

RATIO AND PROPORTION

[INCLUDING SHARING IN A RATIO]

8.1 DEFINITION

A ratio is a relationship between two quantities of the same kind with same unit and is obtained on dividing first quantity by the second.

The symbol for ratio is “ : ” and it is put in between the two quantities to be compared.

Thus, the ratio between 15 kg and 20 kg = $15 \text{ kg} : 20 \text{ kg} = \frac{15}{20} = \frac{3}{4} = 3 : 4$.

1. The two quantities must be of the same kind.

Thus, there can be a ratio between ₹ 50 and ₹ 80, but there can be no ratio between ₹ 50 and 80 kg.

2. The ratio between 3 and 4 is written as $3 : 4$ (read as 3 is to 4) or $\frac{3}{4}$.
3. In the ratio $3 : 4$, the first term (i.e., 3) is called **antecedent** and the second term (i.e., 4) is called **consequent**.
4. A ratio is a pure number.
5. In order to find the ratio between two quantities, both the quantities must be in the same unit, e.g., ratio between 30 cm and 2 metre

$$= 30 \text{ cm} : 200 \text{ cm} \quad [\text{As, 2 metre} = 200 \text{ cm}]$$

$$= \frac{30}{200} = \frac{3}{20} = 3 : 20$$

6. A ratio must always be expressed in its lowest terms in simplest form.
7. A ratio has no unit because it is simply a number.

8.2 TO CONVERT A FRACTIONAL RATIO INTO A WHOLE NUMBER RATIO

Example 1 :

Convert $\frac{1}{3} : \frac{1}{4}$ into whole number ratio, i.e., to ratio in simple form.

Solution :

$$\begin{aligned} \frac{1}{3} : \frac{1}{4} &= \frac{1}{3} \times \frac{4}{1} && [\text{Dividing 1st quantity by the 2nd}] \\ &= \frac{4}{3} = 4 : 3 && (\text{Ans.}) \end{aligned}$$

Alternative method :

- Steps :**
1. Find the L.C.M. of denominators 3 and 4 of the given ratio $\frac{1}{3} : \frac{1}{4}$.
 2. Multiply each term of the ratio by this L.C.M. and simplify.

If each term of a ratio is multiplied or divided by the same number (quantity), the ratio remains the same.

Thus, $\frac{1}{3} : \frac{1}{4} = \frac{1}{3} \times 12 : \frac{1}{4} \times 12$ [Since, L.C.M. of 3 and 4 = 12]
 $= 4 : 3$ (Ans.)

8.3 TO DIVIDE A GIVEN QUANTITY INTO A GIVEN RATIO

Example 2 :

20 sweets are distributed between A and B in the ratio 2 : 3. Find, how many does each get ?

Solution :

A and B get sweets in the ratio 2 : 3 $\Rightarrow$ If A gets 2 parts, then B gets 3 parts.

In other words, if we make $(2 + 3) = 5$ equal parts, then A should get 2 parts out of these 5 equal part

$$\Rightarrow \text{A gets} = \frac{2}{5} \text{ of the total number of sweets} = \frac{2}{5} \text{ of } 20 = \frac{2}{5} \times 20 = 8 \text{ sweets}$$

Similarly, B gets 3 parts out of 5 equal parts

$$\Rightarrow \text{B gets} = \frac{3}{5} \text{ of the total number of sweets} = \frac{3}{5} \text{ of } 20 = \frac{3}{5} \times 20 = 12 \text{ sweets}$$

Thus, **A gets 8 sweets** and **B gets 12 sweets.** (Ans.)

Direct method :

Since, the given ratio = 2 : 3 and $2 + 3 = 5$

$$\therefore \text{A gets} = \frac{2}{5} \text{ of the total no. of sweets}$$

$$= \frac{2}{5} \times 20 \text{ sweets} = \mathbf{8 \text{ sweets}} \quad (\text{Ans.})$$

$$\text{and, B gets} = \frac{3}{5} \times 20 \text{ sweets} = \mathbf{12 \text{ sweets}} \quad (\text{Ans.})$$

Example 3 :

Divide ₹ 260 among A, B and C in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$.

Solution :

First of all convert the given ratio into its simple form.

