POWER AND ROOTS

2.1 ELEMENTARY TREATEMENT

1. **Power** (exponent) :

   (i) We know $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^7$, where **5** is called the **base** and **7** is called the **power** or **exponent** or **index** of 5.

   (ii) $-2 \times -2 \times -2 \times$ .... 20 times $= (-2)^{20}$, in which **20** is the **power** (index or exponent) of **base –2**.

2. **Properties of exponents :**

   First Property (Product Law) :

> For any non-zero integer a :
> $a^m \times a^n = a^{m+n}$, $\quad a^m \times a^n \times a^p = a^{m+n+p}$ and so on.

For example :

(i) $3^4 \times 3^6 = 3^{4+6} = 3^{10}$, $\quad$ (ii) $5^8 \times 5^5 = 5^{8+5} = 5^{13}$,

(iii) $(-2)^6 \times (-2)^7 = (-2)^{6+7} = (-2)^{13}$ and so on.

Thus, **the product of two or more numbers in exponent form** (all having the same base) **is a number with the same base** (as that of the given numbers) **and whose exponent is equal to the sum of the exponents of the numbers multiplied together.**

Second Property (Quotient Law) :

> For any non-zero integer a : $\dfrac{a^m}{a^n} = a^{m-n}$, if $m > n$ and $\dfrac{a^m}{a^n} = \dfrac{1}{a^{n-m}}$, if $n > m$

For example :

(i) $\dfrac{3^8}{3^6} = 3^{8-6} = 3^2$ and $\dfrac{3^6}{3^8} = \dfrac{1}{3^{8-6}} = \dfrac{1}{3^2}$

(ii) $\dfrac{(-5)^4}{(-5)^{10}} = \dfrac{1}{(-5)^{10-4}} = \dfrac{1}{(-5)^6}$ and $\dfrac{(-5)^{10}}{(-5)^4} = (-5)^{10-4} = (-5)^6$ and so on.

Third Property (Power Law) :

> For any non-zero integer a :
> $(a^m)^n = a^{m \times n} = a^{mn}$

For example :

(i) $(2^3)^4 = 2^{3\times4} = 2^{12}$ $\quad$ (ii) $(8^5)^2 = 8^{5\times2} = 8^{10}$

(iii) $(7^4)^5 = 7^{20}$, $[(-7)^4]^5 = (-7)^{20}$ and so on.

Also :

(i) $a^0 = 1 \Rightarrow 2^0 = 1, 32^0 = 1, (-5)^0 = 1$ and so on.

(ii) $a^1 = 1 \Rightarrow 2^1 = 2, 32^1 = 32, (-5)^1 = -5$ and so on.

(iii) $a^{-m} = \frac{1}{a^m} \Rightarrow 2^{-3} = \frac{1}{2^3}, 3^{-2} = \frac{1}{3^2}, (-5)^{-7} = \frac{1}{(-5)^7}$ and so on.

(iv) $\frac{1}{a^{-m}} = a^m \Rightarrow \frac{1}{2^{-5}} = 2^5, \frac{1}{5^{-8}} = 5^8, \frac{1}{(-3)^{-6}} = (-3)^6$ and so on.

(i) If m is even, $(-a)^m = a^m$

i.e., $(-2)^4 = 2^4, (-8)^{10} = 8^{10}, (-5)^6 = 5^6$ and so on

(ii) If m is odd, $(-a)^m = -a^m$

i.e., $(-2)^5 = -2^5, (-8)^9 = -8^9, (-5)^7 = -5^7$ and so on

(iii) Any non-zero number raised to the power zero = 1

i.e., $3^0 = 1, (-3)^0 = 1$, but $-3^0 \neq 1$. Infact, -3^0 means : $-(3)^0 = -1$.

Similarly : $15^0 = 1, (-15)^0 = 1$, but $-15^0 = -1$,

$28^0 = 1, (-28)^0 = 1$, but $-28^0 = -1$ and so on.

