SOLIDS

(Volume and Surface Area)

24.1 SOLID

An object that occupies space is called a **solid**. A book, a brick, a ball, etc. are some examples of a solid.

1. A thin straight line drawn on paper, *i.e.* a line drawn on a plane, has only length

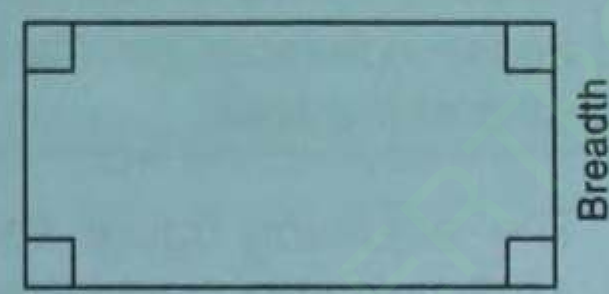
   Thus we say that straight line has only one dimension, namely, a length.

Straight line

2. A rectangle, drawn on paper has length and breadth.

   Thus we say that the figure (rectangle) has two dimensions, namely, length and breadth.

   In fact, each and every figure drawn on a plane is a two-dimensional figure.

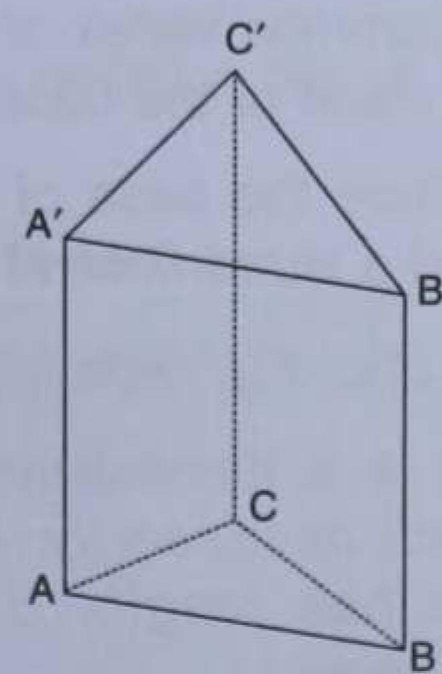


3. Solids have length, breadth and height. For this reason, every solid is a three-dimensional figure.

24.2 RECOGNIZING FACES, EDGES AND VERTICES (CORNERS) OF SOME SOLIDS

(a) Prism :

The adjoining figure shows a prism with :



(i) ***three side faces,*** namely, AA′C′C, ABB′A′ and BB′C′C ; each of these three faces is a parallelogram (or rectangle).

This prism also has two congruent end-faces, *i.e.* bases, namely, triangles ABC and A′B′C′.

The two end-faces (bases) are always parallel to each other.

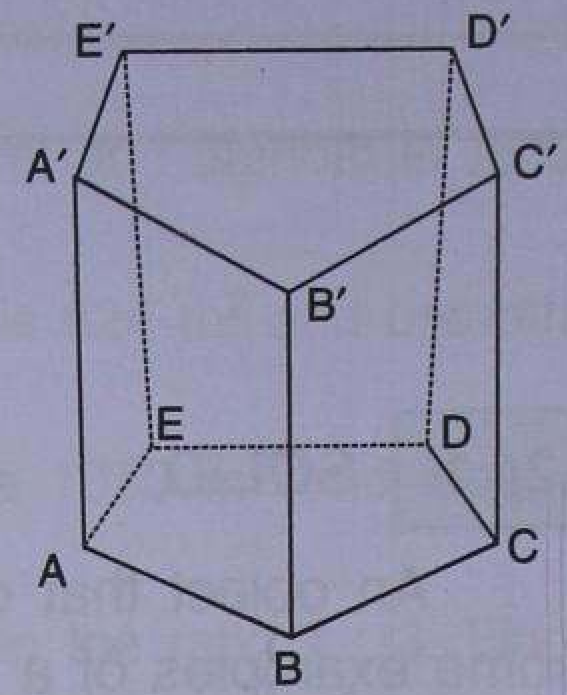
(ii) ***nine edges,*** namely, AB, AC, BC, A′B′, A′C′, B′C′, AA′, BB′ and CC′.