Since, L.C.M. of denominators 2, 3 and 4 is 12.

$$\therefore \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = \frac{1}{2} \times 12 : \frac{1}{3} \times 12 : \frac{1}{4} \times 12 = 6 : 4 : 3$$

$$\text{And, } 6 + 4 + 3 = 13.$$

$$\therefore \text{A's share} = \frac{6}{13} \text{ of ₹ 260} = ₹ \frac{6}{13} \times 260 = ₹ 120$$

$$\text{B's share} = \frac{4}{13} \text{ of ₹ 260} = ₹ \frac{4}{13} \times 260 = ₹ 80$$

$$\text{and C's share} = \frac{3}{13} \text{ of ₹ 260} = ₹ \frac{3}{13} \times 260 = ₹ 60$$

Thus, **A gets ₹ 120, B gets ₹ 80** and **C gets ₹ 60.** (Ans.)

Example 4 :

Two numbers are in the ratio 10 : 13. If the difference between the numbers is 48, find the numbers.

Solution :

Let the two numbers be 10 and 13.

$$\therefore \text{The difference between these numbers} = 13 - 10 = 3$$

Applying unitary method :

When difference between the numbers = 3, 1st number = 10

$$\Rightarrow \text{ " " " " } = 1, \quad \text{1st number} = \frac{10}{3}$$

$$\Rightarrow \text{ " " " " } = 48, \quad \text{1st number} = \frac{10}{3} \times 48 = \mathbf{160}$$

In the same way,

when difference between the numbers = 3, 2nd number = 13

$$\Rightarrow \text{ " " " " } = 48, \quad \text{2nd number} = \frac{13}{3} \times 48 = \mathbf{208}$$

$\therefore$ Required numbers are **160** and **208**

(Ans.)

Alternative method (algebraic method) :

Since, the ratio between the required numbers = 10 : 13

Let the numbers be $10x$ and $13x$.

$$\therefore 13x - 10x = 48 \quad \Rightarrow \quad 3x = 48 \quad \text{and} \quad x = \frac{48}{3} = 16$$

$$\therefore \text{Required numbers} = 10x \quad \text{and} \quad 13x \\ = 10 \times 16 \quad \text{and} \quad 13 \times 16 = \mathbf{160} \quad \text{and} \quad \mathbf{208} \quad (\text{Ans.})$$

EXERCISE 8(A)

1. Express each of the given ratio in its simplest form :

$$(i) \ 22 : 66 \qquad (ii) \ 1.5 : 2.5 \qquad (iii) \ 6\frac{1}{4} : 12\frac{1}{2}$$

$$(iv) \ 40 \text{ kg} : 1 \text{ quintal} \qquad (v) \ 10 \text{ paise} : ₹ 1 \qquad (vi) \ 200 \text{ m} : 5 \text{ km}$$

$$(vii) \ 3 \text{ hours} : 1 \text{ day} \qquad (viii) \ 6 \text{ months} : 1\frac{1}{3} \text{ years} \qquad (ix) \ 1\frac{1}{3} : 2\frac{1}{4} : 2\frac{1}{2}$$

2. Divide 64 cm long string into two parts in the ratio 5 : 3.

3. ₹ 720 is divided between x and y in the ratio 4 : 5. How many rupees will each get ?

4. The angles of a triangle are in the ratio 3 : 2 : 7. Find each angle.

5. A rectangular field is 100 m by 80 m. Find the ratio of :

(i) length to its breadth

(ii) breadth to its perimeter.

