

POINTS TO REMEMBER

1. **Indices** : For any real number 'a' and positive integer 'n', we define

$$a \times a \times a \dots \text{ to } n \text{ factors} = a^n.$$

Here 'a' is called base and n is called index or exponent.

Some Laws of Indices

$$(i) a^0 = 1, \quad (ii) a^{-n} = \frac{1}{a^n} \quad (iii) a^m \times a^n = a^{m+n} \quad (iv) a^m \div a^n = a^{m-n}$$

$$(v) (a^m)^n = a^{mn} \quad (vi) a^{-m} = \frac{1}{a^m} \quad (vii) (ab)^m = a^m \cdot b^m \quad (viii) \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$(ix) \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad (x) \text{ If } a^m = a^n, \text{ then } m = n \text{ where } a > 0 \text{ and } a \neq 1.$$

(xi) If $p^m \times q^n \times r^o = p^a q^b r^c$, then $m = a$, $n = b$ and $o = c$ where p, q and r are different primes.

(xii) If $a^m = b^m$, then $a = b$ where a and b are positive.

$$(xiii) \sqrt{a} = a^{\frac{1}{2}}, \quad \sqrt[3]{a} = a^{\frac{1}{3}}, \quad \sqrt[4]{a} = a^{\frac{1}{4}}$$

Some formulae which are used

$$(i) (a + b)(a - b) = a^2 - b^2 \quad (ii) (a - b)(a^2 + ab + b^2) = a^3 - b^3$$

EXERCISE 8

Evaluate :

Q. 1. (i) $(125)^{\frac{1}{3}}$ (ii) $(8)^{\frac{2}{3}}$

(iii) $\left(\frac{1}{5}\right)^{-2}$ (iv) $(16)^{\frac{-3}{4}}$

(v) $(32)^{\frac{-4}{5}}$ (vi) $\left(\frac{8}{125}\right)^{\frac{-1}{3}}$

(vii) $(-27)^{\frac{2}{3}}$ (viii) $(0.001)^{\frac{-1}{3}}$

(ix) $(0.027)^{\frac{-2}{3}}$

Sol. (i) $(125)^{\frac{1}{3}} = (5 \times 5 \times 5)^{\frac{1}{3}} = (5^3)^{\frac{1}{3}}$

$$= 5^{3 \times \frac{1}{3}} = 5 \text{ Ans.}$$

(ii) $(8)^{\frac{2}{3}} = (2 \times 2 \times 2)^{\frac{2}{3}} = (2^3)^{\frac{2}{3}}$

$$= 2^{3 \times \frac{2}{3}} = 2^2 = 2 \times 2 = 4 \text{ Ans.}$$

$$\{\because (a^m)^n = a^{mn}\}$$

(iii) $\left(\frac{1}{5}\right)^{-2} = \left(\frac{5}{1}\right)^2 = 5 \times 5 = 25 \text{ Ans.}$

(iv) $(16)^{\frac{-3}{4}} = (2 \times 2 \times 2 \times 2)^{\frac{-3}{4}} = (2^4)^{\frac{-3}{4}}$

$$= 2^{\frac{-3}{4} \times 4} = 2^{-3} = \frac{1}{2^3}$$

$$\left\{ \because (a^m)^n = a^{mn} \text{ and } a^{-m} = \frac{1}{a^m} \right\}$$

$$= \frac{1}{2 \times 2 \times 2} = \frac{1}{8} \text{ Ans.}$$

$$(v) (32)^{\frac{-4}{5}} = (2 \times 2 \times 2 \times 2 \times 2)^{\frac{-4}{5}}$$

$$= (2^5)^{\frac{-4}{5}} = 2^{\frac{-4}{5} \times 5} = 2^{-4} = \frac{1}{2^4}$$

$$\left\{ \because (a^m)^n = a^{mn} \text{ and } a^{-m} = \frac{1}{a^m} \right\}$$

$$= \frac{1}{2 \times 2 \times 2 \times 2} = \frac{1}{16} \text{ Ans.}$$

$$(vi) \left(\frac{8}{125} \right)^{\frac{-1}{3}} = \left(\frac{2 \times 2 \times 2}{5 \times 5 \times 5} \right)^{\frac{-1}{3}}$$

$$= \left[\left(\frac{2}{5} \right)^3 \right]^{\frac{-1}{3}}$$

$$= \left(\frac{2}{5} \right)^{\frac{-1}{3} \times 3} = \left(\frac{2}{5} \right)^{-1} = \frac{5}{2} \text{ Ans.}$$

$$\left\{ \because (a^m)^n = a^{mn} \text{ and } a^{-m} = \frac{1}{a^m} \right\}$$

$$(vii) (-27)^{\frac{2}{3}} = [(-3) \times (-3) \times (-3)]^{\frac{2}{3}}$$

$$= (-3)^{3 \times \frac{2}{3}} = (-3)^2 \quad \{ \because (a^m)^n = a^{mn} \}$$

$$= -3 \times -3 = 9 \text{ Ans.}$$

$$(viii) (0.001)^{\frac{-1}{3}} = (0.1 \times 0.1 \times 0.1)^{\frac{-1}{3}}$$

$$= (0.1)^{3 \times \left(\frac{-1}{3} \right)}$$

$$\{ \because (a^m)^n = a^{mn} \}$$

$$= (0.1)^{-1} = \frac{1}{0.1} = \frac{1}{\frac{1}{10}} = \frac{1 \times 10}{1}$$

$$\left\{ \because a^{-m} = \frac{1}{a^m} \right\}$$

= 10 Ans.

