\therefore \angle LBP = \angle MBP$ (C.P.C.T.)

or BP is the bisector of $\angle ABC$. **Q.E.D.**

Q. 3. In the given figure, equal sides BA and CA of ΔABC are produced to Q and P respectively such that $AP = AQ$. Prove that $PB = QC$.



Sol. Given : In ΔABC, $BA = CA$ and BA and CA are produced to P and Q respectively such that $AP = AQ$. PB and QC are produced.

To prove : $PB = QC$.

Proof : In ΔAPB and ΔAQC,

$AP = AQ$ (Given)

$AB = AC$ (Given)

$\angle PAB = \angle QAC$

(Vertically opposite angles)

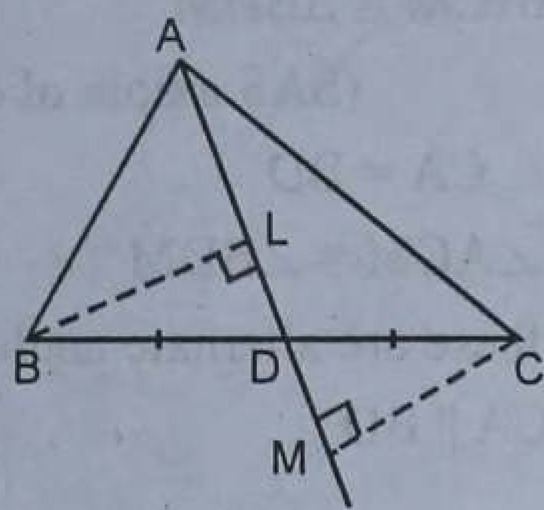
$\therefore \Delta APB \cong \Delta AQC$

(SAS axiom of congruency)

$\therefore$ $PB = QC$ (C.P.C.T.)

Q.E.D.

Q. 4. In the given figure, median AD of ΔABC is produced. If BL and CM are perpendiculars drawn on AD and AD produced, prove that $BL = CM$.



Sol. Given : In ΔABC, AD is the median of BC and it is produced to M.

BL and CM are perpendiculars on AD produced.

To prove : $BL = CM$.

Proof : In Δs BLD and ΔCMD,

$BD = CD$ ($\because$ D is mid-point of BC)

$\angle L = \angle M$ (each 90°)

$\angle BDL = \angle CDM$

(Vertically opposite angles)

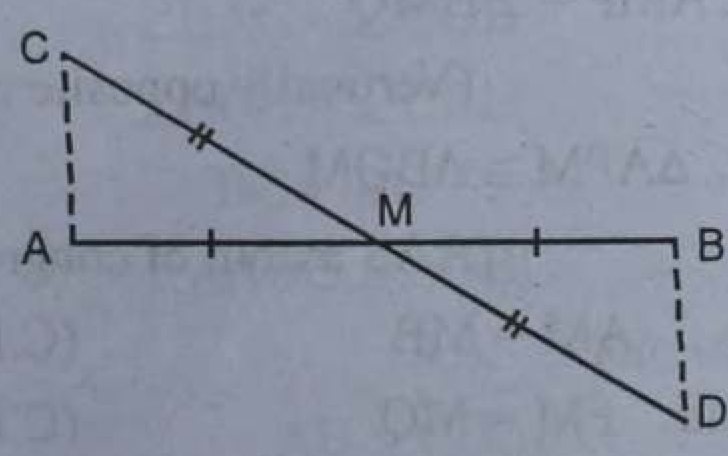
$\therefore \Delta BLD \cong \Delta CMD$

(AAS axiom of congruency)

$\therefore$ $BL = CM$ (C.P.C.T.)

Q.E.D.

Q. 5. In the given figure, M is the mid-point of AB and CD. Prove that $CA = BD$ and $CA \parallel BD$.



Sol. Given : M is mid-point of AB and CD. CA and BD are joined.

To prove : CA = BD and CA || BD.

Proof : In ΔACM and ΔBDM,

CM = DM and AM = BM

{$\because$ M is mid-point of AB and CD}

$\angle AMC = \angle BMD$

(Vertically opposite angles)

$\therefore \Delta ACM \cong \Delta BDM$

(SAS axiom of congruency)

$\therefore$ CA = BD (C.P.C.T.)

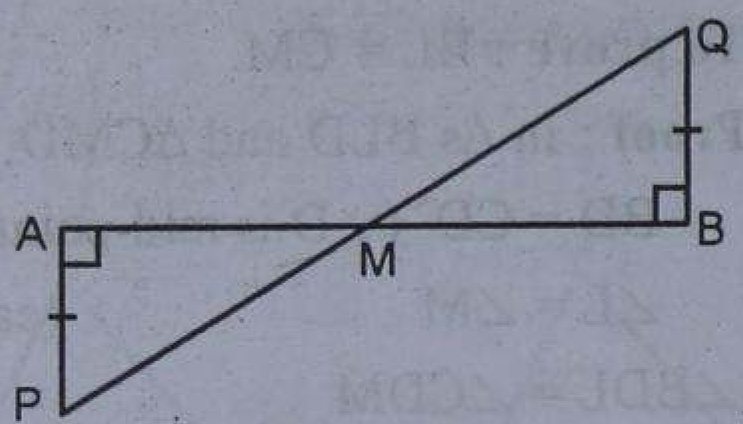
and $\angle ACM = \angle BDM$ (C.P.C.T.)

But these are alternate angles.

$\therefore$ CA || BD

Q.E.D.

Q. 6. In the given figure, PA $\perp$ AB ; QB $\perp$ AB and PA = QB. If PQ intersects AB at M, show that M is the mid-point of both AB and PQ.



Sol. Given : In the figure, PA $\perp$ AB, QB $\perp$ AB and PA = QB. PQ intersects AB at M.

To prove : M is the mid-point of AB and PQ.

Proof : In ΔAPM and ΔBQM,

$\angle A = \angle B$ (each 90^o)

PA = QB (Given)

$\angle AMP = \angle BMQ$

(Vertically opposite angles)

$\therefore \Delta APM \cong \Delta BQM$

(AAS axiom of congruency)

$\therefore$ AM = MB (C.P.C.T.)

and PM = MQ (C.P.C.T.)

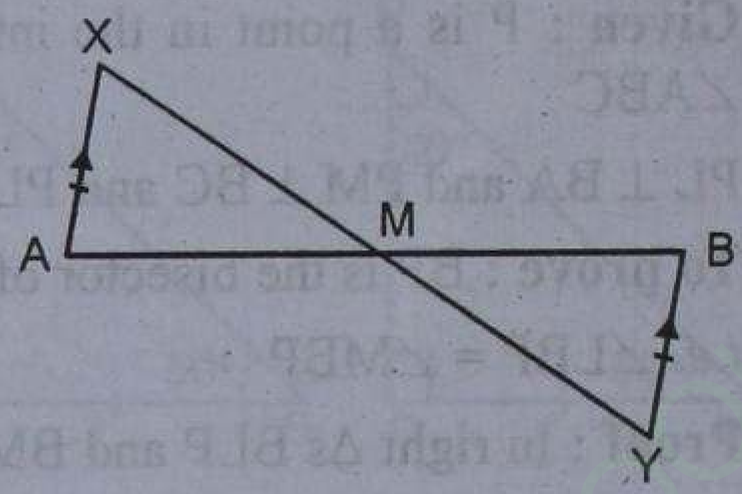
Hence M is the mid-point of AB and PQ.

Q.E.D.

Q. 7. AB is a line segment. AX and BY are two equal line segments drawn on opposite sides of AB such that AX || YB. If AB and XY intersect at M, prove that :

(*i*) $\Delta AMX \cong \Delta BMY$

(*ii*) AB and XY bisect each other at M.



Sol, Given : AB is a line segment. AX || BY and XY meets AB at M and AX = BY.

To prove : (*i*) $\Delta AMX \cong \Delta BMY$.

(*ii*) AB and XY bisect each other at M.

Proof : In ΔAMX and ΔBMY,

AX = BY (Given)

$\angle A = \angle B$ (Alternate angles)

$\angle X = \angle Y$ (Alternate angles)

(*i*) $\therefore \Delta AMX \cong \Delta BMY$

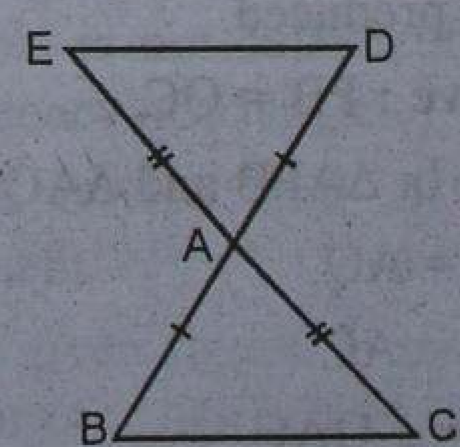
(ASA axiom of congruency)

(*ii*) $\therefore$ AM = MB (C.P.C.T.)

and XM = MY (C.P.C.T.)

Hence AB and XY bisect each other at M. **Q.E.D.**

Q. 8. In the given figure, the sides BA and CA of ΔABC have been produced to D and E such that BA = AD and CA = AE. Prove that, ED || BC.



Sol. Given : Sides BA and CA of ΔABC are produced to D and E such that BA = AD and CA = AE.

To prove : ED ∥ BC.

Proof : In ΔABC and ΔEAD,

$AB = AD$ (Given)

$AC = AE$ (Given)

$\angle BAC = \angle DAE$

(Vertically opposite angles)

$\therefore \Delta ABC \cong \Delta EAD$

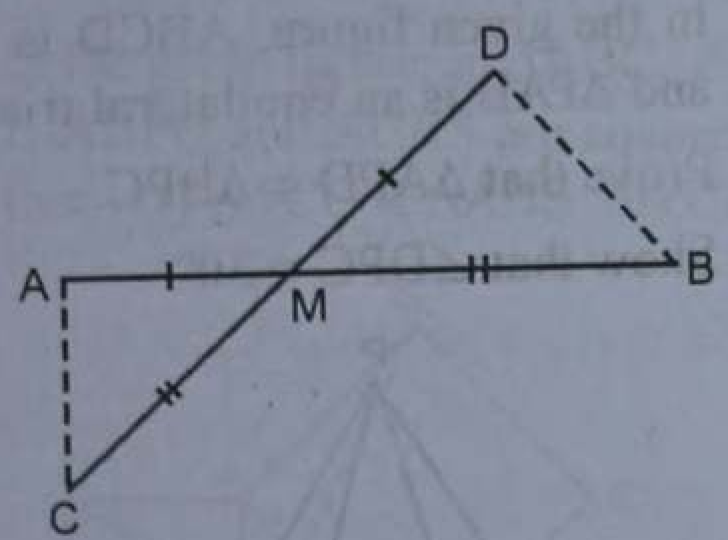
(SAS axiom of congruency)

$\therefore \angle ABC = \angle ADE$ (C.P.C.T.)

But these are alternate angles.

$\therefore ED \parallel BC$. **Q.E.D.**

Q. 9. In the given figure, the line segments AB and CD intersect at a point M in such a way that AM = MD and CM = MB. Prove that, AC = BD but AC may not be parallel to BD.



Sol. Given : Two line segments AB and CD intersect each other at M and AM = MD, CM = MB.

To prove : AC = BD

But AC may not be parallel to BD.

Proof : In ΔAMC and ΔBMD,

$AM = MB$ (Given)

$CM = MD$ (Given)

$\angle AMC = \angle BMD$

(Vertically opposite angles)

$\therefore \Delta AMC \cong \Delta BMD$

(SAS axiom of congruency)

$\therefore \quad AC = BD$ (C.P.C.T.)

But $\angle ACM \neq \angle BDM$

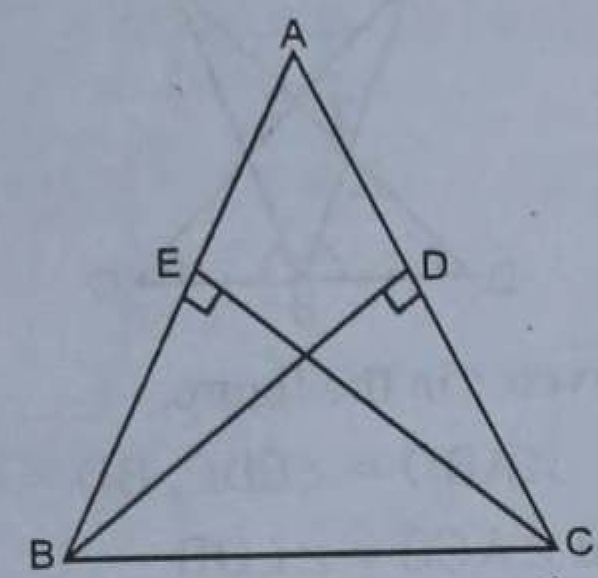
$\therefore$ AC may not be parallel to BD.

Q.E.D.

Q. 10. If two altitudes of a triangle are equal, prove that it is an isosceles triangle.

Sol. Given : In ΔABC,

$BD \perp AC$ and $CE \perp AB$ and $BD = CE$



To prove : ΔABC is an isosceles triangle or $AB = AC$

Proof : In right ΔCBD and BEC

Hyp. $BD = CE$ (given)

Side $BC = BC$ (common)

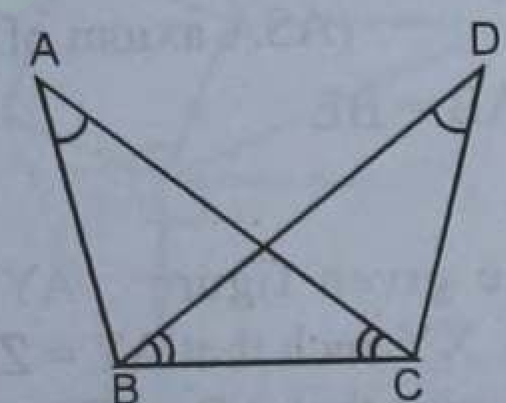
$\therefore \Delta CBD \cong \Delta BEC$ (R.H.S. Axiom)

$\therefore \quad \angle B = \angle C$ (c.p.c.t.)

$\Rightarrow \quad AB = AC$

Hence Δ ABC is an isosceles triangle.

Q. 11. In the given figure : $\angle BAC = \angle CDB$ and $\angle BCA = \angle CBD$. Prove that AB = CD.



Sol. Given : In the figure,

$\angle BAC = \angle CDB$ and $\angle BCA = \angle CBD$

To prove : AB = CD.

Proof : In ΔABC and ΔDBC,

$\angle BAC = \angle CDB$ (Given)

$\angle BCA = \angle CBD$ (Given)

$BC = BC$ (Common)

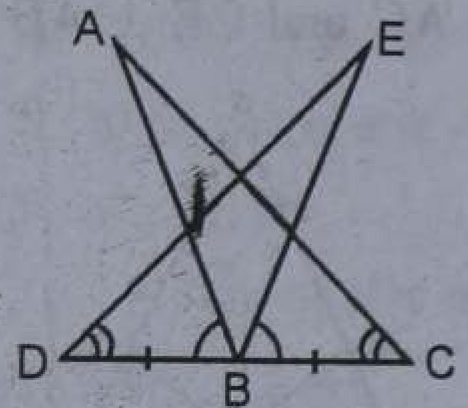
$\therefore \Delta ABC \cong \Delta DBC$

(AAS axiom of congruency)

$\therefore \quad AB = CD$ (C.P.C.T.)

Q.E.D.

Q. 12. In the given figure : $\angle ABD = \angle EBC$, $BD = BC$ and $\angle ACB = \angle EDB$. Prove that $AB = BE$.



Sol. Given : In the figure,

$\angle ABD = \angle EBC$, $BD = BC$

and $\angle ACB = \angle EDB$

To prove : $AB = BE$

Proof : $\angle ABD = \angle EBC$ (Given)

Adding $\angle ABE$ both sides

$\angle ABD + \angle ABE = \angle ABE + \angle EBC$

$\Rightarrow \quad \angle DBE = \angle CBA$

Now in ΔABC and ΔEBD,

$BC = BD$ (Given)

$\angle ABC = \angle EBD$ (Proved)

$\angle ACB = \angle EDB$ (Given)

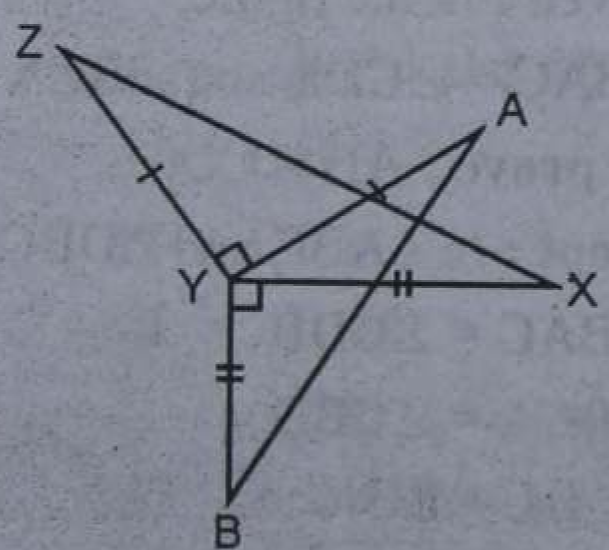
$\therefore ABC \cong \Delta EBD$

(ASA axiom of congruency)

$\therefore \quad AB = BE$ (C.P.C.T.)

Q.E.D.

Q. 13. In the given figure : $AY \perp ZY$ and $BY \perp XY$ such that $AY = ZY$ and $BY = XY$. Prove that $AB = ZX$.



Sol. Given : In the figure,

$AY \perp ZY$ and $BY \perp XY$ and

also $AY = ZY$

and $BY = XY$

To prove : $AB = ZX$

Proof : $\because BY \perp XY$

and $AY \perp ZY$

$\therefore \quad \angle XYB = 90^\circ$ and $\angle AYZ = 90^\circ$

Adding $\angle AYX$ to both sides,

$\angle XYB + \angle AYX = \angle AYZ + \angle AYX$

$\Rightarrow \quad \angle AYB = \angle XYZ$

Now in ΔAYB and XYZ,

$\angle AYB = \angle XYZ$ (Proved)

$AY = ZY$ (Given)

$BY = XY$ (Given)

$\therefore \quad \Delta AYB \cong \Delta XYZ$

(SAS axiom of congruency)

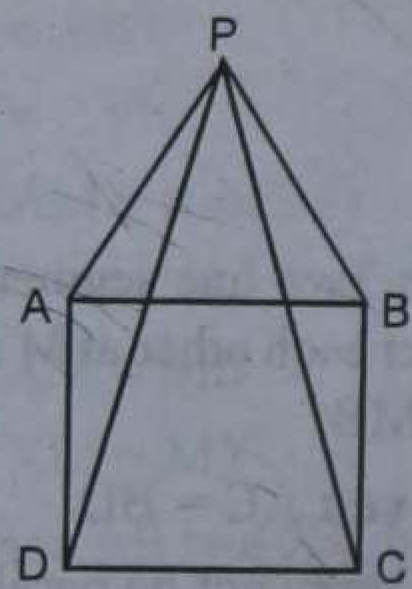
$\therefore \quad AB = XZ$ (C.P.C.T.)

Q.E.D.

Q. 14. In the given figure, ABCD is a square and ΔPAB is an equilateral triangle.

(*i*) Prove that $\Delta APD \cong \Delta BPC$.

(*ii*) Show that $\angle DPC = 30^\circ$.



Sol. Given : ABCD is a square and ΔPAB is an equilateral triangle.

To prove : (*i*) $\Delta APD \cong \Delta BPC$

(*ii*) Show that $\angle DPC = 30^\circ$

Proof : $\because \Delta PAB$ is an equilateral triangle

$\therefore$ Its each angle $= 60^\circ$

$\because$ ABCD is a square

$\therefore$ Its each angle $= 90^\circ$

$\therefore \quad \angle PAD = 90^\circ + 60^\circ = 150^\circ$

and $\angle PBC = 90^\circ + 60^\circ = 150^\circ$

(*i*) Now in ΔPAD and ΔPBC

$\angle PAD = \angle PBC$ (Proved)

PA = PB

(Sides of equilateral triangle)

AD = BC (Sides of a square)

$\therefore \Delta PAD \cong \Delta PBC$ (SAS axiom of congruency)

(*ii*) $\because$ PA = AD = AB

$\therefore \angle APD = \angle ADP$

But $\angle APD + \angle ADP + \angle PAD = 180^\circ$

(Angles of a triangle)

$\Rightarrow \angle APD + \angle APD + 150^\circ = 180^\circ$

$\Rightarrow 2\angle APD = 180^\circ - 150^\circ = 30^\circ$

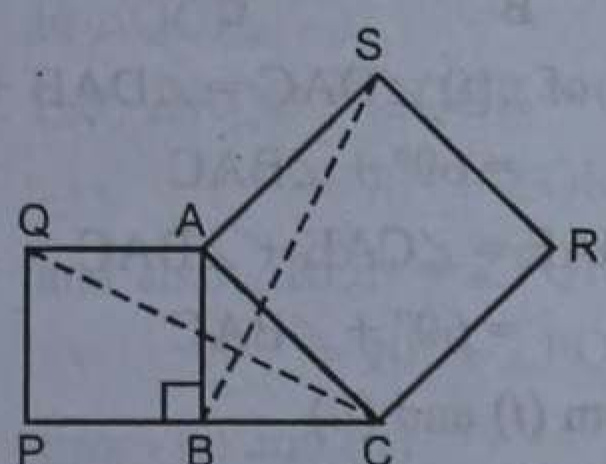
$\Rightarrow \angle APD = \frac{30^\circ}{2} = 15^\circ$

But $\angle APB = 60^\circ$

(Angle of an equilateral triangle)

$\therefore \angle DPC = 60^\circ - 30^\circ = 30^\circ$ **Q.E.D.**

Q. 15. In the given figure, in ΔABC, $\angle B = 90^\circ$. If ABPQ and ACRS are squares, prove that : (*i*) $\Delta ACQ \cong \Delta ABS$ (*ii*) CQ = BS.



Sol. Given : In the figure, in ΔABC, $\angle B = 90^\circ$

ABPQ and ACRS are squares.

To prove : (*i*) $\Delta ACQ \cong \Delta ABS$

(*ii*) CQ = BS.

Proof : $\angle CAQ = 90^\circ + \angle BAC$

and $\angle BAS = 90^\circ + \angle BAC$

$\therefore \angle CAQ = \angle BAS$

(*i*) Now in ΔACQ and ΔABS,

AQ = AB (Sides of a square)

AC = AS (Sides of a square)

$\angle CAQ = \angle BAS$ (Proved)

$\therefore \Delta ACQ \cong \Delta ABS$

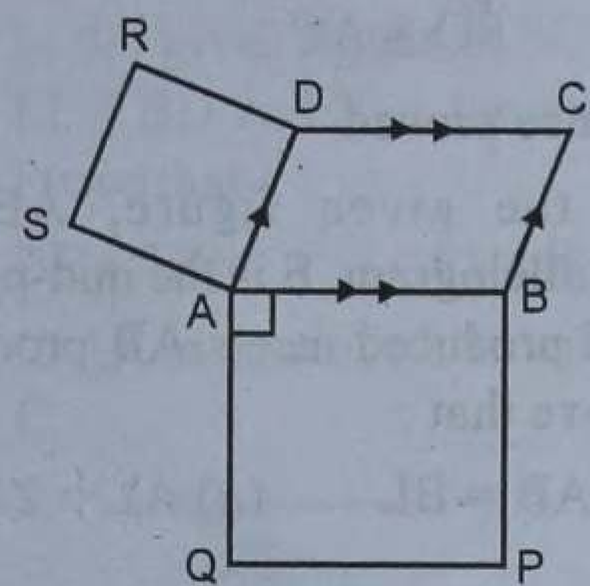
(SAS axiom of congruency)

$\therefore$ CQ = BS (C.P.C.T.)

Hence proved.

Q. 16. Squares ABPQ and ADRS are drawn on the sides AB and AD of a parallelogram ABCD. Prove that :

(*i*) $\angle SAQ = \angle ABC$ (*ii*) SQ = AC.

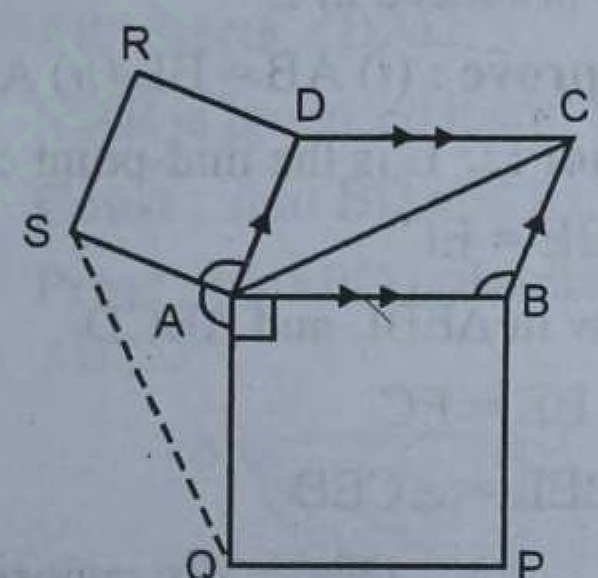


Sol. Given : ABPQ and ADRS are squares which are drawn on the sides of AB and AD of parallelogram ABCD.

To prove :

(*i*) $\angle SAQ = \angle ABC$ (*ii*) SQ = AC

Const : Join SQ and AC.