Also,

(iv) $-2^4 = -2 \times 2 \times 2 \times 2 = -16$, whereas $(-2)^4 = -2 \times -2 \times -2 \times -2 = 16$

or, as exponent 4 is even; $(-2)^4 = 2^4 = 2 \times 2 \times 2 \times 2 = 16$

(v) $(-2)^5 = -2 \times -2 \times -2 \times -2 \times -2 = -32$

or, $(-2)^5 = -2^5 = -2 \times 2 \times 2 \times 2 \times 2 = -32$

Example 1 :

Evaluate : (i) $3^3 \times 3^5 \div 3^6$ (ii) $(-2)^7 \times (-2)^6 \div (-2)^{10}$

Solution :

(i) $3^3 \times 3^5 \div 3^6 = \frac{3^3 \times 3^5}{3^6}$ [Applying BODMAS]

$= \frac{3^8}{3^6} = 3^{8-6} = 3^2 = 3 \times 3 = 9$ **(Ans.)**

(ii) $(-2)^7 \times (-2)^6 \div (-2)^{10} = \frac{(-2)^7 \times (-2)^6}{(-2)^{10}} = \frac{(-2)^{13}}{(-2)^{10}} = (-2)^{13-10}$

$= (-2)^3 = -2 \times -2 \times -2 = -8$ **(Ans.)**

Example 2 :

Evaluate : (i) $3^3 \div 3^5 \times 3^6$ (ii) $(-2)^7 \div (-2)^6 \times (-2)^2$

Solution :

(i) $3^3 \div 3^5 \times 3^6 = \frac{3^3}{3^5} \times 3^6$ [Applying BODMAS]

$= \frac{3^3 \times 3^6}{3^5} = \frac{3^9}{3^5} = 3^{9-5} = 3^4 = 3 \times 3 \times 3 \times 3 = 81$ **(Ans.)**

(ii) $(-2)^7 \div (-2)^6 \times (-2)^2$

$= \frac{(-2)^7}{(-2)^6} \times (-2)^2 = (-2)^1 \times (-2)^2 = (-2)^3 = -2^3 = -8$ **(Ans.)**

Example 3 :

Evaluate : (i) $2^6 - 3^0 \times 2^5$ (ii) $2 \times 3^4 - (-2)^3 + (-4)^2$

Solution :

(i) Since, $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$, $3^0 = 1$ and $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

$\therefore\ 2^6 - 3^0 \times 2^5 = 64 - 1 \times 32$

$= 64 - 32 = 32$ **(Ans.)**

(ii) Since, $3^4 = 3 \times 3 \times 3 \times 3 = 81$

$(-2)^3 = -2 \times -2 \times -2 = -8$

or, $(-2)^3 = -2^3 = -2 \times 2 \times 2 = -8$

and, $(-4)^2 = -4 \times -4 = 16$

$\therefore\ 2 \times 3^4 - (-2)^3 + (-4)^2 = 2 \times 81 - (-8) + 16 = 162 + 8 + 16 = 186$ **(Ans.)**

EXERCISE 2(A)

1. Fill in the blanks :
   (i) In $(5)^{-7}$; 5 is called .... and -7 is called ....
   (ii) In 8^3; 3 is called .... and 8 is called ....
   (iii) $2^5 \times 2^7 =$ ...., $2^5 \times 2^7 \times 2^{-6} =$ .... and $2^5 \times 2^7 \times 2^{-6} \times 2^3 =$ ....
   (iv) $4^8 \div 4^5 =$ ...., $4^8 \times 4^3 \div 4^5 =$ .... and $4^8 \div 4^3 \times 4^5$ ....
   (v) If $3^x = 1$; $x =$ ....
   (vi) For every value of x, $1^x =$ ....
   (vii) $(2^3)^2 =$ .... = ... = ...
   (viii) $(2^5)^0 =$ ...., $(7^3)^0 =$ .... and $(-8)^0 =$ ....
   (ix) $5^0 =$ .... and $-5^0 =$ ....
   (x) $(-8)^0 =$ .... and $-8^0 =$ ....
2. Evaluate :
   (i) $4^8 \times 4^{-6}$ (ii) $3^7 \times 3^{-5} \times 3$ (iii) $5^5 \times 5^4 \times 5^{-6}$
   (iv) $7^4 \times 7^0 \times 7^{-3}$ (v) $8^3 \times 8^2 \times 8^{-5}$
3. Evaluate :
   (i) $2^6 \times 2^4 \div 2^8$ (ii) $2^6 \div 2^7 \times 2^3$ (iii) $3^{10} \div 3^{15} \times 3^6$
   (iv) $5^{10} \times 5^{12} \div 5^{19}$ (v) $(-7)^3 \div (-7)^5 \times (-7)^4$
4. Evaluate :
   (i) $(5^4)^2 \times 5^{-6}$ (ii) $(3^{-2})^4 \times (3)^9$ (iii) $(7^{-3})^4 \times (7^{-2})^{-7}$
   (iv) $(4^{-4})^0 \times (4^3)^2 \times (4^{-2})^2$
5. Evaluate :
   (i) $2^8 - 5^0 \times 2^6$ (ii) $3 \times 4^3 - 4 \times 5^2 + (2)^3 \times 8^0$
   (iii) $7 \times 3^2 + 5 \times 4^3 \times 6^0 - 6 \times 2^6$