$$(ix) (0.027)^{\frac{-2}{3}} = (0.3 \times 0.3 \times 0.3)^{\frac{-2}{3}}$$

$$= (0.3)^{\frac{-2}{3} \times 3}$$

$$\{ \because (a^m)^n = a^{mn} \}$$

$$= (0.3)^{-2} = \frac{1}{(0.3)^2} = \frac{1}{0.3 \times 0.3}$$

$$\left\{ \because a^{-m} = \frac{1}{a^m} \right\}$$

$$= \frac{1}{0.09} = \frac{1}{\frac{9}{100}} = \frac{100}{9} \text{ Ans.}$$

Evaluate the following :

$$\text{Q. 2. (i) } \left(\frac{1}{4} \right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 5^0 + \left(\frac{9}{16} \right)^{-\frac{1}{2}}$$

$$(ii) \sqrt{\frac{1}{4}} + (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}} \times 3^0$$

$$\text{Sol. (i) } \left(\frac{1}{4} \right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 5^0 + \left(\frac{9}{16} \right)^{-\frac{1}{2}}$$

$$= \left(\frac{1}{2} \times \frac{1}{2} \right)^{-2} - 3 \times (2 \times 2 \times 2)^{\frac{2}{3}} \times 5^0 + \left(\frac{3}{4} \times \frac{3}{4} \right)^{-\frac{1}{2}}$$

$$= \left(\frac{1}{2} \right)^{2 \times (-2)} - 3 \times (2^3)^{\frac{2}{3}} \times 5^0 + \left(\frac{3}{4} \right)^{2 \times \left(-\frac{1}{2} \right)}$$

$$\{ \because (a^m)^n = a^{mn} \}$$

$$= \left(\frac{1}{2} \right)^{-4} - 3 \times 2^2 \times 1 + \left(\frac{3}{4} \right)^{-1} \quad \{ \because a^0 = 1 \}$$

$$= (2)^4 - 3 \times 4 \times 1 + \frac{4}{3}$$

$$= (2 \times 2 \times 2 \times 2) - 12 + \frac{4}{3} \quad \left\{ \because a^{-m} = \frac{1}{a^m} \right\}$$

$$= 16 - 12 + \frac{4}{3} = 4 + \frac{4}{3}$$

$$= \frac{12 + 4}{3} = \frac{16}{3} = 5\frac{1}{3} \text{ Ans.}$$

$$(ii) \sqrt{\frac{1}{4}} + (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}} \times 3^0$$

$$= \left[\left(\frac{1}{2} \right)^2 \right]^{\frac{1}{2}} + (0.1 \times 0.1)^{-\frac{1}{2}} - (3 \times 3 \times 3)^{\frac{2}{3}} \times 3^0$$

$$= \left(\frac{1}{2} \right)^{2 \times \frac{1}{2}} + (0.1)^{2 \times \left(-\frac{1}{2} \right)} - (3^3)^{\frac{2}{3}} \times 3^0$$

$$= \left(\frac{1}{2} \right)^1 + (0.1)^{-1} - (3)^2 \times 3^0$$

$$\left\{ \because (a^m)^n = a^{mn} \right\}$$

$$= \frac{1}{2} + \frac{1}{0.1} - 9 \times 1 \left\{ \because a^{-m} = \frac{1}{a^m} \text{ and } a^0 = 1 \right\}$$

$$= \frac{1}{2} + \frac{10}{1} - 9 = 10\frac{1}{2} - 9 = 1\frac{1}{2} \text{ Ans.}$$

$$= \left(\frac{3}{2} \right)^{-3} \times \left[\left(\frac{5}{3} \right)^{-3} \div \left(\frac{2}{5} \right)^3 \right]$$

$$\left\{ \because (a^m)^n = a^{mn} \right\}$$

$$= \left(\frac{2}{3} \right)^3 \times \left[\left(\frac{3}{5} \right)^3 \div \left(\frac{2}{5} \right)^3 \right]$$

$$\left\{ \because a^{-m} = \frac{1}{a^m} \right\}$$

$$= \left[\frac{2 \times 2 \times 2}{3 \times 3 \times 3} \right] \times \left[\left(\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \right) \div \left(\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \right) \right]$$

$$= \frac{8}{27} \times \left[\frac{27}{125} \times \frac{125}{8} \right] = \frac{8}{27} \times \frac{27}{8} = 1 \text{ Ans.}$$

$$(ii) \left[(64)^{\frac{2}{3}} \times 2^{-2} \div 7^0 \right]^{\frac{-1}{2}}$$

$$= \left[(4 \times 4 \times 4)^{\frac{2}{3}} \times \frac{1}{2^2} \div 1 \right]^{\frac{-1}{2}}$$

$$= \left[(4^3)^{\frac{2}{3}} \times \frac{1}{2 \times 2} \div 1 \right]^{\frac{-1}{2}}$$

$$= \left[4 \left(3 \times \frac{2}{3} \right) \times \frac{1}{4} \div 1 \right]^{\frac{-1}{2}}$$

$$= \left[4^2 \times \frac{1}{4} \div 1 \right]^{\frac{-1}{2}} = \left(16 \times \frac{1}{4} \times 1 \right)^{\frac{-1}{2}}$$

$$= (4)^{\frac{-1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{(2 \times 2)^{\frac{1}{2}}} = \frac{1}{2^{2 \times \frac{1}{2}}}$$

$$= \frac{1}{2} \text{ Ans.}$$

$$\text{Q. 3. (i) } \left(\frac{81}{16} \right)^{\frac{-3}{4}} \times \left[\left(\frac{25}{9} \right)^{\frac{-3}{2}} \div \left(\frac{5}{2} \right)^{-3} \right]$$

$$(ii) \left[(64)^{\frac{2}{3}} \times 2^{-2} \div 7^0 \right]^{\frac{-1}{2}}$$

$$\text{Sol. (i) } \left(\frac{81}{16} \right)^{\frac{-3}{4}} \times \left[\left(\frac{25}{9} \right)^{\frac{-3}{2}} \div \left(\frac{5}{2} \right)^{-3} \right]$$

$$= \left(\frac{3 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 2} \right)^{\frac{-3}{4}} \times \left[\left(\frac{5 \times 5}{3 \times 3} \right)^{\frac{-3}{2}} \div \left(\frac{5}{2} \right)^{-3} \right]$$

$$= \left(\frac{3}{2} \right)^{4 \times \left(\frac{-3}{4} \right)} \times \left[\left(\frac{5}{3} \right)^{2 \times \left(\frac{-3}{2} \right)} \div \left(\frac{2}{5} \right)^3 \right]$$

$$\text{Q. 4. (i) } (81)^{\frac{3}{4}} - \left(\frac{1}{32} \right)^{\frac{-2}{5}} + (8)^{\frac{1}{3}} \cdot \left(\frac{1}{2} \right)^{-1} \cdot (2)^0$$