Proof : (*i*) Reflex $\angle SAQ$

$= \angle SAD + \angle DAB + \angle BAQ$

$= 90^\circ + \angle DAB + 90^\circ$

$= 180^\circ + \angle DAB$

$\therefore \angle SAQ = 360^\circ - (180^\circ + \angle DAB)$

$= 180^\circ - \angle DAB$ ...(*i*)

$\because AB \parallel CD$ (Sides of a parallelogram)

$\therefore \angle ABC + \angle DAB = 180^\circ$

$\angle ABC = 180^\circ - \angle DAB$ ...(*ii*)

From (*i*) and (*ii*)

$\angle SAQ = \angle ABC$

(*ii*) Now in ΔSAQ and ΔABC,

$AS = AD = BC$ (Sides of square)

$AQ = AB$ (Sides of a square)

$\angle SAQ = \angle ABC$ (Proved)

$\therefore \Delta SAQ \cong \Delta ABC$

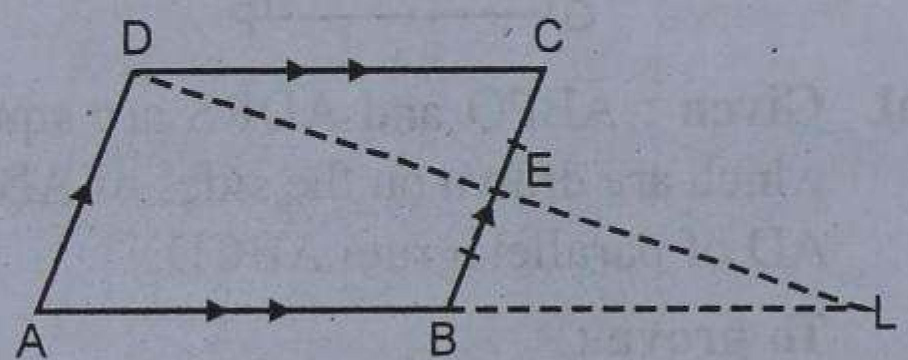
(SAS axiom of congruency)

$\therefore$ $SQ = AC$ (C.P.C.T.)

Hence proved.

Q. 17. In the given figure, ABCD is a parallelogram, E is the mid-point of BC. DE produced meets AB produced at L. Prove that :

(*i*) AB = BL (*ii*) AL = 2 DC.



Sol. Given : In parallelogram ABCD, E is mid-point of BC. DE produced to meet AB produced at L.

To prove : (*i*) AB = BL (*ii*) AL = 2 DC.

Proof : $\because$ E is the mid-point of BC

$\therefore BE = EC$

(*i*) Now in ΔEBL and ΔBCD,

$BE = EC$ (Proved)

$\angle BEL = \angle CED$

(Vertically opposite angles)

$\angle EBL = \angle ECD$ (Alternate angles)

$\therefore \Delta EBL \cong \Delta BCD$

(ASA axiom of congruency)

$\therefore$ $BL = CD$ (C.P.C.T.)

But $AB = CD$

(Opposite sides of parallelogram)

$\therefore$ $AB = BL$ (C.P.C.T.)

(*ii*) Now $AL = AB + BL$

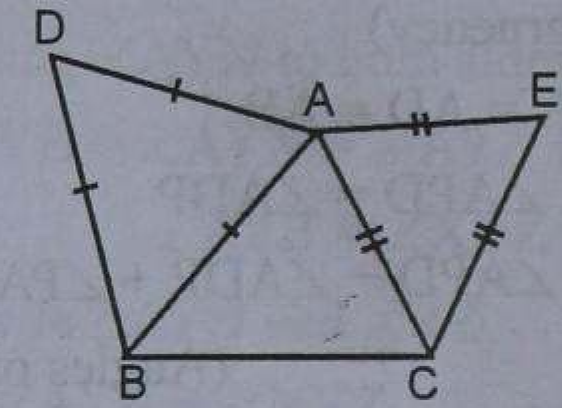
$= AB + AB = 2\ AB$

$= 2\ CD$ ($\because AB = CD$)

Hence proved.

Q. 18. Equilateral triangles ABD and ACE are drawn on the sides AB and AC of ΔABC as shown in the figure. Prove that :

(*i*) $\angle DAC = \angle EAB$ (*ii*) DC = BE.

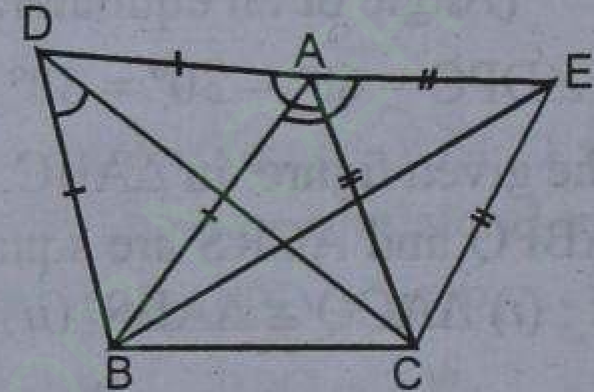


Sol. Given : Equilateral Δs ABD and ACE are on the sides AB and AC of ΔABC.

To prove : (*i*) $\angle DAC = \angle EAB$

(*ii*) DC = BE

Const : Join DC, BE.



Proof : (*i*) $\angle DAC = \angle DAB + \angle BAC$

$= 60^\circ + \angle BAC$ ...(*i*)

$\angle BAE = \angle CAE + \angle BAC$

$= 60^\circ + \angle BAC$ ...(*ii*)

From (*i*) and (*ii*)

$\angle DAC = \angle BAE$ or $\angle DAC = \angle EAB$

(*ii*) Now in ΔDAC and ΔBAE,

$\angle DAC = \angle BAE$ (Proved)

$AD = AB$

(Sides of equilateral triangle)

$AC = AE$

(Sides of equilateral triangle)

$\therefore \Delta DAC \cong \Delta BAE$

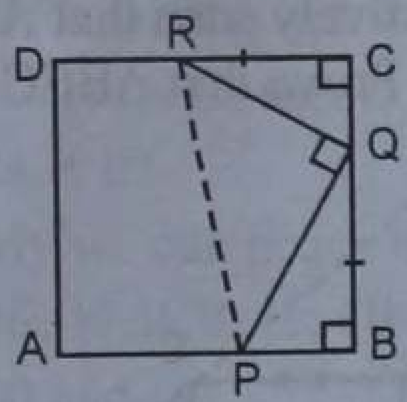
(SAS axiom of congruency)

$\therefore$ $DC = BE$ (C.P.C.T.)

Hence proved.

Q. 19. In the given figure, ABCD is a square and P, Q, R are points on AB, BC and CD respectively such that AP = BQ = CR and $\angle PQR = 90^\circ$. Prove that :

(*i*) PB = QC (*ii*) PQ = QR
(*iii*) $\angle QPR = 45°$.



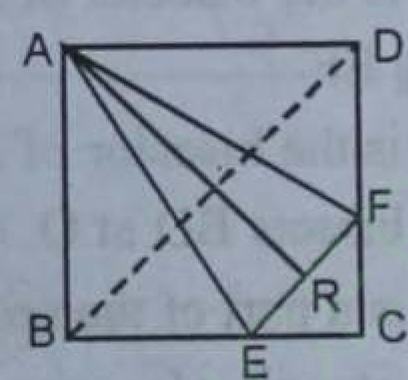
Sol. Given : ABCD is a square. P, Q and R are points on AB, BC and CD respectively such that AP = BQ = CR and $\angle PQR = 90°$.

To prove : (*i*) PB = QC
(*ii*) PQ = QR (*iii*) $\angle QPR = 45°$

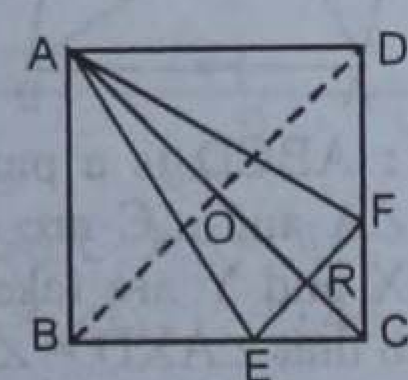
Proof : (*i*) AB = BC (Sides of a square)
and AP = BQ (Given)
Subtracting we get,
$AB - AP = BC - BQ \Rightarrow PB = QC$
In ΔQCR,
Ext. $\angle RQB = \angle QCR + \angle QRC$
$= 90° + \angle QRC$ ...(*i*)
and also $\angle RQB = \angle PQR + \angle PQB$
$= 90° + \angle PQB$ ...(*ii*)
From (*i*) and (*ii*)
$90° + \angle QRC = 90° + \angle PQB$
$\Rightarrow \angle QRC = \angle PQB$
Now in ΔPBQ and ΔQCR,
$\angle PQB = \angle QRC$ (Proved)
$BQ = CR$ (Given)
$\angle PBQ = \angle QCR$ (Each 90°)
$\therefore \Delta PBQ \cong \Delta QCR$
(AAS axiom of congruency)
$\therefore$ PQ = QR.

(*ii*) In ΔPQR,
$\angle PQR = 90°$
$\therefore \angle QPR + \angle QRP = 90°$
But $\angle QPR = \angle QRP$
(Angles opposite to equal sides)
$\therefore \angle QPR + \angle QPR = 90°$
$\Rightarrow 2\angle QPR = 90°$
$\Rightarrow \angle QPR = \frac{90°}{2} = 45°$
Hence $\angle QPR = 45°$
Hence proved.

Q. 20. In the given figure, ABCD is a square, EF || BD and R is the mid-point of EF. Prove that :
(*i*) BE = DF (*ii*) AR bisects $\angle BAD$
(*iii*) If AR is produced, it will pass through C.



Sol. Given : In square ABCD, EF || BD and R is the mid-point of EF. AR is joined.
To prove : (*i*) BE = DF
(*ii*) AR bisects $\angle BAD$
(*iii*) If AR is produced it will pass through C.
Const : Join BD and RC.
Proof : (*i*) $\because$ BD is the diagonal of square ABCD.



$\therefore \angle ABD = \angle ADB = 45°$
$\because$ BD || EF
$\therefore \angle FEC = \angle ABD = 45°$
(Corresponding angles)
Similarly $\angle EFC = \angle ADB = 45°$
$\therefore \angle FEC = \angle EFC$
$\therefore EC = FC$
(Sides opposite to equal angles)
But BC = DC (Sides of a square)

$\therefore \quad BC - EC = DC - FC$

$\Rightarrow \quad DE = DF$

(*ii*) Now in ΔABE and ΔADF

$AB = AD$ (Sides of a square)

$\angle B = \angle D$ (Each $= 90^\circ$)

$BE = DF$ (Proved)

$\therefore \Delta ABE \cong \Delta ADF$

(SAS axiom of congruency)

$\therefore \quad AE = AF$ (C.P.C.T.)

$\therefore \Delta AEF$ is an isosceles triangle.

$\because$ R is the mid-point of base EF.

$\therefore$ AR is the bisector of $\angle EAF$.

$\because BD \parallel EF$ (Given)

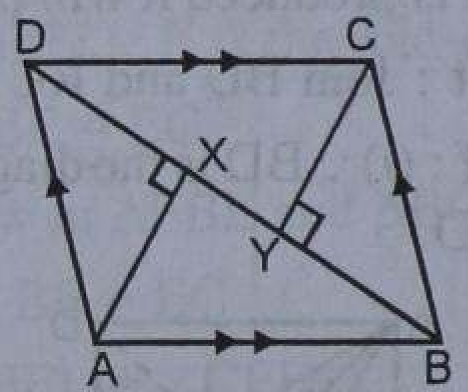
$\therefore$ AR is the bisector of $\angle BAD$

$\therefore$ AR bisects BD at O.

$\therefore$ AR is a part of the second diagonal.

Hence it will also pass through C if produced. **Q.E.D.**

Q. 21. ABCD is a parallelogram in which $\angle A$ and $\angle C$ are obtuse. Points X and Y are taken on diagonal BD such that $\angle AXD = \angle CYB = 90^\circ$. Prove that : XA = YC.



Sol. Given : ABCD is a parallelogram in which $\angle A$ and $\angle C$ are obtuse angles. Points X and Y are taken on diagonal BD such that $\angle AXD = \angle CYB = 90^\circ$.

To prove : XA = YC.

Proof : In ΔADX and ΔBYC

$AD = BC$

(Opposite sides of a parallelogram)

$\angle AXD = \angle CYB$ (Each $= 90^\circ$)

$\angle ADX = \angle YBC$ (Alternate angles)

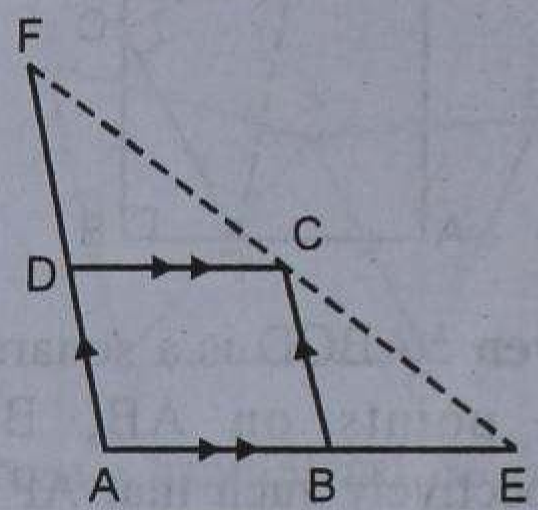
$\therefore \Delta ADX \cong \Delta BYC$

(AAS axiom of congruency)

$\therefore \quad XA = YC$ (C.P.C.T.)

Hence proved.

Q. 22. ABCD is a parallelogram. The sides AB and AD are produced to E and F respectively such that AB = BE and AD = DF. Prove that $\Delta BEC \cong \Delta DCF$.



Sol. Given : ABCD is a parallelogram Sides AB and AD are produced to E and F respectively such that AB=BE and AD= DF.

To prove : $\Delta BEC \cong \Delta DCF$,

Proof : $\because$ AB = BE and AD = DF

$\therefore$ B and D are the mid-points of AE and AF respectively.

Now in ΔBEC and ΔDCF,

$BE = AB = CD$ (Proved)

and $BC = AD = DF$ (Proved)

$\angle CBE = \angle DAB = \angle FDC$

(Corresponding angles)

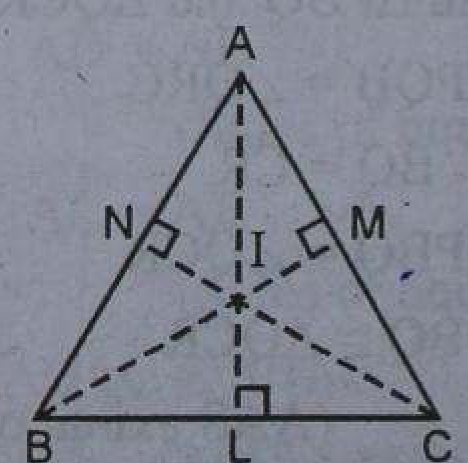
$\therefore \Delta BEC \cong \Delta DCF$

(SAS axiom of congruency)

Hence proved.

Q. 23. The perpendicular bisectors of the sides of a ΔABC meet at I.

Prove that : IA = IB = IC.



Sol. Given : In ΔABC, perpendicular bisectors of sides BC, CA and AB intersect each other at IA, IB and IC are joined.

To prove : IA = IB = IC.

Proof : In ΔAIN and ΔBIN,

$AN = BN$

(N is the mid-point of AB)

$\angle ANI = \angle BNI$ (Each 90°)

$IN = IN$ (Common)

$\therefore \Delta AIN \cong \Delta BIN$

(SAS axiom of congruency)

$\therefore \quad IA = IB$ ...(*i*)

Similarly we can prove that

$\Delta BIL \cong \Delta CIL \ \therefore IB = IC$ ...(*ii*)

From (*i*) and (*ii*)

$IA = IB = IC$ Hence proved.

EXERCISE 10 (B)

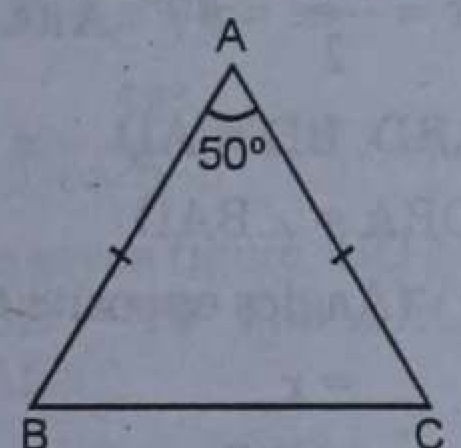
Q. 1. In a ΔABC, $AB = AC$ and $\angle A = 50^\circ$, find $\angle B$ and $\angle C$.

Sol. In ΔABC, $\angle A = 50^\circ$ and

$AB = AC$ (Given)

$\therefore \angle B = \angle C$

(Opposite angles of equal sides)



But $\angle A + \angle B + \angle C = 180^\circ$

(Sum of angles of the triangle)

$\Rightarrow \quad 50^\circ + \angle B + \angle B = 180^\circ$

$\Rightarrow 2\angle B = 180^\circ - 50^\circ = 130^\circ$

$\therefore \quad \angle B = \frac{130^\circ}{2} = 65^\circ$

and $\angle C = \angle B = 65^\circ$ **Ans.**

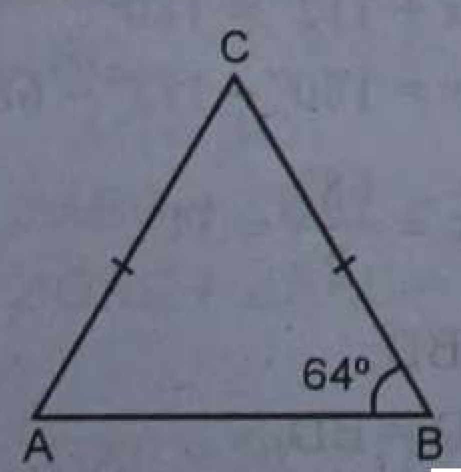
Q. 2. In a ΔABC, $BC = AC$ and $\angle B = 64^\circ$, find $\angle C$.

Sol. In ΔABC, $\angle B = 64^\circ$ and $BC = AC$

$\therefore \quad \angle A = \angle B$ (Angles opposite to equal sides)

$\because \quad \angle B = 64^\circ$ (given)

$\therefore \quad \angle A = 64^\circ$



But $\angle A + \angle B + \angle C = 180^\circ$

(Angles of a triangle)

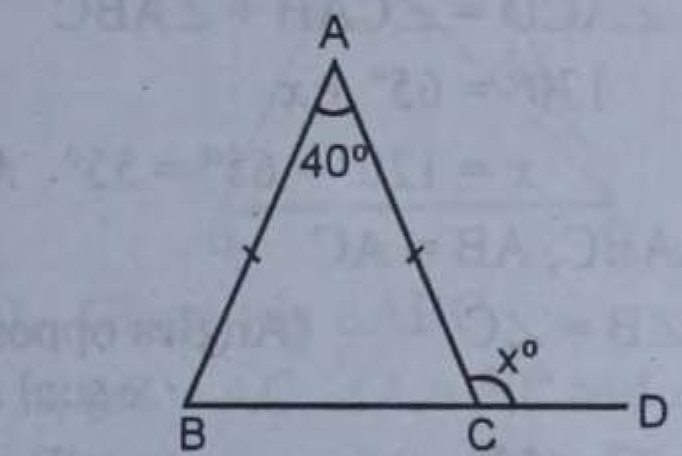
$\Rightarrow \ 64^\circ + 64^\circ + \angle C = 180^\circ$

$\Rightarrow \ 128^\circ + \angle C = 180^\circ$

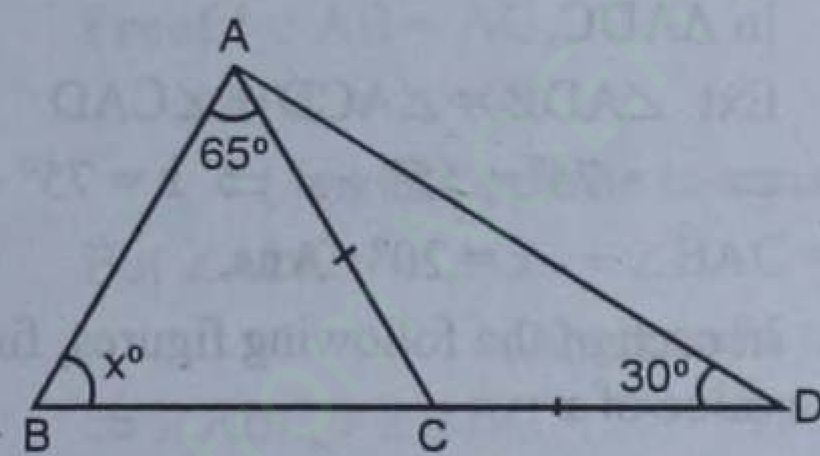
$\Rightarrow \ \angle C = 180^\circ - 128^\circ$

$\therefore \ \angle C = 52^\circ$ **Ans.**

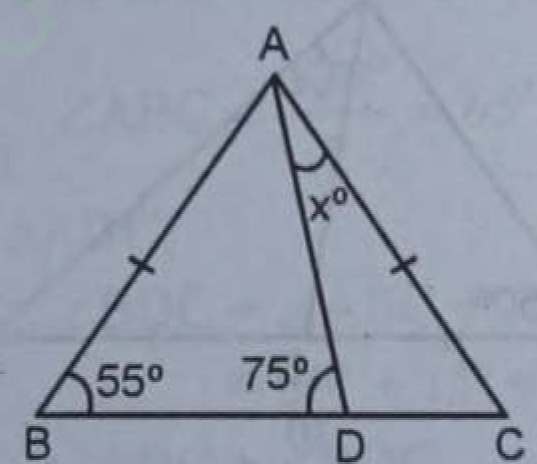
Q. 3. In each of the following figures, find the value of x :



(i)



(ii)



(iii)

Sol. (*i*) In ΔABC, $AB = AC$

$\therefore \ \angle B = \angle C$ (Angles opposite to equal sides)

But $\angle A + \angle B + \angle C = 180^\circ$

(Angles of a triangle)

$\Rightarrow \quad 40^\circ + \angle B + \angle B = 180^\circ$

$\Rightarrow 2\angle B = 180^\circ - 40^\circ = 140^\circ$

$\Rightarrow 2\angle B = 140^\circ \Rightarrow \angle B = \frac{140^\circ}{2} = 70^\circ$

Now ext. $\angle ACD = \angle A + \angle B$

$\Rightarrow x = 40^\circ + 70^\circ = 110^\circ$ **Ans.**

(*ii*) In ΔACD, $AC = CD$

$\therefore \angle CDA = \angle CAD \Rightarrow 30^\circ = \angle CAD$

$\Rightarrow \angle CAD = 30^\circ$

But $\angle CAD + \angle CDA + \angle ACD = 180^\circ$ (Angles of a triangle)

$\Rightarrow 30^\circ + 30^\circ + \angle ACD = 180^\circ$

$\Rightarrow 60^\circ + \angle ACD = 180^\circ$

$\Rightarrow \angle ACD = 180^\circ - 60^\circ = 120^\circ$

But in ΔABC,

Ext. $\angle ACD = \angle CAB + \angle ABC$

$\Rightarrow \quad 120^\circ = 65^\circ + x$

$\Rightarrow \quad x = 120^\circ - 65^\circ = 55^\circ$ **Ans.**

(*iii*) In ΔABC, $AB = AC$

$\therefore \angle B = \angle C$ (Angles opposite to equal sides)

$\Rightarrow \angle C = 55^\circ$ ($\because \angle B = 50^\circ$)

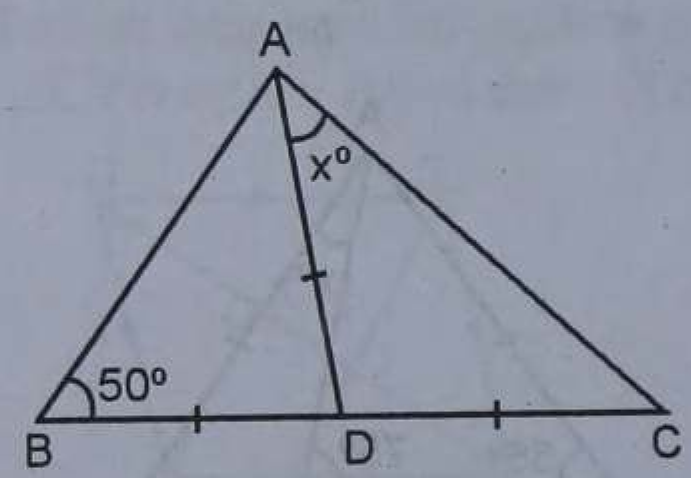
In ΔADC,

Ext. $\angle ADB = \angle ACD + \angle CAD$

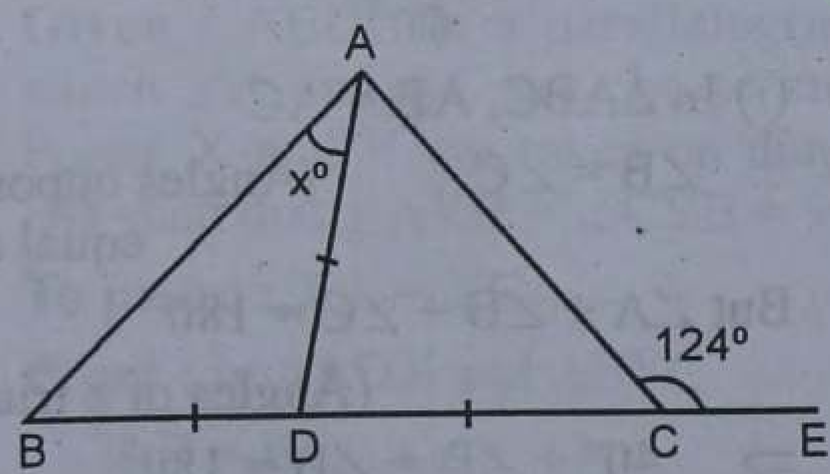
$\Rightarrow \quad 75^\circ = 55^\circ + x \Rightarrow x = 75^\circ - 55^\circ$