Of these nine edges, AA′, BB′ and CC′ are parallel to one another, AB is parallel to A′B′, BC is parallel to B′C′ and AC is parallel to A′C′.

(iii) ***six vertices,*** namely, A, B, C, A′, B′ and C′.

Thus, a prism is a solid, whose side-faces are parallelograms (or rectangles) and whose end-faces, i.e. bases, are two parallel and congruent polygons.

The adjoining figure shows a prism with :



(i) ***five side-faces,*** namely, ABB′A′, BCC′B′, CDD′C′, DEE′D′ and AEE′A′, each of which is a rectangle.

The prism also has two end-faces, ABCDE and A′B′C′D′E′, which are congruent and parallel to each other.

(ii) ***fifteen edges,*** AA′ // BB′ // CC′ // DD′ // EE′,

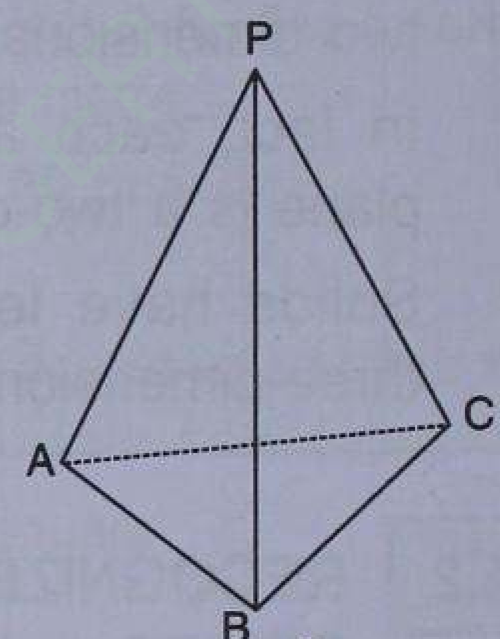
AB // A′ B′, BC // B′C′, CD // C′D′, DE // D′E′ and AE // A′E′.

(iii) ***ten vertices,*** A, B, C, D, E and A′, B′, C′, D′, E′.

(b) Pyramid :

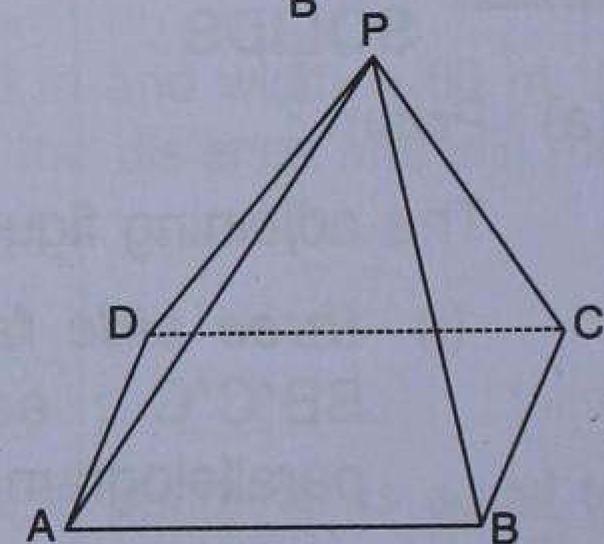
A pyramid is a solid whose base is a plane rectilinear figure, such as a triangle, a quadrilateral, and whose side-faces are triangles with a common vertex. This common vertex must lie outside the plane of the base.

The adjoining figure shows a pyramid with triangular base ABC and side-faces (each of which is also a triangle) PAB, PBC and PAC. Point P is the common vertex of the side-faces.



Since, the base of this pyramid is a triangle, it is called a ***triangular pyramid.***

In the same way, the adjoining figure shows a pyramid whose base is a quadrilateral ABCD and side-faces are ΔPAB, ΔPBC, ΔPCD and ΔPDA. Clearly, P is the common vertex of the side-faces and it does not lie on the plane of the base.