$$(ii) \left(\frac{16}{81}\right)^{-\frac{3}{4}} \times \left(\frac{49}{9}\right)^{\frac{3}{2}} \div \left(\frac{343}{216}\right)^{\frac{2}{3}}$$

$$\begin{aligned} \text{Sol. (i)} & (81)^{\frac{3}{4}} - \left(\frac{1}{32}\right)^{-\frac{2}{5}} + (8)^{\frac{1}{3}} \cdot \left(\frac{1}{2}\right)^{-1} \cdot 2^0 \\ &= (3 \times 3 \times 3 \times 3)^{\frac{3}{4}} - \left(\frac{1}{2 \times 2 \times 2 \times 2 \times 2}\right)^{-\frac{2}{5}} \\ &\quad + (2 \times 2 \times 2)^{\frac{1}{3}} \times \left(\frac{1}{2}\right)^{-1} \times 2^0 \\ &= (3^4)^{\frac{3}{4}} - \left(\frac{1}{2^5}\right)^{-\frac{2}{5}} + (2^3)^{\frac{1}{3}} \times \left(\frac{1}{2}\right)^{-1} \times 2^0 \\ &= 3^{4 \times \frac{3}{4}} - \left(\frac{2^{-5}}{1}\right)^{-\frac{2}{5}} + 2^{3 \times \frac{1}{3}} \times (2)^1 \times 1 \\ &= 3^3 - 2^2 + 2^1 \times 2^1 \times 1 \\ &= 27 - 4 + 2 \times 2 \times 1 = 27 - 4 + 4 = 27 \text{ Ans.} \end{aligned}$$

$$\begin{aligned} (ii) & \left(\frac{16}{81}\right)^{-\frac{3}{4}} \times \left(\frac{49}{9}\right)^{\frac{3}{2}} \div \left(\frac{343}{216}\right)^{\frac{2}{3}} \\ &= \left(\frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3}\right)^{-\frac{3}{4}} \times \left(\frac{7 \times 7}{3 \times 3}\right)^{\frac{3}{2}} \div \left(\frac{7 \times 7 \times 7}{6 \times 6 \times 6}\right)^{\frac{2}{3}} \\ &= \left[\left(\frac{2}{3}\right)^4\right]^{-\frac{3}{4}} \times \left[\left(\frac{7}{3}\right)^2\right]^{\frac{3}{2}} \div \left[\left(\frac{7}{6}\right)^3\right]^{\frac{2}{3}} \\ &= \left(\frac{2}{3}\right)^{4 \times \left(-\frac{3}{4}\right)} \times \left(\frac{7}{3}\right)^{2 \times \frac{3}{2}} \div \left(\frac{7}{6}\right)^{3 \times \frac{2}{3}} \\ &= \left(\frac{2}{3}\right)^{-3} \times \left(\frac{7}{3}\right)^3 \div \left(\frac{7}{6}\right)^2 \\ &= \left(\frac{3}{2}\right)^3 \times \frac{7 \times 7 \times 7}{3 \times 3 \times 3} \div \frac{7 \times 7}{6 \times 6} \\ &= \frac{3 \times 3 \times 3}{2 \times 2 \times 2} \times \frac{343}{27} \div \frac{49}{36} \end{aligned}$$

$$\begin{aligned} &= \frac{27}{8} \times \frac{343}{27} \times \frac{36}{49} \\ &= \frac{63}{2} = 31\frac{1}{2} \text{ Ans.} \end{aligned}$$

$$\text{Q. 5. (i)} \left(\frac{64}{125}\right)^{-\frac{2}{3}} \div \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^0$$

$$(ii) \frac{(32)^{\frac{2}{5}} \times (4)^{-\frac{1}{2}} \times (8)^{\frac{1}{3}}}{(2)^{-2} \div (64)^{-\frac{1}{3}}}$$

$$\text{Sol. (i)} \left(\frac{64}{125}\right)^{-\frac{2}{3}} \div \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^0$$

$$= \left(\frac{4 \times 4 \times 4}{5 \times 5 \times 5}\right)^{-\frac{2}{3}} \div \frac{1}{\left(\frac{4 \times 4 \times 4 \times 4}{5 \times 5 \times 5 \times 5}\right)^{\frac{1}{4}}} + \left(\frac{\sqrt{5^2}}{\sqrt[3]{4^3}}\right)^0$$

$$= \left[\left(\frac{4}{5}\right)^3\right]^{-\frac{2}{3}} \div \frac{1}{\left[\left(\frac{4}{5}\right)^4\right]^{\frac{1}{4}}} + \left(\frac{5}{4}\right)^0$$

$$= \left(\frac{4}{5}\right)^{3 \times \left(-\frac{2}{3}\right)} \div \frac{1}{\left(\frac{4}{5}\right)^{4 \times \frac{1}{4}}} + 1 [\because a^0 = 1]$$

$$= \left(\frac{4}{5}\right)^{-2} \div \frac{1}{\left(\frac{4}{5}\right)^1} + 1 = \left(\frac{5}{4}\right)^2 \div \frac{5}{4} + 1$$

$$\left[\because a^{-m} = \frac{1}{a^m}\right]$$

$$= \frac{25}{16} \times \frac{4}{5} + 1 = \frac{5}{4} + 1 = \frac{9}{4} = 2\frac{1}{4} \text{ Ans.}$$

$$\begin{aligned}
 & \text{(ii)} \quad \frac{(32)^{\frac{2}{5}} \times (4)^{\frac{-1}{2}} \times (8)^{\frac{1}{3}}}{(2)^{-2} \div (64)^{\frac{-1}{3}}} \\
 &= \frac{(2 \times 2 \times 2 \times 2 \times 2)^{\frac{2}{5}} \times (2 \times 2)^{\frac{-1}{2}} \times (2 \times 2 \times 2)^{\frac{1}{3}}}{(2)^{-2} \div (4 \times 4 \times 4)^{\frac{-1}{3}}} \\
 &= \frac{(2^5)^{\frac{2}{5}} \times (2^2)^{\frac{-1}{2}} \times (2^3)^{\frac{1}{3}}}{(2)^{-2} \div (4^3)^{\frac{-1}{3}}} \\
 &= \frac{(2)^{5 \times \frac{2}{5}} \times (2)^{2 \times \left(\frac{-1}{2}\right)} \times (2)^{3 \times \frac{1}{3}}}{(2)^{-2} \div (4)^{3 \times \left(\frac{-1}{3}\right)}} \\
 &= \frac{2^2 \times 2^{-1} \times 2^1}{2^{-2} \div 4^{-1}} = \frac{2^{2-1+1}}{2^{-2} \div (2^2)^{-1}} \\
 &= \frac{2^2}{2^{-2} \div 2^{-2}} = \frac{2 \times 2}{1} = 4 \text{ Ans.}
 \end{aligned}$$

