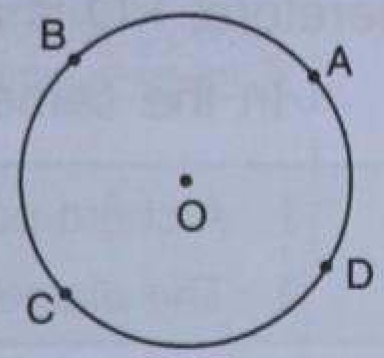
CHAPTER 21

THE CIRCLE

21.1 CIRCLE :

A circle is a closed curve such that all the points on its circumference (boundary) are equidistant from a fixed point inside it.



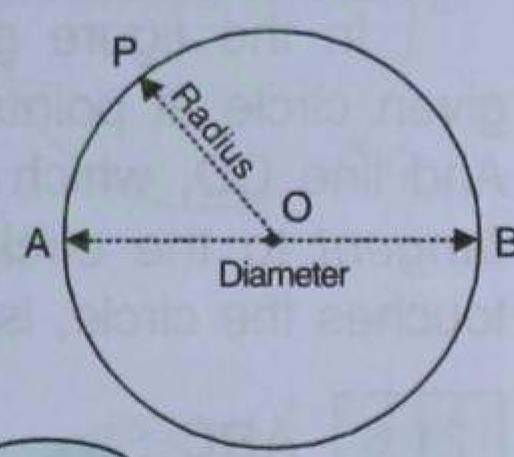
The given figure shows a circle and a fixed point O inside the circle. The points A, B, C and D, which are marked on the circumference (boundary) of the circle, are all at the same distance from the fixed point O, *i.e.* OA = OB = OC = OD and so on.

In fact, a **circle** is defined as the ***path traced*** **by a *moving point* that always remains at a *fixed distance* from a given *fixed point.***

The path so traced by a moving point is called the ***circumference*** of a circle. The fixed point is called the ***centre*** of the circle and *the fixed distance* from this fixed point to any point on the circumference is called ***radius.***

21.2 RADIUS :

The radius of a circle is the length of the line segment joining the centre of the circle with any point on its circumference.



In the adjacent figure, OP is the line that joins the centre O of the circle with a point P on its circumference; therefore OP = radius of the circle.

Each of OA and OB is a radius.

21.3 DIAMETER :

A ***straight line*** *passing* ***through the centre*** *of a circle and with both its* ***ends on the circumference*** *of the circle is called the* ***diameter*** *of the circle.*

In the figure given above, AB is the diameter.

1. In a circle, the length of the diameter is always double the length of the radius, *i.e.* Diameter = 2 × Radius and Radius = $\frac{\text{Diameter}}{2}$
2. The diameter of a circle always divides the circle into two equal parts, each of these two equal parts is called a **semi-circle**.

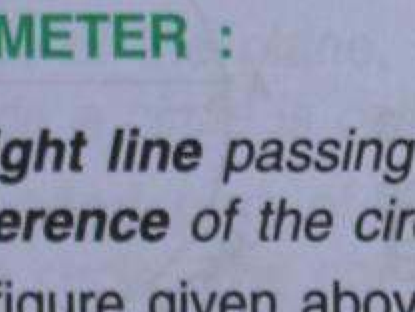
IMPORTANT : Let a circle with centre O and a point P lie in the same plane such that the distance between O and P is :

(i) ***greater than the radius***, the point P lies outside the circle.

(ii) ***equal to the radius***, the point P lies on the circumference of the circle.

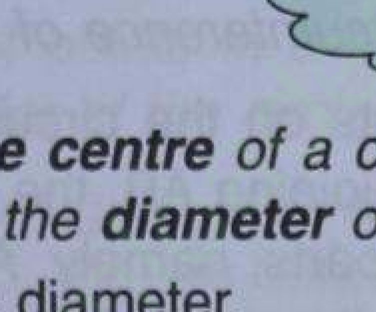
(iii) ***less than the radius***, the point P lies inside the circle.