2.2 SQUARE OF A NUMBER

If a number be multiplied by itself, the product obtained is called the square of the number.

Thus, (i) **square of 4** = 4×4 = **16**

(ii) **square of –5** = -5×-5 = **25** and so on.

Since, (i) square of 4 is 16; we write : $(4)^2 = 16$

(ii) square of –5 is 25; we write : $(-5)^2 = 25$ and so on.

> **Whether the** ***number is positive or negative; its square is always positive.***
>
> e.g., (i) $\mathbf{(-5)^2} = -5 \times -5 = \mathbf{25}$ and $\mathbf{(5)^2} = 5 \times 5 = \mathbf{25}$
>
> (ii) $\mathbf{(7)^2} = 7 \times 7 = \mathbf{49}$ and $\mathbf{(-7)^2} = -7 \times -7 = \mathbf{49}$ and so on.
>
> But; $\mathbf{-5^2} = -5 \times 5 = \mathbf{-25}$; as the square is for 5 only.
>
> $\mathbf{-7^2} = -7 \times 7 = \mathbf{-49}$; as the square is for 7 only.

2.3 CUBE OF A NUMBER

If a number be multiplied by itself three times, the product obtained is called the cube of the number.

Thus, (i) **cube of 3** = $3 \times 3 \times 3$ = **27**, *i.e.,* $(3)^3 = 27$

(ii) **cube of –4** = $-4 \times -4 \times -4$ = **–64**, *i.e.,* $(-4)^3 = -64$

(iii) **cube of** $\mathbf{\frac{3}{5}} = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \mathbf{\frac{27}{125}}$, *i.e.,* $\left(\frac{3}{5}\right)^3 = \frac{27}{125}$ and so on.

> **Cube of a positive number is positive and cube of a negative number is negative.**
>
> *e.g.* $(6)^3 = 6 \times 6 \times 6 = 216$ and
>
> $(-6)^3 = -6 \times -6 \times -6 = -216$.

Consider the following table :

	Number	Its square	Its cube
(i)	**0**	$0^2 = 0 \times 0 = \mathbf{0}$	$0^3 = 0 \times 0 \times 0 = \mathbf{0}$
(ii)	**1**	$1^2 = 1 \times 1 = \mathbf{1}$	$1^3 = 1 \times 1 \times 1 = \mathbf{1}$
(iii)	**2**	$2^2 = 2 \times 2 = \mathbf{4}$	$2^3 = 2 \times 2 \times 2 = \mathbf{8}$
(iv)	**8**	$8^2 = 8 \times 8 = \mathbf{64}$	$8^3 = 8 \times 8 \times 8 = \mathbf{512}$
(v)	**–3**	$(-3)^2 = (-3) \times (-3) = \mathbf{9}$	$(-3)^3 = (-3) \times (-3) \times (-3) = \mathbf{-27}$
(vi)	$\mathbf{\frac{2}{3}}$	$\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3} = \mathbf{\frac{4}{9}}$	$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \mathbf{\frac{8}{27}}$
(vii)	$\mathbf{2\frac{1}{2}}$	$\left(\frac{5}{2}\right)^2 = \frac{5}{2} \times \frac{5}{2} = \frac{25}{4} = \mathbf{6\frac{1}{4}}$	$\left(\frac{5}{2}\right)^3 = \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{125}{8} = \mathbf{15\frac{5}{8}}$
(viii)	$\mathbf{-1\frac{2}{3}}$	$\left(-\frac{5}{3}\right)^2 = \left(-\frac{5}{3}\right) \times \left(-\frac{5}{3}\right) = \frac{25}{9} = \mathbf{2\frac{7}{9}}$	$\left(-\frac{5}{3}\right)^3 = \left(-\frac{5}{3}\right) \times \left(-\frac{5}{3}\right) \times \left(-\frac{5}{3}\right) = \mathbf{-4\frac{17}{27}}$

EXERCISE 2(B)

1. Fill in the blanks :

   (i) $5^2 =$ .... (ii) $-5^2 =$ .... (iii) $(-5)^2 =$ ....