$$\text{Q. 6. (i)} \quad (27)^{\frac{4}{3}} + (32)^{0.8} + (0.8)^{-1} + (0.8)^0$$

$$\text{(ii)} \quad \left[\frac{(27)^{-3}}{(9)^{-3}} \right]^{\frac{1}{3}}$$

$$\begin{aligned}
 \text{Sol. (i)} & \quad (27)^{\frac{4}{3}} + (32)^{0.8} + (0.8)^{-1} + (0.8)^0 \\
 &= (3 \times 3 \times 3)^{\frac{4}{3}} + (2 \times 2 \times 2 \times 2 \times 2)^{\frac{4}{5}} + \frac{1}{0.8} + 1 \\
 &= 3^{3 \times \frac{4}{3}} + 2^{5 \times \frac{4}{5}} + \frac{5}{4} + 1 = 3^4 + 2^4 + \frac{5}{4} + 1 \\
 &= 81 + 16 + \frac{5}{4} + 1 = 99 \frac{1}{4} = 99.25 \text{ Ans.}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \left[\frac{(27)^{-3}}{(9)^{-3}} \right]^{\frac{1}{3}} &= \left[\left(\frac{27}{9} \right)^{-3} \right]^{\frac{1}{3}} = \left(\frac{27}{9} \right)^{-3 \times \frac{1}{3}} \\
 &= \left(\frac{27}{9} \right)^{-1} = \frac{9}{27} = \frac{1}{3} \text{ Ans.}
 \end{aligned}$$

$$\text{Q. 7. (i)} \quad (\sqrt{32} - \sqrt{5})^{\frac{1}{3}} (\sqrt{32} + \sqrt{5})^{\frac{1}{3}}$$

$$\text{(ii)} \quad (9)^{\frac{5}{2}} - 3 \cdot (4)^0 - \left(\frac{1}{81} \right)^{\frac{-1}{2}}$$

$$\text{Sol. (i)} \quad (\sqrt{32} - \sqrt{5})^{\frac{1}{3}} (\sqrt{32} + \sqrt{5})^{\frac{1}{3}} \quad [\because a^m b^m = (ab)^m]$$

$$\begin{aligned}
 &= [(\sqrt{32} - \sqrt{5})(\sqrt{32} + \sqrt{5})]^{\frac{1}{3}} \\
 &= [(\sqrt{32})^2 - (\sqrt{5})^2]^{\frac{1}{3}} \\
 &\quad \{ \because (a-b)(a+b) = a^2 - b^2 \} \\
 &= (32 - 5)^{\frac{1}{3}} = (27)^{\frac{1}{3}} \\
 &= (3 \times 3 \times 3)^{\frac{1}{3}} = (3^3)^{\frac{1}{3}} = 3 \text{ Ans.}
 \end{aligned}$$

$$\text{(ii)} \quad 9^{\frac{5}{2}} - 3 \cdot (4)^0 - \left(\frac{1}{81} \right)^{\frac{-1}{2}}$$

$$\begin{aligned}
 &= (3 \times 3)^{\frac{5}{2}} - 3 \times 1 - \left(\frac{1}{9 \times 9} \right)^{\frac{-1}{2}} \\
 &\quad [\because a^0 = 1]
 \end{aligned}$$

$$\begin{aligned}
 &= 3^{2 \times \frac{5}{2}} - 3 - \left(\frac{1}{9} \right)^{2 \times \left(-\frac{1}{2} \right)} \\
 &= 3^5 - 3 - \left(\frac{1}{9} \right)^{-1} \\
 &= 3 \times 3 \times 3 \times 3 \times 3 - 3 - 9 \\
 &= 243 - 12 = 231 \text{ Ans.}
 \end{aligned}$$

$$\text{Q. 8. Simplify : } \frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$$

$$\begin{aligned}
 \text{Sol.} \quad \frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}} &= \frac{3^n \times (3^2)^{n+1}}{3^{n-1} \times (3^2)^{n-1}} \\
 &= \frac{3^n \times 3^{2n+2}}{3^{n-1} \times 3^{2n-2}} \\
 &= \frac{3^{n+2n+2}}{3^{n-1+2n-2}} = \frac{3^{3n+2}}{3^{3n-3}} \\
 &= 3^{3n+2-3n+3} = 3^5 \text{ Ans.}
 \end{aligned}$$

Q. 9. $\frac{(27)^{\frac{2n}{3}} \times (8)^{-\frac{n}{6}}}{(18)^{-\frac{n}{2}}}$

Sol. $\frac{(27)^{\frac{2n}{3}} \times (8)^{-\frac{n}{6}}}{(18)^{-\frac{n}{2}}} = \frac{(3^3)^{\frac{2n}{3}} \times (2^3)^{-\frac{n}{6}}}{(2 \times 3^2)^{-\frac{n}{2}}}$

$$= \frac{3^{3 \times \frac{2n}{3}} \times 2^{3 \times (-\frac{n}{6})}}{2^{-\frac{n}{2}} \times 3^{2 \times (-\frac{n}{2})}} = \frac{3^{2n} \times 2^{-\frac{n}{2}}}{2^{-\frac{n}{2}} \times 3^{-n}}$$

$$= 3^{2n+n} \times 2^{-\frac{n}{2} + \frac{n}{2}} = 3^{3n} \times 2^0$$

$$= 3^{3n} \times 1 = 3^{3n} \text{ Ans.}$$

Q. 10. $\frac{5^{2(n+6)} \times (25)^{-7+2n}}{(125)^{2n}}$

Sol. $\frac{5^{2(n+6)} \times (25)^{-7+2n}}{(125)^{2n}}$

$$= \frac{5^{2n+12} \times (5^2)^{-7+2n}}{(5^3)^{2n}}$$

$$= \frac{5^{2n+12} \times 5^{-14+4n}}{5^{6n}} = \frac{5^{2n+12-14+4n}}{5^{6n}}$$