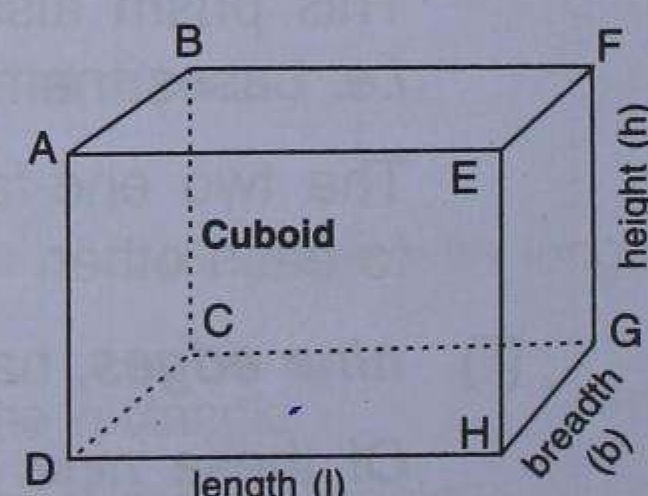


Since the base of this pyramid is a quadrilateral, it is called a ***quadrilateral pyramid.***

(c) Cuboid (a rectangular solid) :

It is a three-dimensional solid all of whose sides are not necessarily equal. That is, in general, a cuboid has length, breadth and height of different values (sizes).

The figure given alongside shows a cuboid. It is clear from the figure that a cuboid has :



(i) ***six faces,*** namely, ABCD, ABFE, AEHD, CGHD, CGFB and EFGH.

Each face of a cuboid is a rectangle.

(ii) ***twelve edges,*** namely, AB, BC, CD, DA, AE, EH, HD, EF, FG, GH, BF and CG.

(iii) ***eight vertices*** (corners), namely, A, B, C, D, E, F, G and H.

Also,

(i) length (l) of the cuboid = AE = DH = CG = BF

(ii) breadth (b) of the cuboid = AB = DC = HG = EF

(iii) height (h) of the cuboid = AD = BC = EH = FG

(d) Cube :

A cube is a cuboid with all sides equal, i.e. length = breadth = height.

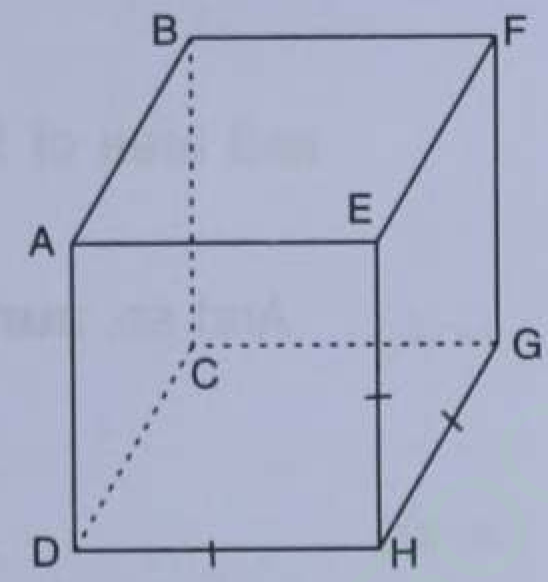
The adjoining figure shows a cube.

Since a cube is a cuboid, it also has :

(i) ***six faces :*** ABCD, ABFE, AEHD, CGHD, CGFB and EFGH.

(ii) ***twelve edges :*** AB, BC, CD, DA, AE, EH, HD, EF, FG, GH, BF and CG.

(iii) ***eight corners :*** A, B, C, D, E, F, G and H.



Each face of a cube is a square in shape, and all the six faces of a cube are congruent (equal).

24.3 VOLUME AND SURFACE OF CUBOID AND CUBE

(a) Volume :

The volume of a solid is the measure of the space occupied by it.

