

PERIMETER AND AREA OF PLANE FIGURES

23.1 INTRODUCTION

In mensuration, we deal with measurements of length, area, volume, surface area, etc. Knowledge of mensuration is of great use in our day-to-day life, specially, for instance, when we buy :

- (i) **cloth** for shirts by **length**,
- (ii) **a plot of land** by **area**
- (iii) **milk, petrol**, etc., by **volume** and so on.

23.2 SOME DEFINITIONS

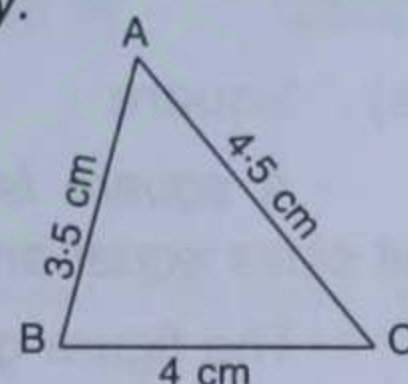
(a) (i) **Perimeter :**

The perimeter of a closed figure is the length of its boundary.

For example :

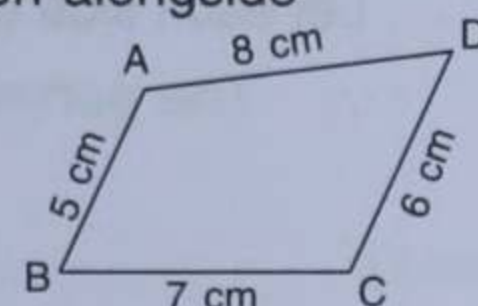
1. **Perimeter of ΔABC** given alongside

$$\begin{aligned} &= \text{Length of the boundary of } \Delta ABC \\ &= \text{Length of } AB + \text{length of } BC + \text{length of } CA \\ &= 3.5 \text{ cm} + 4 \text{ cm} + 4.5 \text{ cm} = \mathbf{12 \text{ cm}} \end{aligned}$$



2. **Perimeter of the plane figure (quadrilateral) ABCD** given alongside

$$\begin{aligned} &= AB + BC + CD + DA \\ &= 5 \text{ cm} + 7 \text{ cm} + 6 \text{ cm} + 8 \text{ cm} = \mathbf{26 \text{ cm}} \end{aligned}$$



(ii) **Unit of Perimeter :**

The unit of perimeter is the same as the unit of length, i.e. centimetre (cm), metre (m), etc.

$$1. \quad 1 \text{ cm} = \frac{1}{100} \text{ m} \quad \text{and} \quad 1 \text{ m} = 100 \text{ cm}$$

2. *For finding the perimeter of any plane-figure convert each length into the same unit,*

e.g. if the lengths of the sides of a triangular figure are 80 cm, 1.2 m and 95 cm,

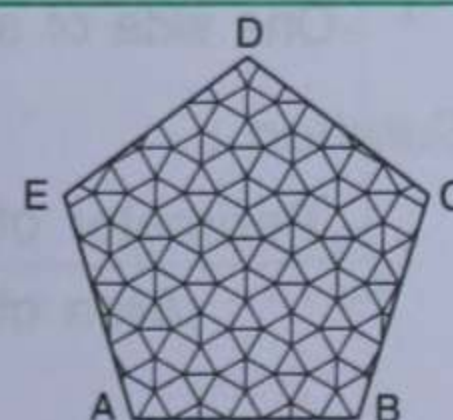
$$\begin{aligned} \text{its perimeter} &= 80 \text{ cm} + 1.2 \text{ m} + 95 \text{ cm} \\ &= 80 \text{ cm} + 120 \text{ cm} + 95 \text{ cm} \\ &= \mathbf{295 \text{ cm}} \end{aligned}$$

$$1.2 \text{ m} = 1.2 \times 100 \text{ cm} = 120 \text{ cm}$$

$$\begin{aligned} \text{OR, perimeter of the given triangle} &= 80 \text{ cm} + 1.2 \text{ m} + 95 \text{ cm} \\ &= 0.8 \text{ m} + 1.2 \text{ m} + 0.95 \text{ m} = \mathbf{2.95 \text{ m}} \end{aligned}$$

(b) (i) **Area :**

The area of a plane figure is the measure of the size of the surface enclosed by its boundary.