$$= \frac{5^{6n-2}}{5^{6n}} = 5^{6n-2-6n} = 5^{-2}$$

$$= \frac{1}{5^2} = \frac{1}{5 \times 5} = \frac{1}{25} \text{ Ans.}$$

Q. 11. $\frac{5^{n+3} - 16 \times 5^{n+1}}{12 \times 5^n - 2 \times 5^{n+1}}$

Sol. $\frac{5^{n+3} - 16 \times 5^{n+1}}{12 \times 5^n - 2 \times 5^{n+1}}$

$$= \frac{5^n (5^3 - 16 \times 5^1)}{5^n (12 - 2 \times 5^1)} = \frac{125 - 80}{12 - 10}$$

$$= \frac{45}{2} = 22.5 \text{ Ans.}$$

Q. 12. $\frac{3 \times (27)^{n+1} + 9 \times 3^{(3n-1)}}{8 \times 3^{3n} - 5 \times (27)^n}$

Sol. $\frac{3 \times (27)^{n+1} + 9 \times 3^{(3n-1)}}{8 \times 3^{3n} - 5 \times (27)^n}$

$$= \frac{3 \times (3^3)^{n+1} + 9 \times 3^{3n-1}}{8 \times 3^{3n} - 5 \times (3^3)^n}$$

$$= \frac{3 \times 3^{3n+3} + 9 \times 3^{3n-1}}{8 \times 3^{3n} - 5 \times 3^{3n}}$$

$$= \frac{3^{3n} (3 \times 3^3 + 9 \times 3^{-1})}{3^{3n} (8 - 5)}$$

$$= \frac{3 \times 27 + 9 \times \frac{1}{3}}{3} = \frac{81 + 3}{3} = \frac{84}{3}$$

$$= 28 \text{ Ans.}$$

Q. 13. $\frac{5 \times (25)^{n+1} - 25 \times 5^{2n}}{5 \times 5^{(2n+3)} - (25)^{n+1}}$

Sol. $\frac{5 \times (25)^{n+1} - 25 \times 5^{2n}}{5 \times 5^{2n+3} - (25)^{n+1}}$

$$= \frac{5 \times (5^2)^{n+1} - 25 \times 5^{2n}}{5 \times 5^{2n+3} - (5^2)^{n+1}}$$

$$= \frac{5 \times 5^{2n+2} - 25 \times 5^{2n}}{5 \times 5^{2n+3} - 5^{2n+2}}$$

$$= \frac{5^{2n} (5 \times 5^2 - 25)}{5^{2n} (5 \times 5^3 - 5^2)} = \frac{5 \times 25 - 25}{5 \times 125 - 25}$$

$$= \frac{125 - 25}{625 - 25} = \frac{100}{600} = \frac{1}{6} \text{ Ans.}$$

$$\frac{7^{(2n+3)} - (49)^{n+2}}{2}$$

Q. 14. $\frac{7^{(2n+3)} - (49)^{n+2}}{[(343)^{n+1}]^{\frac{2}{3}}}$

$$\frac{7^{(2n+3)} - (49)^{n+2}}{2}$$

Sol. $\frac{7^{(2n+3)} - (49)^{n+2}}{[(343)^{n+1}]^{\frac{2}{3}}}$

$$= \frac{7^{2n+3} - (7^2)^{n+2}}{[(7^3)^{n+1}]^{\frac{2}{3}}} = \frac{7^{2n+3} - 7^{2n+4}}{7^{3(n+1) \times \frac{2}{3}}}$$

$$= \frac{7^{2n+3} - 7^{2n+4}}{7^{2n+2}} = \frac{7^{2n} (7^3 - 7^4)}{7^{2n} \times 7^2}$$

$$= \frac{7^3 - 7^4}{7^2} = \frac{7^3(1-7)}{7^2}$$

$$= 7^{3-2}(1-7) = 7(-6) = -42 \text{ Ans.}$$

Q. 15. $(x^{\frac{1}{3}} - x^{-\frac{1}{3}})(x^{\frac{2}{3}} + 1 + x^{-\frac{2}{3}})$

Sol. $(x^{\frac{1}{3}} - x^{-\frac{1}{3}})(x^{\frac{2}{3}} + 1 + x^{-\frac{2}{3}})$

Let $x^{\frac{1}{3}} = a$ and $x^{-\frac{1}{3}} = \frac{1}{a}$, then

Given expression

$$= \left(a - \frac{1}{a}\right) \left(a^2 + 1 + \frac{1}{a^2}\right)$$

$$= a^3 - \frac{1}{a^3} = (x^{\frac{1}{3}})^3 - (x^{-\frac{1}{3}})^3$$

$$= x^1 - x^{-1} = x - \frac{1}{x} \text{ Ans.}$$

Q. 16. Simplify :

(i) $a^7 \times a^4 \times a^{-6} \times a^0$

(ii) $a^{\frac{4}{3}} \div a^{\frac{-2}{3}}$

(iii) $(a^{-1} + b^{-1}) \div (a^{-2} - b^{-2})$

(iv) $(a^{-1} + b^{-1}) \div (ab)^{-1}$

(v) $(a^{-1} \times b^{-1}) \div (a^{-1} + b^{-1})$

(vi) $(a+b)^{-1} \times (a^{-1} + b^{-1})$

(vii) $\frac{a+b+c}{(a^{-1}b^{-1} + b^{-1}c^{-1} + c^{-1}a^{-1})}$

Sol. (i) $a^7 \times a^4 \times a^{-6} \times a^0$

$$= a^{7+4-6+0} = a^{11-6} = a^5 \text{ Ans.}$$

(ii) $a^{\frac{4}{3}} \div a^{\frac{-2}{3}} = a^{\frac{4}{3} - (-\frac{2}{3})}$

$$= a^{\frac{4}{3} + \frac{2}{3}} = a^{\frac{6}{3}} = a^2 \text{ Ans.}$$

(iii) $(a^{-1} + b^{-1}) \div (a^{-2} - b^{-2})$

$$= \left(\frac{1}{a} + \frac{1}{b}\right) \div \left(\frac{1}{a^2} - \frac{1}{b^2}\right)$$