$x = 20^\circ$ **Ans.**

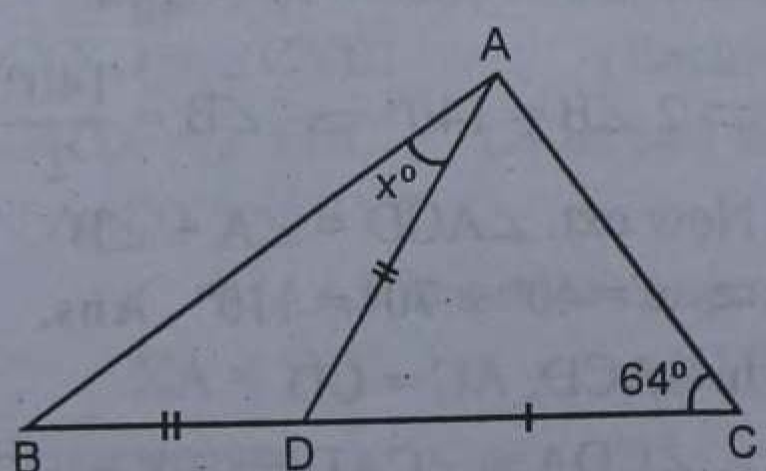
Q. 4. In each of the following figures, find the value of x :



(i)



(ii)



(iii)

Sol. (*i*) In ΔABD, $AD = BD$

$\therefore \angle B = \angle BAD = 50^\circ$ ($\because \angle B = 50^\circ$)

Again in ΔADC,

$AD = DC$

$\therefore \angle DAC = \angle DCA$ (Angles opposite to equal sides)

$= x$ ($\because \angle DAC = x$)

In ΔABC,

$\angle B + \angle BAC + \angle C = 180^\circ$ (Angles of a triangle)

$\Rightarrow \angle B + \angle BAD + \angle DAC + \angle C = 180^\circ$

$\Rightarrow 50^\circ + 50^\circ + x + x = 180^\circ$

$\Rightarrow 100^\circ + 2x = 180^\circ$

$\Rightarrow 2x = 180^\circ - 100^\circ = 80^\circ$

$\therefore \quad x = \frac{80^\circ}{2} = 40^\circ$ **Ans.**

(*ii*) In ΔABD, $BD = AD$

$\therefore \angle DBA = \angle BAD$

(Angles opposite to equal sides)

$= x$

Again in ΔADC,

$AD = DC$

$\therefore \angle DCA = \angle DAC$

(Angles opposite to equal sides)

But $\angle ACE + \angle ACD = 180^\circ$

(Linear pair)

$\Rightarrow \quad 124^\circ + \angle ACD = 180^\circ$

$\Rightarrow \angle ACD = 180^\circ - 124^\circ = 56^\circ$

$\therefore \quad \angle DAC = 56^\circ$

Now in ΔABC,

$\angle B + \angle BAC + \angle BCA = 180^\circ$

$\Rightarrow \angle B + \angle BAD + \angle DAC + \angle BCA = 180^\circ$

$\Rightarrow x + x + 56^\circ + 56^\circ = 180^\circ$

$\Rightarrow 2x + 112^\circ = 180^\circ$

$\Rightarrow 2x = 180^\circ - 112^\circ = 68^\circ$

$\therefore \quad x = \frac{68^\circ}{2} = 34^\circ$

(*iii*) In ΔABD,

$AD = BD$

$\therefore\ \angle B = \angle BAD$

(Angles opposite to equal sides

$= x$

Again in ΔADC,

$AC = DC$

$\therefore \angle ADC = \angle ACD$

(Angles of opposite to equal sides)

But $\angle ADC + \angle ACD + \angle C = 180^\circ$

(Angles in a triangle)

$\Rightarrow \angle ADC + \angle ADC + 64^\circ = 180^\circ$

$\Rightarrow 2\angle ADC = 180^\circ - 64^\circ = 116^\circ$

$\therefore\quad \angle ADC = \dfrac{116^\circ}{2} = 58^\circ$

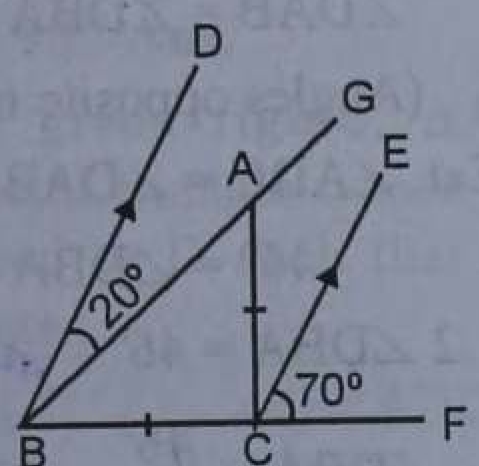
In ΔABD,

Ext. $\angle ADC = \angle B + \angle BAD$

$\Rightarrow 58^\circ = x + x \Rightarrow\quad 2x = 58^\circ$

$\Rightarrow\quad x = \dfrac{58^\circ}{2} = 29^\circ$ **Ans.**

Q. 5. In the given figure, $BD \parallel CE$; $AC = BC$, $\angle ABD = 20^\circ$ and $\angle ECF = 70^\circ$. Find $\angle GAC$.



Sol. In the figure, $BD \parallel CE$, $AC = BC$.

$\angle ABD = 20^\circ$, $\angle ECF = 70^\circ$

$\because\ BD \parallel CE$

$\therefore \angle DBC = \angle ECF$

(Corresponding angles)

But $\angle ABC + \angle ABD = 70^\circ$

$\Rightarrow\quad \angle ABC + 20^\circ = 70^\circ$

$\Rightarrow \angle ABC = 70^\circ - 20^\circ = 50^\circ$

In ΔABC,

$AC = BC$

$\therefore\ \angle ABC = \angle BAC = 50^\circ$

But $\angle GAC + \angle BAC = 180^\circ$

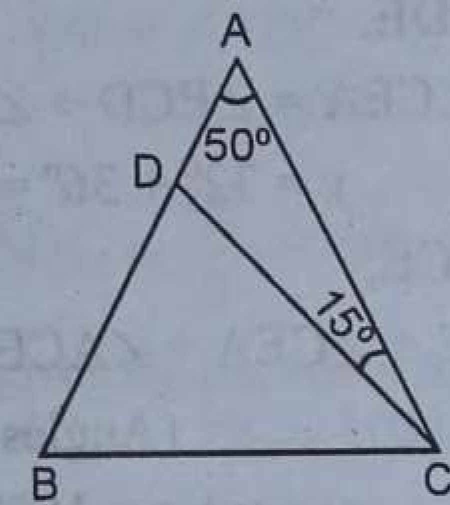
(Linear pair)

$\Rightarrow \angle GAC + 50^\circ = 180^\circ$

$\Rightarrow \angle GAC = 180^\circ - 50^\circ$

$\therefore\ \angle GAC = 130^\circ$ **Ans.**

Q. 6. In the given figure, $AB = AC$; $\angle A = 50^\circ$ and $\angle ACD = 15^\circ$. Show that $BC = CD$.



Sol. Given : In ΔABC,

$AB = AC$, $\angle A = 50^\circ$ and $\angle ACD = 15^\circ$

To prove : $BC = CD$

Proof : $\because AB = AC$

$\therefore\ \angle ABC = \angle ACB$

(Angles opposite to equal sides)

But $\angle ABC + \angle ACB + \angle BAC = 180^\circ$

(Angles of a triangle)

$\Rightarrow \angle ABC + \angle ABC + 50^\circ = 180^\circ$

$\Rightarrow 2\angle ABC = 180^\circ - 50^\circ = 130^\circ$

$\therefore\quad \angle ABC = \dfrac{130^\circ}{2} = 65^\circ$

In ΔADC,

Ext. $\angle BDC = \angle ABC + \angle DCA$

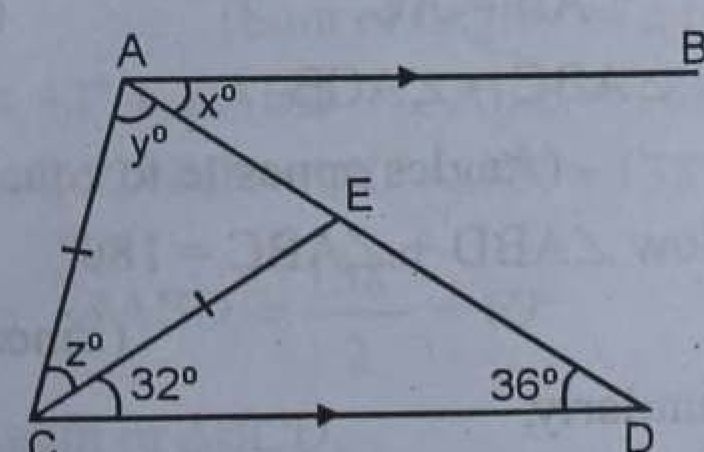
$= 50^\circ + 15^\circ = 65^\circ$

$\therefore\quad \angle ABC = \angle BDC$ (Each 65°)

Hence $BC = CD$

Q.E.D.

Q. 7. In the given figure, $AB \parallel CD$ and $CA = CE$. Find the values of x, y and z.



Sol. In the figure,

$AB \parallel CD$, $CA = CE$

$\because AB \parallel CD$

$\therefore \angle BAD = \angle ADC$ (Alternate angles)

$\Rightarrow \quad x = 36^\circ$

In ΔACE, $AC = CE$

$\therefore \angle CAE = \angle CEA \Rightarrow \angle CEA = y$

In ΔCDE,

Ext. $\angle CEA = \angle ECD + \angle EDC$

$\Rightarrow \quad y = 32^\circ + 36^\circ = 68^\circ$

In ΔACE,

$\angle CAE + \angle CEA + \angle ACE = 180^\circ$

(Angles of a triangle)

$\Rightarrow \quad y + y + z = 180^\circ$

$\Rightarrow \quad 68^\circ + 68^\circ + z = 180^\circ$

$\Rightarrow \quad 136^\circ + z = 180^\circ$

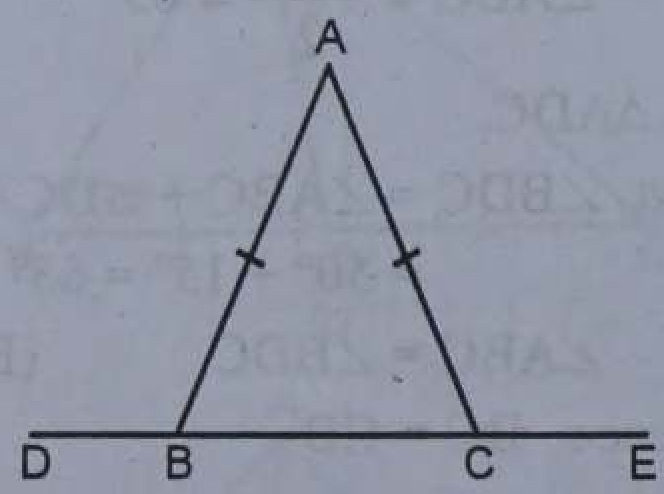
$\therefore \quad z = 180^\circ - 136^\circ = 44^\circ$

Hence $x = 36^\circ, y = 68^\circ, z = 44^\circ$ **Ans.**

Q. 8. If the base of an isosceles triangle is produced on both sides, prove that the exterior angles so formed are equal to each other.

Sol. Given : In ΔABC, $AB = AC$

BC is produced on both sides to D and E respectively.



To prove : $\angle ABD = \angle ACE$.

Proof : In ΔABC,

$AB = AC$ (Given)

$\therefore \angle ABC = \angle ACB$

(Angles opposite to equal sides)

Now $\angle ABD + \angle ABC = 180^\circ$ ...(*i*)

(Linear pair)

Similarly,

$\angle ACE + \angle ACB = 180^\circ$ ...(*ii*)

From (*i*) and (*ii*)

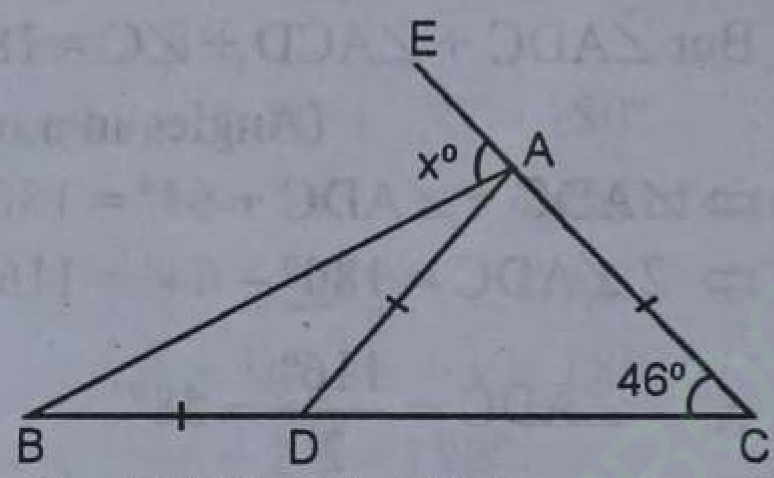
$\angle ABD + \angle ABC = \angle ACE + \angle ACE$

But $\angle ABC = \angle ACB$ (Proved)

$\therefore \angle ABD = \angle ACE$

Hence proved.

Q. 9. In the given figure, side CA of ΔABC has been produced to E. If $AC = AD = BD$; $\angle ACD = 46^\circ$ and $\angle BAE = x^\circ$; find the value of x.



Sol. In ΔABC, side CA is produced to E

$AC = AD = BD, \angle ACD = 46^\circ$

and $\angle BAE = x^\circ$

In ΔADC,

$\therefore \quad \angle C = \angle ADC = 46^\circ$

In ΔADB,

$AD = BD$

$\therefore \quad \angle DAB = \angle DBA$

(Angles opposite to equal sides)

But Ext. $\angle ADC = \angle DAB + \angle DBA$

$\Rightarrow \quad 46^\circ = \angle DBA + \angle DBA$

$\Rightarrow \quad 2\angle DBA = 46^\circ$

$\Rightarrow \quad \angle DBA = \frac{46^\circ}{2} = 23^\circ$

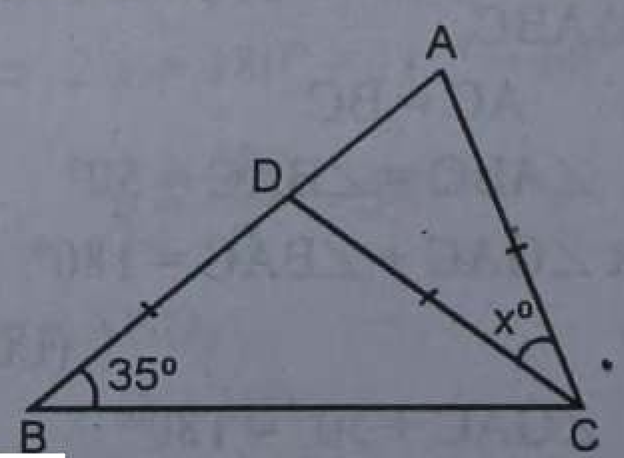
Now in ΔABC,

Ext. $\angle BAE = \angle B + \angle C$

$\Rightarrow \quad x = 23^\circ + 46^\circ = 69^\circ$

Hence $x = 69^\circ$ **Ans.**

Q. 10. In the given figure, $CA = CD = BD$; $\angle DBC = 35^\circ$ and $\angle DCA = x^\circ$. Find the value of x.



Sol. In the figure,

$CA = CD = BD$

$\angle DBC = 35^\circ, \angle DCA = x^\circ$

In ΔDBC,

$BD = CD$

$\therefore \angle DBC = \angle DCB = 35^\circ$

In ΔACD,

$CA = CD$

$\therefore \angle CAD = \angle CDA$

(Angles opposite to equal sides)

But ext. $\angle CDA = \angle DBC + \angle DCB$

$= 35^\circ + 35^\circ = 70^\circ$

$\therefore \quad \angle CAD = 70^\circ$

But in ΔACD,

$\angle CDA + \angle CAD + \angle ACD = 180^\circ$

(Angles of a triangle)

$\Rightarrow 70^\circ + 70^\circ + x^\circ = 180^\circ$

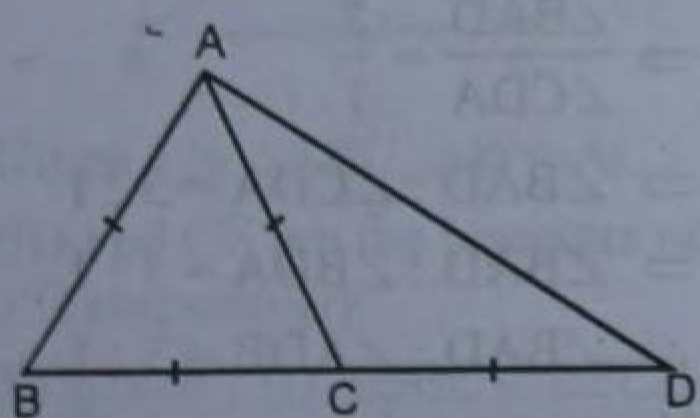
$\Rightarrow 140^\circ + x^\circ = 180^\circ$

$\Rightarrow x = 180^\circ - 140^\circ$

$\therefore \ x = 40^\circ$ **Ans.**

Q. 11. In the given figure, ΔABC is an equilateral triangle whose base BC is produced to D such that BC = CD. Calculate :

(*i*) $\angle ACD$ (*ii*) $\angle ADC$.



Sol. ΔABC is an equilateral triangle.

$\therefore AB = BC = CA$

Base BC is produced to D such that BC = CD.

$\because \Delta ABC$ is an equilateral triangle.

$\therefore \angle BAC = \angle ABC = \angle ACB = 60^\circ$

(*i*) and ext. $\angle ACD = \angle ABC + \angle BAC$

$= 60^\circ + 60^\circ = 120^\circ$

(*ii*) In ΔACD,

$CD = BC = AC$

$\therefore \angle CDA = \angle CAD$

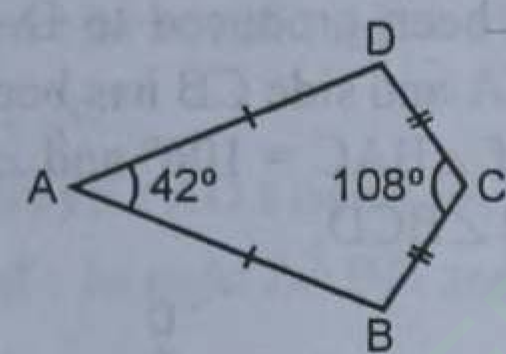
Ext. $\angle ACB = \angle ADC + \angle CAD$

$\Rightarrow \quad 60^\circ = \angle ADC + \angle ADC$

$= 2 \angle ADC$

$\therefore \ \angle ADC = \frac{60^\circ}{2} = 30^\circ$ **Ans.**

Q. 12. In the given figure, AB = AD ; CB = CD ; $\angle A = 42^\circ$ and $\angle C = 108^\circ$, find $\angle ABC$.

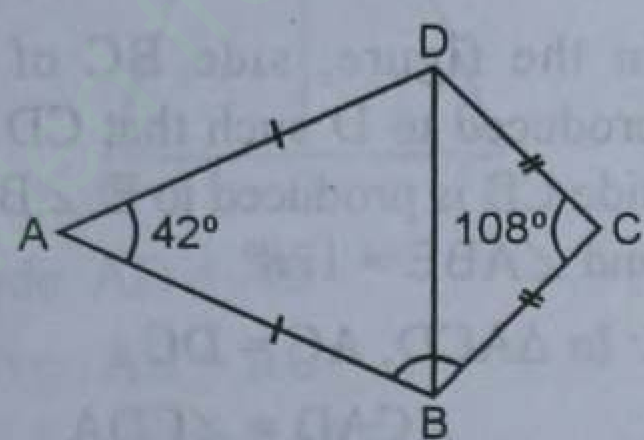


Sol. In the figure,

$AB = AD, CB = CD,$

$\angle A = 42^\circ, \angle C = 108^\circ$

Join BD



In ΔABD

$AB = AD$

$\therefore \angle ABD = \angle ADB$

(Angles opposite to equal sides)

But $\angle A + \angle ABD + \angle ADB = 180^\circ$

(Sum of angles of a triangle)

$\Rightarrow 42^\circ + \angle ABD + \angle ABD = 180^\circ$

$\Rightarrow 2 \angle ABD = 180^\circ - 42^\circ = 138^\circ$

$\Rightarrow \quad \angle ABD = \frac{138^\circ}{2} = 69^\circ \quad ...(i)$

Again in ΔBCD,

$CB = CD$

$\therefore \angle DBC = \angle BDC$

(Angles opposite to equal sides)

Similarly $\angle DBC + \angle BDC + \angle C = 180^\circ$

$\Rightarrow \angle DBC + \angle DBC + 108^\circ = 180^\circ$

$\Rightarrow 2\angle DBC + 108^\circ = 180^\circ$

$\Rightarrow 2\angle DBC = 180^\circ - 108^\circ = 72^\circ$

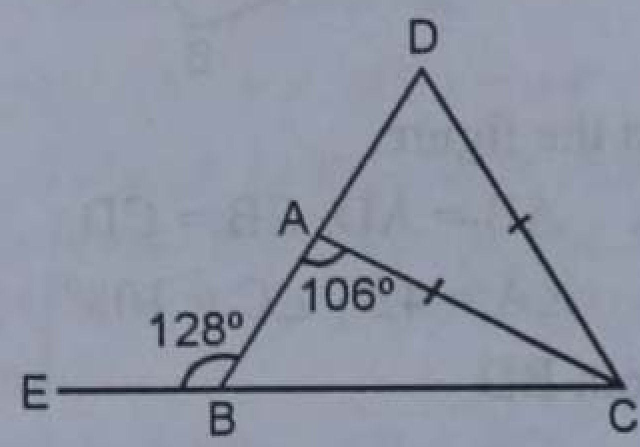
$\therefore \quad \angle DBC = \frac{72^\circ}{2} = 36^\circ \qquad ...(ii)$

Adding (*i*) and (*ii*)

$\angle ABD + \angle DBC = 69^\circ + 36^\circ$

$\Rightarrow \quad \angle ABC = 105^\circ$ **Ans.**

Q. 13. In the given figure, side BA of ΔABC has been produced to D such that CD = CA and side CB has been produced to E. If $\angle BAC = 106^\circ$ and $\angle ABE = 128^\circ$, find $\angle BCD$.



Sol. In the figure, side BC of ΔABC is produced to D such that CD = CA and side CB is produced to E. $\angle BAC = 106^\circ$ and $\angle ABE = 128^\circ$.

$\because$ In ΔACD, AC = DC

$\therefore \quad \angle CAD = \angle CDA$

(Angles opposite to equal sides)

But $\angle BAC + \angle CAD = 180^\circ$

(Linear pair)

$\Rightarrow 106^\circ + \angle CAD = 180^\circ$

$\Rightarrow \angle CAD = 180^\circ - 106^\circ = 74^\circ$

$\Rightarrow \angle CDA = 74^\circ$ or $\angle CDB = 74^\circ$

Now in ΔDBC,

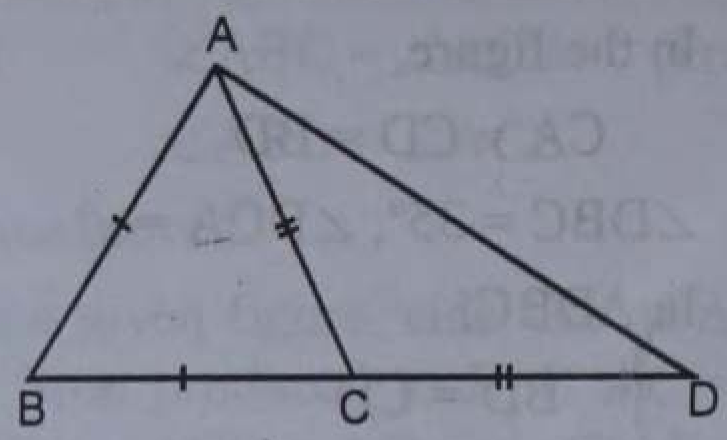
Ext. $\angle DBE = \angle BCD + \angle CDB$

$\Rightarrow \quad 128^\circ = \angle BCD + 74^\circ$

$\Rightarrow \angle BCD = 128^\circ - 74^\circ$

$= 54^\circ$ **Ans.**

Q. 14. In the given figure, AB = BC and AC = CD. Show that : $\angle BAD : \angle ADB = 3 : 1$.



Sol. Given : In the figure,

AB = BC and AC = CD.