1. ***Volume of a cuboid***

$$= \text{Its length} \times \text{breadth} \times \text{height},$$

i.e. $\mathbf{V} = \boldsymbol{l \times b \times h}$ unit^3

2. ***Volume of a cube***

Since a cube is a cuboid in which length = breadth = height = say, a units,

$\therefore$ **Volume of cube** = length × breadth × height

$= a \times a \times a = \boldsymbol{a^3}$ cubic unit (unit^3)

1. *The formula $V = l \times b \times h$ for the volume of a cuboid can be re-written as :*

   (i) *Length of the cuboid* $l = \frac{V}{b \times h}$

   (ii) *Breadth of the cuboid* $b = \frac{V}{l \times h}$ and

   (iii) *Height of the cuboid* $h = \frac{V}{l \times b}$

2. *When the dimensions of a cuboid or a cube are in centimetre (cm), the volume is in cubic centimetre (cm^3).*

   Similarly, when the dimensions of a cuboid or a cube are in metre (m), the volume is in cubic metre (m^3) and so on.

3. $1 \text{ m} = 100 \text{ cm}$, $1 \text{ m}^2 = 100 \times 100 \text{ cm}^2$ and $1 \text{ m}^3 = 100 \times 100 \times 100 \text{ cm}^3$

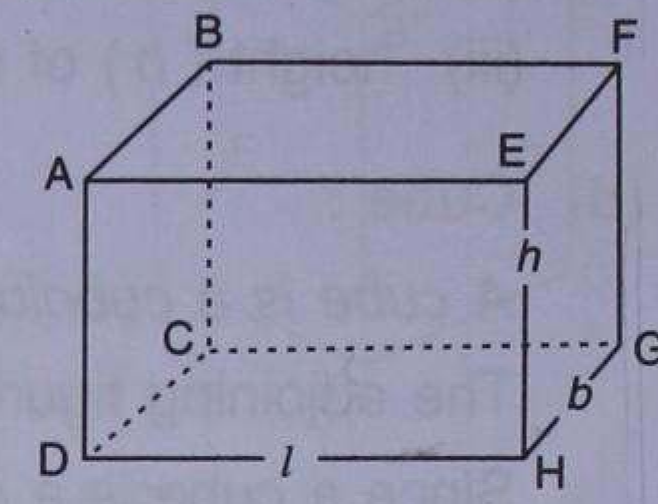
4. $1 \text{ cm} = \frac{1}{100} \text{ m}$, $1 \text{ cm}^2 = \frac{1}{100 \times 100} \text{ m}^2$ and $1 \text{ cm}^3 = \frac{1}{100 \times 100 \times 100} \text{ m}^3$

(b) Surface Area :

The surface area of a solid is the sum of the areas of all its faces.

1. A cuboid has six faces in which opposite faces are equal in area.

   Thus, for the cuboid shown alongside :



Opposite faces of a cuboid are always equal

$$\text{area of face ABFE} = \text{area of face DCGH} = l \times b \text{ sq. units,}$$

$$\text{area of face ABCD} = \text{area of face EFGH} = b \times h \text{ sq. units,}$$

$$\text{and area of face BCGF} = \text{area of face AEHD} = h \times l \text{ sq. units.}$$

And so, **surface area of cuboid**

$$= 2 \times \text{area of ABFE} + 2 \times \text{area of ABCD} + 2 \times \text{area of BCGF}$$
$$= 2\,(l \times b) + 2\,(b \times h) + 2\,(h \times l)$$
$$= \mathbf{2\,(l \times b + b \times h + h \times l)}$$

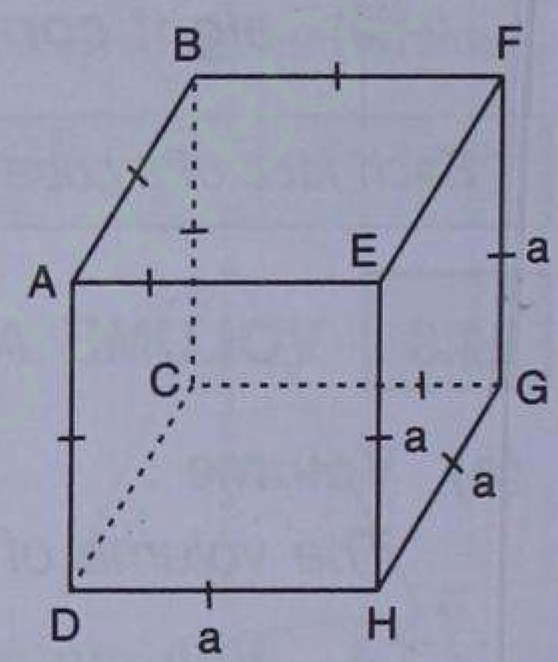
2. A cube has six faces, all of them equal in area.

