H.C.F. AND L.C.M.

3.1 ELEMENTARY TREATMENT

1. Factors :

Each of the natural numbers that divides a given number exactly (completely) is called a factor of the given number.

For example :

(a) Each of the natural numbers 1, 2, 3 and 6 divides the number 6 completely; therefore each of 1, 2, 3 and 6 is a factor of 6.

(b) Each of the natural numbers 1, 2, 4, 8 and 16 divides the number 16 exactly; therefore 1, 2, 4, 8 and 16 are factors of 16.

For the same reason :

(c) Factors of 7 are : 1 and 7

(d) Factors of 35 are : 1, 5, 7 and 35

(e) Factors of 20 are : 1, 2, 4, 5, 10 and 20 and so on.

Moreover :

Since, 7 divides 56 exactly; **7 is** called **a factor** (or divisor) **of 56** and **56 is called a multiple of 7**.

Similarly, 8 divides 72 exactly; so **8 is a factor of 72** whereas **72 is a multiple of 8**.

Remember :

(a) 1 (one) is a factor of every number.

(b) Every non-zero number is factor as well as multiple of itself.
e.g., number 5 is a factor of itself as 5 divides 5 completely and 5 is also a multiple of 5.

(c) Except one (1), every natural number has **atleast** two factors.
e.g., (i) Factors of 2 are : 1 and 2
(ii) Factors of 3 are : 1 and 3
(iii) Factors of 4 are : 1, 2 and 4 and so on

(d) Every natural number has an infinity number of multiples.
e.g., (i) Multiples of 1 are : 1, 2, 3, 4, 5, ...
(ii) Multiples of 2 are : 2, 4, 6, 8, 10, ...
(iii) Multiples of 3 are : 3, 6, 9, 12, ...

1. **Every prime number has two factors,** *i.e.*, 1 (unity) and the number itself.
e.g., (i) 5 has only two factors : 1 and 5
(ii) 17 has only two factors : 1 and 17 and so on.
2. Two (2) is the only prime number which is even also.
3. Every number, which is greater than 1 and is not prime, is called a **composite number** and **every composite number has three or more factors.**
e.g., (i) 4 is a composite number as it has three factors : 1, 2 and 4.
(ii) since, 10 has four factors : 1, 2, 5 and 10; 10 is a composite number and so on.
4. The number 1 (unity) has only one factor, *i.e.*, 1 itself. So, **the number one (1) is neither prime nor composite.**

2. Common factors :

A common factor of two or more numbers is a number which divides each of the given numbers exactly.

For example :

(i) ∵ Factors of 6 = 1, 2, 3 and 6 and factors of 8 = 1, 2, 4 and 8

∴ Common factors of 6 and 8 = 1 and 2

It can easily be shown that each common factor (1 and 2) divides the given numbers 6 and 8 exactly.

(ii) ∵ Factors of 8 = 1, 2, 4 and 8, factors of 12 = 1, 2, 3, 4, 6 and 12 and factors of 16 = 1, 2, 4, 8 and 16.

∴ Common factors of 8, 12 and 16 = 1, 2 and 4.

[Check whether each common factor divides each given number (8, 12 and 16) exactly or not].

Example 1 :

Find the common factors of :

(i) 10 and 15 (ii) 14, 21 and 42

Solution :

(i) ∵ Factors of 10 = 1, 2, 5 and 10

and, factors of 15 = 1, 3, 5 and 15

∴ **Common factors** of 10 and 15 = **1 and 5** **(Ans.)**

(ii) ∵ Factors of 14 = 1, 2, 7 and 14

factors of 21 = 1, 3, 7 and 21

and, factors of 42 = 1, 2, 3, 6, 7, 14, 21 and 42

∴ **Common factors** of 14, 21 and 42 = **1 and 7** **(Ans.)**

3. Prime factor :

When a factor of a given number is a prime number also, it is called a ***prime factor*** *of the given number.*

Since, 3 is a factor of 12 and 3 is a prime number also; therefore 3 is a prime factor of 12.

In the same way.