   (iv) $4^3 =$ .... (v) $-4^3 =$ .... (vi) $(-4)^3 =$ ....

   (vii) $10^0 =$ .... (viii) $-10^0 =$ .... (ix) $(-10)^0 =$ ....

2. Find the squares of :

   (i) all natural numbers smaller than 5, (ii) all natural numbers between 5 and 9,

   (iii) even natural numbers from 6 to 10, (iv) odd natural numbers between 3 and 8,

   (v) multiples of 3 upto 10, (vi) multiples of 5 upto 20,

   (vii) integers between –2 and 3.

3. Find the cubes of :

   (i) all natural numbers between 2 and 6, (ii) even natural numbers less than 8,

   (iii) odd natural numbers less than 8, (iv) multiples of 3 less than 10,

   (v) integers between –3 and 2.

4. Find the square of :

   (i) 6 (ii) –1 (iii) $-\frac{4}{7}$ (iv) $2\frac{1}{3}$ (v) $-1\frac{1}{5}$

5. Find the cube of :

   (i) –7 (ii) 11 (iii) $-\frac{3}{4}$ (iv) $1\frac{1}{3}$ (v) $-2\frac{1}{3}$

6. Evaluate :

   (i) $6^2 - 4^2$ (ii) $5^3 - 8^2$ (iii) $7^2 - 3^3$ (iv) $8^3 - 10^2$ (v) $7^3 - 12^2$

2.4 SQUARE ROOT

The square root of a number is that number, whose square is equal to the given number. The sign for square root is the radical sign '$\sqrt{\ }$'.

e.g., (i) Since $(5)^2 = 25$; the square root of 25 is 5; and we write : $\sqrt{25} = 5$.

(ii) Since $(0.8)^2 = 0{\cdot}64$; the square root of 0.64 is 0.8; *i.e.*, $\sqrt{0{\cdot}64} = 0.8$.

2.5 METHODS OF FINDING SQUARE ROOT OF A GIVEN NUMBER

1. Prime Factor Method :

Steps :

1. Find all the prime factors of the given number.
2. Write these factors in pairs such that both the factors in each pair are equal.
3. Take one factor from each pair and remove the sign of square root.
4. Multiply the factors, so obtained, to get the square root of the given number.

e.g., **Square root of 144** $= \sqrt{144}$

$= \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3}$ [Step 1]

$= \sqrt{(2 \times 2) \times (2 \times 2) \times (3 \times 3)}$ [Step 2]

$= 2 \times 2 \times 3$ [Step 3]

$= \mathbf{12}$ [Step 4]

Note : Instead of writing the prime factors of the given number in pairs, we can write them in *index form.*

For example :

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$$

Now for finding the square root, take the *half* of each index value

i.e., **square root of 144** $= 2^2 \times 3^1$ [Half of index 4 is 2 and half of index 2 is 1]

$= 2 \times 2 \times 3 = \mathbf{12}$

Example 4 :

Find the square root of : (i) $6\frac{1}{4}$ (ii) 2·25

Solution :

Square root of a fraction $= \dfrac{\text{Square root of its numerator}}{\text{Square root of its denominator}}$

$\therefore$ (i) **Square root of** $\mathbf{6\frac{1}{4}} = \sqrt{6\frac{1}{4}} = \sqrt{\frac{25}{4}} = \sqrt{\frac{5 \times 5}{2 \times 2}} = \frac{5}{2} = 2\frac{1}{2}$ **(Ans.)**

And, (ii) $\sqrt{\mathbf{2 \cdot 25}} = \sqrt{\frac{225}{100}}$ [Removing decimal]

$= \sqrt{\frac{3 \times 3 \times 5 \times 5}{2 \times 2 \times 5 \times 5}} = \frac{3 \times 5}{2 \times 5} = \frac{15}{10} = 1{\cdot}5$ **(Ans.)**

2. Division Method :

If it is not convenient to find the prime factors of the given number, then this method is applied.