   Each face of a cube is a square.

   Therefore, area of each face of a cube

   $= a^2$ square units

   Surface area of a cube = $\mathbf{6\ a^2}$ sq. units (where a is one side of the cube).



Example 1 :

The length, breadth and height of a cuboid are 15 cm, 8 cm and 6 cm respectively. Find : (i) its volume (ii) its surface area.

Solution :

Since $l = 15$ cm, $b = 8$ cm and $h = 6$ cm

(i) ∴ **Volume of the cuboid** $= l \times b \times h$

$= 15 \text{ cm} \times 8 \text{ cm} \times 6 \text{ cm} = \mathbf{720 \text{ cm}^3}$ **(Ans.)**

(ii) **Surface area of the cuboid** $= 2\,(l \times b + b \times h + h \times l)$

$= 2(15 \times 8 + 8 \times 6 + 6 \times 15) \text{ cm}^2$

$= 2\,(120 + 48 + 90) \text{ cm}^2 = \mathbf{516 \text{ cm}^2}$ **(Ans.)**

Example 2 :

The volume of a cuboid is 240 cm^3. If its length is 8 cm and height 5 cm, find its breadth.

Solution :

The **breadth of a cuboid,** $b = \dfrac{V}{l \times h} = \dfrac{240}{8 \times 5}$ cm = **6 cm** **(Ans.)**

Example 3 :

One side of a cube is 8 cm. Find : (i) its volume (ii) its surface area.

Solution :

(i) Since each side of the cube = 8 cm, *i.e.* $a = 8$ cm,

$\therefore$ Its **volume** $= a^3 = 8^3 \text{ cm}^3 = \mathbf{512 \text{ cm}^3}$ **(Ans.)**

(ii) Its **surface area** $= 6\,a^2 = 6 \times (8)^2 \text{ cm}^3 = \mathbf{384 \text{ cm}^2}$ **(Ans.)**

Example 4 :

The surface area of a cube is 96 cm^2. Find :

(i) the length of one of its sides (edges) (ii) its volume.

Solution :

(i) $6\,(\text{side})^2 = 96$

$\Rightarrow (\text{side})^2 = \frac{96}{6} = 16$ and **side = 4 cm** **(Ans.)**

(ii) **Volume** $= (\text{side})^3$

$\Rightarrow = (4)^3 \text{ cm}^3 = \mathbf{64 \text{ cm}^3}$ **(Ans.)**

EXERCISE 24(A)

1. Fill in the following blanks :

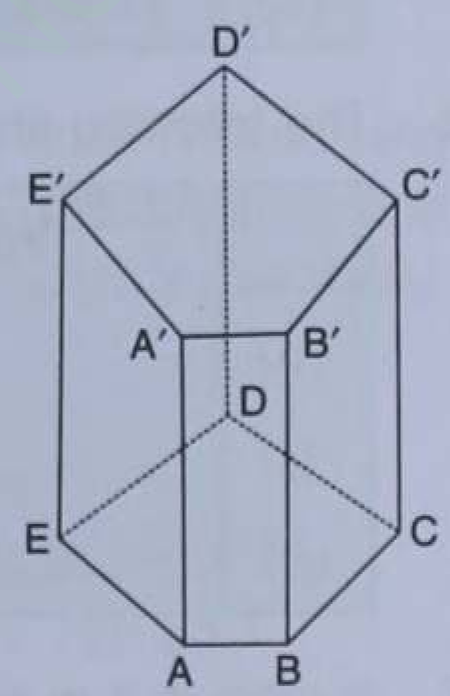
   (i) the name of the solid drawn alongside is ....

   (ii) the name of each of the edges is ....

   (iii) the name of each of the edges parallel to AA′ is ....

   (iv) the name of a side-face is ....

   (v) the name of the vertices is ....