(i) Factors of 14 are 1, 2, 7 and 14, out of these 2 and 7 are prime numbers; therefore, 2 and 7 are prime factors of 14.

(ii) Factors of 42 are 1, 2, 3, 6, 7, 14, 21 and 42; out of these factors 2, 3 and 7 are prime numbers also, therefore prime factors of 42 are 2, 3 and 7.

4. Repeated prime factors :

Since, $8 = 2 \times 2 \times 2$; 2 is said to be repeated prime factor of 8.

A number can have two or more different repeated prime factors.

(i) $36 = 2 \times 2 \times 3 \times 3$

(ii) $108 = 2 \times 2 \times 3 \times 3 \times 3$

(iii) $225 = 3 \times 3 \times 5 \times 5$ and so on

3.2 H.C.F. (HIGHEST COMMON FACTOR)

Highest common factor (H.C.F.) of two or more given numbers is the greatest number which divides each of the given numbers exactly.

For example :

H.C.F. of 18, 24 and 36 is 6, as 6 is the greatest number which divides each of 18, 24 and 36 exactly.

In other way :

$\because$ Factors of 18 = 1, 2, 3, 6, 9 and 18,

factors of 24 = 1, 2, 3, 4, 6, 8, 12 and 24

and, factors of 36 = 1, 2, 3, 4, 6, 9, 12, 18 and 36

$\therefore$ Common factors of 18, 24 and 36 = 1, 2, 3 and 6

$\Rightarrow$ **Highest common factor** of 18, 24 and 36 = **6**

3.3 METHODS OF FINDING H.C.F.

1. Common factor method :

Steps : 1. Find all the factors of each given numbers.

2. Find common factors of the given numbers.

3. The greatest of all the factors obtained in Step 2, is the required H.C.F.

Example 2 :

Find, using common factor method, H.C.F. of 27, 54 and 81

Solution :

$\because$ Factors of 27 = 1, 3, 9 and 27;

factors of 54 = 1, 2, 3, 6, 9, 18, 27 and 54

and, factors of 81 = 1, 3, 9, 27 and 81

$\therefore$ Common factors of 27, 54 and 81 = 1, 3, 9 and 27

$\Rightarrow$ Required **H.C.F.** = 27 **(Ans.)**

2. Prime factor method

Steps : 1. Express each given number as the product of its prime factors.

2. From the result of step 1, find out all the prime factors which are common and then multiply these common prime factors to get the required H.C.F.

Example 3 :

Find, using prime factor method, the H.C.F. of :

(i) 84 and 105 (ii) 124, 296 and 228

Solution :

(i) **Step 1 :** $84 = 2 \times 2 \times \mathbf{3} \times \mathbf{7}$ and $105 = \mathbf{3} \times 5 \times \mathbf{7}$

Step 2 : **H.C.F.** = 3 × 7 = 21 **(Ans.)**

(ii) **Step 1 :** $124 = 2 \times 2 \times 31$, $296 = 2 \times 2 \times 2 \times 37$ and $228 = 2 \times 2 \times 3 \times 19$

Step 2 : **H.C.F.** = 2 × 2 = 4 **(Ans.)**

Alternative method :

(i) **Step 1 :** Express each given number as the product of its prime factors and then in the exponent form.

Step 2 : Then required H.C.F. is the product of all the common prime factors with lowest powers of them.

(ii) **Step 1 :** $124 = 2 \times 2 \times 31 = 2^2 \times 31$,

$296 = 2 \times 2 \times 2 \times 37 = 2^3 \times 37$

and, $228 = 2 \times 2 \times 3 \times 19 = 2^2 \times 3 \times 19$

Step 2 : Since, common prime factor with lowest power of it is 2^2.

$\therefore$ **H.C.F.** $= 2^2 = 2 \times 2 = 4$ **(Ans.)**

3. Division method :

Steps : 1. Divide the greater number by the smaller number.