Example 5 :

Find the square root of : (i) 1369 (ii) 64009

(iii) 535·9225 (iv) 0·001849

Solution :

(i)

$$\begin{array}{r|l}
 & 3\ \ 7 \\
\hline
3 & \overline{13}\ \overline{69} \\
 & \underline{9} \\
67 & 4\ 69 \\
 & \underline{4\ 69} \\
 & \ \ \times
\end{array}$$

[Step 1]

[Step 2]

[Step 3]

Steps :

1. Mark the digits of the given number in pairs starting from right to left., *i.e.,* $\overline{13}\ \overline{69}$

   Take the first pair (*i.e.,* 13) and find the largest whole number whose square is either equal to or less than 13. Such a whole number is 3. Write 3 in the quotient and also in the divisor.
2. Subtract $3 \times 3 = 9$ from 13 (the first pair). The remainder is 4.
3. Bring down the second pair of digits (*i.e.,* 69)

   Double the quotient (*i.e.,* 3) and write the result on the left of 469. Add the largest possible digit on the right of 6 so that the product of the two digit number obtained and this number does not exceed 469.

   Such a digit, in this example is 7, since $67 \times 7 = 469$.

   Write 7 in the quotient also.

   Thus, $\sqrt{1369} = 37$ **(Ans.)**

(ii) Pairing the digits from right to left, we get $64009 = 6\ \overline{40}\ \overline{09}$

$$\begin{array}{r|l}
 & \;\;2\ \ 5\ \ 3 \\ \hline
2 & 6\ \overline{40}\ \overline{09} \\
 & \underline{4} \\
45 & 2\ 40 \\
 & \underline{2\ 25} \\
503 & \;\;15\ 09 \\
 & \;\;\underline{15\ 09} \\
 & \quad\ \times
\end{array}$$

[Step 1]
[Step 2]

[Step 3]

Bring down the next pair 09 and proceed further as Step 3.

$\therefore \quad \sqrt{64009} = 253$ **(Ans.)**

(iii) For finding the square root of a decimal number, start marking pairs of digits from the decimal point to the left and to the right of it. Now proceed in exactly the same way as explained above. Just remember to put a decimal in the quotient as the decimal point in the dividend is crossed.

$\therefore\ 535{\cdot}9225 = 5\ \overline{35}{\cdot}\overline{92}\ \overline{25} \quad \Rightarrow$

$$\begin{array}{r|l}
 & \;\;2\ \ 3\ \cdot\ 1\ \ 5 \\ \hline
2 & 5\ \overline{35}{\cdot}\overline{92}\ \overline{25} \\
 & \underline{4} \\
43 & 1\ 35 \\
 & \underline{1\ 29} \\
461 & \quad 6\ 92 \\
 & \quad \underline{4\ 61} \\
4625 & \quad 2\ 31\ 25 \\
 & \quad \underline{2\ 31\ 25} \\
 & \qquad\ \times
\end{array}$$

$\therefore \quad \sqrt{535{\cdot}9225} = 23{\cdot}15$ **(Ans.)**

(iv) $0.001849 = 0.\overline{00}\ \overline{18}\ \overline{49}$ $\Rightarrow$

$$\begin{array}{r|l} & 0.0\ 4\ 3 \\ \hline 04 & 0.\overline{00}\ \overline{18}\ \overline{49} \\ & \quad\ \ 16 \\ \hline 83 & \quad\ \ 2\ 49 \\ & \quad\ \ 2\ 49 \\ \hline & \quad\quad \times \end{array}$$

Hence, $\sqrt{0.001849} = \mathbf{0.043}$ **(Ans.)**

2.6 PERFECT SQUARE

A number, whose exact square root can be obtained, is called a perfect square.

Thus, (i) 16 is a perfect square, since $\sqrt{16} = 4$

(ii) 1·44 is a perfect square, since $\sqrt{1.44} = 1.2$ and so on.

Example 6 :

Find the least number by which 180 should be multiplied to make it a perfect square.

