

CARDINAL PROPERTY OF A SET

29.1 CARDINAL NUMBER OF A SET

The number of elements in a set is called its **cardinal number**.

The elements in the set must not be repeated

For example :

- (i) Set $P = \{ 2, 9, 11, 14 \}$ has 4 elements $\therefore$ Cardinal number of set $P = 4$.
- (ii) Set $M = \{ x, y, z \}$ has 3 elements $\therefore$ Cardinal number of set $M = 3$.
- (iii) Set $E = \{ \}$ has no element $\therefore$ Cardinal number of set $E = 0$ and so on.

The symbol used for showing the cardinal number is the small letter 'n' attached before the name of the set that is written inside brackets.

Thus, the cardinal number of set A is represented by $n(A)$.

Conversely, $n(B)$ represents the cardinal number of set B.

Thus, for the sets P, M and E given above :

- (i) $n(P) = 4$, as set P has only four elements; 2, 9, 11 and 14.
- (ii) $n(M) = 3$, as set M has only three elements; x, y and z.
- (iii) $n(E) = 0$, as set E is the empty set and so on.

1. Cardinal number of the empty set 0.
2. Cardinal number of infinite set is not defined.

Example 1 :

Write the cardinal number of each of the following sets :

- (i) $A = \{ 2, 3, 5, 5, 3, 3 \}$ (ii) $B = \{ \text{letters in the word "NOORJAHAN"} \}$
- (iii) $P = \{ \text{counting numbers between 10 and 30; that are divisible by 5} \}$

Solution :

- (i) Since $A = \{ 2, 3, 5, 5, 3, 3 \} = \{ 2, 3, 5 \}$
 $\therefore$ Cardinal number of set $A = 3$, i.e. $n(A) = 3$ (Ans.)
- (ii) Since $B = \{ n, o, r, j, a, h \}$
 $\therefore$ Cardinal number of set $B = 6$, i.e. $n(B) = 6$ (Ans.)
- (iii) Since $P = \{ 15, 20, 25 \}$, $n(P) = 3$ (Ans.)

Example 2 :

Let $A = \{ 5, 6, 8, 9 \}$, $B = \{ 3, 6, 9, 12, 15 \}$ and $C = \{ 2, 4, 6, 8, 10, 12 \}$. Find :

- (i) $n(A)$ (ii) $n(A \cup B)$
- (iii) $n(B \cap C)$ (iv) $n(B) + n(A \cup C)$

Solution :

- (i) Set A has 4 elements; $\therefore n(A) = 4$ (Ans.)
- (ii) Since $A \cup B = \{ 3, 5, 6, 8, 9, 12, 15 \}$
 $\therefore n(A \cup B) = 7$ (Ans.)

(iii) Since $B \cap C = \{6, 12\} \therefore n(B \cap C) = 2$ (Ans.)

(iv) Set B has five elements $\therefore n(B) = 5$

Since $A \cup C = \{2, 4, 5, 6, 8, 9, 10, 12\}$

$\therefore n(A \cup C) = 8$

and $n(B) + n(A \cup C) = 5 + 8 = 13$ (Ans.)

EXERCISE 29

1. Write the cardinal number of each of the following sets :

(i) $A = \{0, 1, 2, 4\}$

(ii) $B = \{-3, -1, 1, 3, 5, 7\}$

(iii) $C = \{ \}$

(iv) $D = \{3, 2, 2, 1, 3, 1, 2\}$

(v) $E = \{\text{Natural numbers between 15 and 20}\}$

(vi) $F = \{\text{Whole numbers from 8 to 14}\}$.

2. Given : $A = \{\text{Natural numbers less than 10}\}$

$B = \{\text{Letters of the word 'PUPPET'}\}$

$C = \{\text{Squares of the first four whole numbers}\}$

$D = \{\text{Odd numbers divisible by 2}\}$

Find :

(i) $n(A)$

(ii) $n(B)$

(iii) $n(C)$

(iv) $n(D)$

(v) $A \cup B$ and $n(A \cup B)$

(vi) $A \cap C$ and $n(A \cap C)$

(vii) $n(B \cup D)$

(viii) $n(C \cap D)$

(ix) $n(B \cup C)$

(x) $n(A \cup D)$.

3. If $A = \{5, 7, 8, 9\}$, $B = \{3, 4, 5, 6\}$ and $C = \{2, 4, 6, 8, 10\}$; find :

(i) $n(A) + n(B)$

(ii) $n(A \cup B)$

(iii) $n(A \cap B)$

(iv) $n(A \cup B) + n(A \cap B)$

(v) $n(B \cup C)$

Is $n(A) + n(B) = n(A \cup B) + n(A \cap B)$?

4. Given : $P = \{\text{Set of letters in the word 'ALLAHABAD'}\}$ and $Q = \text{Empty set}$,

find :

(i) $n(P)$

(ii) $n(Q)$

(iii) $n(P \cup Q)$

(iv) $n(P \cap Q)$

Is $n(P) = n(P \cap Q)$?

Is $n(Q) = n(P \cup Q)$?

Is $n(P) = n(P \cup Q)$?

5. State **true** or **false** for each of the following. Correct the wrong statement.

(i) If $A = \{0\}$, then $n(A) = 0$.

(ii) $n(\emptyset) = 1$.

(iii) If $T = \{a, l, a, h, b, d, h\}$; then $n(T) = 5$

(iv) If $B = \{1, 5, 51, 15, 5, 1\}$, then $n(B) = 6$.

Revision Exercise (Chapter 29)

1. For each of the following statements, write **true** or **false**.

- (i) If $n(A) = n(B)$, sets A and B are equal.
- (ii) If sets A and B are equal, $n(A) = n(B)$.
- (iii) If set $A = \{3, 4, 4, 4, 3, 2, 7\}$, $n(A) = 4$.
- (iv) If set $B = \{5, 5, 5, 4\}$, $n(B) = 4$.
- (v) If P = set of natural numbers less than 6, $n(P) = 6$.
- (vi) If M = set of whole numbers between 10 and 15, $n(M) = 4$.

2. If $A = \{5, 6, 7, 8, 9\}$ and $B = \{7, 8, 9, 10\}$

Find : (i) $A \cup B$ and $n(A \cup B)$

(ii) $A \cap B$ and $n(A \cap B)$

3. If $n(D) = 0$, set D is the empty set. Is this statement true ?

4. If $M = \{2, 4, 6, 8, 10, 12\}$ and

$N = \{4, 8, 12, 16, 18\}$

Find : (i) $M \cup N$

(ii) $M \cap N$

(iii) $n(M)$ and $n(N)$

(iv) $n(M \cup N)$ and $n(M \cap N)$

(v) $n(M) + n(N)$

(vi) $n(M \cup N) + n(M \cap N)$

5. If $A = \{10, 20, 30, 40\}$ and B = The empty set.

Find : (i) $A \cup B$

(ii) $A \cap B$

(iii) $n(A \cup B)$

(iv) $n(A \cap B)$

6. If set $P = \{x : x \text{ is a natural number between } 15 \text{ and } 23\}$, write :

- (i) set P in roster form
- (ii) cardinal number of set P.

7. If set $M = \{x : x \text{ is a whole number less than } 8\}$, write :

- (i) set M in tabular form
- (ii) cardinal number of set M.

8. If set S contains all values of x where x is a whole number and $5 \leq x < 15$, write :

- (i) set S in roster form
- (ii) cardinal number of set S.