2. By the remainder of division in Step 1; divide the smaller number.

3. By the remainder in Step 2, divide the remainder of Step 1.

4. Continue in the same way, till no remainder is left. **The last divisor is the required H.C.F.**

Example 4 :

Find, using division method, the H.C.F. of :

(i) 180 and 270 (ii) 852 and 1065

Solution :

(i) **Step 1 :** 180) 270 (1
180

Step 2 : 90)180 (2
180
x

$\therefore$ **H.C.F.** = 90 **(Ans.)**

(ii) 852) 1065 (1
852

213) 852 (4
852
x

$\therefore$ **H.C.F.** = 213 **(Ans.)**

Example 5 :

Find, using division method, the H.C.F. of :

(i) 18 and 30 (ii) 75 and 180

Solution :

(i) **Step 1 :** 18) 30 (1
18

Step 2 : 12)18 (1
12

Step 3 : 6) 12 (2
12
x

$\therefore$ **H.C.F.** = 6 **(Ans.)**

(ii) 75) 180 (2
150

30) 75 (2
60

15) 30 (2
30
x

$\therefore$ **H.C.F.** = 15 **(Ans.)**

Example 6 :

Find the H.C.F. of 184, 230 and 276.

Solution :

To find the H.C.F. of three numbers by division method :

1. first of all find the H.C.F. of any two of the given numbers.
2. then find the H.C.F. of the third given number and the H.C.F. obtained in Step 1.

Let us find the H.C.F. of 184 and 230.

```
184 ) 230 (1
      184
      ---------
      46) 184 ( 4
          184
          ---
           x
```

H.C.F. of 184 and 230 = 46

Now find the H.C.F. of 276 and 46

```
46 ) 276 ( 6
     276
     ---
      x
```

H.C.F. of 276 and 46 = 46

∴ Required **H.C.F.** = 46 **(Ans.)**

In order to find the H.C.F. of four numbers by division method :

1. Find the H.C.F. of any three given numbers by the method given in Example 6.
2. Then find the H.C.F. of the fourth given number and the H.C.F. obtained in Step 1.

Similarly H.C.F. of five or more numbers can be obtained.

Example 7 :

Find the greatest number which divides 288 and 420 leaving no remainder.

Solution :

The greatest number that divides 288 and 420.

= H.C.F. of 288 and 420

= 12 **(Ans.)**

```
288 ) 420 ( 1
      288
      ----------
      132 ) 288 ( 2
            264
            ---------
            24 ) 132 ( 5
                 120
                 --------
                 12 ) 24 ( 2
                      24
                      --
                       x
```

3.4 IMPORTANT POINTS

1. *Two numbers which do not have any common prime factor are called co-prime numbers.*

   Since, $36 = 2 \times 2 \times 3 \times 3$ and $175 = 5 \times 5 \times 7$

   ⇒ 36 and 175 have no common factor.

   ∴ 36 and 175 are co-prime numbers.

   For the same reason; each of the following pairs of numbers are co-prime :

   (i) 16 and 25 (ii) 18 and 49 (iii) 27 and 64, etc.

2. *The H.C.F. of two co-prime numbers is always* 1 (unity).

   ∴ H.C.F. of 16 and 25 = 1, H.C.F. of 18 and 49 = 1 and so on.

EXERCISE 3 (A)