For example :

The area of the given figure ABCDE is the measure of the size of shaded portion that is enclosed by its boundary.

(ii) Unit of Area :

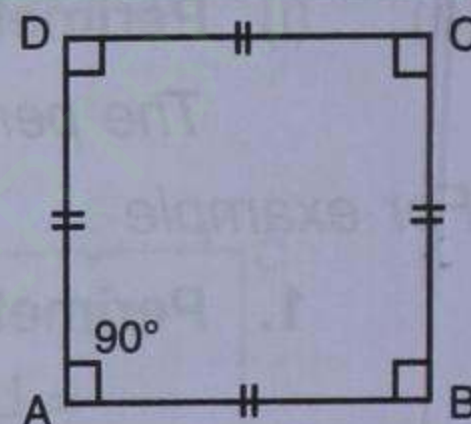
If the unit of the **length** of each side of a plane figure is **centimetre** (cm), the unit of its area will be **square-centimetre** (sq. cm, i.e. cm^2). In the same way, if the **length** of each side of a plane figure is **metre** (m), the unit of its **area** will be **square-metre** (sq. m, i.e. m^2).

$1 \text{ m} = 100 \text{ cm}$ and $1 \text{ m}^2 = 100 \times 100 \text{ cm}^2 = 10,000 \text{ cm}^2$ $1 \text{ cm} = \frac{1}{100} \text{ m}$ and $1 \text{ cm}^2 = \frac{1}{100} \times \frac{1}{100} \text{ m}^2 = \frac{1}{10,000} \text{ m}^2$	Some other units in use :	
	For length	For area
	1. Millimetre (mm) 2. Kilometre (km)	Square millimetre (mm^2) Square kilometre (km^2)

23.3 SOME IMPORTANT PLANE FIGURES

(a) Square :

A square is a four-sided closed figure with all sides equal and each angle 90° .



The figure given alongside shows a square ABCD in which $AB = BC = CD = DA$. And $\angle A = \angle B = \angle C = \angle D = 90^\circ$.

Let each side of the square be of length a units, i.e. $AB = BC = CD = DA = a$ units.

$$\begin{aligned} \therefore \text{The perimeter of the square} &= AB + BC + CD + DA \\ &= a + a + a + a \\ &= 4a = 4 \times \text{side of the square} \end{aligned}$$

$$\begin{aligned} \text{and area of the square} &= \text{its length} \times \text{its breadth} \\ &= a \times a = a^2 = (\text{side})^2. \end{aligned}$$

1. Since the perimeter (P) of a square is given by the formula :

$$P = 4 \times \text{length of its side}$$

$$\therefore \text{Length of each side of the square} = \frac{\text{Its perimeter}}{4}$$

2. Since the area A of a square $= (\text{side})^2$

$$\therefore \text{Length of its each side} = \sqrt{A}$$

Example 1 :

One side of a square is 6 cm. Find its perimeter and area.

Solution :

$$\text{Perimeter of the square} = 4 \times \text{side} = 4 \times 6 \text{ cm} = 24 \text{ cm} \quad (\text{Ans.})$$

$$\text{Area of the square} = (\text{side})^2 = (6 \text{ cm})^2 = 36 \text{ cm}^2 \quad (\text{Ans.})$$

Example 2 :

The perimeter of a square field is 96 m.

Find : (i) the length of its each side, (ii) the area of the square field.