To prove : $\angle BAD : \angle ADB = 3 : 1$

Proof : In ΔABC,

AB = BC

$\therefore \angle BAC = \angle BCA \qquad ...(i)$

(Angle opposite to equal sides)

Similarly in ΔACD,

AC = CD

$\therefore \angle CAD = \angle CDA \qquad ...(ii)$

and ext. $\angle ACB = \angle CAD + \angle CDA$

$\Rightarrow \angle ACB = \angle CDA + \angle CDA$

$\Rightarrow \angle ACB = 2\angle CDA$ [From (*ii*)]

$\Rightarrow \angle BAC = 2\angle CDA$ [From (*i*)]

Adding $\angle CAD$ both sides

$\angle BAC + \angle CAD = 2\angle CDA + \angle CAB$

$= 2\angle CDA + \angle CDA$

[From (*ii*)]

$\therefore \angle BAD = 3\angle CDA$

$\Rightarrow \frac{\angle BAD}{\angle CDA} = \frac{3}{1}$

$\Rightarrow \angle BAD : \angle CDA = 3 : 1$

$\Rightarrow \angle BAD : \angle BDA = 3 : 1$

$\therefore \angle BAD : \angle ADB = 3 : 1$

Hence proved.

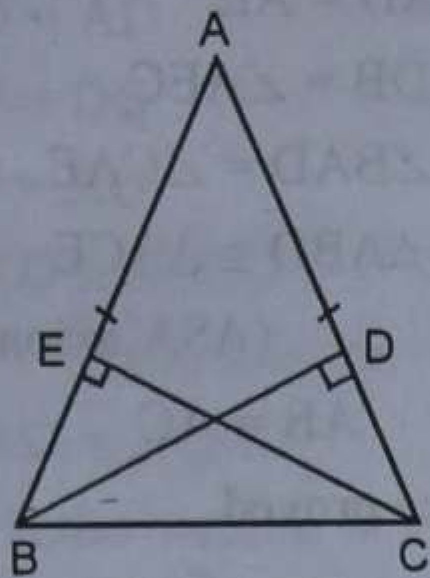
Q. 15. Show that the perpendiculars drawn from the extremities of the base of an isosceles triangle to the opposite sides are equal.

Sol. Given : In ΔABC, AB = AC

$BD \perp AC$ and $CE \perp AB$

To prove : BD = CE

Proof : In ΔBDC and ΔBEC,



BC = BC (Common)

$\angle D = \angle E$ (Each 90°)

$\angle C = \angle B$ ($\because$ AB = AC)

$\therefore \Delta BDC \cong \Delta BEC$

(AAS axiom of congruency)

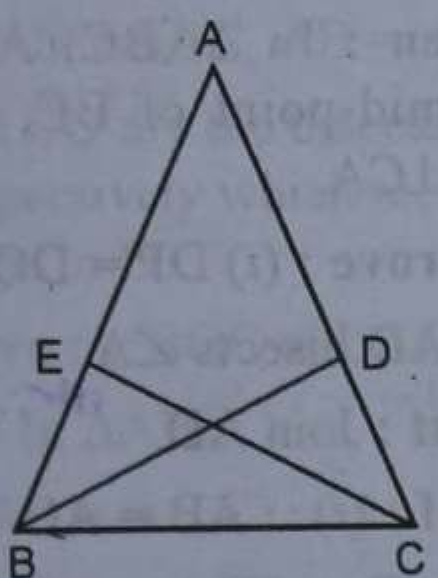
$\therefore$ BD = CE (C.P.C.T.)

Hence proved.

Q. 16. In a ΔABC, AB = AC. If the bisectors of $\angle B$ and $\angle C$ meet AC and AB at points D and E respectively, show that :

(*i*) $\Delta DBC \cong \Delta ECB$

(*ii*) BD = CE.



Sol. Given : In ΔABC, AB = AC

BD and CE are the bisectors of $\angle B$ and $\angle C$ respectively.

To prove : (*i*) $\Delta DBC \cong \Delta ECB$

(*ii*) BD = CE.

Proof : In ΔABC,

AB = AC (Given)

$\therefore \angle B = \angle C$

(Angles opposite to equal sides)

But BD and CE are the bisector of $\angle B$ and $\angle C$

$\therefore \angle DBC = \angle ECB$

Now in ΔDBC and $\angle ECB$

BC = BC (Common)

$\angle DBC = \angle ECB$ (Proved)

$\angle C = \angle B$ (Proved)

(*i*) $\therefore \Delta DBC \cong \Delta ECB$

(ASA axiom of congruency)

(*ii*) $\therefore$ BD = CE (C.P.C.T.)

Q.E.D.

Q. 17. In an isosceles triangle, prove that the altitude from the vertex bisects the base.

Sol. Given : In ΔABC, AB = AC

AD $\perp$ BC.

To prove : AD bisects BC.

Proof : In right ΔABD and ΔACD

Side AD = AD (Common)

Hyp. AB = AC (Given)

$\therefore \Delta ABD \cong \Delta ACD$

(R.H.S. axiom of congruency)

$\therefore$ BD = DC (C.P.C.T.)

Hence AD bisects BC.

Hence proved.

Q. 18. If the altitude from one vertex of a triangle bisects the opposite side, prove that the triangle is isosceles.

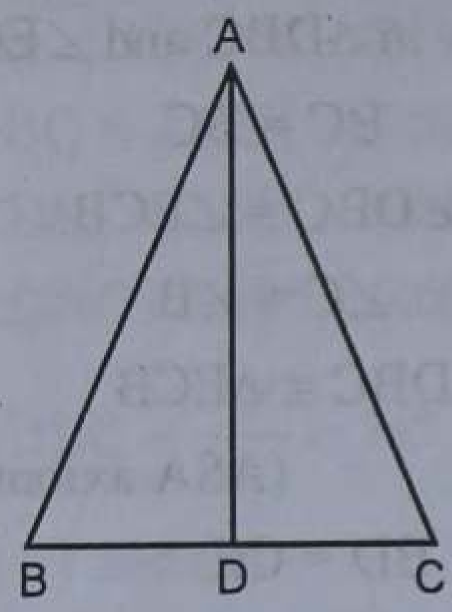
Sol. Given : In ΔABC, AD $\perp$ BC and BD = DC.

To prove : ΔABC is an isosceles triangle.

Proof : In ΔABD and ΔACD,

AD = AD (Common)

$\angle ADB = \angle ADC$ (Each 90°)



$BD = DC$ (Given)

$\therefore \Delta ABD \cong \Delta ACD$

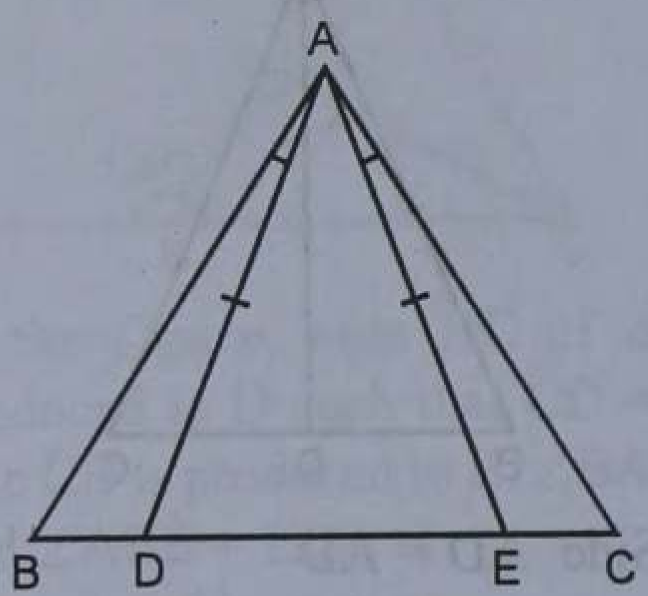
(SAS axiom of congruency)

$\therefore \quad AB = AC$ (C.P.C.T.)

Hence ΔABC is an isosceles triangle.

Hence proved.

Q. 19. In the given figure, $AD = AE$ and $\angle BAD = \angle CAE$. Prove that : $AB = AC$.



Sol. Given : In the figure, $AD = AE$ and $\angle BAD = \angle CAE$.

To prove : $AB = AC$

Proof : In ΔADE,

$AD = AE$ (Given)

$\therefore \angle ADE = \angle AED$

(Angles opposite to equal sides)

But $\angle ADE + \angle ADB = 180^\circ$

(Linear pair)

Similarly $\angle AED + \angle AEC = 180^\circ$

$\therefore \angle ADE + \angle ADB = \angle AED + \angle AEC$

But $\angle ADE = \angle AED$ (Proved)

$\therefore \angle ADB = \angle AEC$

Now in ΔABD and ΔACE

$AD = AE$ (Proved)

$\angle ADB = \angle AEC$ (Proved)

and $\angle BAD = \angle CAE$ (Given)

$\therefore \quad \Delta ABD \cong \Delta ACE$

(ASA axiom of congruency)

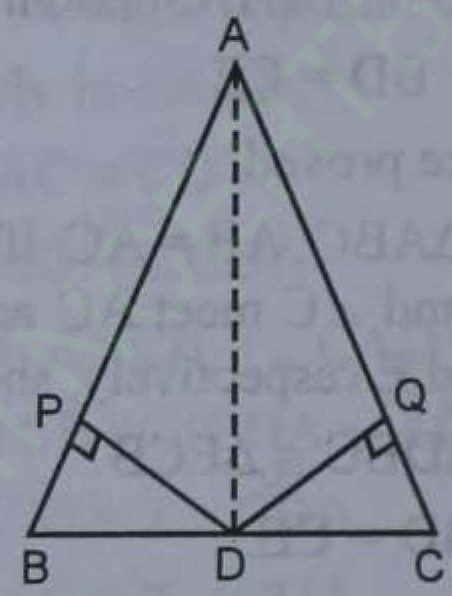
$\therefore \quad AB = AC$ (C.P.C.T.)

Hence proved.

Q. 20. In the given figure, $AB = AC$; D is the mid-point of BC ; $DP \perp BA$ and $DQ \perp CA$.

Prove that : (*i*) $DP = DQ$

(*ii*) $AP = AQ$ (*iii*) AD bisects $\angle A$.



Sol. Given : In ΔABC, $AB = AC$, D is the mid-point of BC, $DP \perp BA$ and $DQ \perp CA$.

To prove : (*i*) $DP = DQ$ (*ii*) $AP = AQ$ (*iii*) AD bisects $\angle A$.

Const : Join AD.

Proof : (*i*) $\because AB = AC$

$\therefore \quad \angle B = \angle C$

(Angles opposite to equal sides)

$\angle BPD = \angle CQD$ (Each $= 90^\circ$)

$BD = DC$ ($\because$ D is mid point)

$\therefore \Delta BPD \cong \Delta COD$

(AAS axiom of congruency)

$\therefore DP = DQ$ (C.P.C.T.)

(*ii*) Also $BP = CQ$ (C.P.C.T.)

$\because AB = AC$ (Given)

$\therefore AB - BP = AC - CQ$

$\Rightarrow \quad AP = AQ$

Now in ΔAPD and ΔAQD,

$AD = AD$ (Common)

$DP = DQ$ (Proved)

$AP = AQ$ (Proved)

$\therefore \Delta APD \cong \Delta AQD$

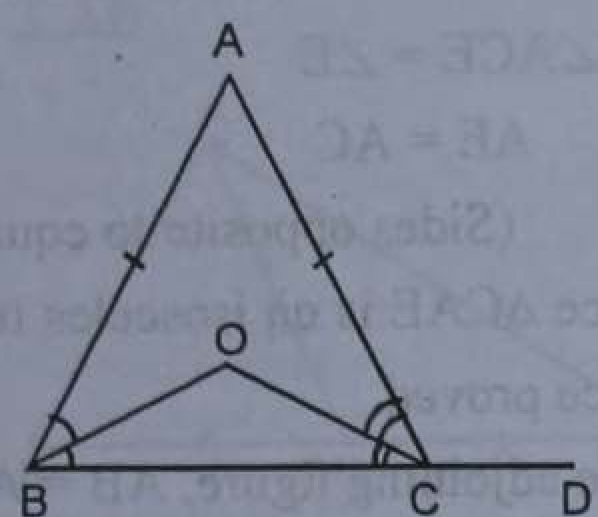
(SSS axiom of congruency)

$\therefore \angle PAD = \angle QAD$ (C.P.C.T.)

$\therefore$ AD is the bisector of $\angle A$.

Hence proved.

Q. 21. In the given figure, AB = AC. If BO and CO, the bisectors of $\angle B$ and $\angle C$ respectively meet at O and BC is produced to D, prove that $\angle BOC = \angle ACD$.



Sol. Given : In ΔABC, $AB = AC$

BO and CO are the bisectors of $\angle B$ and $\angle C$ respectively which meet at O. BC is produced to D.

To prove : $\angle BOC = \angle ACD$

Proof : In ΔABC,

$AB = AC$ (Given)

$\therefore \angle ABC = \angle ACB$

(Angles opposite to equal sides)

$\because$ BO and CO are the bisectors of $\angle B$ and $\angle C$ respectively.

$\therefore \angle OBC = \angle OCB$

(Half of equal angles)

In ΔOBC,

$\angle BOC = 180° - (\angle OBC + \angle OCB)$

$= 180° - 2\angle OCB$

$[\because \angle OBC = \angle OCB]$

$= 180° - \angle ACB$ ...(*i*)

$[\because$ OC is the bisector of $\angle C]$

But $\angle ACD + \angle ACB = 180°$

(Linear pair)

$\Rightarrow \angle ACD = 180° - \angle ACB$ ...(*ii*)

From (*i*) and (*ii*)

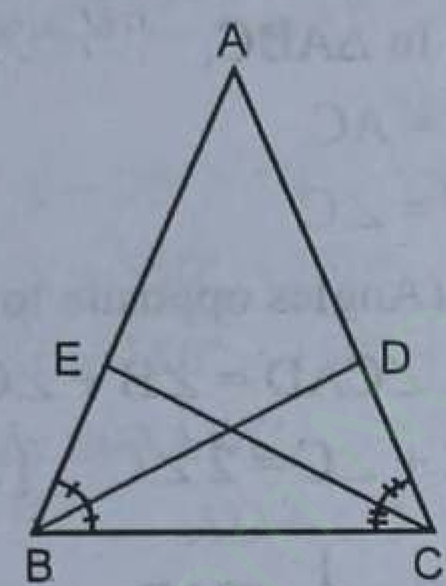
$\angle BOC = \angle ACD$

Hence proved.

Q. 22. Prove that the bisectors of the base angles of an isosceles triangle are equal.

Sol. Given : In ΔABC, $AB = AC$

BD and CE are the bisectors of $\angle B$ and $\angle C$ respectively.



To prove : $BD = CE$

Proof : In ΔABC,

$AB = AC$

$\therefore \angle B = \angle C$

(Angles opposite to equal sides)

$\because$ BD and CE are the bisectors of $\angle B$ and $\angle C$ respectively.

$\therefore \angle DBC = \angle ECB$

Now in ΔBCD and ΔBCE,

$BC = BC$ (Common)

$\angle B = \angle C$ (Proved)

$\angle DBC = \angle ECB$ (Proved)

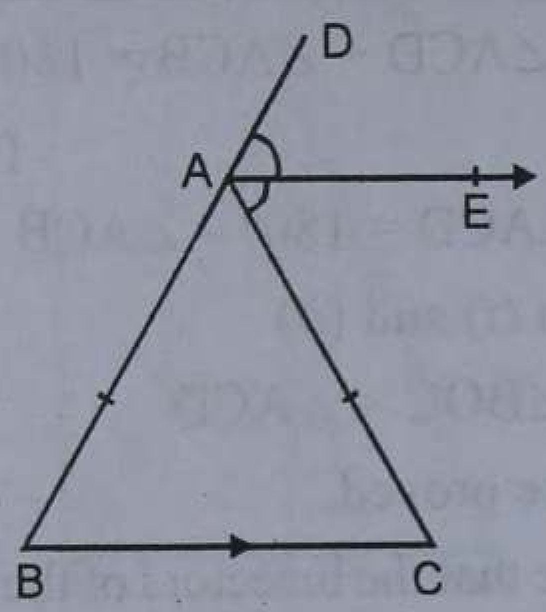
$\therefore \Delta BCD \cong \Delta BCE$

(ASA axiom of congruency)

$\therefore BD = CE$ (C.P.C.T.)

Hence proved.

Q. 23. In the given figure, AB = AC and side BA has been produced to D. If AE is the bisector of $\angle CAD$, prove that AE $\parallel$ BC.



Sol. Given : In ΔABC, $AB = AC$.

BA is produced to D and AE is the bisector of $\angle CAD$.

To prove : $AE \parallel BC$

Proof : In ΔABC,

$AB = AC$ (Given)

$\therefore \angle B = \angle C$

(Angles opposite to equal sides)

and ext. $\angle CAD = \angle B + \angle C$

$\Rightarrow \angle C + \angle C = 2\angle C \quad [\because \angle B = \angle C]$

$\Rightarrow \angle C = \frac{1}{2}\angle CAD$ ...(*i*)

But AE is the bisector of $\angle CAD$

$\therefore \angle EAC = \frac{1}{2}\angle CAD$ ...(*ii*)

From (*i*) and (*ii*)

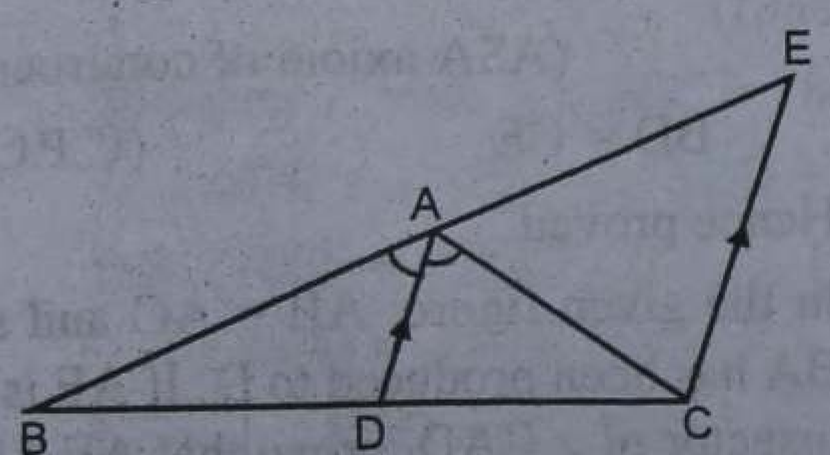
$\angle C = \angle EAC$

But these are alternate angles.

$\therefore AE \parallel BC$

Hence proved.

Q. 24. In the given figure, AD is the internal bisector of $\angle A$ and $CE \parallel DA$. If CE meets BA produced at E, prove that ΔCAE is isosceles.



Sol. Given : In ΔABC,

AD is the bisector of $\angle A$ meeting BC at D.

BA is produced to E and $CE \parallel DA$.

To prove : ΔCAE is an isosceles triangle.

Proof : In ΔACE,

Ext. $\angle CAB = \angle E + \angle ACE$

$AD \parallel CE$

$\therefore \angle DAC = \angle ACE$

(Alternate angles)

and $\angle BAD = \angle E$ (Corresp. angles)

But $\angle BAD = \angle DAC$

($\because$ AD is the bisector)

$\therefore \angle ACE = \angle E$

$\therefore AE = AC$

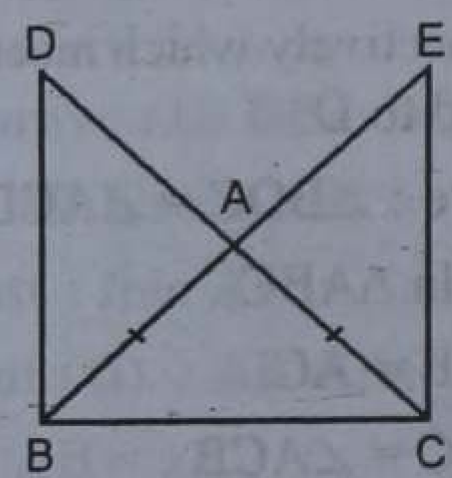
(Sides opposite to equal angles)

Hence ΔCAE is an isosceles triangle.

Hence proved.

Q. 25. In the adjoining figure, $AB = AC$. If $DB \perp BC$ and $EC \perp BC$, prove that :

(*i*) $BD = CE$ (*ii*) $AD = AE$.



Sol. Given : In the figure,

$AB = AC$, $DB \perp BC$, $EC \perp BC$.

To prove : (*i*) $BD = CE$

(*ii*) $AD = AE$.

Proof : In ΔABC, $AB = AC$

$\therefore \angle ABC = \angle ACB$

(Angles opposite to equal sides)

Now $\because DB \perp BC$ and $EC \perp BC$

$\therefore BD \parallel CE$

$\therefore \angle ABD = \angle AEC$

(Alternate angles)

Now in ΔABD and ΔACE,

$AB = AC$ (Given)

$\angle BAD = \angle CAE$ (Vertically opposite angles)

$\angle ABD = \angle AEC$ (Proved)

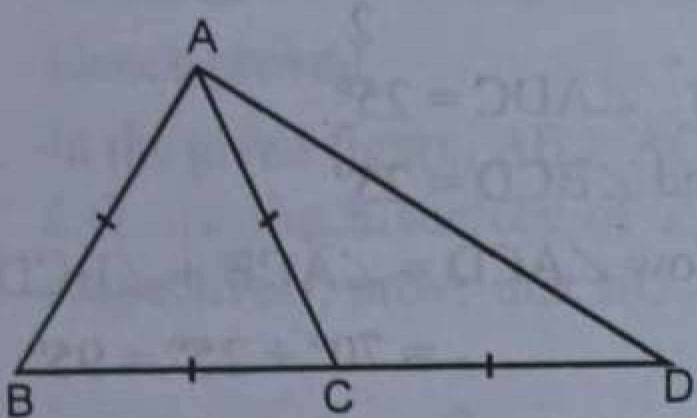
$\therefore \angle ABD \cong \Delta ACE$ (ASA axiom of congruency)

(*i*) $\therefore$ $BD = CE$ (C.P.C.T.)

$AD = AE$ (C.P.C.T.)

Hence proved.

Q. 26. In the given figure, ΔABC is an equilateral triangle and BC is produced to D such that $BC = CD$. Prove that $AD \perp AB$.



Sol. Given : ΔABC is an equilateral triangle BC is produced to D such that $BC = CD$.

To prove : $AD \perp AB$

Proof : In ΔABC,

$AB = AC = BC$

$\therefore \angle A = \angle B = \angle C = 60^\circ$

In ΔACD,

Ext. $\angle ACB = \angle CAD + \angle CDA$

But $\angle CAD = \angle CDA$ $(\because CD = BC = AC)$

But $\angle ACB = 60^\circ$

$\therefore \angle CAD = \frac{1}{2}\angle ACB = \frac{1}{2} \times 60^\circ = 30^\circ$

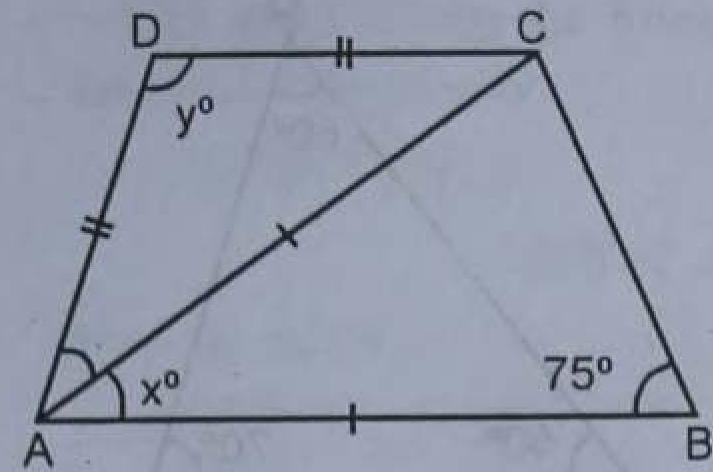
$\therefore \angle BAD = \angle BAC + \angle CAD$

$= 60^\circ + 30^\circ = 90^\circ$

Hence $AD \perp AB$

Q.E.D.

Q. 27. In the given figure, AC is the bisector of $\angle A$. If $AB = AC$, $AD = CD$ and $\angle ABC = 75^\circ$, find the values of x and y.



Sol. AC is the bisector of $\angle A$

$AB = AC$, $AD = CD$ and $\angle ABC = 75^\circ$

In ΔABC,

$\because$ $AB = AC$ (Given)

$\therefore \angle ABC = \angle ACB = 75^\circ$

But $\angle BAC + \angle ABC + \angle ACB = 180^\circ$ (Angles of a triangle)

$\Rightarrow x + 75^\circ + 75^\circ = 180^\circ$

$\Rightarrow x + 150^\circ = 180^\circ$

$\Rightarrow x = 180^\circ - 150^\circ = 30^\circ$

Now in ΔADC,

$AD = CD$ (Given)

$\therefore$ $\angle DAC = \angle DCA$

But AC is the bisector of $\angle A$

$\therefore \angle DAC = \angle CAB = x$

$\therefore \angle DCA = x$

Now in ΔADC,

$\angle DAC + \angle DCA + \angle ADC = 180^\circ$

$\Rightarrow$ $x + x + y = 180^\circ$

$\Rightarrow$ $2x + y = 180^\circ$

$\Rightarrow 2 \times 30^\circ + y = 180^\circ$

$\Rightarrow$ $60^\circ + y = 180^\circ$

$\Rightarrow$ $y = 180^\circ - 60^\circ = 120^\circ$ **Ans.**

EXERCISE 10 (C)

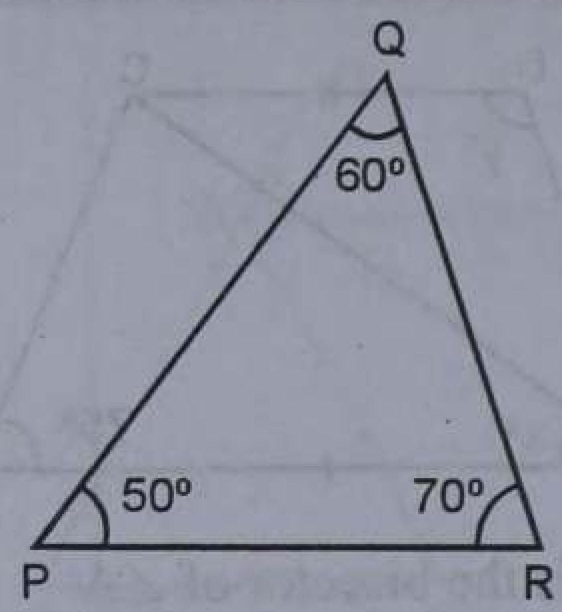
Q. 1. In ΔPQR, $\angle P = 50^\circ$ and $\angle R = 70^\circ$ Name (*i*) the shortest side (*ii*) the longest side.