$$= \frac{b+a}{ab} \div \frac{b^2 - a^2}{a^2b^2}$$

$$= \frac{(b+a)}{ab} \times \frac{a^2b^2}{b^2 - a^2} = \frac{(b+a)a^2b^2}{ab(b+a)(b-a)}$$

$$= \frac{ab}{b-a} \text{ Ans.}$$

(iv) $(a^{-1} + b^{-1}) \div (ab)^{-1}$

$$= \left(\frac{1}{a} + \frac{1}{b}\right) \div \frac{1}{ab} = \frac{b+a}{ab} \times \frac{ab}{1} = b+a$$

$$= a+b \text{ Ans.}$$

(v) $(a^{-1} \times b^{-1}) \div (a^{-1} + b^{-1})$

$$= \left(\frac{1}{a} \times \frac{1}{b}\right) \div \left(\frac{1}{a} + \frac{1}{b}\right) = \frac{1}{ab} \div \left(\frac{b+a}{ab}\right)$$

$$= \frac{1}{ab} \times \frac{ab}{(b+a)} = \frac{1}{(b+a)} = \frac{1}{(a+b)} \text{ Ans.}$$

(vi) $(a+b)^{-1} \times (a^{-1} + b^{-1})$

$$= \left(\frac{1}{a+b}\right) \times \left(\frac{1}{a} + \frac{1}{b}\right) = \frac{1}{a+b} \times \frac{b+a}{ab}$$

$$= \frac{1}{a+b} \times \frac{a+b}{ab} = \frac{1}{ab} \text{ Ans.}$$

(vii) $\frac{a+b+c}{a^{-1}b^{-1} + b^{-1}c^{-1} + c^{-1}a^{-1}}$

$$= \frac{a+b+c}{\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}} = \frac{a+b+c}{\frac{c+a+b}{abc}}$$

$$= \frac{(a+b+c)(abc)}{(a+b+c)} = abc \text{ Ans.}$$

Q. 17. Prove that

(i) $\left(\frac{x^a}{x^b}\right)^{(a+b)} \cdot \left(\frac{x^b}{x^c}\right)^{(b+c)} \cdot \left(\frac{x^c}{x^a}\right)^{(c+a)} =$

(ii) $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \cdot \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = 1.$

(iii) $\left(\frac{x^a}{x^b}\right)^{(a+b-c)} \cdot \left(\frac{x^b}{x^c}\right)^{(b+c-a)} \cdot \left(\frac{x^c}{x^a}\right)^{(c+a-b)} = 1.$

Sol. (i) L.H.S.

$$= \left(\frac{x^a}{x^b}\right)^{(a+b)} \cdot \left(\frac{x^b}{x^c}\right)^{(b+c)} \cdot \left(\frac{x^c}{x^a}\right)^{(c+a)}$$

$$\begin{aligned}
 &= x^{(a-b)(a+b)} \cdot x^{(b-c)(b+c)} \cdot x^{(c-a)(c+a)} \\
 &= x^{a^2-b^2} \cdot x^{b^2-c^2} \cdot x^{c^2-a^2} \\
 &= x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^0 = 1 \\
 &= \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) L.H.S.} &= \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \cdot \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} \\
 &= x^{(a-b)\frac{1}{ab}} \cdot x^{(b-c)\frac{1}{bc}} \cdot x^{(c-a)\frac{1}{ca}} \\
 &= x^{\frac{a-b}{ab} + \frac{b-c}{bc} + \frac{c-a}{ca}} \\
 &= x^{\frac{c(a-b) + a(b-c) + b(c-a)}{abc}} \\
 &= x^{\frac{ca-bc+ab-ca+bc-ab}{abc}} = x^0 = 1 \\
 &= \text{R.H.S.}
 \end{aligned}$$

(iii) L.H.S.

$$\begin{aligned}
 &= \left(\frac{x^a}{x^b}\right)^{(a+b-c)} \cdot \left(\frac{x^b}{x^c}\right)^{(b+c-a)} \cdot \left(\frac{x^c}{x^a}\right)^{(c+a-b)} \\
 &= x^{(a-b)(a+b-c)} \cdot x^{(b-c)(b+c-a)} \cdot x^{(c-a)(c+a-b)} \\
 &= x^{a^2-b^2-ac+bc+b^2-c^2-ab+ca+c^2-a^2-bc+ab} \\
 &= x^0 = 1 = \text{R.H.S.}
 \end{aligned}$$

Q. 18. Prove that :

$$\frac{a^{-1}}{(a^{-1}+b^{-1})} + \frac{a^{-1}}{(a^{-1}-b^{-1})} = \frac{2b^2}{b^2-a^2}$$

$$\text{Sol. L.H.S.} = \frac{a^{-1}}{(a^{-1}+b^{-1})} + \frac{a^{-1}}{(a^{-1}-b^{-1})}$$

$$= \frac{\frac{1}{a}}{\frac{1}{a} + \frac{1}{b}} + \frac{\frac{1}{a}}{\frac{1}{a} - \frac{1}{b}}$$

$$= \frac{\frac{1}{a}}{\frac{b+a}{ab}} + \frac{\frac{1}{a}}{\frac{b-a}{ab}}$$

$$= \frac{1}{a} \times \frac{ab}{b+a} + \frac{1}{a} \times \frac{ab}{b-a}$$

$$\begin{aligned}
 &= \frac{b}{b+a} + \frac{b}{b-a} \\
 &= \frac{b^2-ab+b^2+ab}{(b+a)(b-a)} = \frac{2b^2}{b^2-a^2} \\
 &= \text{R.H.S.}
 \end{aligned}$$

Q. 19. Prove that :

$$\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} = 1.$$

Sol. L.H.S.