Solution :

$$(i) \text{ Length of each side of the square} = \frac{\text{its perimeter}}{4} = \frac{96}{4} \text{ m} = 24 \text{ m} \quad (\text{Ans.})$$

$$(ii) \text{ Area of the square field} = (\text{side})^2 = (24 \text{ m})^2 = 576 \text{ m}^2 \quad (\text{Ans.})$$

Example 3 :

The area of a square is 144 m^2 .

Find : (i) its side (ii) its perimeter.

Solution :

$$(i) \text{ Side of the square} = \sqrt{A} = \sqrt{144} \text{ m} \\ = \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3} \text{ m} \\ = 2 \times 2 \times 3 \text{ m} = 12 \text{ m} \quad (\text{Ans.})$$

$$(ii) \text{ Perimeter of the square} = 4 \times \text{side} = 4 \times 12 \text{ m} = 48 \text{ m} \quad (\text{Ans.})$$

Example 4 :

Each side of a square field is 36 m. Find :

(i) its perimeter

Length of fencing = Perimeter of the field.

(ii) its area

(iii) the cost of fencing the field at the rate of ₹ 20 per metre.

(iv) the cost of ploughing the field at the rate of ₹ 1.50 per m^2 .

Solution :

$$(i) \text{ Perimeter of the square field} = 4 \times \text{its side} \\ = 4 \times 36 \text{ m} = 144 \text{ m} \quad (\text{Ans.})$$

$$(ii) \text{ Area of the square field} = (\text{side})^2 \\ = (36 \text{ m})^2 = 1,296 \text{ m}^2 \quad (\text{Ans.})$$

$$(iii) \text{ Cost of fencing the field} = \text{Rate of fencing} \times \text{perimeter} \\ = ₹ 20 \times 144 = ₹ 2,880 \quad (\text{Ans.})$$

$$(iv) \text{ Cost of ploughing the field} = \text{Rate of ploughing} \times \text{area} \\ = ₹ 1.50 \times 1296 = ₹ 1,944 \quad (\text{Ans.})$$

(b) Rectangle :

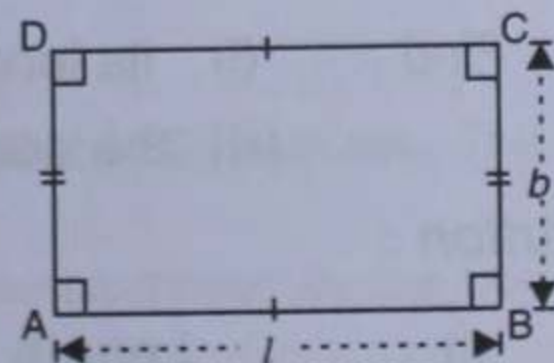
A rectangle is a four-sided closed figure of which the opposite sides are equal and each angle is 90° .

The adjacent figure shows a rectangle ABCD.

Clearly, $AB = CD = \text{length } (l) \text{ of the rectangle}$

and, $AD = BC = \text{breadth } (b) \text{ of the rectangle.}$

Also, $\angle A = \angle B = \angle C = \angle D = 90^\circ$.



∴ **Perimeter of rectangle ABCD** = Length of its boundary

$$= AB + BC + CD + DA$$

$$= l + b + l + b$$

$$= 2(l + b) \quad \text{i.e. } P = 2(l + b)$$

Opposite sides of a rectangle are equal

And **area of rectangle** = its length $\times$ its breadth

$$= l \times b \quad \text{i.e. } A = l \times b$$

Example 5 :

The length and the breadth of a rectangle are 10 cm and 8 cm, respectively. Find its perimeter and area.