1. Fill in the blanks :
   - (i) Factors of 2 = ...
   - (ii) Factors of 5 = ...
   - (iii) Factors of 62 = ...
   - (iv) Factors of 90 = ...
   - (v) Common factors of 4 and 6 = ...
   - (vi) Common factors of 18 and 36 = ...
   - (vii) Prime factors of 54 = ...
   - (viii) Prime factors of 6 = ...
   - (ix) H.C.F. of 5 and 15 = ...
   - (x) H.C.F. of 5, 15 and 35 = ...
   - (xi) If x divides y exactly; H.C.F. of x and y = ...
   - (xii) H.C.F. of 1, 4, 9 and 18 = ...
2. State ***True*** or ***False*** :
   - (i) 1 is a factor of every number.
   - (ii) Every non-zero number is a factor of itself.
   - (iii) Every number (greater than 1) has atleast two factors.
   - (iv) Prime factors of 9 are 1 and 3.
   - (v) H.C.F. of 2 and 4 = 4.
   - (vi) H.C.F. of 3 and 6 = 3.
   - (vii) H.C.F. of 18 and 25 = 18.
   - (viii) 18 and 25 are co-prime numbers.
   - (ix) If H.C.F. of numbers x and y is y; the number x is exactly divisible by y.
3. Use the *common factor method* to find the H.C.F. of the following :
   - (i) 35 and 40 (ii) 55 and 84 (iii) 28 and 70
   - (iv) 18, 27 and 36 (v) 48, 60 and 84
4. Use the *prime factor method* to find the H.C.F. of :
   - (i) 30 and 75 (ii) 12, 8 and 16 (iii) 108 and 144
   - (iv) 90, 126 and 198 (v) 56, 140, 168 and 224
5. Use the *division method* to find the H.C.F. of :
   - (i) 175 and 275 (ii) 324, 630 and 342 (iii) 135, 441 and 576
   - (iv) 143, 169, 221 and 299 (v) 512, 456, 344 and 296.
6. Find the greatest number which divides 533 and 287 leaving no remainder.
7. Use division method to find the largest number which may divide numbers 675, 825 and 450 completely.
8. Find the H.C.F. of :
   - (i) $2^3 \times 5^2$ and $4^2 \times 5^3$ (ii) $3^2 \times 4^5 \times 7^3 \times 17$ and $3 \times 4^2 \times 5^3 \times 19^2$

3.5 MULTIPLES AND COMMON MULTIPLES

1. Multiples :

Multiples of a given number are those numbers which can be divided completely by the given number.

For example :

Multiples of 4 are 4, 8, 12, 16, ..., etc., and each of these numbers is exactly divisible by 4.

Similarly, multiples of 5 are : 5, 10, 15, 20, 35, ..., etc.;

multiples of 7 are : 7, 14, 21, 28, 35, ..., etc., and so on.

2. Common multiples :

Common multiples of two or more given numbers are the numbers which can exactly be divided by each of the given numbers.

For example :

(a) Multiples of 3 are : 3, 6, 9, 12, 15, 18, 21, 24, ..., etc.

Multiples of 4 are : 4, 8, 12, 16, 20, 24, 28, ..., etc.

$\therefore$ Common multiples of 3 and 4 = 12, 24, 36, ..., etc.

> It can easily be seen that each of the common multiples 12, 24, 36, etc., is exactly divisible by both 3 and 4.

(b) Multiples of 2 are : 2, 4, 6, 8, 10, 12, 14, 16, 18, ..., etc.

Multiples of 3 are : 3, 6, 9, 12, 15, 18, 21, ..., etc.

Multiples of 6 are : 6, 12, 18, 24, 30, ..., etc.

$\therefore$ Common multiples of 2, 3 and 6 = 6, 12, 18, etc.

> Here also, each common multiple 6, 12, 18, etc., is exactly divisible by 2, 3 and 4.

3.6 LEAST COMMON MULTIPLE (L.C.M.)

The least common multiple of two or more given numbers is the least number which is exactly divisible by each of the given numbers.

In Example (*a*), given above, 12 is the **least common multiple** (L.C.M.) **of 3 and 4.**

And, in Example (*b*), given above, 6 is the L.C.M. of 2, 3 and 6.

3.7 METHODS OF FINDING L.C.M.

1. Prime factor method :

Steps :

1. Resolve each given number into its prime factors and express the factors obtained in exponent form.
2. Find the product of the highest powers of all the factors that occur in any of the given numbers.
3. The product obtained in Step 2 is the required L.C.M.

Example 8 :

Find the L.C.M. of 18, 24 and 60.

Solution :

Step 1 : Resolving each given number into its prime factors.

$18 = 2 \times 3 \times 3 = 2 \times 3^2$,

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$ and

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

Step 2 : The product of all the factors with highest powers

$= 2^3 \times 3^2 \times 5 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

Step 3 : Required **L.C.M. = 360** **(Ans.)**

Example 9 :

Find the L.C.M. of 60, 32, 45 and 80.