Solution :

$$\begin{aligned} 180 &= 2 \times 2 \times 3 \times 3 \times 5 \\ &= (2 \times 2) \times (3 \times 3) \times 5 \end{aligned}$$

Since, the factor 5 is not in pair.

$\therefore$ **The given number should be multiplied by 5.** **(Ans.)**

$[180 \times \mathbf{5} = 2 \times 2 \times 3 \times 3 \times 5 \times \mathbf{5}$ and $\sqrt{180 \times \mathbf{5}} = 2 \times 3 \times 5 = 30]$

Example 7 :

By what least number must 48 be divided to make it a perfect square ?

Solution :

$48 = 2 \times 2 \times 2 \times 2 \times 3 = (2 \times 2) \times (2 \times 2) \times 3$

Since, the factor 3 is not in pair.

$\therefore$ **The given number must be divided by 3.** **(Ans.)**

$$\frac{48}{3} = \frac{2 \times 2 \times 2 \times 2 \times 3}{3} = 2 \times 2 \times 2 \times 2 \text{ and } \sqrt{\frac{48}{3}} = 2 \times 2 = 4$$

Example 8 :

484 students are to stand in such a way that there are as many students in each row as there are number of rows. Find the number of students in each row.

Solution :

Since, the number of rows = No. of students in each row

And, no. of rows × no. of students in each row = Total no. of students = 484

$\therefore$ **No. of students in each row** $= \sqrt{484}$

$= \sqrt{2 \times 2 \times 11 \times 11} = 2 \times 11 = \mathbf{22}$ **(Ans.)**

Alternative Method :

Let the no. of rows $= x$

$= $ No. of students in each row

Since, no. of rows $\times$ no. of students in each row $=$ Total no. of students

$\Rightarrow \quad x \times x = 484$

$\Rightarrow \quad x^2 = 484$

and, $\quad x = \sqrt{484} = \sqrt{2 \times 2 \times 11 \times 11} = 2 \times 11 = 22$

$\therefore$ **No. of students in each row = 22** **(Ans.)**

Example 9 :

A gardener wanted to plant 375 plants in such a way that there are as many plants in each row, as there are number of rows. He finds that 14 plants are extra. How many plants did he want to put in each row ?

Solution :

Clearly, no. of rows $\times$ no. of plants in each row $= 375 - 14 = 361$

Hence, **no. of plants in each row** $= \sqrt{361}$

$= \sqrt{19 \times 19} = 19$ **(Ans.)**

EXERCISE 2(C)

1. Write the square root of :

(i) 12^2 (ii) 3^4 (iii) 5^6 (iv) $2^4 \times 6^2$

(v) $3^2 \times 5^4 \times 2^6$ (vi) $\dfrac{8^2 \times 3^4}{6^4}$ (vii) $16 \times 25 \times 4 \times 64$ (viii) $\dfrac{81 \times 225}{100}$

2. Use the prime factor method to find the square root of :

(i) 400 (ii) 2500 (iii) 625 (iv) 784

(v) 1024 (vi) 2916 (vii) 2025 (viii) 5184

(ix) 15876 (x) 11025 (xi) 12544 (xii) 16900

(xiii) 69696 (xiv) 1·96 (xv) 7·29 (xvi) 31·36

(xvii) 0·0169 (xviii) 0·0081 (xix) $\dfrac{1 \cdot 089}{0 \cdot 121}$ (xx) $\dfrac{57 \cdot 6}{44 \cdot 1}$

(xxi) $3\dfrac{13}{36}$

3. Find by division method the square root of :

(i) 529 (ii) 2209 (iii) 43264 (iv) 64009

(v) 165649 (vi) 3621409 (vii) 56644 (viii) 8472·466116

(ix) 12·96 (x) 20·8849 (xi) 1528·81 (xii) 149·5729

(xiii) 1606·4064 (xiv) 0·0225 (xv) 1·1236

4. What is the least number by which 396 should be multiplied to make it a perfect square ?

5. By what least number must 5202 be multiplied to get a perfect square ?

6. By what least number must 18050 be divided to make it a perfect square ?

7. Ganesh plants total 6561 plants in such a way that there are as many rows as there are plants in each row. How many rows are there ?