Sol. In ΔPQR,

$\angle P = 50^\circ$ and $\angle R = 70^\circ$

But $\angle P + \angle Q + \angle R = 180^\circ$

$\Rightarrow$ $50^\circ + \angle Q + 70^\circ = 180^\circ$



$\Rightarrow \angle Q + 120° = 180°$

$\Rightarrow \angle Q = 180° - 120° = 60°$

We know that side opposite to smaller angle is shortest and opposite to greater angle is greatest (longest).

$\therefore$ In ΔPQR,

$\because \angle P = 50°$, the shortest angle

$\therefore$ QR is the shortest side

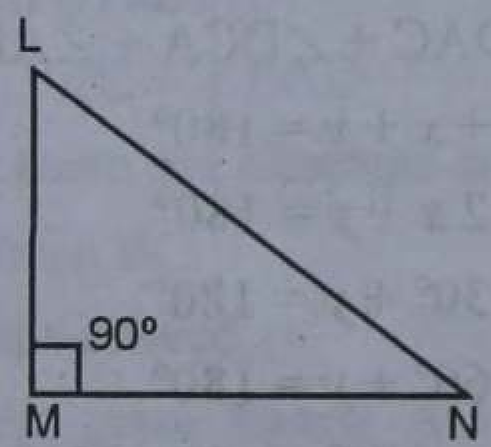
$\because \angle R = 70°$, the greatest angle

$\therefore$ PQ is the longest side.

Q. 2. In ΔLMN, if $\angle M = 90°$, name the longest side of the triangle.

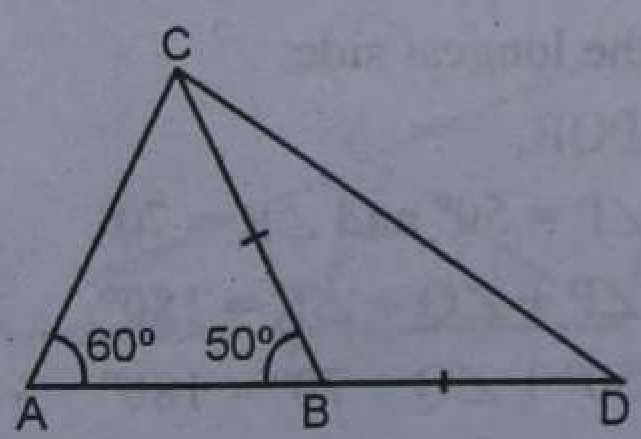
Sol. In ΔLMN, $\angle M = 90°$

$\therefore$ ΔLMN is the right angled triangle and in right angled triangle, the side opposite to 90° is the longest. Hence LN is the longest side.



Q. 3. In the given figure, side AB of ΔABC is produced to D such that BD = BC. If $\angle A = 60°$ and $\angle B = 50°$, prove that

(*i*) AD > CD (*ii*) AD > AC.



Sol. Given : Side AB of ΔABC is produced to D such that BD = BC. $\angle A = 60°$ and $\angle B = 50°$.

To prove : (*i*) AD > CD (*ii*) AD > AC.

Proof : In ΔABC, $\angle A = 60°$, $\angle B = 50°$

$\therefore \angle C = 180° - (\angle A + \angle B)$

$= 180° - (60° + 50°)$

$= 180° - 110° = 70°$

In ΔBCD,

Ext. $\angle CBA = \angle BCD + \angle BDC$

$\Rightarrow \angle CBA = \angle BDC + \angle BDC$ $\quad (\because BD = BC)$

$\Rightarrow \quad 50° = 2\angle BDC$

$\therefore \quad \angle BDC = \frac{50°}{2} = 25°$

or $\angle ADC = 25°$

and $\angle BCD = 25°$

Now $\angle ACD = \angle ACB + \angle BCD$

$= 70° + 25° = 95°$

Now we can conclude that

(*i*) $\because \angle ACD > \angle CAB$

$\therefore \quad AD > CD$

(Side opposite to greater angle is longer)

(*ii*) $\angle ACD > \angle ADC$

$\therefore$ AD > AC

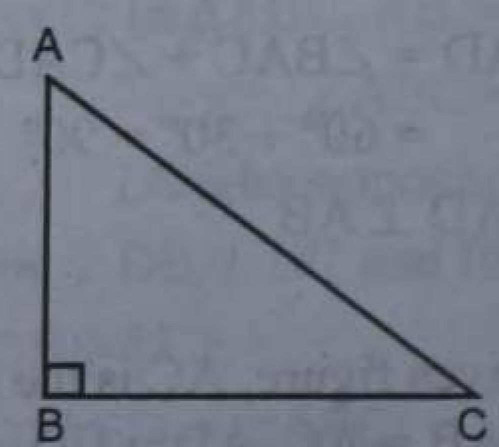
(Side opposite to greater angle is longer)

Hence proved.

Q. 4. In a right angled triangle, prove that hypotenuse is the longest side.

Sol. Given : A right angled triangle ABC in which $\angle B = 90°$.

To prove : AC is the longest side.



Proof : In ΔABC,

$\angle B = 90^\circ$

$\therefore \angle A + \angle C = 180^\circ - 90^\circ = 90^\circ$

In other words, we can say that

$\angle C < \angle B$ and $\angle A < \angle B$

(*i*) If $\angle C < \angle B$ or $\angle B > \angle C$

$\therefore AC > AB$

(Side opposite to greater angle is longer)

(*ii*) If $\angle A < \angle B$ or $\angle B > \angle A$

$\therefore AC > AC$

(Side opposite to greater angle is longer)

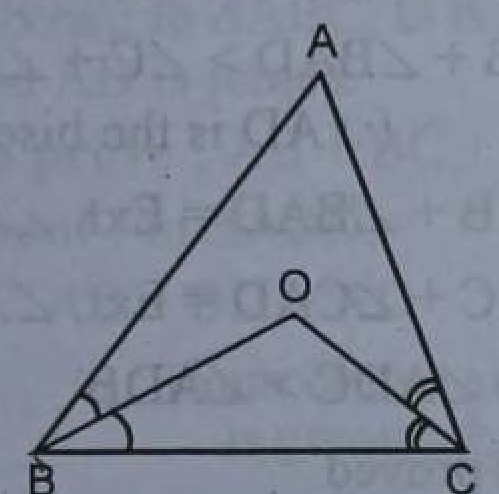
From (*i*) and (*ii*)

$AC > AB$ and also $AC > AC$

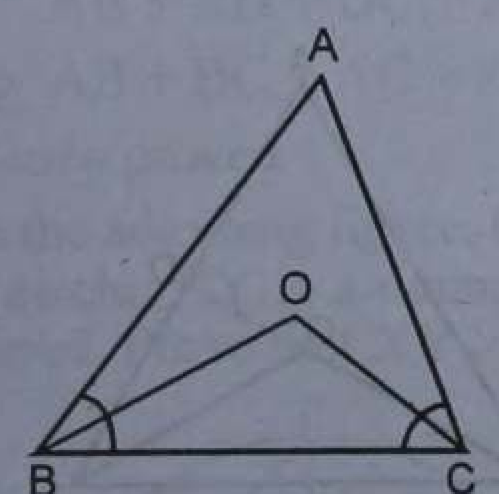
Hence AC is the longest side.

Hence proved.

Q. 5. In the given figure, $AB > AC$. If BO and CO are the bisectors of $\angle B$ and $\angle C$ respectively, prove that $BO > CO$.



Sol. Given : In ΔABC, OB and OC are the bisectors of $\angle B$ and $\angle C$ respectively and $AB > AC$.



To prove : $BO > CO$

Proof : In ΔABC,

$AB > AC$

$\therefore \angle C > \angle B$

(Angle opposite to longer side is greater)

$\because$ OB and OC are the bisectors of $\angle B$ and $\angle C$ respectively.

$\therefore \angle OCB > \angle OBC$

(Half of $\angle B$ and $\angle C$)

Now in ΔOBC,

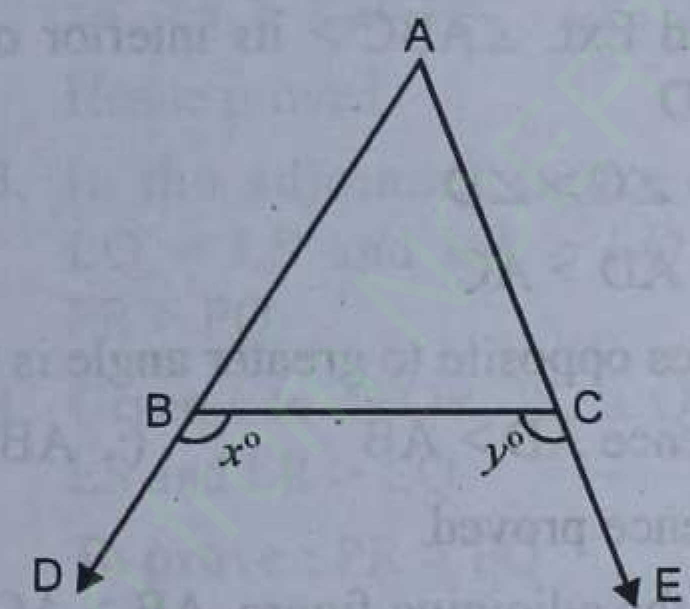
$\because \angle OCB > \angle OBC$ (Proved)

$\therefore BO > CO$

(Side opposite to greater angle is longer)

Hence proved.

Q. 6. In the given figure, sides AB and AC of ΔABC have been produced to D and E respectively. If $\angle CBD = x^\circ$ and $\angle BCE = y^\circ$ such that $x > y$, show that $AB > AC$.



Sol. Given : In ΔABC, AB and AC are produced to D and E respectively forming exterior angles x and y such that $x > y$.

To prove : $AB > AC$

Proof : $\angle ABC + \angle CBD = 180^\circ$

(Linear pair)

$\Rightarrow \angle ABC + x = 180^\circ$ ...(*i*)

Similarly $\angle ACB + y = 180^\circ$ ...(*ii*)

From (*i*) and (*ii*)

$\angle ABC + x = \angle ACB + y$

$\because x > y$

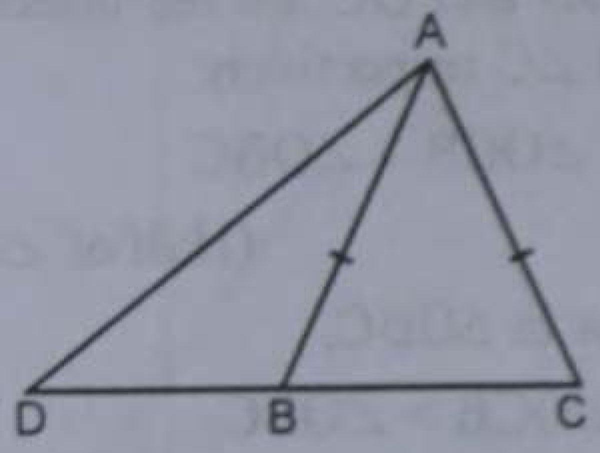
$\therefore \angle ABC < \angle ACB$

$\therefore AC < AB$ or $AB > AC$

(Sides opposite to greater angle is longer)

Hence proved.

Q. 7. In the given figure, $AB = AC$. Show that $AD > AB$.



Sol. **Given :** In ΔABC,

$AB = AC$

CB is produced to D and AD is joined.

To prove : $AD > AB$

Proof : Ext. $\angle ABD >$ Interior opposite $\angle C$

and $\angle B = \angle C$ $(\because AB = AC)$

and Ext. $\angle ABC >$ its interior opposite $\angle D$

$\Rightarrow \angle C > \angle D$

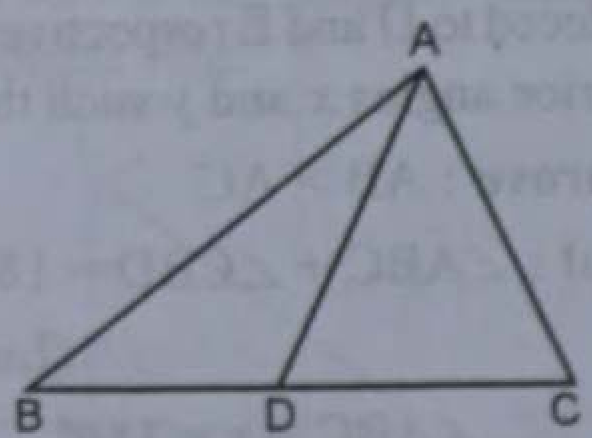
$\therefore AD > AC$

(Sides opposite to greater angle is longer)

Hence $AD > AB$ $(\because AB = AC)$

Hence proved.

Q. 8. In the adjoining figure, $AB > AC$ and D is any point on BC. Show that $AB > AD$.



Sol. **Given :** In ΔABC, $AC > AB$ and D is any point on BC. AD is joined.

To prove : $AB > AD$

Proof : In ΔADC,

Ext. $\angle ADB > \angle C$

But $\angle C > \angle B$ $(\because AB > AC)$

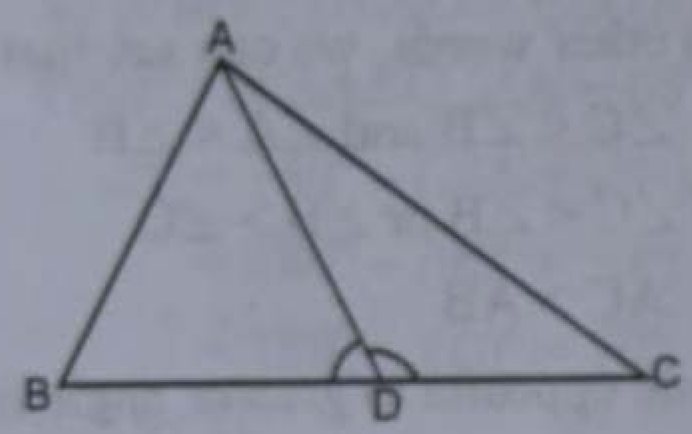
$\therefore \angle ADB > \angle B$

Hence $AB > AD$

(Side opposite to greater angle is longer)

Hence proved.

Q. 9. In the adjoining figure, $AC > AB$ and AD is the bisector of $\angle A$. Show that $\angle ADC > \angle ADB$.



Sol. **Given :** In ΔABC,

$AC > AB$

and AD is the bisector of $\angle A$ which meets BC at D.

To prove : $\angle ADC > \angle ADB$

Proof : $\because$ $AC > AB$ (Given)

$\therefore$ $\angle B > \angle C$

(Angle opposite to longer side is greater)

Adding $\angle BAD$ both sides,

$\angle B + \angle BAD > \angle C + \angle BAD$

$\Rightarrow \angle B + \angle BAD > \angle C + \angle CAD$

$(\because$ AD is the bisector of $\angle A)$

But $\angle B + \angle BAD =$ Ext. $\angle ADC$

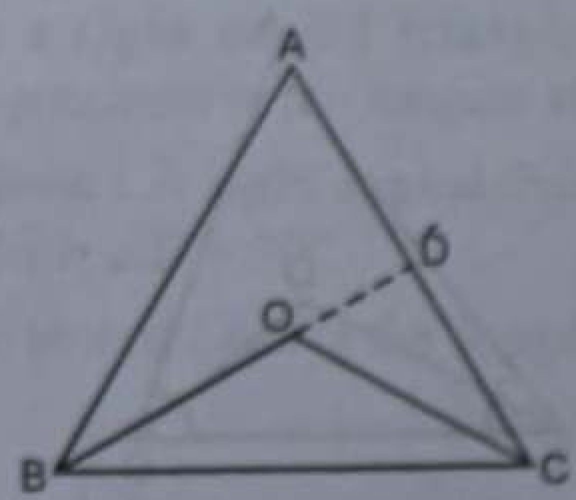
and $\angle C + \angle CAD =$ Ext. $\angle ADB$

Hence $\angle ADC > \angle ADB$

Hence proved.

Q. 10. In the adjoining figure, in ΔABC, O is any point in its interior. Show that :

$OB + OC < AB + AC$.



Sol. **Given :** In ΔABC, O is any point inside the ΔABC, OB and OC are joined.

To prove : $OB + OC < AB + AC$

Const : Produce BO to meet AC in D.

Proof : In ΔABD,

$AB + AD > BD$

(Sum of two sides of a triangle is greater than its third side)

$\Rightarrow AB + AD > OB + OD$ ...(*i*)

Similarly in ΔCD

$OD + DC > OC$ ...(*ii*)

Adding (*i*) and (*ii*)

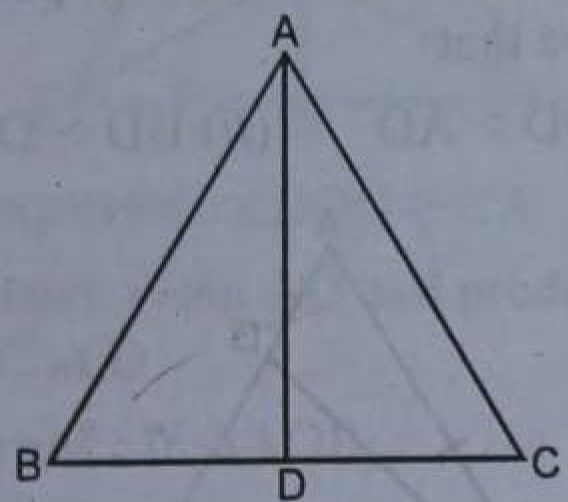
$AB + OD + AD + DC > OB + OD + OC$

$\Rightarrow AB + OD + AC > OB + OD + OC$

$\Rightarrow AB + AC > OB + OC$

Hence proved.

Q. 11. In ΔABC, D is any point on BC. Prove that : $AB + BC + AC > 2\ AD$.



Sol. Given : In ΔABC, D is any point on BC and AD is joined.

To prove : $AB + BC + AC > 2\ AD$

Proof : In ΔABD,

$AB + BD > AD$ ...(*i*)

(Sum of two sides of a triangle is greater than its third side)

Similarly in ΔADC

$AC + DC > AD$ ...(*ii*)

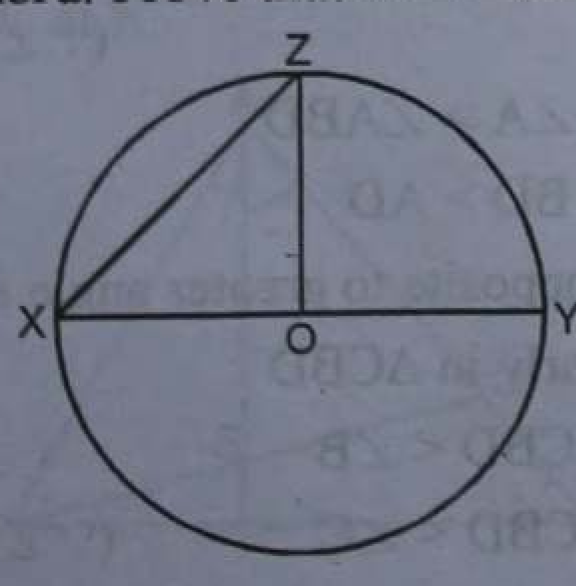
Adding (*i*) and (*ii*)

$AB + BD + DC + AC > AD + AD$

$\Rightarrow AB + BC + AC > 2\ AD$

Hence proved.

Q. 12. In the adjoining figure, O is the centre of a circle, XY is a diameter and XZ is a chord. Prove that $XY > XZ$.



Sol. Given : O is the centre of the circle whose XY is the diameter and XZ is a chord.

To prove : $XY > XZ$

Const : Join ZO

Proof : In ΔOXZ,

$OX + OZ > XZ$

(Sum of two sides of a triangle is greater than its third side)

But $OX = OZ = OY$

(Radii of the same circle)

$\therefore\ OX + OX > XZ$

$\Rightarrow OX + OY > XZ$

$\Rightarrow XY > XZ$

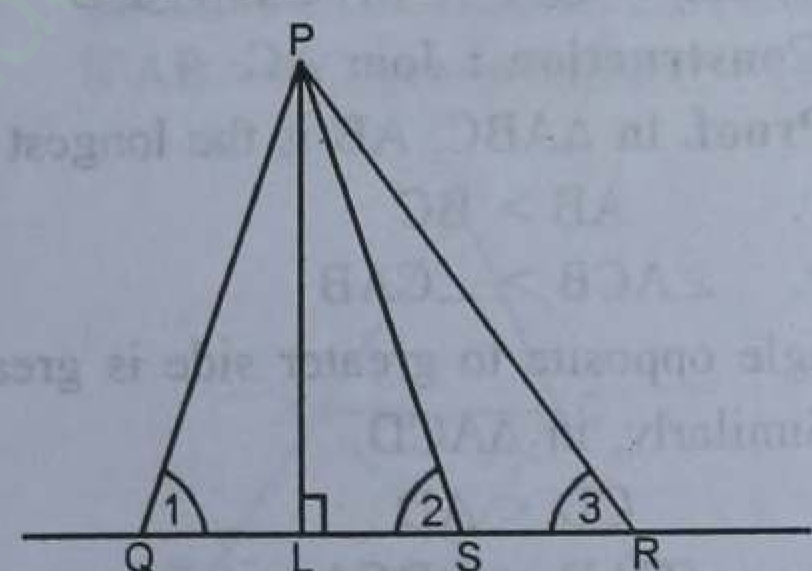
Hence proved.

Q. 13. In the adjoining figure, $PL \perp QR$, $LQ = LS$ and $LR > LQ$, show that $PR > PQ$.

Sol. Given : In ΔPQR, $PL \perp QR$ and $LQ = LS$ and $LR > LQ$

To prove : $PR > PQ$

Const : Join PS



Proof : In ΔPQL and ΔPSL,

$PL = PL$ (Common)

$\angle PLQ = \angle PLS$ (Each 90°)

$QL = LS$ (Given)

$\therefore \Delta PQL \cong \Delta PSL$

(SAS axiom of congruency)

$\therefore\ \angle 1 = \angle 2$ (C.P.C.T.)

But in ΔPSR,

Ext. $\angle 2 > \angle 3$

$\Rightarrow\ \angle 1 > \angle 3$ $(\angle 2 = \angle 1)$

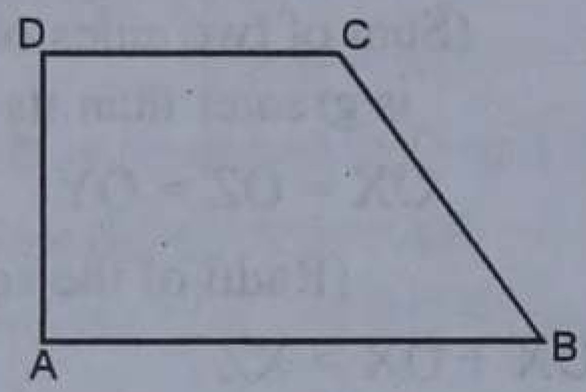
$\therefore$ PR > PQ

(Sides opposite to greater angle is longer)

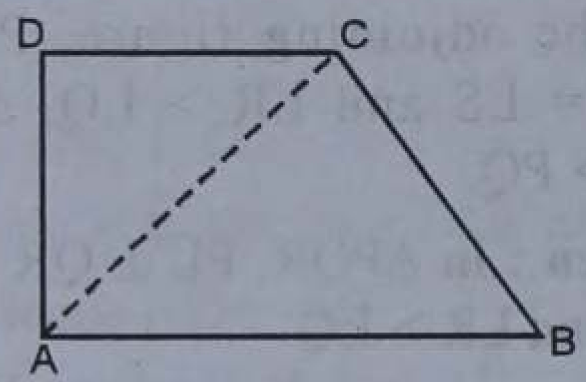
Hence proved.

Q. 14. In the adjoining quadrilateral ABCD, AB is the longest side and DC is the shortest side. Prove that :

(*i*) $\angle C > \angle A$ (*ii*) $\angle D > \angle B$



Sol. Given : In quadrilateral ABCD, AB is the longest side and CD is the shortest side.



To prove :

(*i*) $\angle C > \angle A$ (*ii*) $\angle D > \angle B$

Construction : Join AC.

Proof. In ΔABC, AB is the longest side

$\therefore$ AB > BC

$\therefore$ $\angle ACB > \angle CAB$ ...(*i*)

(Angle opposite to greater side is greater)

Similarly, in ΔACD,

CD < AD

$\therefore$ $\angle CAD < \angle DCA$

$\Rightarrow$ $\angle DCA > \angle CAD$ ...(*ii*)

Joining (*i*) and (*ii*),

$\angle ACB + \angle DCA > \angle CAB + \angle CAD$

$\Rightarrow$ $\angle C > \angle A$

Similarly by joining BD, we can prove that $\angle D > \angle B$

Hence proved.