$$\begin{aligned}
 &= \frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} \\
 &\quad + \frac{1}{1+x^{b-c}+x^{a-c}} \\
 &= \frac{1}{x^{a-a}+x^{b-a}+x^{c-a}} \\
 &\quad + \frac{1}{x^{b-b}+x^{a-b}+x^{c-b}} \\
 &\quad + \frac{1}{x^{c-c}+x^{b-c}+x^{a-c}} \\
 &= \frac{1}{x^{-a}(x^a+x^b+x^c)} \\
 &\quad + \frac{1}{x^{-b}(x^b+x^a+x^c)} \\
 &\quad + \frac{1}{x^{-c}(x^c+x^b+x^a)} \\
 &= \frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^a+x^b+x^c} + \frac{x^c}{x^a+x^b+x^c} \\
 &= \frac{x^a+x^b+x^c}{x^a+x^b+x^c} = \text{R.H.S.}
 \end{aligned}$$

Q. 20. If $abc = 1$, prove that :

$$\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} = 1.$$

Sol. $\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} = 1$

$\therefore abc = 1, a = \frac{1}{bc}$

$a^{-1} = \left(\frac{1}{bc}\right)^{-1} = bc$

$\therefore \text{L.H.S.} = \frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}$

Putting values of a and a^{-1}

$\text{L.H.S.} = \frac{1}{1+\frac{1}{bc}+\frac{1}{b}} + \frac{1}{1+b+\frac{1}{c}} + \frac{1}{1+c+bc}$

$= \frac{1}{\left(\frac{bc+1+c}{bc}\right)} + \frac{1}{\left(\frac{c+bc+1}{c}\right)} + \frac{1}{1+c+bc}$

$= \frac{bc}{bc+1+c} + \frac{c}{c+bc+1} + \frac{1}{1+c+bc}$
 $= \frac{(bc+c+1)}{(bc+c+1)} = 1 = \text{R.H.S.}$

Hence the result is proved.

Q. 21. If a, b, c are positive real numbers, show that: $\sqrt{a^{-1}b} \cdot \sqrt{b^{-1}c} \cdot \sqrt{c^{-1}a} = 1$.

Sol. $\text{L.H.S.} = \sqrt{a^{-1}b} \cdot \sqrt{b^{-1}c} \cdot \sqrt{c^{-1}a}$

$= \sqrt{\frac{b}{a}} \cdot \sqrt{\frac{c}{b}} \cdot \sqrt{\frac{a}{c}}$

$= \sqrt{\frac{b}{a} \times \frac{c}{b} \times \frac{a}{c}} = \sqrt{\frac{abc}{abc}}$

$= \sqrt{1} = 1 = \text{R.H.S.}$

($\because a, b, c$ are positive real numbers)

Q. 22. If $\frac{9^n \times 3^2 \times 3^n - (27)^n}{3^{3m} \times 2^3} = 3^{-3}$,

prove that $(m - n) = 1$.

Sol. $\frac{9^n \times 3^2 \times 3^n - (27)^n}{3^{3m} \times 2^3} = 3^{-3}$

$\Rightarrow \frac{[(3)^2]^n \times 3^2 \times 3^n - (3^3)^n}{3^{3m} \times 2^3} = 3^{-3}$

$\Rightarrow \frac{3^{2n} \times 3^2 \times 3^n - 3^{3n}}{3^{3m} \times 8} = 3^{-3}$

$\Rightarrow \frac{3^{2n+2+n} - 3^{3n}}{8 \cdot 3^{3m}} = 3^{-3}$

$\Rightarrow \frac{3^{3n+2} - 3^{3n}}{8 \cdot 3^{3m}} = 3^{-3}$

$\Rightarrow \frac{3^{3n}(3^2 - 1)}{8 \cdot 3^{3m}} = 3^{-3}$

$\Rightarrow \frac{3^{3n} \times (9 - 1)}{8 \times 3^{3m}} = 3^{-3}$

$\Rightarrow \frac{8 \cdot 3^{3n}}{8 \cdot 3^{3m}} = 3^{-3}$

$\Rightarrow 3^{3n-3m} = 3^{-3}$

$\Rightarrow 3^{3(n-m)} = 3^{-3}$

Comparing we get,

$\therefore 3(n-m) = -3 \Rightarrow n-m = -1$

$\Rightarrow m-n = 1$ Hence proved.

Q. 23. If $21168 = x^4 \times y^3 \times z^2$, find the values of x, y, z .

Sol. $21168 = x^4 \times y^3 \times z^2$

$\Rightarrow 2^4 \times 3^3 \times 7^2 = x^4 \times y^3 \times z^2$

2	21168
2	10584
2	5292
2	2646
3	1323
3	441
3	147
7	49
7	7
	1

Comparing, we get

$x^4 = 2^4 \Rightarrow x = 2$

$y^3 = 3^3 \Rightarrow y = 3$

and $z^2 = 7^2 \Rightarrow z = 7$

Q. 24. If $1960 = 2^a \times 5^b \times 7^c$, find the values of a, b, c .

Hence calculate the value of

$$(2^{-a} \times 5^{-c} \times 7^b).$$

Sol. $1960 = 2^a \times 5^b \times 7^c$

$$\Rightarrow 2^3 \times 5^1 \times 7^2 = 2^a \times 5^b \times 7^c$$

2	1960
2	980
2	490
5	245
7	49
7	7
	1

Comparing, we get

$$a = 3, b = 1, c = 2.$$

Now $2^{-a} \times 5^{-c} \times 7^b$

$$= 2^{-3} \times 5^{-2} \times 7^1 = \frac{1}{2^3 \times 5^2} \times 7^1$$

$$= \frac{7}{8 \times 25} = \frac{7}{200} \text{ Ans.}$$

Solve for x :

Q. 25. $3^x = \frac{1}{3}$

Sol. $3^x = \frac{1}{3} = 3^{-1} \quad \left(\because \frac{1}{3} = 3^{-1} \right)$

Comparing, we get,

$$\therefore x = -1 \text{ Ans.}$$

Q. 26. $3^{2x+1} = 1$

Sol. $3^{2x+1} = 1 = 3^0 \quad (\because 3^0 = 1)$

Comparing, we get,

$$\therefore 2x + 1 = 0 \Rightarrow 2x = -1$$

$$x = \frac{-1}{2} \text{ Ans.}$$

Q. 27. $\sqrt{\frac{a}{b}} = \left(\frac{b}{a}\right)^{1-3x}$

Sol. $\sqrt{\frac{a}{b}} = \left(\frac{b}{a}\right)^{1-3x}$

$$\left(\frac{a}{b}\right)^{\frac{1}{2}} = \left(\frac{a}{b}\right)^{-(1-3x)} = \left(\frac{a}{b}\right)^{3x-1}$$