Solution :

Since the length of the rectangle (l) = 10 cm

and the breadth of the rectangle (b) = 8 cm

$$\begin{aligned} \therefore \quad \text{Perimeter (P)} &= 2(l + b) \\ &= 2(10 + 8) \text{ cm} = 36 \text{ cm} \end{aligned} \quad (\text{Ans.})$$

$$\begin{aligned} \text{Area of rectangle (A)} &= l \times b \\ &= 10 \text{ cm} \times 8 \text{ cm} = 80 \text{ cm}^2 \end{aligned} \quad (\text{Ans.})$$

1. Since the perimeter of a rectangle is given by the formula $P = 2(l + b)$

$$\therefore \quad \text{Its length, } l = \frac{P}{2} - b \text{ and its breadth, } b = \frac{P}{2} - l$$

2. Since the area of a rectangle is given by $A = l \times b$

$$\therefore \quad \text{Its length, } l = \frac{A}{b} \text{ and its breadth, } b = \frac{A}{l}$$

Example 6 :

The perimeter of a rectangle is 30 cm and its length is 8 cm.

Find : (i) its breadth (ii) its area

Solution :

Given : $P = 30 \text{ cm}$ and $l = 8 \text{ cm}$

$$\therefore \quad \text{(i) Breadth, } b = \frac{P}{2} - l = \frac{30}{2} \text{ cm} - 8 \text{ cm} = (15 - 8) \text{ cm} = 7 \text{ cm} \quad (\text{Ans.})$$

$$\text{and (ii) Area, } A = l \times b = 8 \text{ cm} \times 7 \text{ cm} = 56 \text{ cm}^2 \quad (\text{Ans.})$$

Example 7 :

The area of a rectangular field is 450 m^2 and its width is 25 m.

Find : (i) its length (ii) its perimeter

(iii) the cost of fencing the field at the rate of ₹ 35.50 per metre.

Solution :

$$\text{(i) Length} = \frac{\text{Area}}{\text{Breadth}} = \frac{450}{25} \text{ m} = 18 \text{ m} \quad (\text{Ans.})$$

(ii) **Perimeter** = $2(l + b) = 2(18 + 25) \text{ m} = 2 \times 43 \text{ m} = 86 \text{ m}$ (Ans.)

(iii) **Cost of fencing** = Length of fence $\times$ Rate

= $86 \times ₹ 35.50$

= ₹ 3,053

Length of fence = Perimeter = 86 m

(Ans.)

EXERCISE 23(A)

- The sides of a triangle are 3 cm, 4 cm and 5 cm long. Find its perimeter.
- The lengths of the sides of a triangular field are 15 m, 20 m and 24 m. Find the total distance travelled by a boy moving along its boundary in making :
 - one complete round
 - 7 complete rounds.
- The perimeter of a square is 52 cm. Find the length of one of its sides and also its area.
- The area of a square is 225 m^2 . Find the length of one of its sides and also its perimeter.
- A square field has a side of 160 m length. Find its area and the cost of levelling it at the rate of ₹ 5 per square metre.
- The area of a square field is 576 m^2 . Find :
 - the length of its side.
 - the length of its perimeter.
 - the cost of fixing a fence along the boundary of the field at the rate of ₹ 3.20 per metre.
- A rectangular carpet is 4.5 m long and 3.2 m wide; find :
 - its area.
 - the cost of the carpet at the rate of ₹ 36 per square metre.
- The perimeter of a rectangular field is 64 m. If its length is 20 m, find :
 - its breadth
 - its area
- The perimeter of a rectangular field is 86 m. If its breadth is 25 m, find :
 - its length
 - its area.
- The area of a rectangle is 260 cm^2 . If its length is 20 cm, find :
 - its breadth
 - its perimeter
- The area of a rectangular field is 300 m^2 . If its breadth is 15 m, find :
 - its length
 - its perimeter
- The floor of a room is square in shape. If the length of one side of the room is 2.6 m, find :
 - the area of the floor.
 - the cost of carpeting the room at the rate of ₹ 40 per square metre.
- An agricultural field is rectangular in shape. If its length is 200 m and width 125 m, find :
 - its area.
 - the cost of ploughing the field at the rate of 60 paise per square metre.
- A playground is rectangular in shape. If its length is 60 m and width 45 m, find :
 - its perimeter.
 - the cost of fencing it at the rate of ₹ 2.50 per metre.
- A boy makes one round along the boundary of a rectangular field in 20 minutes. The field is 160 m long and 124 m wide. Find :
 - the perimeter of the field
 - the distance moved by the boy
 - the speed (in ms^{-1}) at which the boy runs.