Q. 15. Can you construct a ΔABC in which AB = 5 cm, BC = 4 cm and AC = 9 cm ? Give reason.

Sol. In ΔABC,

AB = 5 cm, BC = 4 cm, AC = 9

We know that in a triangle, it is possible to construct if sum of any two sides is greater than its third side. Here

AB + BC = 5 + 4 cm = 9 cm

and AC = 9 cm

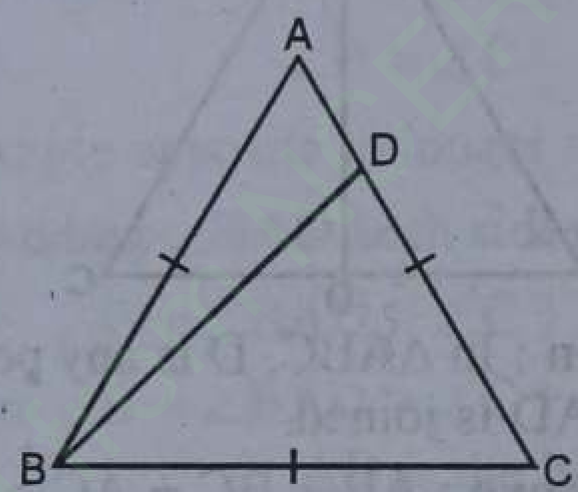
$\therefore$ AB + BC = AC

or AB + BC $\not>$ AC

Hence it is not possible to construct this triangle.

Q. 16. In the adjoining figure, ΔABC is equilateral and D is any point on AC. Prove that

(*i*) BD > AD (*ii*) BD > DC.



Sol. Given : In ΔABC, AB = BC = CA

D is any point on AC and BD is joined.

To prove : (*i*) BD > AD (*ii*) BD > DC.

Proof : In ΔABC,

$\because$ AB = BC = CD

(Sides of equilateral triangle)

$\therefore \angle A = \angle B = \angle C = 60°$

$\therefore \angle B = \angle ABD + \angle CBD$

or $\angle ABD < \angle B$ and $\angle CBD < \angle C$

In ΔABD,

$\because \angle ABD < \angle B \Rightarrow \angle ABD < \angle A$

($\because \angle A = \angle B$)

$\Rightarrow$ $\angle A > \angle ABD$

$\therefore$ BD > AD

(Sides opposite to greater angle is longer)

Similarly in ΔCBD

$\angle CBD < \angle B$

$\Rightarrow \angle CBD < \angle C$ ($\because \angle B = \angle C$)

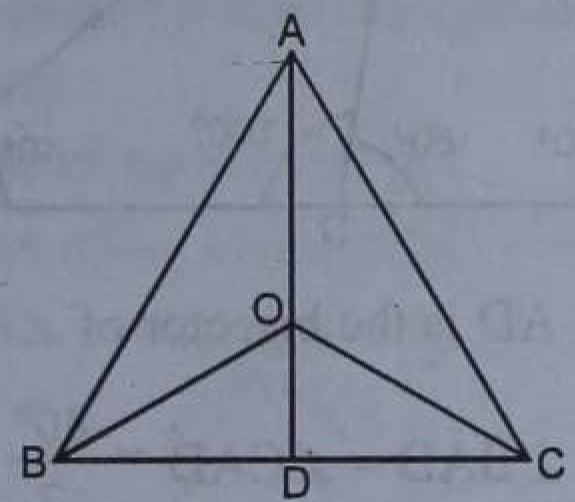
$\Rightarrow \quad \angle C > \angle CBD$

$\therefore \quad BD > DC$

Hence proved.

Q. 17. If O is any point inside ΔABC, prove that $\angle BOC > \angle A$.

Sol. Given : In ΔABC, O is any point BO and CO are joined.



To prove : $\angle BOC > \angle A$

Const : Join AO and produce it to meet BC at D.

Proof : In ΔAOB,

Ext. $\angle BOD > \angle BAD$ ...(*i*)

Similarly in ΔAOC

Ext. $\angle COD > \angle CAO$ ...(*ii*)

Adding (*i*) and (*ii*)

$\angle BOD + \angle COD > \angle BAO + \angle CAO$

$\Rightarrow \angle BOC > \angle BAC$

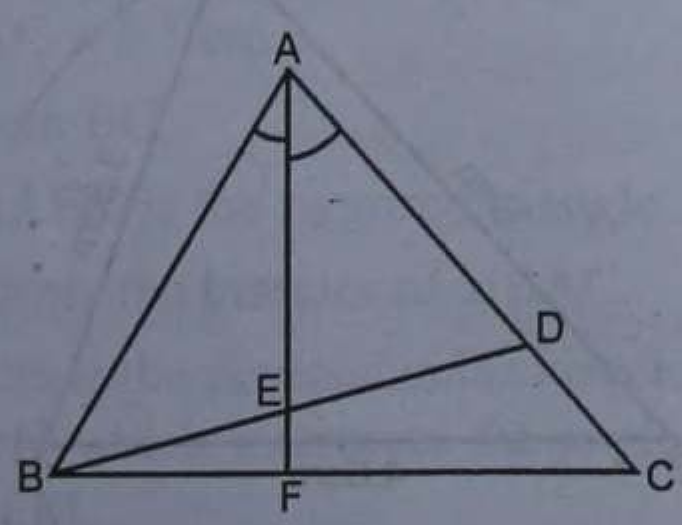
$\angle BOC > \angle A$

Hence proved.

Q. 18. In the given figure, AD = AB and AE bisects $\angle A$. Prove that :

(*i*) BE = ED (*ii*) $\angle ABD > \angle BCA$.

Sol. Given : In ΔABC, D is any point on AC such that AD = AB. AE is the bisector of $\angle A$ intersecting BD at E and meeting BC at F.



To prove : (*i*) BE = ED

(*ii*) $\angle ABD > \angle BCA$

Proof : In ΔABE and ΔADE,

AE = AE (Common)

$\angle BAE = \angle DAE$

($\because$ AE is the bisector of $\angle A$)

AB = AD (Given)

$\therefore \Delta ABE \cong \Delta ADE$

(SAS axiom of congruency)

$\therefore$ BE = ED (C.P.C.T.)

and $\angle ABE = \angle ADE$ (C.P.C.T.)

$\Rightarrow \angle ABD = \angle ADB$

Now in ΔBCD,

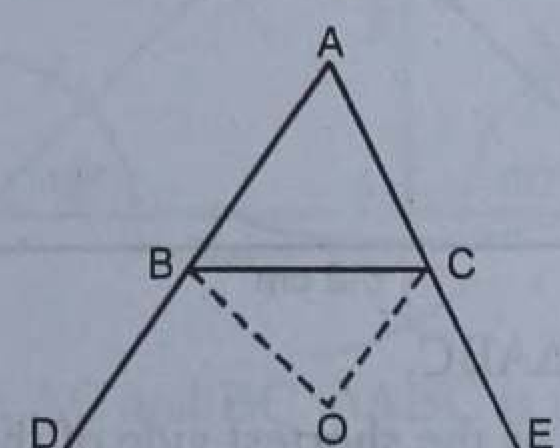
Ext. $\angle BDA > \angle BCA$

$\Rightarrow \angle ABD > \angle BCA$

($\because \angle ABD = \angle ADB$)

Hence proved.

Q. 19. The sides AB and AC of ΔABC are produced to D and E respectively and the bisectors of $\angle CBD$ and $\angle BCE$ and at O. If AB > AC, prove that OC > OB.



Sol. Given : In ΔABC, AB and AC are produced to D and E respectively. BO and CO are the bisectors of $\angle CBD$ and $\angle BCE$ respectively meeting at O. AB > AC.

To prove : OC > OB

Proof : In ΔABC, AB > AC

$\therefore \quad \angle ACB > \angle ABC$

(Angle opposite to longer side)

But $\angle ACB + \angle BCE = 180^\circ$

(Linear pair)

Similarly $\angle ABC + \angle CBD = 180^\circ$

$\therefore \angle ACB + \angle BCE = \angle ABC + \angle CBD$

But $\angle ACB > \angle ABC$

$\therefore \angle BCE < \angle CBD$

$\Rightarrow \frac{1}{2}\angle BCE < \frac{1}{2}\angle CBD$

($\because$ OC and OB are the bisectors of angles)

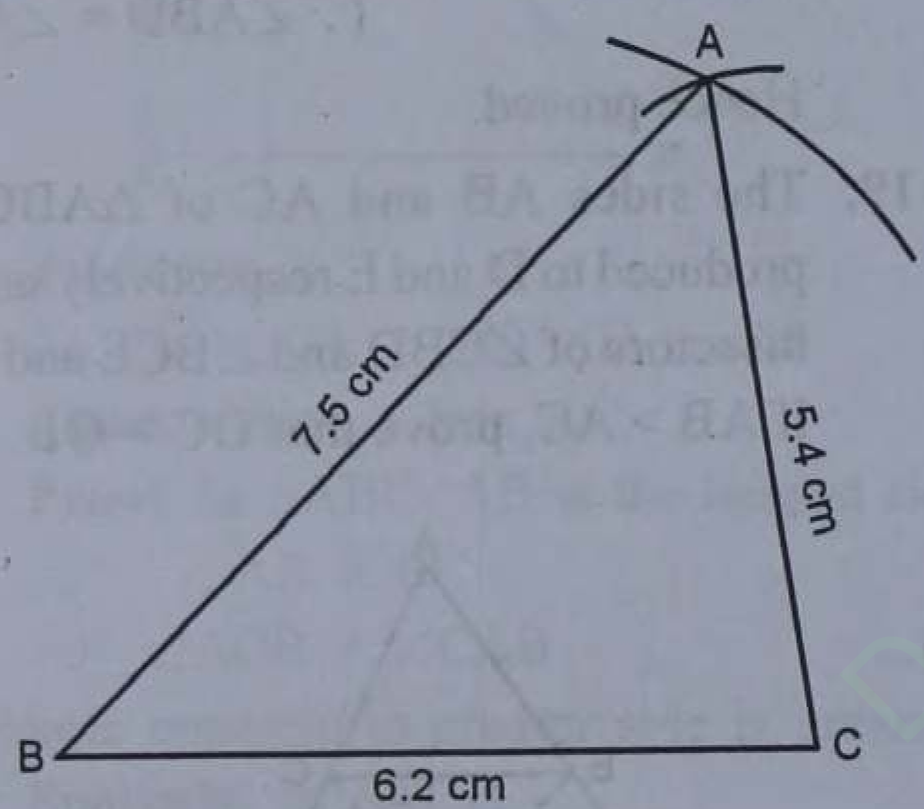
$\Rightarrow \angle BCO < \angle CBO \Rightarrow \angle CBO > \angle BCO$

$\therefore \quad OC > OB$

(Sides opposite to greater angles)

Hence proved.

Q. 20. In ΔABC, AB = 7·5 cm, BC = 6·2 cm and AC = 5·4 cm. Name : (*i*) The least angle (*ii*) The greatest angle of the triangle.



Sol. In ΔABC,

AC is the shortest side of the triangle.

$\therefore \angle B$ is the least angle

(Angle opposite to the smallest side)

Similarly AB is the largest side of the triangle.

$\therefore \angle C$ is the greatest angle.

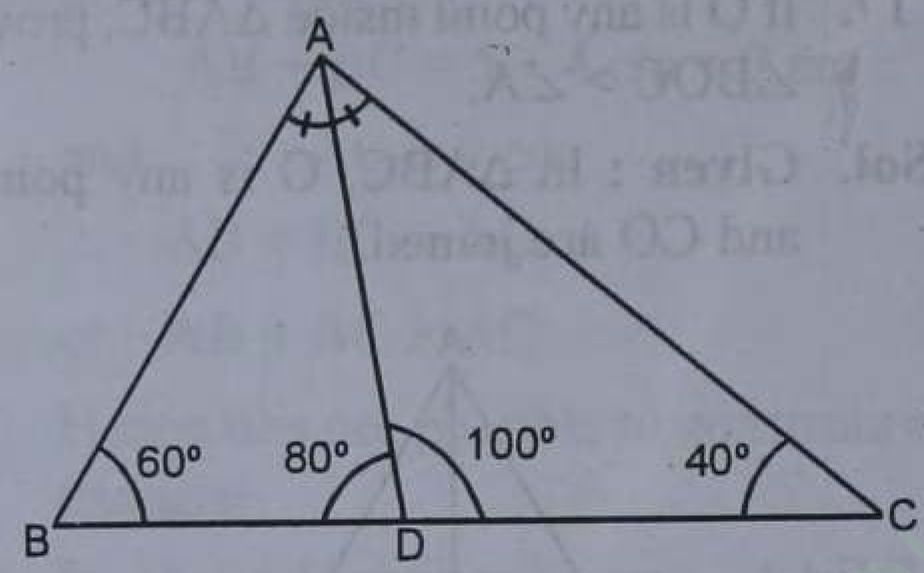
Q. 21. In the given figure, AD bisects $\angle A$. If $\angle B = 60^\circ$, $\angle C = 40^\circ$, then arrange AB, BD and DC in ascending order of their lengths.

Sol. In ΔABC, $\angle B = 60^\circ$, $\angle C = 40^\circ$

$\therefore \angle A = 180^\circ - (\angle B + \angle C)$

$= 180^\circ - (60^\circ + 40^\circ)$

$= 180^\circ - 100^\circ = 80^\circ$



$\because$ AD is the bisector of $\angle A$

$\therefore \angle BAD = \angle CAD = \frac{80^\circ}{2} = 40^\circ$

Now $\angle C = 40^\circ$, $\angle BAD = 40^\circ$

and $\angle CAD = 40^\circ$

Now $\angle ADB = 180^\circ - (\angle B + \angle BAD)$

$= 180^\circ - (60^\circ + 40^\circ) = 180^\circ - 100^\circ = 80^\circ$

and $\angle ADC = 180^\circ - (\angle C + \angle CAD)$

$= 180^\circ - (40^\circ + 40^\circ) = 180^\circ - 80^\circ = 100^\circ$

$\therefore$ In ΔABD,

AB > BD and AB > AD

and in ΔACD,

DC = AD ($\because \angle C = \angle CAD = 40^\circ$)

$\therefore$ AB > BD and AB > DC

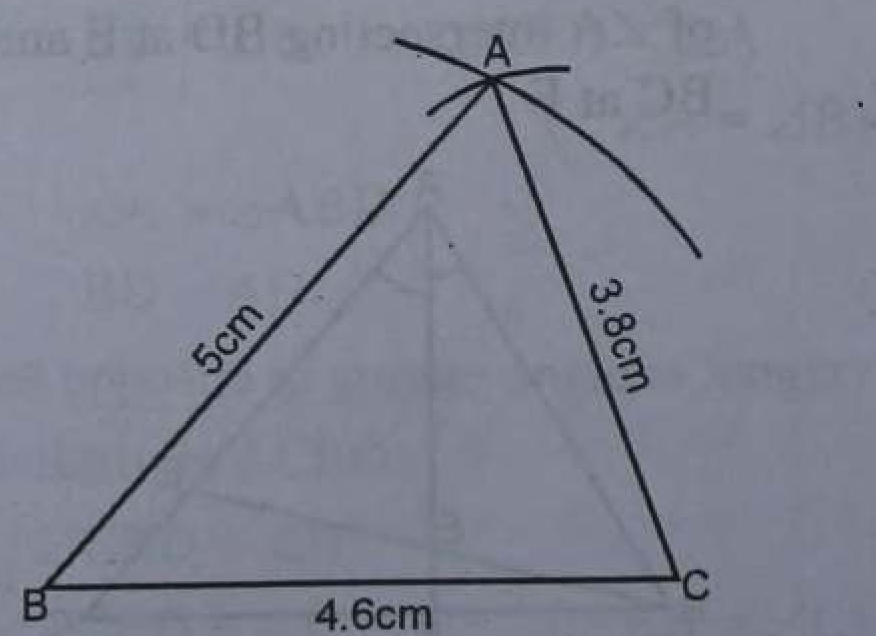
$\therefore$ BD = DC < AB Hence proved.

EXERCISE 10 (D)

Q. 1. Construct a ΔABC in which AB = 5 cm, BC = 4·6 cm and CA = 3·8 cm.

Sol. Steps of construction :

(*i*) Draw a line segment BC = 4·6 cm.



(*ii*) With centre B and radius 5 cm draw an arc.

(*iii*) With centre C and radius 3·8 cm draw another arc intersecting the first arc at A.

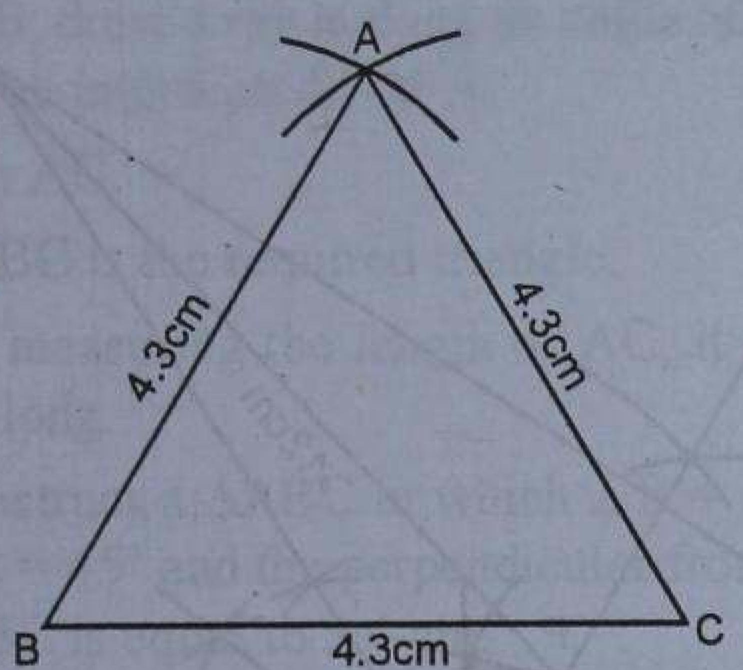
(*iv*) Join AB and AC.

Then ΔABC is the required triangle.

Q. 2. Construct an equilateral triangle of side 4·3 cm.

Sol. Steps of construction :

(*i*) Draw a line segment BC = 4·3 cm.



(*ii*) With centres B and C and radius 4·3 cm each, draw two arcs intersecting each other at A.

(*iii*) Join AB and AC.

Then ΔABC is the required equilateral triangle.

Q. 3. Construct a ΔABC in which AB = 4·5 cm, AC = 3·5 cm and ∠BAC = 75°. Draw the bisector of ∠BAC and the perpendicular bisector of BC to meet at a point M. Measure ∠BMC.

Sol. Steps of construction :

(*i*) Draw a line segment AB = 4·5 cm.

(*ii*) At A, construct ∠BAC = 75° and cut off AC = 3·5 cm.

(*iii*) Join BC.

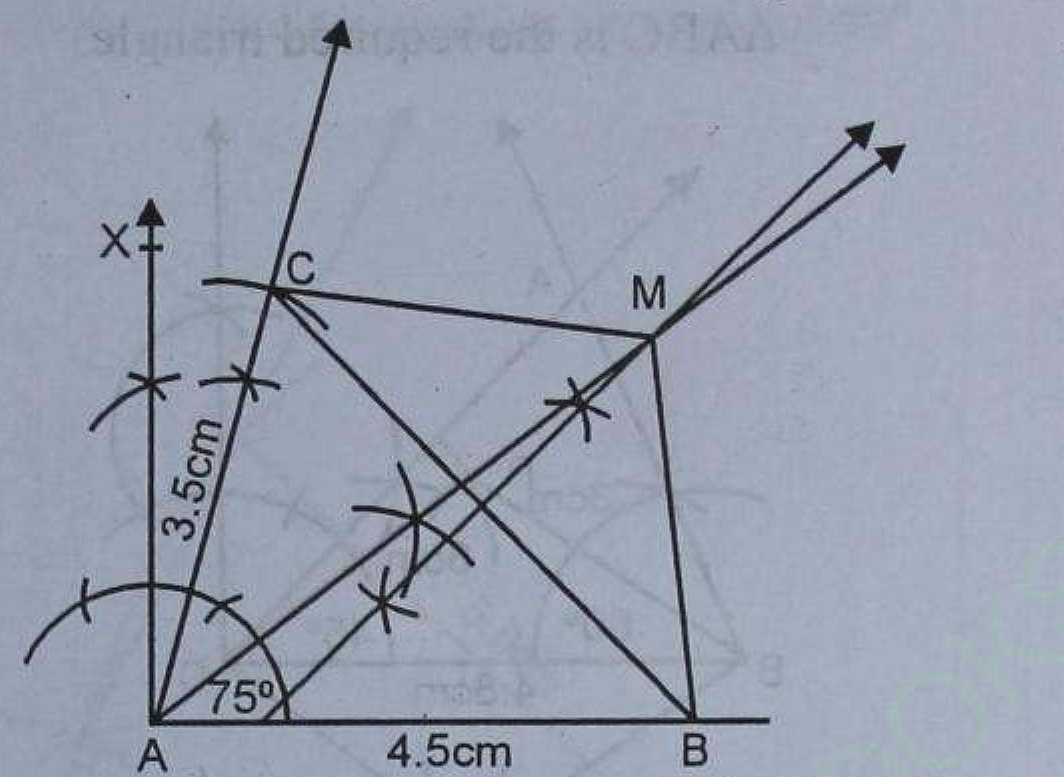
ΔABC is the required triangle.

(*iv*) Draw the bisector of ∠BAC.

(*v*) Draw the perpendicular bisector of side BC which intersects the angle bisector at M.

(*vi*) Join BM and CM.

On measuring ∠BMC, it is equal to 135°.



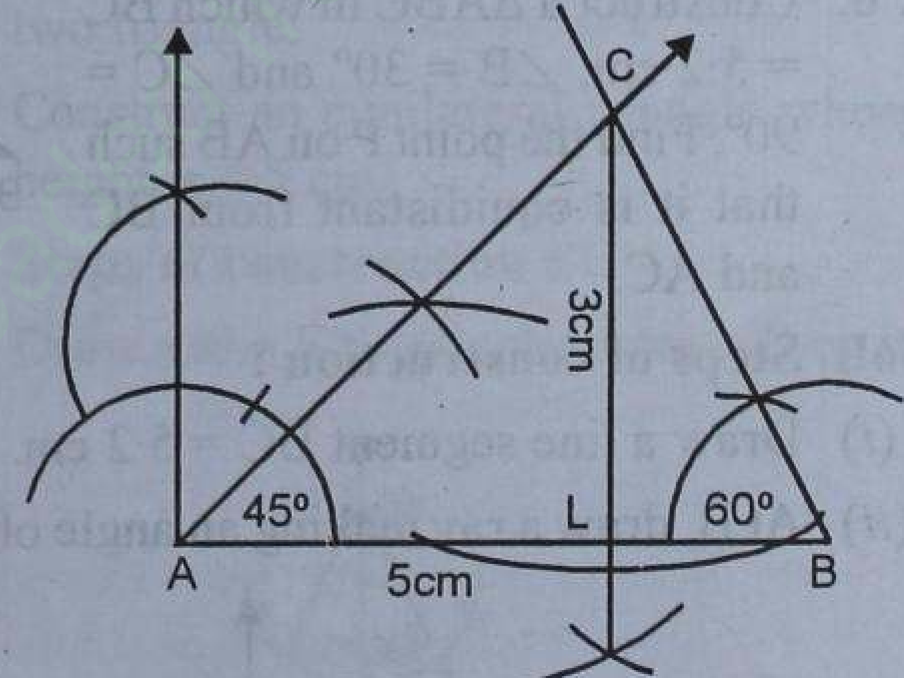
Q. 4. Construct a ΔABC in which AB = 5 cm, ∠A = 45° and ∠B = 60°. Draw CL ⊥ AB. Measure the length of CL.

Sol. Steps of construction :

(*i*) Draw a line segment AB = 5 cm.

(*ii*) At A, draw a ray making an angle of 45°.

(*iii*) AT B, draw a ray making an angle of 60° which intersects the first ray at C.



(*iv*) Join AC and BC. ΔABC is the required triangle.

(*v*) From C, draw C ⊥ AB which meet AB at L.

On measuring CL, it is 3 cm long.

Q. 5. Construct a ΔABC in which BC = 4·8 cm, ∠B = 60° and ∠C = 45°. Locate the point P on the side of BC opposite to A such that BP = CP = 3 cm.

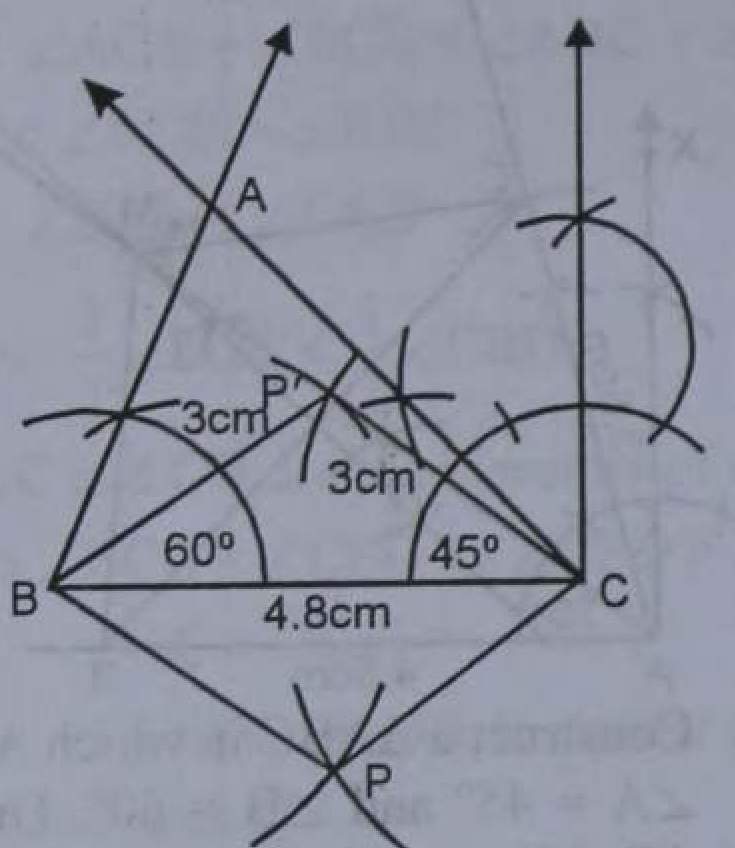
Sol. Steps of construction :

(*i*) Draw a line segment BC = 4·8 cm.