Solution :

$\because 60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$, $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

$45 = 3 \times 3 \times 5 = 3^2 \times 5$ and $80 = 2 \times 2 \times 2 \times 2 \times 5 = 2^4 \times 5$

$\therefore$ Required **L.C.M.** $= 2^5 \times 3^2 \times 5 = 32 \times 9 \times 5 =$ **1440** **(Ans.)**

2. Common division method :

Steps :
1. Write the given numbers in a horizontal line, separating them by commas.
2. Divide them by a suitable prime number, which exactly divides atleast two of the given numbers.
3. Write down the quotients and the undivided numbers (if any) in Step 2, in a line below the first.
4. Repeat the process until you get prime numbers as dividend.
5. The product of all the divisors and the numbers in the last line (prime dividends) will be the required L.C.M.

Example 10 :

Find the L.C.M. of :

(i) 18, 24 and 60 (ii) 60, 32, 45 and 80

Solution :

(i)

2	18, 24, 60
2	9, 12, 30
3	9, 6, 15
	3, 2, 5

$\therefore$ **L.C.M.** $= 2 \times 2 \times 3 \times 3 \times 2 \times 5$

$=$ **360** **(Ans.)**

(ii)

2	60, 32, 45, 80
2	30, 16, 45, 40
2	15, 8, 45, 20
2	15, 4, 45, 10
5	15, 2, 45, 5
3	3, 2, 9, 1
	1, 2, 3, 1

$\therefore$ **L.C.M.** $= 2 \times 2 \times 2 \times 2 \times 5 \times 3 \times 2 \times 3$

$=$ **1440** **(Ans.)**

Example 11 :

Find the smallest number which when divided by 36, 54 and 90 leaves no remainder.

Solution :

According to the definition of least (smallest) common multiple,

Required number = L.C.M. of 36, 54 and 90

Since, $36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$,

$54 = 2 \times 3 \times 3 \times 3 = 2^1 \times 3^3$

and, $90 = 2 \times 3 \times 3 \times 5 = 2^1 \times 3^2 \times 5^1$

$\therefore$ **Required smallest number**

$= 2^2 \times 3^3 \times 5^1 = 540$ **(Ans.)**

Example 12 :

(i) Find the least number which when divided by 12, 21 and 35 will leave the same remainder 6 in each case.

(ii) Find the least number which when increased by 8 is exactly divisible by 24, 32 and 36.

Solution :

(i) Required number = (L.C.M. of 12, 21 and 35) + 6.

$\because$ $12 = 2 \times 2 \times 3 = 2^2 \times 3$, $21 = 3 \times 7$ and $35 = 5 \times 7$

$\Rightarrow$ L.C.M. of 12, 21 and 35 $= 2^2 \times 3 \times 7 \times 5 = 420$

$\therefore$ **Required least number** $= 420 + 6 = 426$ **(Ans.)**

(ii) Required number = (L.C.M. of 24, 32 and 36) – 8.

$\because$ $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$, $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

and, $36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

$\Rightarrow$ L.C.M. of 24, 32 and 36 $= 2^5 \times 3^2 = 288$

$\therefore$ **Required least number** $= 288 - 8 = 280$ **(Ans.)**

Example 13 :

Show that the product of the numbers 60 and 84 is equal to the product of their H.C.F. and their L.C.M.

Solution :

Since, $60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$ and $84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$

$\therefore$ Their **H.C.F.** $= 2^2 \times 3 = $ **12** and their **L.C.M.** $= 2^2 \times 3 \times 5 \times 7 = $ **420**

The product of the given numbers $= 60 \times 84 = $ **5040**

And, **product of their H.C.F. and L.C.M.** $= 12 \times 420 = $ **5040**

$\Rightarrow$ **The product of the given two numbers = Product of their H.C.F. and L.C.M.**

Since, the product of two given numbers = the product of their H.C.F. and L.C.M.

i.e. for any two given numbers :

One number × another number = Their H.C.F. × their L.C.M.

(i) H.C.F. of given two numbers = $\frac{\text{Product of the numbers}}{\text{Their L.C.M.}}$.

(ii) L.C.M. of given two numbers = $\frac{\text{Product of the numbers}}{\text{Their H.C.F.}}$.