Example 8 :

A square and a rectangle have equal areas. If each side of the square is 18 m and the width of the rectangle is 12 m, find :

- (i) the area of the square
- (ii) the length of the rectangle
- (iii) the perimeter of the rectangle

Solution :

(i) Since each side of the square = 18 m

$$\begin{aligned}\therefore \text{Area of square} &= (\text{side})^2 \\ &= (18 \text{ m})^2 = 324 \text{ m}^2 \quad (\text{Ans.})\end{aligned}$$

(ii) Given, area of rectangle = area of square

$$\therefore \text{Area of rectangle} = 324 \text{ m}^2,$$

$$\text{i.e. length of rectangle} \times \text{its breadth} = 324 \text{ m}^2$$

$$\text{Area} = \text{length} \times \text{breadth}$$

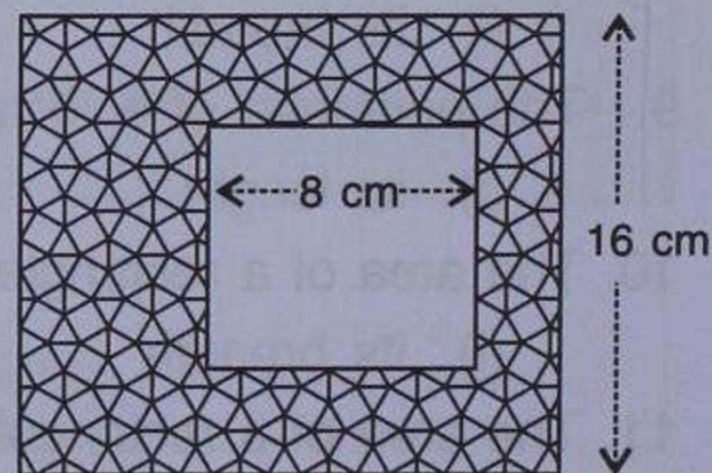
$$\Rightarrow \text{length of rectangle} \times 12 \text{ m} = 324 \text{ m}^2$$

$$\text{So, length of rectangle} = \frac{324}{12} \text{ m} = 27 \text{ m} \quad (\text{Ans.})$$

$$\begin{aligned}\text{(iii) Perimeter of rectangle} &= 2(\text{length} + \text{breadth}) \\ &= 2(27 \text{ m} + 12 \text{ m}) \\ &= 78 \text{ m} \quad (\text{Ans.})\end{aligned}$$

Example 9 :

The adjoining figure shows a shaded portion, enclosed by two squares. If the sides of the squares are 16 cm and 8 cm, respectively, find the area of the shaded portion.



Solution :

$$\begin{aligned}\text{Since area of the bigger (outer) square} &= (\text{side})^2 \\ &= (16 \text{ cm})^2 = 256 \text{ cm}^2\end{aligned}$$