(*ii*) At B, draw a ray making an angle of 60°.

(*iii*) AC, draw another ray making an angle of 45° which intersects the first ray at A.

ΔABC is the required triangle.



(*iv*) With centre B and C and radius 3 cm each draw arcs intersecting each other at P and P'.

(*v*) Join PB, PC and P'B and P'C.

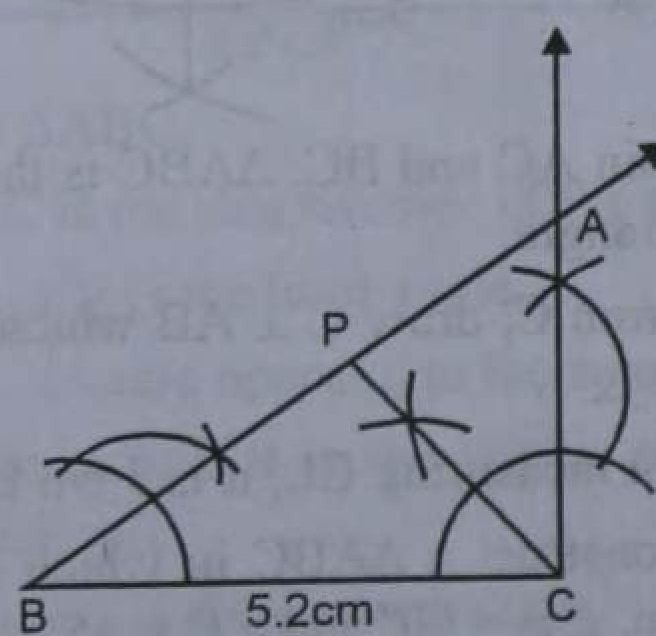
P and P' are the required points.

Q. 6. Construct a ΔABC in which BC = 5·2 cm, $\angle B = 30°$ and $\angle C = 90°$. Find the point P on AB such that it is equidistant from BC and AC.

Sol. Steps of construction :

(*i*) Draw a line segment BC = 5·2 cm.

(*ii*) At B, draw a ray making an angle of 30°.



(*iii*) At C, draw another ray making an angle of 90° which intersects the first ray at A.

Then ΔABC is the required triangle.

(*iv*) Draw the bisector of $\angle BCA$ which meets AB at P.

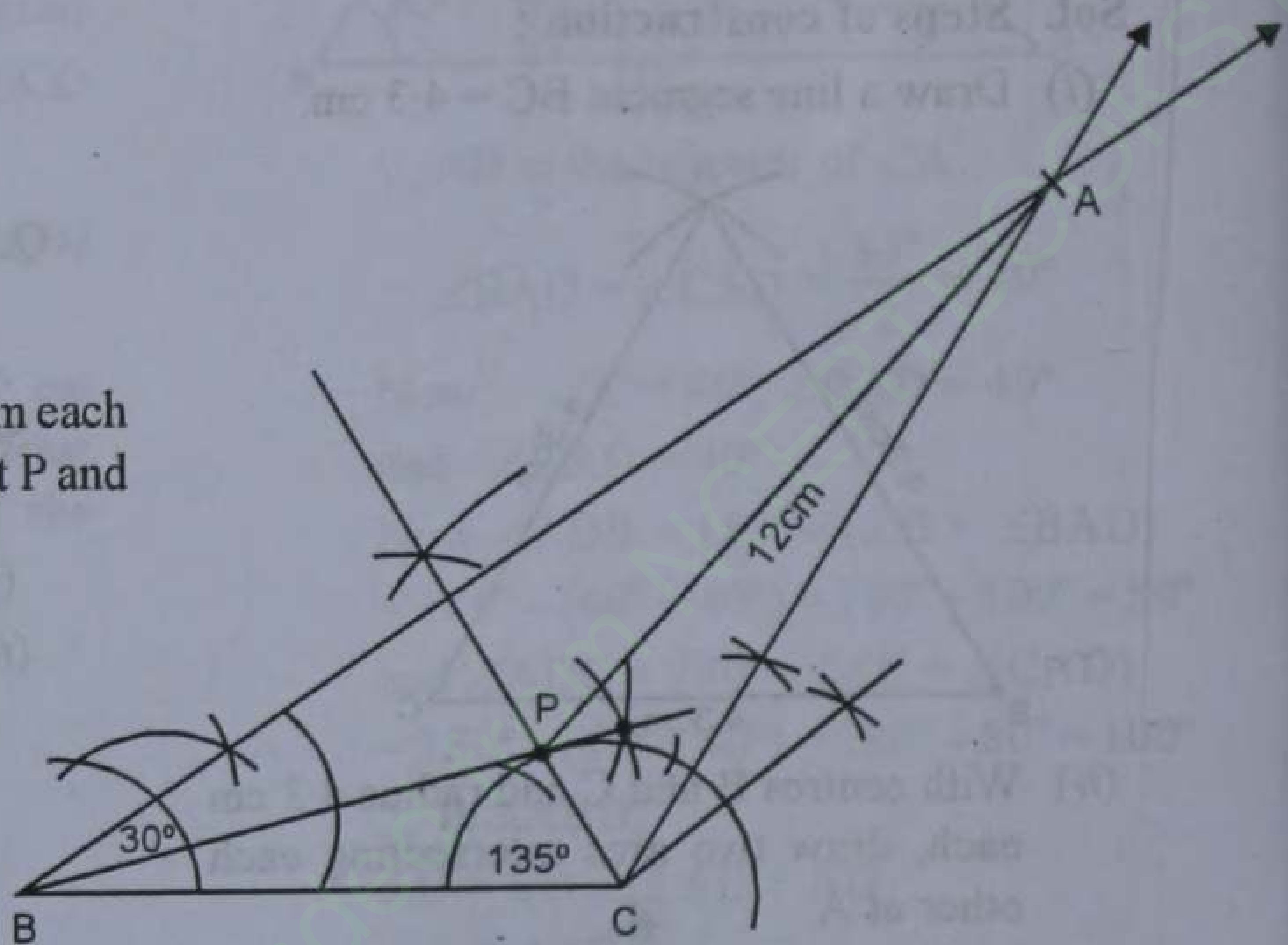
P is the required point which is equidistant from BC and AC.

Q. 7. Construct a ΔABC in which BC = 6 cm, $\angle B = 30°$ and $\angle C = 135°$. Bisect $\angle B$ and $\angle C$. Let these bisectors meet at P. Measure distance PA.

Sol. Steps of construction :

(*i*) Draw a line segment BC = 6 cm.

(*ii*) At B, draw a ray making an angle of 30°.



(*iii*) At C, draw another ray making an angle of 135° which intersects the ray at A.

Then ΔABC is the required triangle.

(*iv*) Draw the bisectors of $\angle B$ and $\angle C$ intersecting each other at P.

(*v*) Join PA.

Measuring PA, it is 12 cm (approximately).

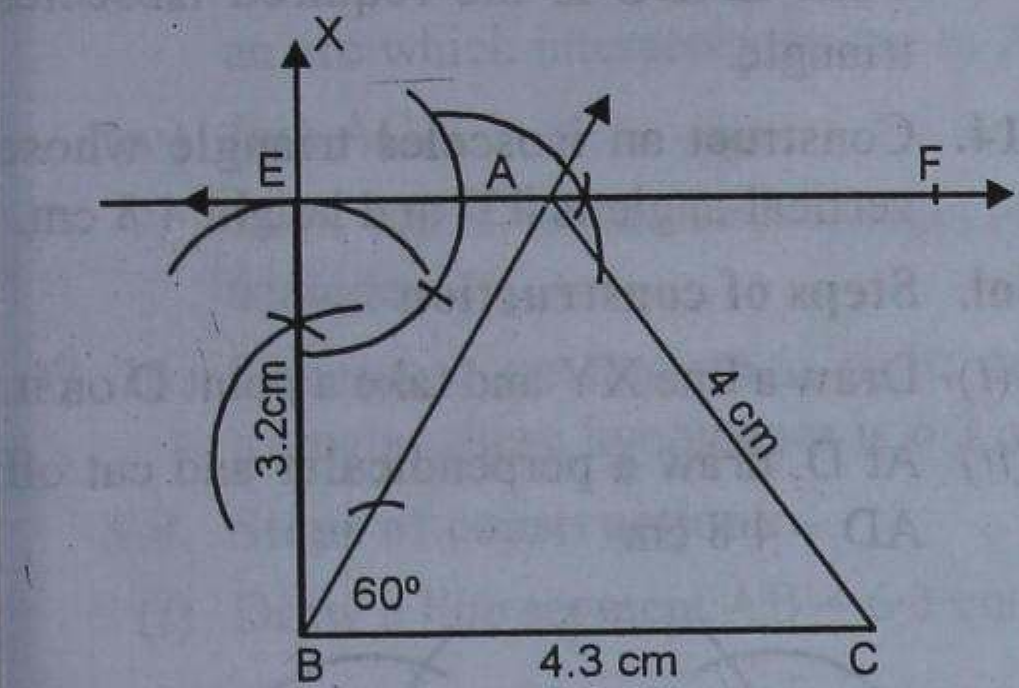
Q. 8. Construct a ΔABC in which BC = 4·3 cm, $\angle B = 60°$ and length of perpendicular from vertex A to the base is 3·2 cm. Measure AC.

Sol. Steps of construction :

(*i*) Draw a line segment BC = 4·3 cm.

(*ii*) Draw a ray at B, making an angle of 60°.

(*iii*) Draw a perpendicular at B and cut off BE = 3·2 cm.



(*iv*) From E, draw a line EF parallel to BC.

(*v*) At B, draw a ray making an angle of 60° which intersects EF at A.

(*vi*) Join AC.

ΔABC is the required triangle.

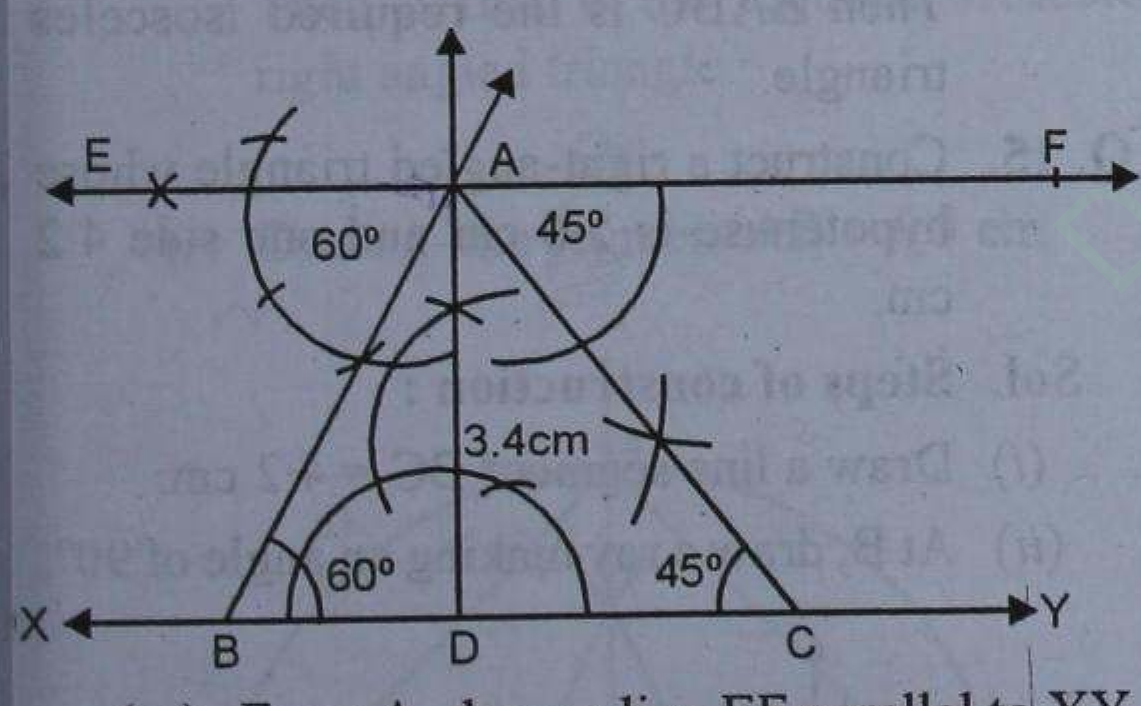
On measuring the length of AC, it is 4 cm long.

Q. 9. Construct a ΔABC in which $\angle B = 60^\circ$, $\angle C = 45^\circ$ and the perpendicular from A to BC is equal to 3·4 cm.

Sol. Steps of construction :

(*i*) Draw a line XY and take a point D on it.

(*ii*) At D, draw a perpendicular and cut off DA = 3·4 cm.



(*iii*) From A, draw a line EF parallel to XY.

(*iv*) At A draw an angle EAB equal to 60° and $\angle FAC = 45^\circ$ meeting XY at B and C respectively.

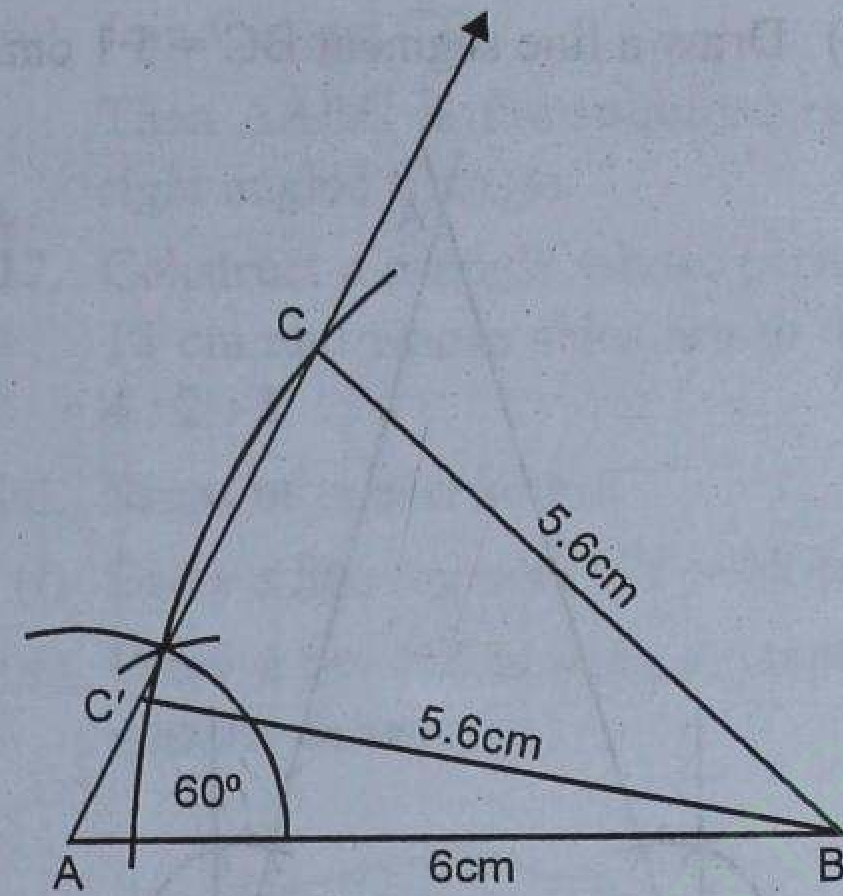
ΔABC is the required triangle.

Q. 10. Construct a ΔABC in which AB = 6 cm, BC = 5·6 cm and $\angle CAB = 60^\circ$.

Sol. Steps of construction :

(*i*) Draw a line segment AB = 6 cm.

(*ii*) At A, draw a ray making an angle of 60°.



(*iii*) With centre B and radius 5·6 cm draw an arc which intersects the ray at C and C′.

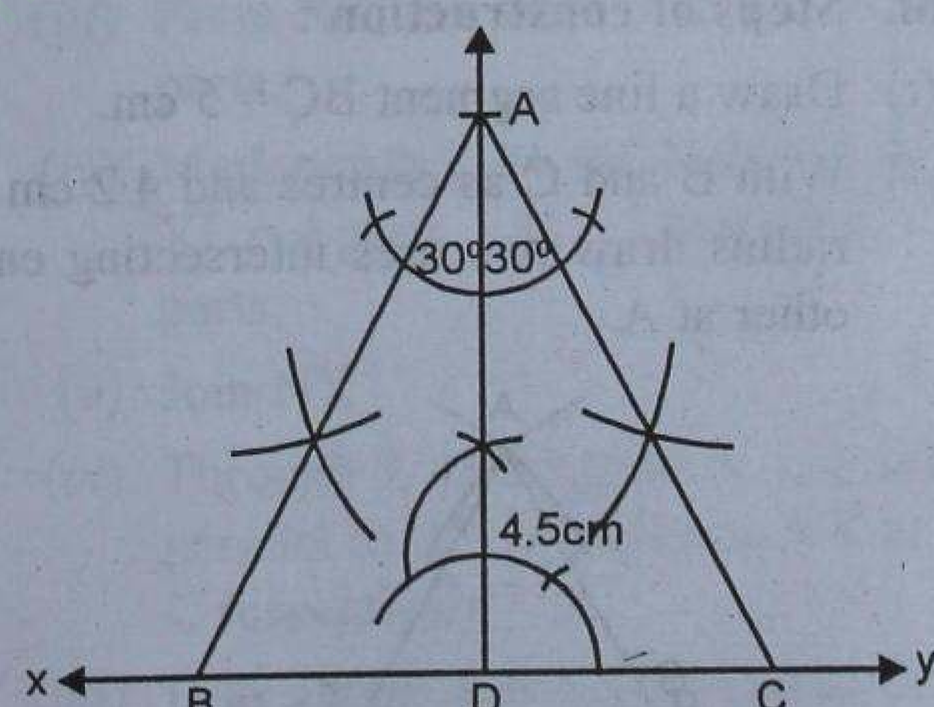
(*iv*) Join BC and BC′.

Then ΔABC and $\Delta ABC'$ are the required two triangle.

Q. 11. Construct an equilateral triangle whose height is 4·5 cm.

Sol. Steps of construction :

(*i*) Draw a line XY and take a point D on it.



(*ii*) At D, draw a perpendicular and cut off DA = 4·5 cm.

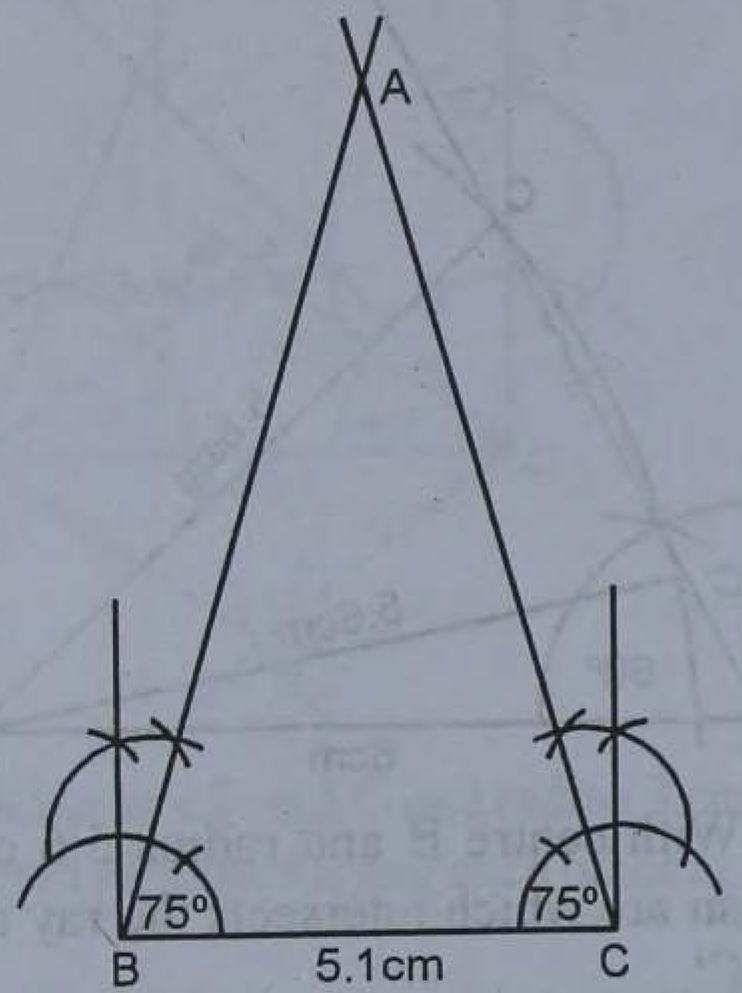
(*iii*) At A draw two rays making an angle of 30° on each side of AD which meet XY at B and C respectively.

Then ΔABC is the required equilateral triangle.

Q. 12. Construct an isosceles triangle ABC in which base BC = 5·1 cm and ∠B = 75°.

Sol. Steps of construction :

(*i*) Draw a line segment BC = 5·1 cm.



(*ii*) At B and C, draw two rays making an angle of 75° on each point which meet each other at A.

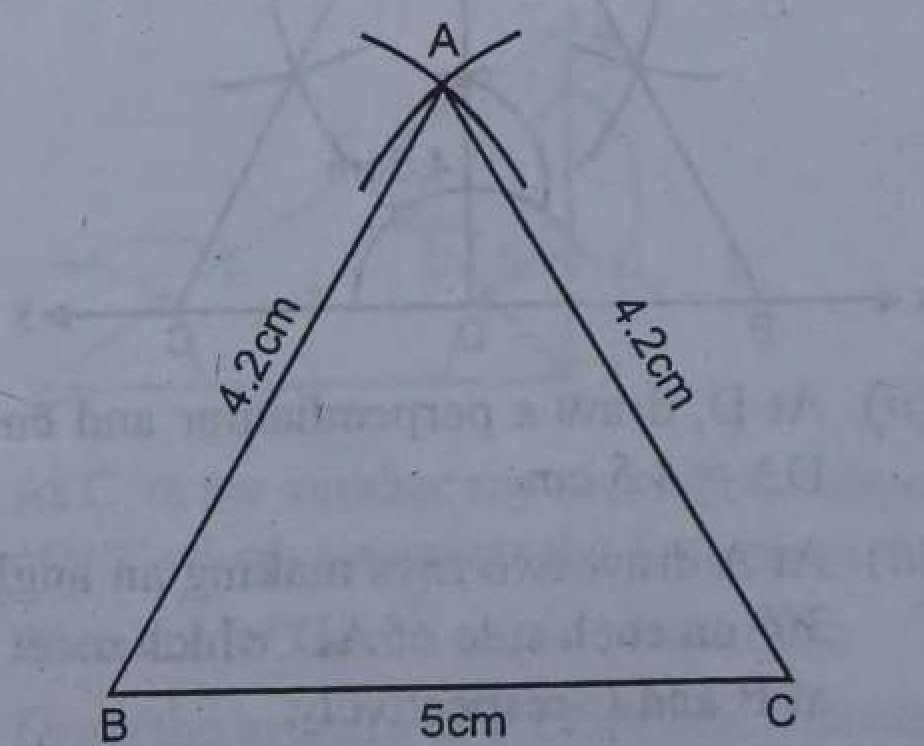
Then ΔABC is the required isosceles triangle.

Q. 13. Construct an isosceles triangle ABC in which base BC = 5 cm and AB = 4·2 cm.

Sol. Steps of construction :

(*i*) Draw a line segment BC = 5 cm.

(*ii*) With B and C as centres and 4·2 cm as radius draw two arcs intersecting each other at A.



(*iii*) Join AB and AC.

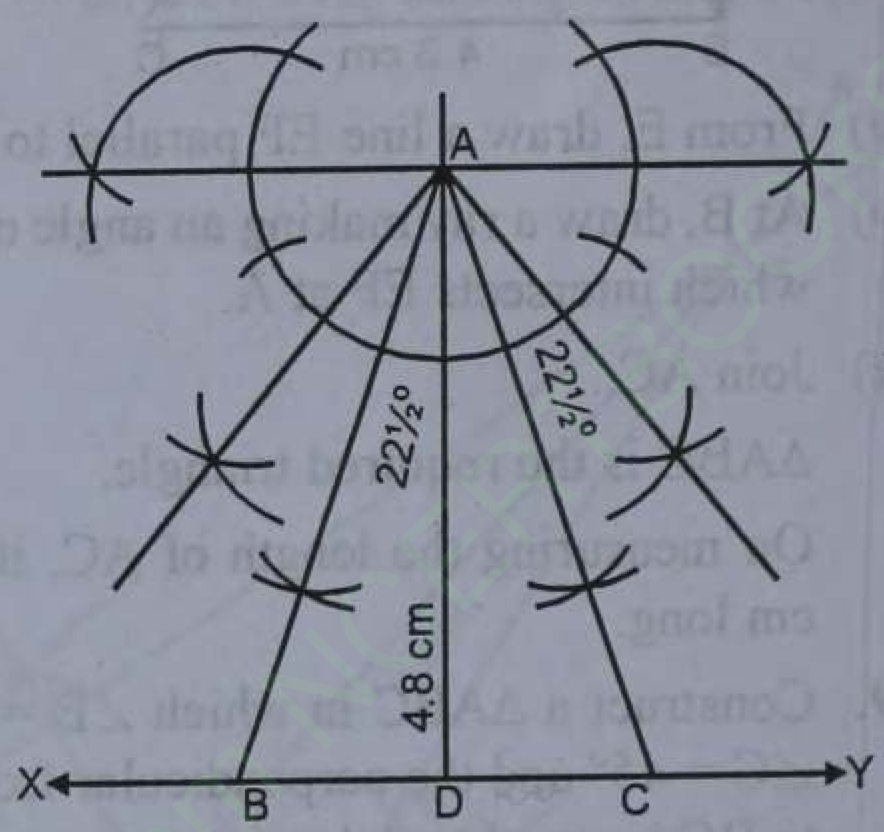
Then ΔABC is the required isosceles triangle.