(iii) One of the two given numbers = $\frac{\text{Their H.C.F.} \times \text{their L.C.M.}}{\text{The other number}}$.

Example 14 :

Two numbers are 32 and 48. Find their H.C.F. Use the H.C.F. obtained to find their L.C.M.

Solution :

$\because$ $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$ and $48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$

$\therefore$ **H.C.F.** $= 2^4 =$ **16** **(Ans.)**

Since, the product of the given two numbers

= Product of their H.C.F. and L.C.M.

$\therefore$ $32 \times 48 = 16 \times$ L.C.M.

L.C.M. $= \frac{32 \times 48}{16} =$ **96** **(Ans.)**

Directly : **Required L.C.M.** $= \frac{\text{Product of the two numbers}}{\text{Their H.C.F.}}$

$= \frac{32 \times 48}{16} =$ **96** **(Ans.)**

ACTIVITY – 1 :

1. Express 28 and 45 as the product of their prime factors :
   $28 = 2 \times 2 \times 7$ and $45 = 3 \times 3 \times 5$.
2. Is there any prime factor common to both these numbers? Obviously; No.
   Here, 28 and 45 are called **co-prime numbers**.

Two natural numbers, which do not have any common prime factor are called **co-prime numbers**.

For example : 8 and 15, 35 and 36, 18 and 125, etc.

ACTIVITY – 2 :

1. Find the L.C.M. and the H.C.F. of natural numbers 28 and 45.

   Since, $28 = 2 \times 2 \times 7$ and $45 = 3 \times 3 \times 5$

   $\therefore$ **L.C.M.** $= 2 \times 2 \times 3 \times 3 \times 5 \times 7 =$ **1260**

   and, **H.C.F.** = **1**, as no prime factor is common and every natural number is always divisible by 1.

   $\therefore$ L.C.M. $\times$ H.C.F. $= 1260 \times 1 = 1260$

2. Find the product of the given natural numbers 28 and 45.

Their product = 28×45

= **1260**

= Product of L.C.M. and H.C.F. as obtained above.

CONCLUSION :

1. When two natural numbers do not have any common prime factor; they are called **co-prime numbers**.
2. For any two co-prime numbers, their :
   (i) L.C.M. = The product of the numbers.
   (ii) H.C.F. = 1.

EXERCISE 3 (B)

1. Find the least common multiple (L.C.M.) of :
   (i) 90 and 120,
   (ii) 63 and 70,
   (iii) 35, 49 and 56,
   (iv) 208, 225 and 240,
   (v) 91, 65, 39 and 130.
2. Find L.C.M. of :
   (i) 18, 63, 30 and 45,
   (ii) 14, 56, 91 and 84,
   (iii) 64, 56, 72 and 224.
3. What is the smallest number, which is exactly divisible by 36, 45 and 63 ?
4. What is the least number, which when divided by 98 and 105 has, in each case, 10 as remainder ?
5. Find the least number that, on being increased by 8, is divisible by 21, 35 and 49 ?
6. Find the least number that, on being diminished by 8, is exactly divisible by 32, 36 and 40.
7. The product of two numbers is 20736 and their H.C.F. is 54. Find their L.C.M.
8. The product of two numbers is 396×576 and their L.C.M. is 6336. Find their H.C.F.
9. The L.C.M. of two numbers is 2079 and their H.C.F. is 27. If one of the numbers is 189, find the other.
10. The H.C.F. of two numbers is 119 and their L.C.M. is 11781. If one of the numbers is 1071, find the other.
11. Find which of the following pairs of natural numbers are co-prime :
    (i) 24 and 150
    (ii) 51 and 64
    (iii) 105 and 88.
12. (i) Are the numbers 32 and 45 co-prime ?
    (ii) If the numbers 32 and 45 are co-prime, write the values of their H.C.F. and L.C.M.
13. Find the H.C.F. and the L.C.M. of 96 and 60. Then show that the L.C.M. is exactly divisible by the H.C.F.

Infact for any two natural numbers, their L.C.M. is always divisible by their H.C.F.