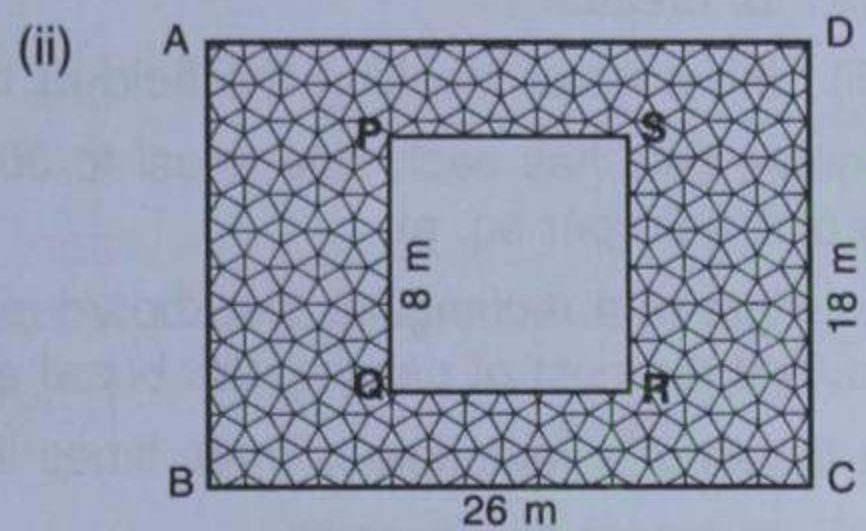
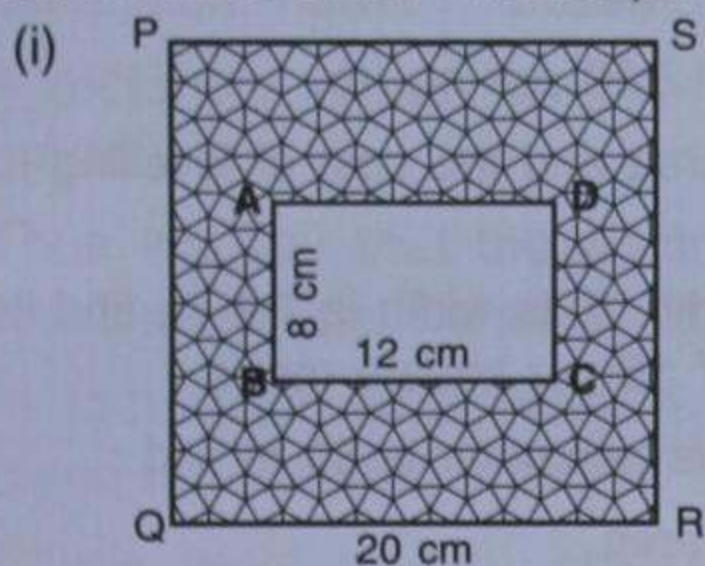
$$\begin{aligned}\text{And, area of the smaller (inner) square} &= (8 \text{ cm})^2 \\ &= 64 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\therefore \text{Area of the shaded portion} &= \text{Area of the bigger square} - \text{Area of the smaller square} \\ &= 256 \text{ cm}^2 - 64 \text{ cm}^2 = 192 \text{ cm}^2 \quad (\text{Ans.})\end{aligned}$$

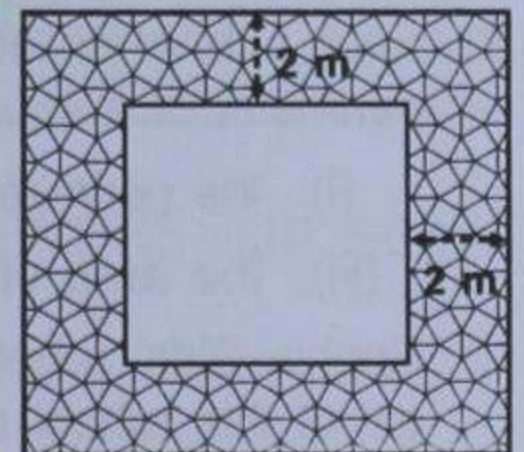
EXERCISE 23(B)

1. The area of a square is the same as the area of a rectangle. If each side of the square is 24 cm and the breadth of the rectangle is 18 cm, find :
 - (i) the area of the rectangle
 - (ii) the length of the rectangle
 - (iii) the perimeter of the rectangle