2. With reference to the adjoining figure, fill in the following blanks :

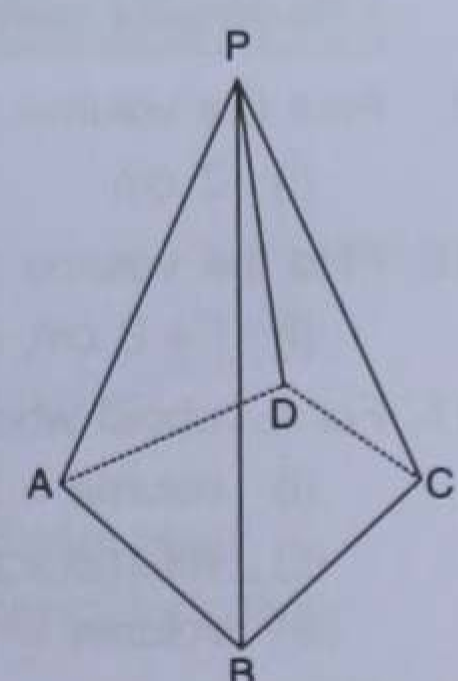
   (i) the name of the given solid is ....

   (ii) it has .... side-faces; each of which is a...............

   (iii) the name of each of the side-faces is ....

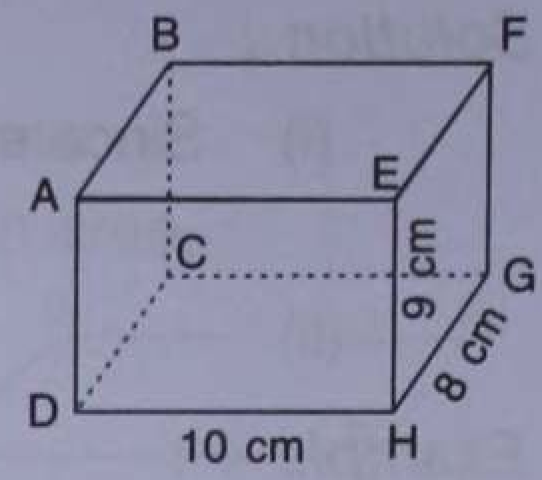
   (iv) the base of the given solid is a ...., and so such a solid is called a ....

   (v) the point P is the .... of the side-faces.

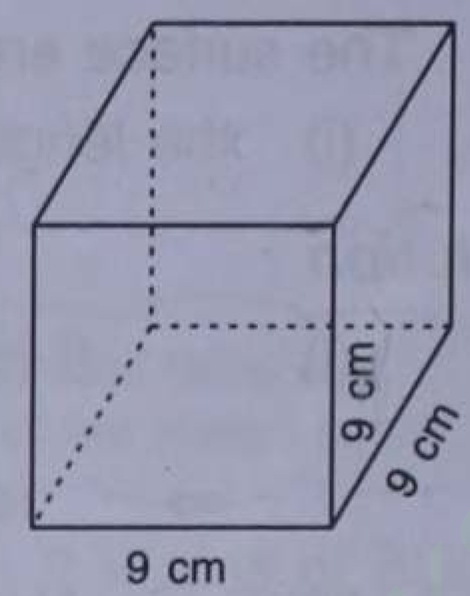


3. The figure given alongside shows a cuboid. Find the
   (i) area of face DCGH.
   (ii) area of face EFGH.
   (iii) area of face AEHD.

   Also write the areas of faces ABFE, ABCD and BCGF.



4. The figure given alongside shows a cube. Find the
   (i) area of one face of the cube.
   (ii) total surface area of the cube.



5. The following are the measurements for a cuboid. Fill in the blanks :

	Volume (V)	Length *(l)*	Breadth *(b)*	Height *(h)*	Surface Area (P)
(i)	160 cm^3	....	5 cm	4 cm	....
(ii)	24 m^3	3 m	....	2 m	....
(iii)	72 cm^3	6 cm	2·5 cm	....	....

6. The following are the measurements for a cube. Fill in the blanks :

	Volume (V)	One side *(a)*	Surface Area (P)
(i)	64 cm^3	....	....
(ii)	....	2·4 m	....
(iii)	....	....	150 cm^2

7. A room is 4 m in length, 3·2 m in breadth and 3 m in height. Find the volume of air in it.
8. Find the space occupied by a rectangular solid whose length is 12 cm, breadth 8 cm and height 10 cm. Also find the surface area of the solid.