Comparing, we get

$$3x - 1 = \frac{1}{2} \Rightarrow 3x = \frac{1}{2} + 1 = \frac{3}{2}$$

$$\therefore x = \frac{3}{2} \times \frac{1}{3} = \frac{1}{2} \text{ Ans.}$$

Q. 28. $\left(\sqrt[3]{\frac{2}{3}}\right)^{x-1} = \frac{27}{8}$

Sol. $\left(\sqrt[3]{\frac{2}{3}}\right)^{x-1} = \frac{27}{8} = \frac{(3)^3}{(2)^3} = \left(\frac{3}{2}\right)^3$

$$\Rightarrow \left(\frac{2}{3}\right)^{\frac{x-1}{3}} = \left(\frac{2}{3}\right)^{-3}$$

Comparing, we get

$$\frac{x-1}{3} = -3 \Rightarrow x-1 = -9$$

$$\Rightarrow x = -9 + 1 = -8$$

$$\therefore x = -8 \text{ Ans.}$$

Q. 29. $\left(\sqrt{\frac{3}{5}}\right)^{x-1} = \left(\frac{27}{125}\right)^{-1}$

Sol. $\left(\sqrt{\frac{3}{5}}\right)^{x-1} = \left(\frac{27}{125}\right)^{-1}$

$$\Rightarrow \left(\frac{3}{5}\right)^{\frac{x-1}{2}} = \left(\frac{3^3}{5^3}\right)^{-1} = \left[\left(\frac{3}{5}\right)^3\right]^{-1}$$

$$\Rightarrow \left(\frac{3}{5}\right)^{\frac{x-1}{2}} = \left(\frac{3}{5}\right)^{-3}$$

Comparing, we get

$$\frac{x-1}{2} = -3 \Rightarrow x-1 = -6$$

$$\Rightarrow x = -6 + 1 = -5$$

$$\therefore x = -5 \text{ Ans.}$$

Q. 30. $9 \times 3^x = (27)^{2x-5}$

Sol. $9 \times 3^x = (27)^{2x-5}$

$$\Rightarrow 3^2 \times 3^x = (3^3)^{2x-5}$$

$$\Rightarrow 3^{x+2} = 3^{6x-15}$$

Comparing, we get

$$x + 2 = 6x - 15$$

$$\Rightarrow -6x + x = -15 - 2$$

$$\Rightarrow -5x = -17$$

$$\Rightarrow x = \frac{-17}{-5} = \frac{17}{5} = 3\frac{2}{5} \text{ Ans.}$$

31. $2^{(5x-1)} = 4 \times 2^{(3x+1)}$

Sol. $2^{(5x-1)} = 4 \times 2^{(3x+1)}$

$$\Rightarrow 2^{5x-1} = 2^2 \times 2^{(3x+1)} = 2^{3x+1+2}$$

$$\Rightarrow 2^{5x-1} = 2^{3x+3}$$

Comparing, we get

$$5x - 1 = 3x + 3$$

$$\Rightarrow 5x - 3x = 3 + 1$$

$$\Rightarrow 2x = 4$$

$$\therefore x = \frac{4}{2} = 2 \text{ Ans.}$$

Q. 32. $5^{x-3} \times 3^{2x-8} = 225$

Sol. $5^{x-3} \times 3^{2x-8} = 225$

$$\Rightarrow 5^x \times 5^{-3} \times 3^{2x} \times 3^{-8} = 225$$

$$\Rightarrow 5^x \times \frac{1}{5^3} \times (3^2)^x \times \frac{1}{3^8} = 225$$

$$\Rightarrow 5^x \times (9)^x \times \frac{1}{5^3} \times \frac{1}{3^8} = 225$$

$$\begin{aligned} \Rightarrow (45)^x &= 225 \times 5^3 \times 3^8 \\ &= 3 \times 3 \times 5 \times 5 \times 5^3 \times 3^8 \\ &= 5^{1+1+3} \times 3^{8+1+1} = 5^5 \times 3^{10} \\ &= (5^5) \times (3^2)^5 = (5)^5 \times (9)^5 \end{aligned}$$

$$\Rightarrow 45^x = (5 \times 9)^5 = (45)^5$$

Comparing, we get

$$x = 5 \text{ Ans.}$$

Q. 33. If $2^x = 3^y = 12^z$, show that :

$$\frac{1}{z} = \frac{1}{y} + \frac{2}{x}$$

Sol. Let $2^x = 3^y = 12^z = k$, then

$$2^x = k \Rightarrow 2 = k^{1/x}$$

$$3^y = k \Rightarrow 3 = k^{1/y}$$

$$12^z = k \Rightarrow 12 = k^{1/z}$$

$$\text{Now } 12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$\Rightarrow k^{\frac{1}{z}} = (k^{\frac{1}{x}})^2 \times k^{\frac{1}{y}}$$

$$\Rightarrow k^{\frac{1}{z}} = k^{\frac{2}{x}} \times k^{\frac{1}{y}}$$

$$\Rightarrow k^{\frac{1}{z}} = k^{\frac{2}{x} + \frac{1}{y}}$$

Comparing, we get

$$\frac{1}{z} = \frac{2}{x} + \frac{1}{y}$$

Hence proved.

Q. 34. If $2^x = 3^y = 6^{-z}$ show that :

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

Sol. Let $2^x = 3^y = 6^{-z} = k$, then

$$2 = k^{\frac{1}{x}}, 3 = k^{\frac{1}{y}}, 6 = k^{-\frac{1}{z}}$$

$$\text{Now } 2 \times 3 = 6$$

$$\Rightarrow k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{-\frac{1}{z}}$$

$$\Rightarrow k^{\frac{1}{x} + \frac{1}{y}} = k^{-\frac{1}{z}}$$

Comparing, we get

$$\frac{1}{x} + \frac{1}{y} = -\frac{1}{z}$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

Hence proved.