2. The area of a rectangle is the same as the area of a square. If the length and the breadth of the rectangle are 8 cm and 4.5 cm, respectively, find :
- the area of the square.
 - the side of the square.
 - the perimeter of the square
3. The length of a rectangle is 24 cm and breadth 16 cm. Find its perimeter;
Also, if the perimeter of a square is the same as the perimeter of this rectangle, find :
- the length of a side of the square.
 - the area of the square.
4. The length and the breadth of a rectangle are 24 cm and 18 cm, respectively, whereas the length of each side of a square is 20 cm. Find :
- the difference between the perimeters of the given rectangle and square.
 - the difference between their areas.
5. In each of the following figures, ABCD is a rectangle and PQRS is a square. Find, in each case, the area of the shaded portion :

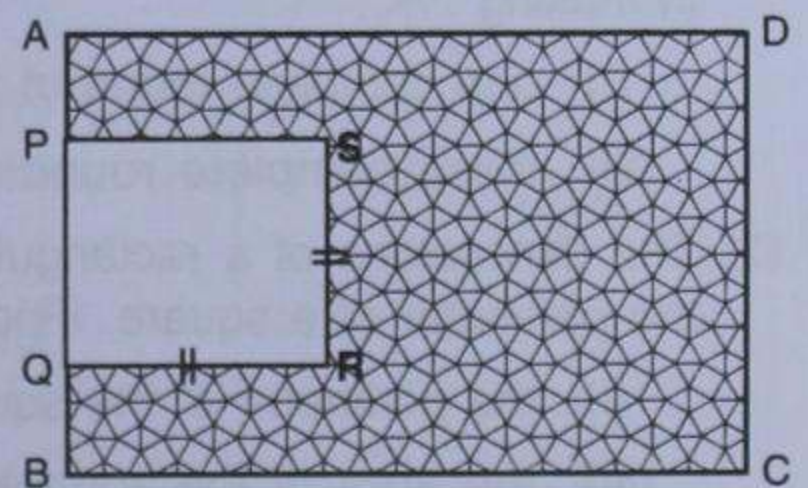


6. The shaded portion in the adjoining figure has uniform width of 2 m and is enclosed by two squares. If each side of the outer (bigger) square is 9 m, find the area of the shaded portion.

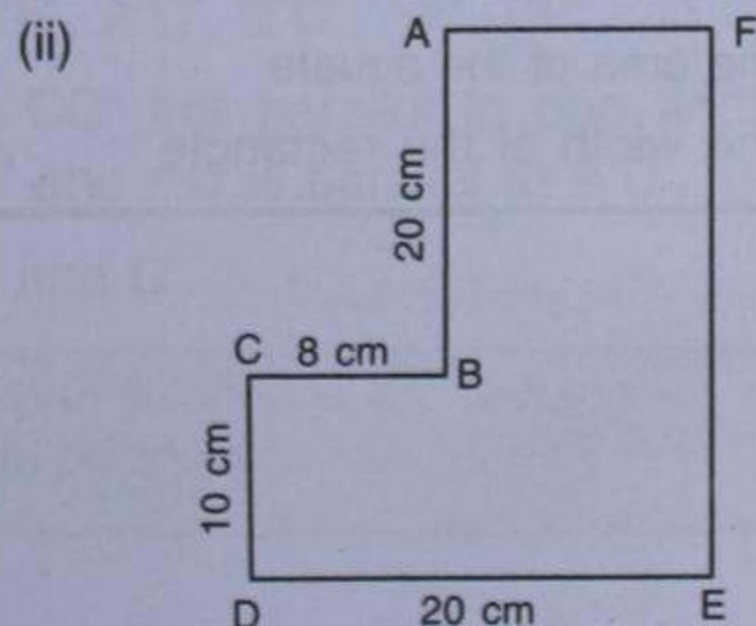
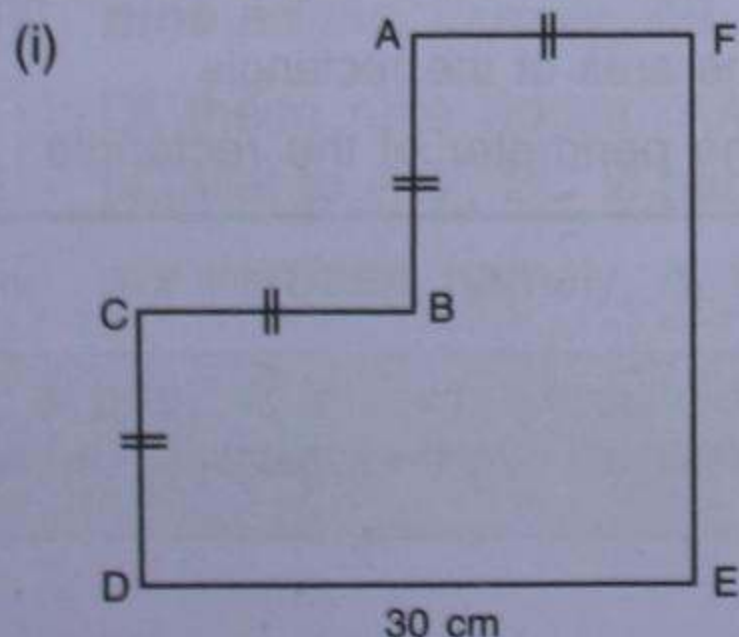