Q. 14. Construct an isosceles triangle whose vertical angle is 45° and height 4·8 cm.

Sol. Steps of construction :

(*i*) Draw a line XY and take a point D on it.

(*ii*) At D, draw a perpendicular and cut off AD = 4·8 cm.



(*iii*) At A, draw two rays making an angle of $22\frac{1^{\circ}}{2}$ on each side of AD which meet XY at B and C respectively.

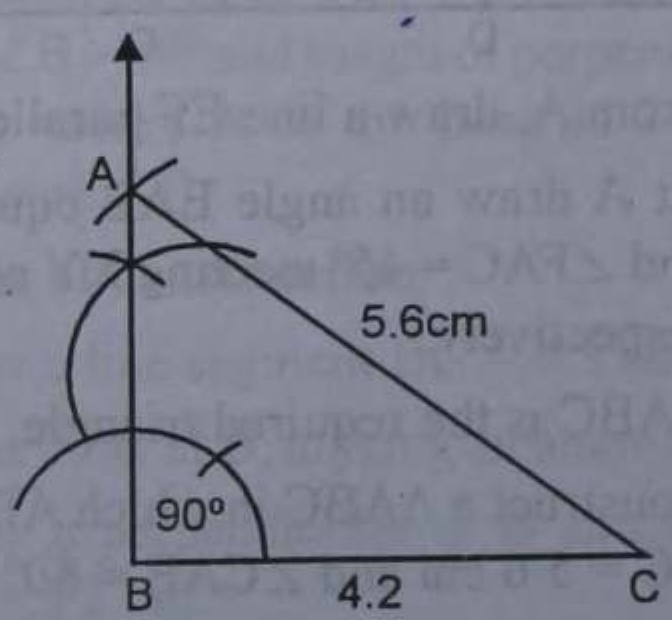
Then ΔABC is the required isosceles triangle.

Q. 15. Construct a right-angled triangle whose hypotenuse is 5·6 cm and one side 4·2 cm.

Sol. Steps of construction :

(*i*) Draw a line segment BC = 4·2 cm.

(*ii*) At B, draw a ray making an angle of 90°.



(*iii*) With centre C and radius 5·6 cm draw an arc which intersects the ray at A.

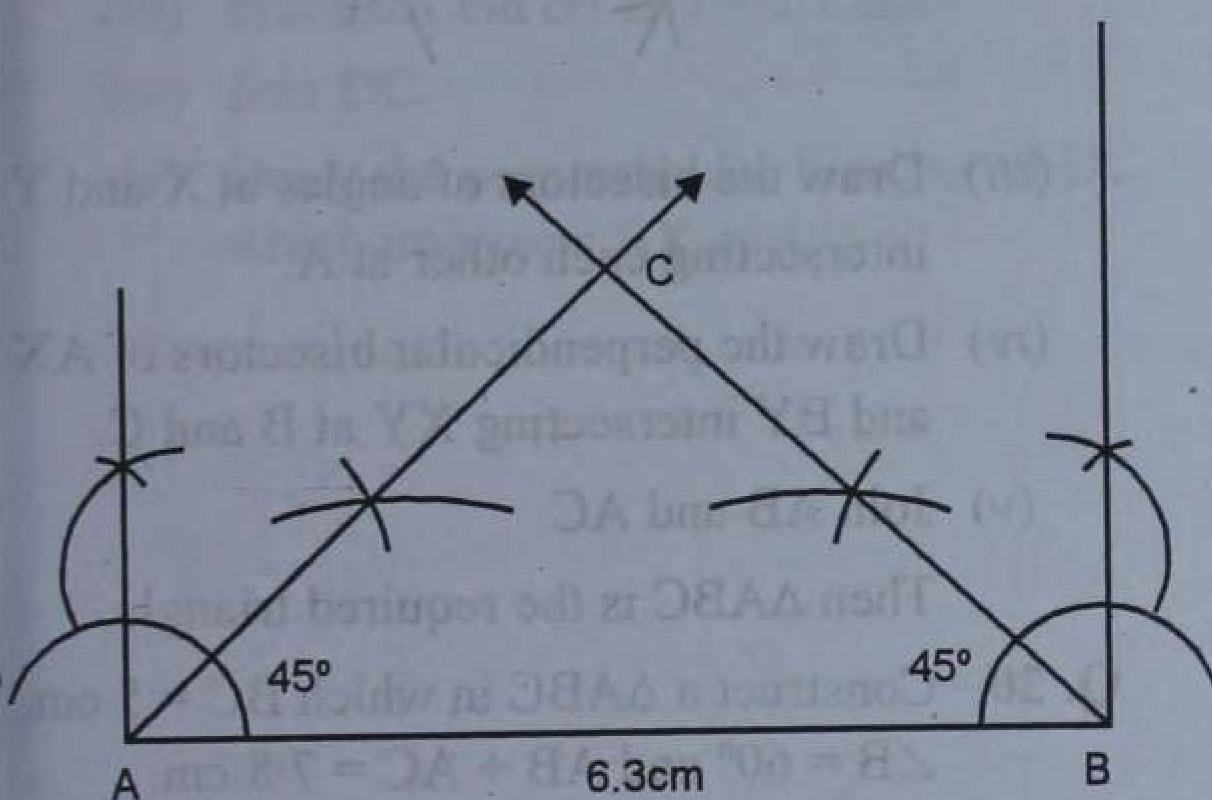
(*iv*) Join AC.

Then ΔABC is the required right angled triangle.

Q. 16. Construct an isosceles right-angled triangle whose hypotenues is 6·3 cm.

Sol. Steps of construction :

(*i*) Draw a line segment AB = 6·3 cm.

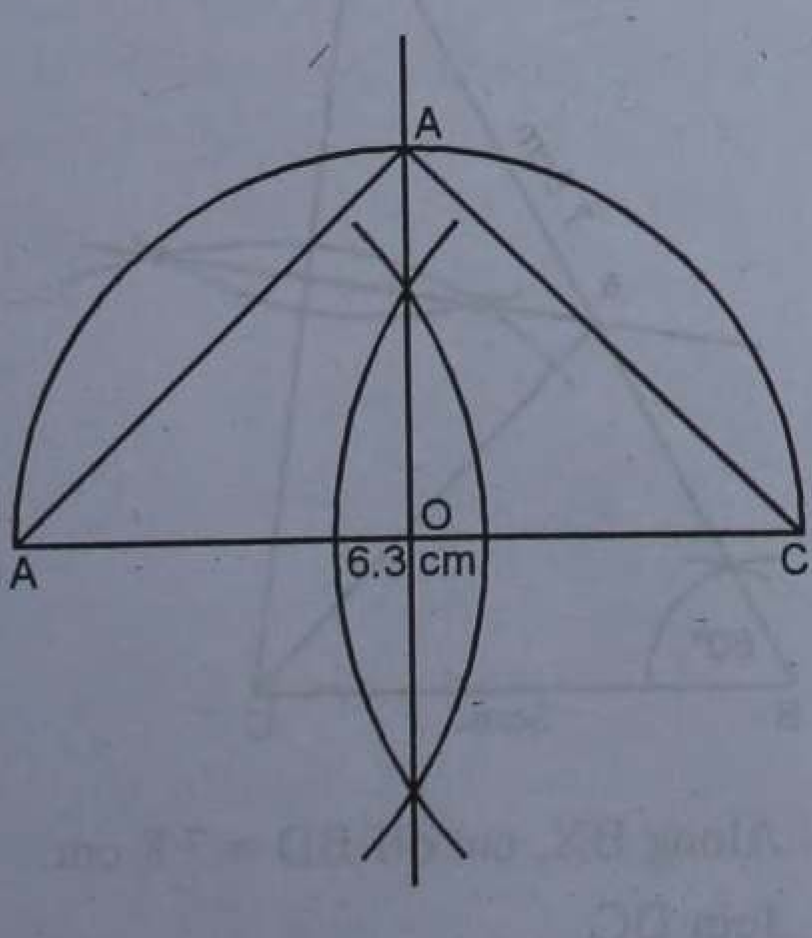


(*ii*) At A and B, draw two rays making an angle of 45° on each point intersecting each other at C.

Then ΔABC is the required isosceles right angled triangle.

Or

(*i*) Draw a line segment AB = 6·3 cm.



(*ii*) Draw a semi-circle on AB as diameter.

(*iii*) Draw the perpendicular bisector of AB intersecting the semi-circle at C.

(*iv*) Join CA and CB.

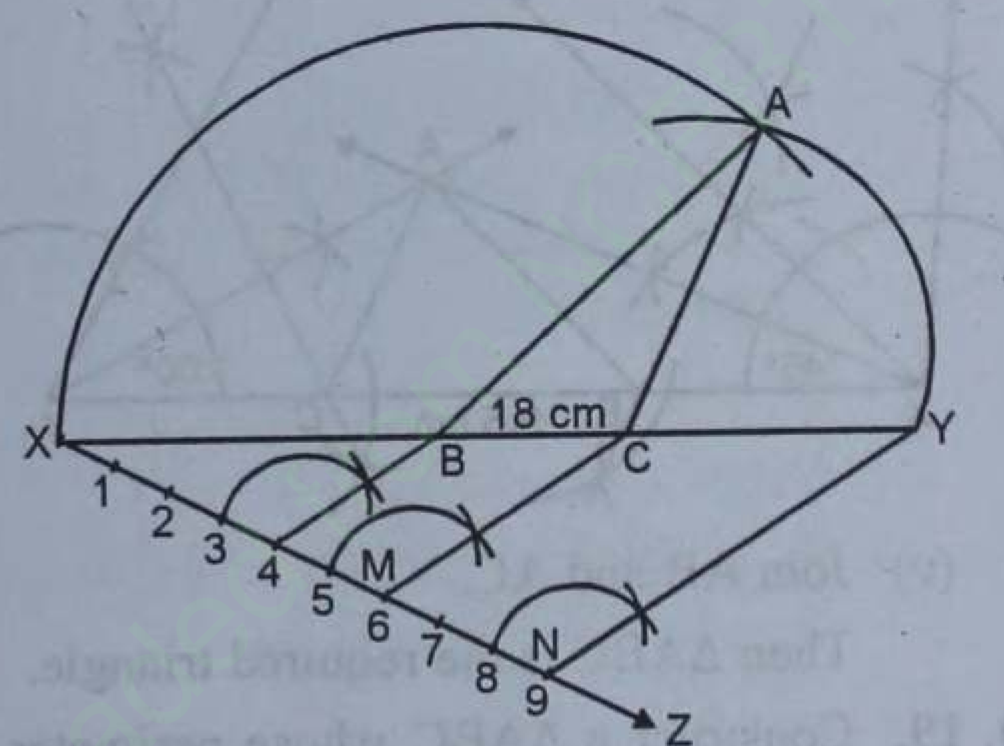
Then ΔABC is the required isosceles right angled triangle.

Q. 17. Construct a triangle whose perimeter is 18 cm and whose sides are in the ratio 4 : 2 : 3.

Sol. Steps of construction :

(*i*) Draw a line segment XY = 18 cm.

(*ii*) Draw a ray XZ making an acute angle with its downward.



(*iii*) From XZ, cut off 4 + 2 + 3 = 9 equal parts.

(*iv*) Mark points L, M and N on XZ such that XL = 4 parts, LM = 2 parts and MN = 3 parts.

(*v*) Join NY.

(*vi*) Through L and M, draw LB and MC parallel to NY intersecting XY at B and C respectively.

(*vii*) B as centre and BX as radius draw an arc.

(*viii*) C as centre and CY as radius draw another arc intersecting the first arc at A.

(*ix*) Join AB and AC.

Then ΔABC is the required triangle.

Q. 18. Construct a ΔABC whose perimeter is 9·5 cm and base angles 45° and 60°.

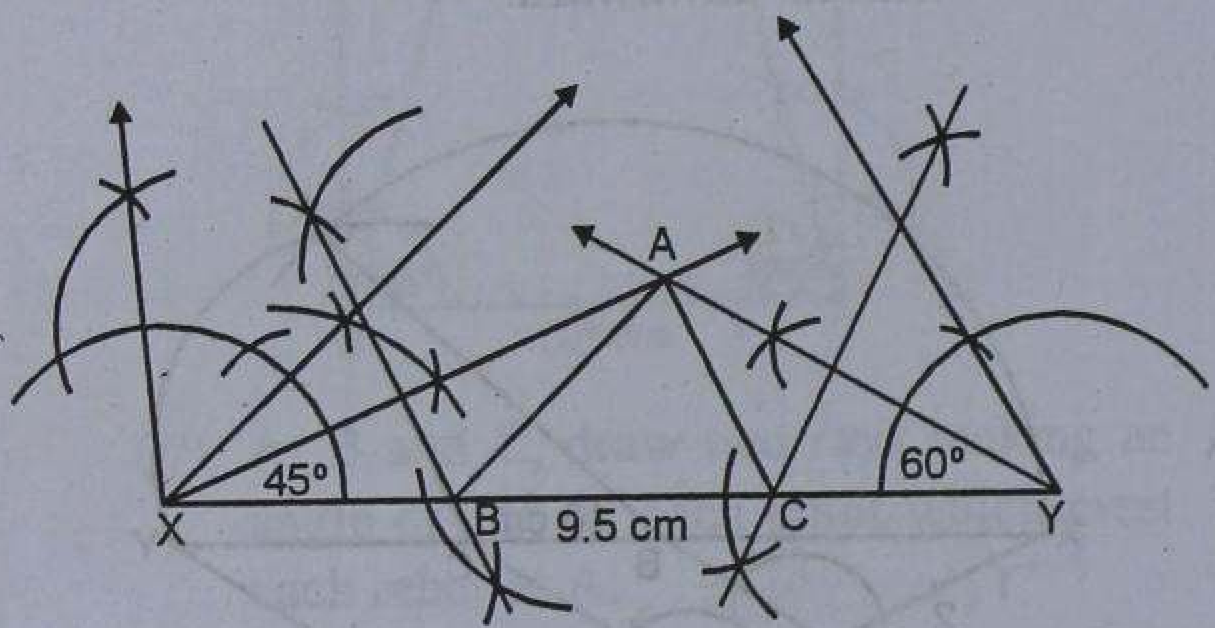
Sol. Steps of construction :

(*i*) Draw a line segment XY = 9·5 cm.

(*ii*) Draw a ray at X making an angle of 45° and another ray at Y making an angle of 60°.

(*iii*) Draw the bisectors of angles at X and Y intersecting each other at A.

(*iv*) Draw perpendicular bisectors of AX and AX to intersect XY at B and C respectively.



(*v*) Join AB and AC.

Then ΔABC is the required triangle.

Q. 19. Construct a ΔABC whose perimeter is 12 cm and the angles are in the ratio 3 : 4 : 5.

Sol. Ratio of angles = 3 : 4 : 5

Let $\angle A = 3x$, $\angle B = 4x$ and $\angle C = 5x$

$\therefore\ 3x + 4x + 5x = 180°$

(Sum of angles of a triangle)

$$\Rightarrow 12x = 180° \Rightarrow x = \frac{180°}{12} = 15°$$

$\therefore\ \angle A = 3x = 3 \times 15° = 45°$

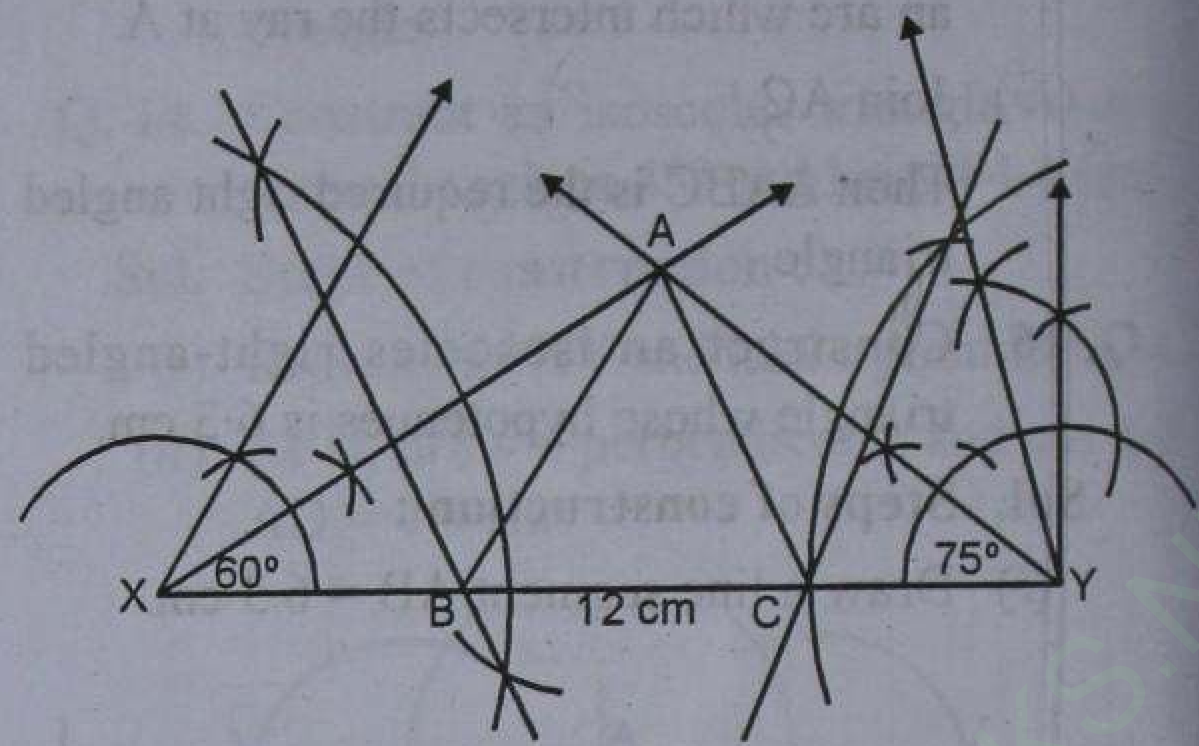
$\angle B = 4x = 4 \times 15° = 60°$

and $\angle C = 5x = 5 \times 15° = 75°$

Steps of construction :

(*i*) Draw a line segment XY = 12 cm.

(*ii*) At X, draw a ray making an angle of 60° and at Y, draw another ray making an angle of 75°.



(*iii*) Draw the bisectors of angles at X and Y intersecting each other at A.

(*iv*) Draw the perpendicular bisectors of AX and BY intersecting XY at B and C.

(*v*) Join AB and AC.

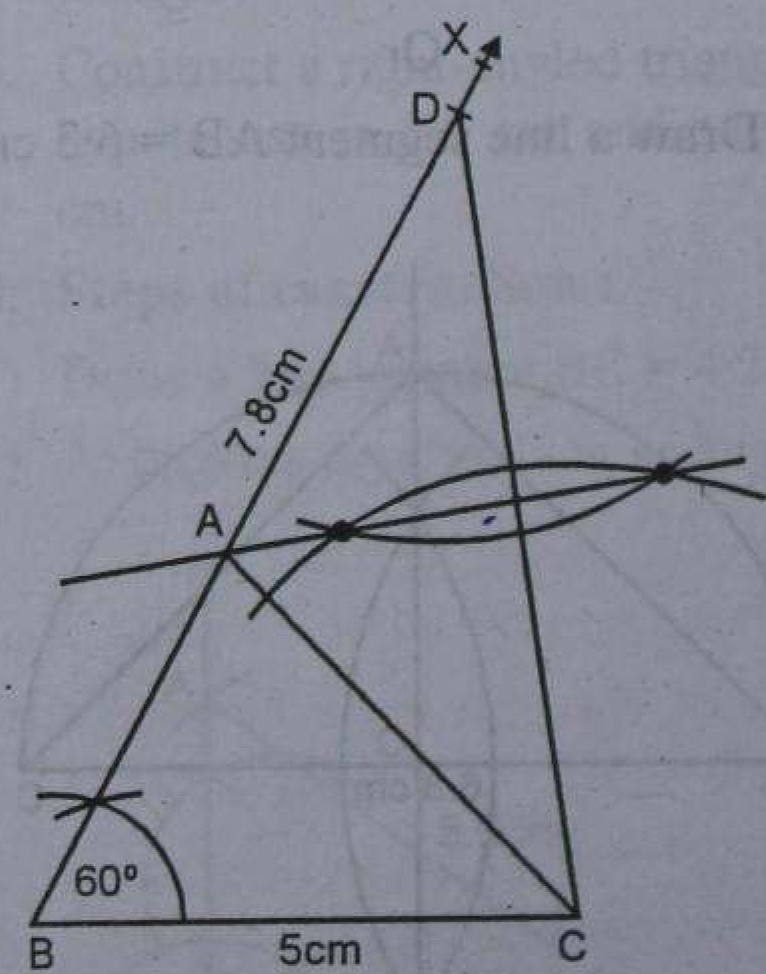
Then ΔABC is the required triangle.

Q. 20. Construct a ΔABC in which BC = 5 cm, $\angle B = 60°$ and AB + AC = 7·8 cm.

Sol. Steps of construction :

(*i*) Draw a line segment BC = 5 cm.

(*ii*) At B, draw a ray BX making an angle of 60°.



(*iii*) Along BX, cut off BD = 7·8 cm.

(*iv*) Join DC.

(*v*) Draw the perpendicular bisector of DC which intersects BP at A.

(*vi*) Join AC.

Then ΔABC is the required triangle.

Q. 21. Construct a ΔABC in which $BC = 6$ cm, $\angle B = 30^\circ$ and $AB - AC = 3{\cdot}1$ cm.

Sol. Steps of construction :

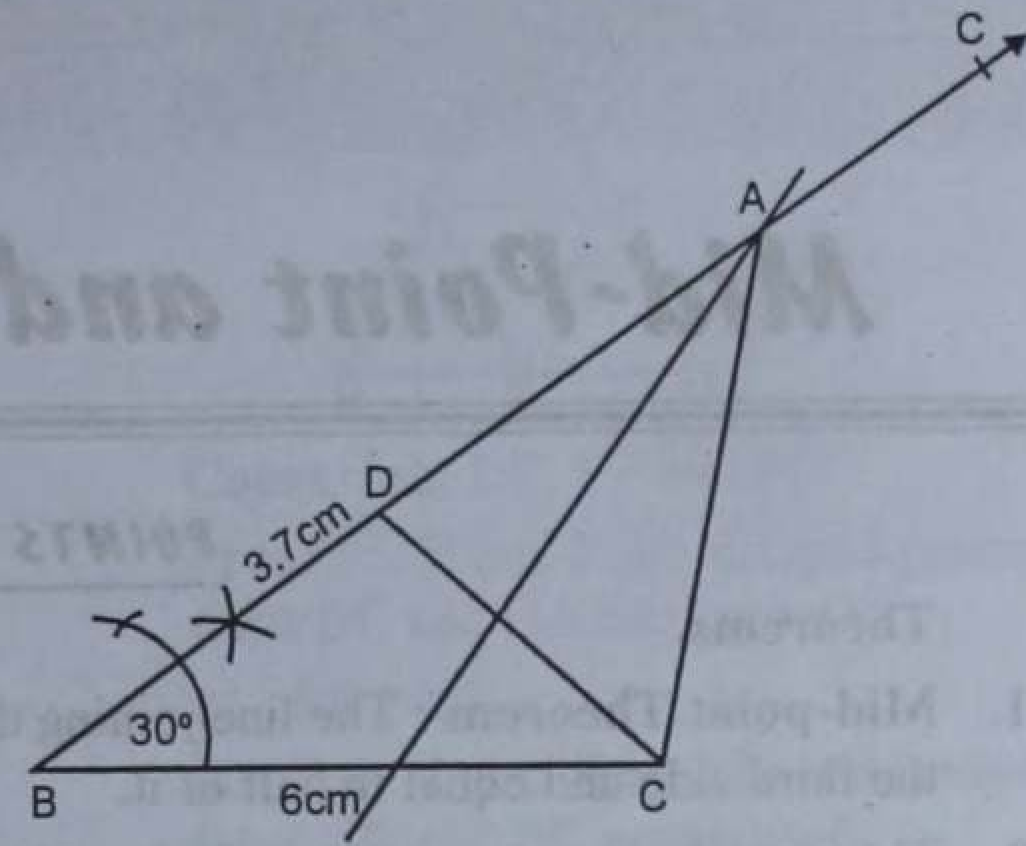
(*i*) Draw a line segment $BC = 6$ cm.

(*ii*) At B, draw a ray BX making an angle of 30°.

(*iii*) From BX cut off $BD = 3{\cdot}1$ cm.

(*iv*) Join DC.

(*v*) Draw the perpendicular bisector of DC which intersects BX at A.



(*vi*) Join AC.

Then ΔABC is the required triangle.