   Rectangular solid means cuboid and space occupied means volume.

9. Find the volume and the surface area of a cube whose one side is :
   (i) 7 cm (ii) 2·1 cm (iii) 1·5 m
10. Find the volume and the surface area of a cuboid whose :
   (i) $l = 3$ cm, $b = 2{\cdot}4$ cm and $h = 1{\cdot}5$ cm (ii) $l = 7{\cdot}5$ m, $b = 6$ m and $h = 8$ m
11. For a cuboid whose :
   (i) volume = 300 cm^3, $l = 15$ cm and $b = 5$ cm, find h.
   (ii) volume = 60 cm^3, $l = 5$ cm and $h = 4$ cm, find b.
   (iii) volume = 150 cm^3, $b = 5$ cm and $h = 5$ cm, find l.
12. For a cube, whose surface area = 216 cm^2, find (i) length of one side (ii) volume.

Example 5 :

A rectangular tank has length = 6 m, width = 2·4 m and depth = 1 m. Find :

(i) the capacity of the tank.

(ii) the volume of the water in the tank if half of it is filled with water.

(iii) the volume of the water in litre that this tank can hold. [1 m^3 = 1000 litre]

Solution :

(i) **The capacity of the tank** = its length × its width × its depth

$= 6\text{ m} \times 2{\cdot}4\text{ m} \times 1\text{ m} = 14{\cdot}4\text{ m}^3$ **(Ans.)**

(ii) Since the tank is half filled with water;

volume of the water in the tank $= \frac{1}{2} \times 14{\cdot}4\text{ m}^3 = 7{\cdot}2\text{ m}^3$ **(Ans.)**

(iii) **The total volume of water** that the tank can hold

= volume of the tank

$= 14{\cdot}4\text{ m}^3$

1 m^3 = 1000 litres

$= 14{\cdot}4 \times 1000$ litres

= **14400 litres** **(Ans.)**

Example 6 :

A rectangular solid is 16 cm long, 9 cm wide and 5 cm high. It is melted and smaller rectangular solids, all of equal size, are made. If the length, the breadth and the height of each of the smaller rectangular solids is 4 cm, 3 cm and 2 cm, respectively, find how many of them were made.

Solution :

Volume of the given rectangular solid = its length × its width × its height

$= 16\text{ cm} \times 9\text{ cm} \times 5\text{ cm} = 720\text{ cm}^3$

= vol. of all the smaller rectangular solids

Since the volume of each smaller rectangular solid

= its length × its width × its height

$= 4\text{ cm} \times 3\text{ cm} \times 2\text{ cm} = 24\text{ cm}^3$

∴ **Number of smaller rectangular solids formed**

$$= \frac{\text{Volume of rectangular solid melted}}{\text{Volume of each rectangular solid formed}} = \frac{720\text{ cm}^3}{24\text{ cm}^3} = 30$$ **(Ans.)**

Direct Method :

Number of smaller rectangular solids formed

$$= \frac{\text{Volume of rectangular solid melted}}{\text{Volume of each smaller rectangular solid}}$$

$$= \frac{16\text{ cm} \times 9\text{ cm} \times 5\text{ cm}}{4\text{ cm} \times 3\text{ cm} \times 2\text{ cm}} = 30$$ **(Ans.)**

Example 7 :

A solid metal, cuboid in shape, has dimensions 18 cm, 12 cm and 9 cm. It is melted and recast into identical cubes of side 3 cm. Find the number of cubes obtained.

Solution :

Number of cubes obtained

$$= \frac{\text{Volume of cuboid melted}}{\text{Volume of each cube formed}}$$

$$= \frac{18\text{ cm} \times 12\text{ cm} \times 9\text{ cm}}{(3)^3\text{ cm}^3} = \frac{18 \times 12 \times 9}{3 \times 3 \times 3} = \mathbf{72} \qquad \textbf{(Ans.)}$$

Example 8 :

A wall is 4·5 m long, 3·6 m high and 25 cm thick.

(i) Find the volume of the wall in cubic centimetre ?