7. In the figure given alongside ABCD is a rectangle and PQRS is a square. Find the area of the shaded portion if :

$$\begin{aligned} BC &= 16 \text{ m}, \\ AB &= 12 \text{ m}, \\ \text{and } AP &= BQ = 3 \text{ m}. \end{aligned}$$



8. Find the area and the perimeter of each figure, given below in which all the lengths are in cm and the angle at each vertex is 90° .



Revision Exercise (Chapter 23)

1. Find the perimeter and the area of a rectangle whose length is 20 cm, and breadth is 75% of its length.
 2. Find the length and the perimeter of a rectangular field whose area is 624 m^2 and breadth is 24 m.
 3. Find the breadth and the perimeter of a rectangular field with area 1350 m^2 and length 45 m.
 4. The area of a rectangle is 768 cm^2 and its length and breadth are in the ratio 4 : 3. Find its length, breadth and perimeter.
 5. Find the cost of fencing a rectangular field with length 32 m and breadth 25 m if the rate of fencing is ₹ 40 per m.
 6. A rectangular field has perimeter 152 m and length 48 m. Find :
 - (i) its breadth
 - (ii) its area
 - (iii) the cost of levelling the field at the rate of ₹ 12.50 per sq. m.
 7. A square field has each side equal to 36 m. Find its area and the cost of levelling it at the rate of ₹ 5.60 per sq. m.
 8. The length of a rectangular blackboard is double its width. If its width is 1.2 m, find its area. Also, find the cost of painting the board at the rate of ₹ 15 per square metre.
 9. The length of a rectangle is three times its breadth. If its perimeter is 80 m find :
 - (i) its length and breadth.
 - (ii) its area
 10. Each side of an equilateral triangle is 20 cm. If the perimeter of this triangle is equal to the perimeter of a square, find :
 - (i) the perimeter of the square
 - (ii) each side of the square
 - (iii) the area of the square.
 11. Sophia Girls' school has a rectangular playground with length = 80 m and width = 60 m. A girl of this school runs along the boundary of the playground. Find the distance she will run in making :
 - (i) one complete round of the playground
 - (iii) three complete rounds of the playground.
 12. The dimensions of a rectangular body are 25 cm and 15 cm, and its perimeter is equal to the perimeter of a square. Find :
 - (i) the perimeter of the square
 - (ii) each side of the square
 - (iii) the area of the square.
 13. Each side of a square is 20 cm and its area is equal to the area of a rectangle with length 25 cm. Find :
 - (i) the area of the square
 - (ii) the area of the rectangle
 - (iii) the width of the rectangle
 - (iv) the perimeter of the rectangle.
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