8. Find the value of each of the following :

(i) $\sqrt{144} + \sqrt{81}$ (ii) $\sqrt{64} - \sqrt{121}$ (iii) $\dfrac{\sqrt{49}}{\sqrt{16} + \sqrt{9}}$ (iv) $\dfrac{\sqrt{121}}{\sqrt{64} - \sqrt{36}}$

(v) $\dfrac{\sqrt{64} + \sqrt{49}}{\sqrt{81} - \sqrt{25}}$ (vi) $\dfrac{\sqrt{25} - \sqrt{9}}{\sqrt{36} + \sqrt{25}}$ (vii) $\dfrac{8 + \sqrt{81}}{\sqrt{121} - 9}$ (viii) $\dfrac{\sqrt{144} - 5}{7 + \sqrt{25}}$

9. A gardener wanted to plant 450 plants in such a way that there are as many plants in each row as there are number of rows. He finds that there are 9 extra plants. How many plants he wished to put in each row ?

10. A tourist spends daily, as many rupees as the number of days of his total tour. His total expenses were ₹ 289. Find, in how many days did his tour last ?

2.7 CUBE ROOT OF A GIVEN NUMBER

The cube root of a given number is that number which on multiplying by itself three times gives the given number.

e.g., (i) 5 multiplied by itself three times = $5 \times 5 \times 5 = 125$

$\therefore$ Cube root of 125 = 5 *i.e.* $\sqrt[3]{125} = 5$

(ii) Since, –8 multiplied by itself three times = $-8 \times -8 \times -8 = -512$

$\therefore$ Cube root of –512 = –8 *i.e.* $\sqrt[3]{-512} = -8$

Alternative method :

The cube root of a given number is that number which when raised to the power three produces the given number.

e.g., (i) cube root of 8 is 2; as $2^3 = 2 \times 2 \times 2 = 8$

(ii) cube root of –125 is –5; as $(-5)^3 = (-5) \times (-5) \times (-5) = -125$

(iii) cube root of $\dfrac{27}{64}$ is $\dfrac{3}{4}$; as $\left(\dfrac{3}{4}\right)^3 = \dfrac{3}{4} \times \dfrac{3}{4} \times \dfrac{3}{4} = \dfrac{27}{64}$ and so on.

The symbol for cube root is '$\sqrt[3]{\ }$'.

Thus, (i) the cube root of 8 is written as $\sqrt[3]{8}$ and so $\sqrt[3]{8} = 2$.

(ii) the cube root of $-27 = -3$ and so $\sqrt[3]{-27} = -3$

In the same way :

$\sqrt[3]{27} = 3$, $\sqrt[3]{-125} = -5$, $\sqrt[3]{125} = 5$, $\sqrt[3]{64} = 4$ and so on.

2.8 METHODS OF FINDING CUBE ROOT OF A GIVEN NUMBER

1. Using Prime Factor Method

Steps :

1. Express the given number as the product of its prime factors.
2. Make groups, each consisting three identical prime factors.
3. From each group, obtained in Step 2, take one prime factor and then multiply them.
4. The product, so obtained, is the cube root of the given number.

Example 10 :

Find the cube root of : (i) 216 (ii) 1000 (iii) 1728.

Solution :

(i) Since, $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = (2 \times 2 \times 2) \times (3 \times 3 \times 3)$

$\therefore$ **Cube root of 216** $= 2 \times 3 = 6$ **(Ans.)**

(ii) $\because$ $1000 = (2 \times 2 \times 2) \times (5 \times 5 \times 5)$

Cube root of 1000 $= 2 \times 5 = 10$ **(Ans.)**

or, directly,

$1000 = 10 \times 10 \times 10$

$\Rightarrow$ **Cube root of 1000** $= 10$ **(Ans.)**

(iii) Since, $1728 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3)$

$\therefore$ **Cube root of 1728** $= 2 \times 2 \times 3 = 12$ **(Ans.)**

Example 11 :

Find the cube root of : (i) -27 (ii) -3375

Solution :

(i) Since, $-27 = -(3 \times 3 \times 3)$

$\therefore$ **Cube root of** $-27 = -3$ **(Ans.)**

(ii) Since, $-3375 = -3 \times 3 \times 3 \times 5 \times 5 \times 5$

$= -(3 \times 3 \times 3) \times (5 \times 5 \times 5)$

$\therefore$ **Cube root of** $-3375 = -3 \times 5 = -15$ **(Ans.)**

Cube root of a positive number is always positive and cube root of a negative number is always negative.

e.g. $\sqrt[3]{125} = 5$ and $\sqrt[3]{-125} = -5$

2. Using index form :

Steps :

1. If required, express the given number in the index form.
2. Keeping the base same, divide each index (power) by 3.

This method can be used only when each index is exactly divisible by 3.