(ii) How many bricks of length 30 cm, width 12 cm and thickness 10 cm are required to make this wall ?

Solution :

(i) Since the length of the wall = 4·5 m = 4·5 × 100 cm = 450 cm,

its height = 3·6 m = 3·6 × 100 cm = 360 cm, and thickness = 25 cm,

∴ The **volume of the wall** = Its length × thickness × height

= 450 cm × 25 cm × 360 cm

= **40,50,000 cm^3** **(Ans.)**

(ii) ∵ The volume of each brick = 30 cm × 12 cm × 10 cm = 3,600 cm^3

∴ **Number of bricks required to make the wall**

$$= \frac{\text{Volume of the wall}}{\text{Volume of each brick}}$$

$$= \frac{40{,}50{,}000\text{ cm}^3}{3{,}600\text{ cm}^3} = \mathbf{1125} \qquad \textbf{(Ans.)}$$

Direct method :

$$\textbf{Number of bricks required} = \frac{\text{Volume of the wall}}{\text{Volume of each brick}}$$

$$= \frac{450\text{ cm} \times 360\text{ cm} \times 25\text{ cm}}{30\text{ cm} \times 12\text{ cm} \times 10\text{ cm}} = \mathbf{1125} \qquad \textbf{(Ans.)}$$

EXERCISE 24(B)

1. A water tank is 2·4 m in length, 1·5 m in breadth and 1 m in depth. Find how many litres of water can it hold. $\boxed{1\text{ m}^3 = 1000\text{ litres}}$

2. A container is 15 cm long, 12 cm wide and 30 cm high. Find how many litres of milk it can hold ? $\boxed{1000\text{ cm}^3 = 1\text{ litre}}$

3. A wooden block measures 30 cm × 24 cm × 18 cm.
How many cubes, each of edge 6 cm, can be cut out of this block ?

> 30 cm × 24 cm × 18 cm means; l = 30 cm, b = 24 cm and h = 18 cm

4. A brick measures 20 cm × 10 cm × 7·5 cm. How many bricks are required to make a rectangular pile 40 cm long, 30 cm wide and 20 cm high ?
5. A cuboid of dimensions 25 cm × 16 cm × x cm has the same volume as that of a cube of edge 20 cm. Find :
   (i) the volume of the cube
   (ii) the volume of the cuboid
   (iii) the value of x.
6. A cube and a cuboid have equal volumes. If the cuboid is 25 cm long, 15 cm wide and 9 cm high, find :
   (i) the volume of the cuboid
   (ii) the volume of the cube
   (iii) each side of the cube

Revision Exercise (Chapter 24)

1. A solid cuboid is 36 cm long, 30 cm broad and 24 cm high. Find :
   (i) the surface area of the cuboid.
   (ii) the cost of painting it at the rate of ₹ 0·45 per sq. cm.
2. The volume of a cuboid is 187·5 m^3. If its length and breadth are 10 m and 5 m, respectively, find its height and surface area.
3. A rectangular water tank contains 10·5 m^3 water upto a depth of 2 m. If the breadth of the tank is 1·75 m, find its length.
4. The base of a rectangular pool is horizontal. If the depth of the pool is 60 cm and its length and breadth are 8·5 m and 5·6 m, respectively, find the volume of water required to fill the pool completely.
5. A solid cuboidal metal has length = 72 cm, breadth = 50 cm and height = 36 cm. It is melted and recast into identical solid cubes, each of edge 6 cm; find the number of cubes so obtained.
6. The solid cube with each side 24 cm is melted and recast into identical solid cuboidals, each with length = 8 cm, breadth = 6 cm and height = 4 cm. Find the number of the solid cuboidals formed.
7. Find the least internal volume of a box that can hold 8 boxes, each with dimensions 15 cm × 24 cm × 16 cm.
8. A birthday cake, rectangular in shape, has dimensions 45 cm × 30 cm × 18 cm. If each person attending the birthday party consumes 150 cm^3 of cake, for how many persons will the cake be sufficient.
9. Find the least number of bricks required to make a wall 3·2 m long, 36 cm broad and 4·2 m high, if each brick is 24 cm × 12 cm × 7·5 cm.