6. The sum of three numbers, whose ratios are $3\frac{1}{3} : 4\frac{1}{5} : 6\frac{1}{8}$, is 4917. Find the numbers.

7. The ratio between two quantities is 3 : 4. If the first is ₹ 810, find the second.

8. Two numbers are in the ratio 5 : 7. Their difference is 10. Find the numbers.

9. Two numbers are in the ratio 10 : 11. Their sum is 168. Find the numbers.

10. A line is divided in two parts in the ratio 2.5 : 1.3. If the smaller one is 35.1 cm, find the length of the line.

11. In a class, the ratio of boys to the girls is 7 : 8. What part of the whole class are girls ?

12. The population of a town is 180,000, out of which males are $\frac{1}{3}$ of the whole population. Find the number of females. Also, find the ratio of the number of females to the whole population.
13. Ten gram of an alloy of metals A and B contains 7.5 gm of metal A and the rest is metal B. Find the ratio between :
 (i) the weights of metals A and B in the alloy.
 (ii) the weight of metal B and the weight of the alloy.
14. The ages of two boys A and B are 6 years 8 months and 7 years 4 months respectively. Divide ₹ 3,150 in the ratio of their ages.
15. Three persons start a business and spend ₹ 25,000; ₹ 15,000 and ₹ 40,000 respectively. Find the share of each out of a profit of ₹ 14,400 in a year.
16. A plot of land, 600 sq m in area, is divided between two persons such that the first person gets three-fifth of what the second gets. Find the share of each.
17. Two poles of different heights are standing vertically on a horizontal field. At a particular time, the ratio between the lengths of their shadows is 2 : 3. If the height of the smaller pole is 7.5 m, find the height of the other pole.

8.4 PROPORTION

When *four quantities* are so related that the *ratio between the first and the second quantity is equal to the ratio between the third and the fourth quantity*, the quantities are said to be in **proportion**.

Thus, **proportion is equality of two ratios**.

In order to represent a proportion, either put the sign of equality (=) between the two ratios or put a double colon (: :).

Consider four quantities (numbers) 15, 20, 9 and 12.

The ratio between the first and the second quantity is $15 : 20 = \frac{15}{20} = \frac{3}{4} = 3 : 4$.

Whereas, the ratio between the third and the fourth quantity is $9 : 12 = \frac{9}{12} = \frac{3}{4} = 3 : 4$.

Since, the ratio between the first and the second quantity is same as the ratio between the third and the fourth quantity, we say the four quantities 15, 20, 9 and 12 are in proportion and we write : $15 : 20 = 9 : 12$ or $15 : 20 :: 9 : 12$.

- (i) Each quantity in a proportion is called its **term** or its **proportional**.
- (ii) In a proportion, the **first** and the **last terms** are called the **extremes**, whereas the **second** and the **third terms** are called the **means**.
- (iii) For every proportion, **the product of the extremes is always equal to the product of the means**.
 e.g., In proportion $15 : 20 :: 9 : 12$; product of extremes = $15 \times 12 = 180$
 and, product of means = $20 \times 9 = 180$.
- (iv) The **fourth term** of a proportion is called **fourth proportional**.

Example 5 :

Find the fourth proportional of 3, 4 and 18.

Solution :

Let the fourth proportional be $x \Rightarrow 3, 4, 18$ and x are proportion.

$$\therefore 3 : 4 = 18 : x$$

$$\Rightarrow 3 \times x = 4 \times 18$$

[Product of extremes = Product of means]

$$\Rightarrow x = \frac{72}{3} = 24$$

(Ans.)

8.5 CONTINUED PROPORTION

Three quantities are said to be in *continued* proportion, if the ratio between the first and the second quantity is equal to the ratio between the second and the third quantity.

i.e., a, b and c are in continued proportion, if $a : b = b : c$.

The **second quantity** is called the **mean proportional** between the first and the third.

i.e., in $a : b = b : c$, b is the *mean proportional* between a and c .

The **third quantity** is called the **third proportional** to the first and the second.

i.e., in $a : b = b : c$, c is the *third proportional* to a and b .

Example 6 :

(i) Find the mean proportion between 4 and 9.

(ii) Find the third proportional to 12 and 30.

Solution :

(i) Let the mean proportion be $x \Rightarrow 4, x$ and 9 are in continued proportion.

$$\therefore 4 : x = x : 9 \Rightarrow x \times x = 4 \times 9 \Rightarrow x^2 = 36 \Rightarrow x = 6 \quad (\text{Ans.})$$

(ii) Let x be the third proportional $\Rightarrow 12, 30$ and x are in continued proportion.

$$\therefore 12 : 30 = 30 : x \Rightarrow 12 \times x = 30 \times 30 \Rightarrow x = \frac{900}{12} = 75 \Rightarrow x = 75 \quad (\text{Ans.})$$

Example 7 :

If $a : b = 4 : 5$ and $b : c = 6 : 7$, find $a : c$.

Solution :

$$\text{Since, } a : b = 4 : 5 \Rightarrow \frac{a}{b} = \frac{4}{5} \quad \text{and} \quad b : c = 6 : 7 \Rightarrow \frac{b}{c} = \frac{6}{7}$$

$$\therefore \frac{a}{b} \times \frac{b}{c} = \frac{4}{5} \times \frac{6}{7} \Rightarrow \frac{a}{c} = \frac{24}{35} \quad \text{i.e.,} \quad a : c = 24 : 35 \quad (\text{Ans.})$$

Example 8 :

If $a : b = 4 : 5$ and $b : c = 6 : 7$, find $a : b : c$.