Example 12 :

Find the cube root of : (i) 3^6 (ii) 2^9 (ii) 6^3

Solution :

(i) **Cube root of** $3^6 = 3^{6 \div 3} = 3^2 = 3 \times 3 = 9$ **(Ans.)**

or, directly, dividing the power (index) by 3, we get :

Cube root of $3^6 = 3^2 = 3 \times 3 = 9$ **(Ans.)**

(ii) **Cube root of** $2^9 = 2^3 = 2 \times 2 \times 2 = 8$ **(Ans.)**

(iii) **Cube root of** $6^3 = 6^1 = 6$ **(Ans.)**

Example 13 :

Find the cube root of : (i) $3^6 \times 2^9$ (ii) $2^6 \times 3^9 \times 5^3$

Solution :

Dividing each power by 3, we get :

(i) **Cube root of $3^6 \times 2^9$** $= 3^2 \times 2^3 = 3 \times 3 \times 2 \times 2 \times 2 = 72$ **(Ans.)**

(ii) **Cube root of $2^6 \times 3^9 \times 5^3$** $= 2^2 \times 3^3 \times 5^1 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 540$ **(Ans.)**

1. Cube root of a fraction $= \dfrac{\text{Cube root of its numerator}}{\text{Cube root of its denominator}}$

e.g., (i) $\sqrt[3]{\dfrac{8}{27}} = \dfrac{\sqrt[3]{8}}{\sqrt[3]{27}} = \dfrac{\mathbf{2}}{\mathbf{3}}$

(ii) $\sqrt[3]{1\dfrac{61}{64}} = \sqrt[3]{\dfrac{125}{64}} = \dfrac{\sqrt[3]{125}}{\sqrt[3]{64}} = \dfrac{5}{4} = \mathbf{1\dfrac{1}{4}}$ and so on.

2. $\sqrt[3]{48 \times 90 \times 50} = \sqrt[3]{(2\times2\times2\times2\times3)\times(2\times3\times3\times5)\times(2\times5\times5)}$

$= \sqrt[3]{(2\times2\times2)\times(2\times2\times2)\times(3\times3\times3)\times(5\times5\times5)}$

$= 2 \times 2 \times 3 \times 5 = \mathbf{60}$

EXERCISE 2(D)

1. Fill in the blanks :
   (i) If cube of $x = y$, the cube root of $y =$ ....
   (ii) If cube root of $x = y$, the cube of $y =$ ....
   (iii) $2^3 = x \Rightarrow x =$ ....
   (iv) $(-1)^3 =$ ....

2. Find the cube root of :
   (i) 1 (ii) 27 (iii) 1331
   (iv) 4096 (v) 46656 (vi) 8000

3. Find the cube root of :
   (i) -1 (ii) -64 (iii) -1728
   (iv) -729 (v) -216000 (vi) -5832

4. Find the cube root of :
   (i) $\dfrac{108}{500}$ (ii) $3\dfrac{3}{8}$ (iii) $-15\dfrac{5}{8}$
   (iv) 0·008 (v) 0·125 (vi) $-$ 0·064
   (vii) $-$ 0·001 (viii) $4\dfrac{17}{27} \times 15\dfrac{5}{8}$ (ix) $2^3 \times 3^6 \times 4^9$
   (x) $(0{\cdot}8)^6 \times (5)^3 \times 2^9$

5. Find the value of each of the following :
   (i) $\sqrt[3]{3^3 \times 7^6}$ (ii) $\sqrt[3]{5^6 \div 3^9}$ (iii) $\sqrt[3]{2^3} + \sqrt[3]{3^6}$
   (iv) $\sqrt[3]{5^6} - \sqrt[3]{4^3}$ (v) $\dfrac{\sqrt[3]{27} + \sqrt[3]{64}}{\sqrt[3]{125} + 5}$ (vi) $\dfrac{8 - \sqrt[3]{8}}{4 + \sqrt[3]{64}}$