Solution :

$$a : b = 4 : 5 = \frac{4}{5} : \frac{5}{5} \text{ or } \frac{4}{5} : 1 \quad [\text{Dividing each term by 5}]$$

$$b : c = \frac{6}{6} : \frac{7}{6} \text{ or } 6 : 7 = 1 : \frac{7}{6} \quad [\text{Dividing each term by 6}]$$

In both the given ratios, the quantity b is common, so we have made the value of b same i.e., one (1).

$$\begin{aligned} \text{Clearly, } a : b : c &= \frac{4}{5} : 1 : \frac{7}{6} = \frac{4}{5} \times 30 : 1 \times 30 : \frac{7}{6} \times 30 \quad [\text{L.C.M. of 5 and 6} = 30] \\ &= 24 : 30 : 35 \quad (\text{Ans.}) \end{aligned}$$

Alternative method :

We know that if both the terms of a ratio are multiplied by the same number, the ratio remains the same.

So, multiply each ratio by such a number that the value of b (the common term in both the given ratios) acquires the same value.

$$\therefore a : b = 4 : 5 = 24 : 30$$

[Multiplying both the terms by 6]

$$\text{and, } b : c = 6 : 7 = 30 : 35$$

[Multiplying both the terms by 5]

$$\text{Clearly, } a : b : c = 24 : 30 : 35$$

(Ans.)

EXERCISE 8(B)

1. Find the value of x if :

(i) $x : 3 = 15 : 9$

(ii) $4 : x = 12 : 45$

(iii) $13 : 11 = x : 88$

(iv) $1.7 : 5.1 = 1.9 : x$

(v) $\frac{1}{13} : x = \frac{1}{2} : \frac{1}{5}$

2. Check whether the following quantities form a proportion or not :

(i) $3x, 7x, 24$ and 56

(ii) $0.8, 3, 2.4$ and 9

(iii) $1\frac{1}{2}, 3\frac{1}{4}, 4\frac{1}{2}$ and $9\frac{3}{4}$

(iv) $0.4, 0.5, 2.9$ and 3.5

(v) $2\frac{1}{2}, 5\frac{1}{2}, 3.0$ and 6.0

(i) Ratio between first two quantities $= 3x : 7x = \frac{3x}{7x} = \frac{3}{7} = 3 : 7$ and,

ratio between last two quantities $= 24 : 56 = \frac{24}{56} = \frac{3}{7} = 3 : 7$

$\Rightarrow$ the given four quantities are in proportion.

3. Find the fourth proportional of :

(i) $3, 12$ and 4

(ii) $5, 9$ and 45

(iii) $2.1, 1.5$ and 8.4

(iv) $\frac{1}{3}, \frac{2}{5}$ and 8.4

(v) 4 hours 40 minutes, 1 hour 10 minutes and 16 hours

4. Find the third proportional of :

(i) 27 and 9

(ii) $2\text{m } 40\text{cm}$ and 40cm

(iii) 1.8 and 0.6

(iv) $\frac{1}{7}$ and $\frac{3}{14}$

(v) 1.6 and 0.8

5. Find the mean proportional between :

(i) 16 and 4

(ii) 3 and 27

(iii) 0.9 and 2.5

(iv) 0.6 and 9.6

(v) $\frac{1}{4}$ and $\frac{1}{16}$

6. (i) If $A : B = 3 : 5$ and $B : C = 4 : 7$, find $A : B : C$.

(ii) If $x : y = 2 : 3$ and $y : z = 5 : 7$, find $x : y : z$.

(iii) If $m : n = 4 : 9$ and $n : s = 3 : 7$, find $m : s$.

(iv) If $P : Q = \frac{1}{2} : \frac{1}{3}$ and $Q : R = 1\frac{1}{2} : 1\frac{1}{3}$, find $P : R$.

(v) If $a : b = 1.5 : 3.5$ and $b : c = 5 : 6$, find $a : c$.

(vi) If $1\frac{1}{4} : 2\frac{1}{3} = p : q$ and $q : r = 4\frac{1}{2} : 5\frac{1}{4}$, find $p : r$.