Thus, (i)



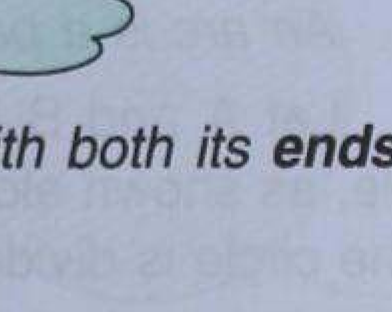
[P lies outside the circle ⇒ OP > radius]

(ii)



[P lies on the circumference ⇒ OP = radius]

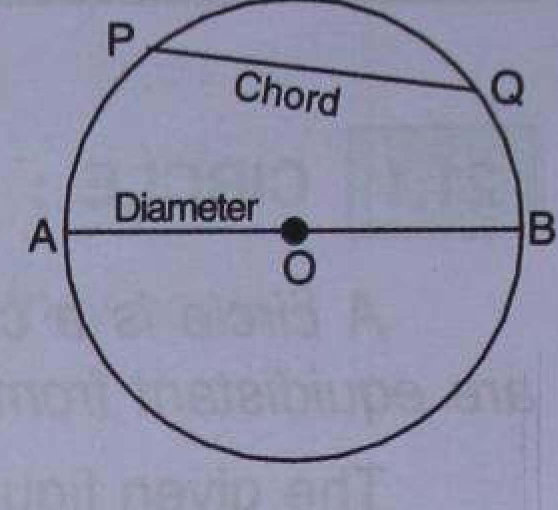
(iii)



[P lies inside the circle ⇒ OP < radius]

21.4 CHORD :

*A straight line joining any two points on the circumference of a circle, is called a **chord**.*



In figure given alongside, the straight line PQ is obtained by joining the points P and Q lying on the circumference of the circle; therefore, PQ is a chord of the circle.

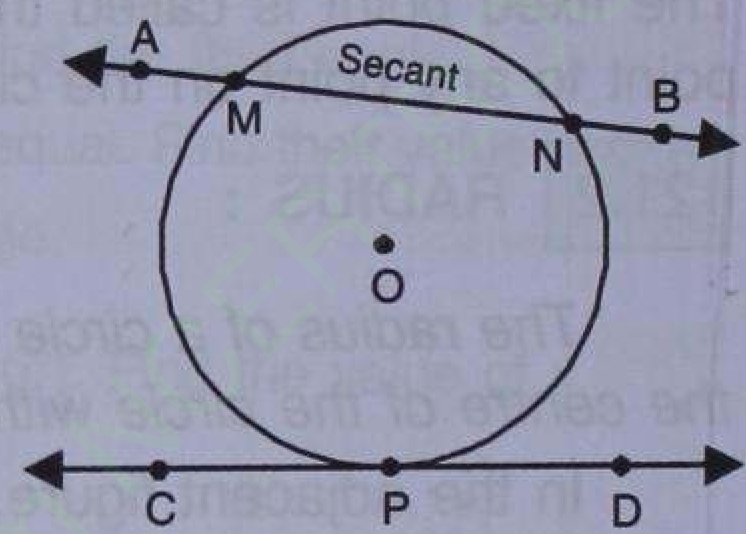
In the same way, diameter AB, too is a chord of the circle.

1. A chord passing through the centre of a circle is also its diameter; viz. AB.
2. The diameter of a circle is the longest chord of that circle.

21.5 SECANT AND TANGENT :

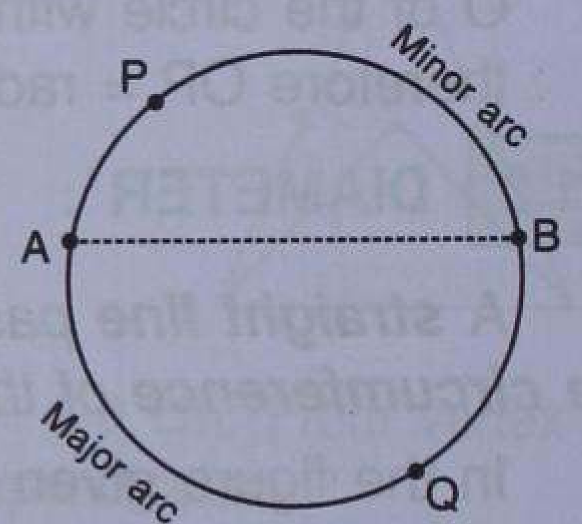
*A straight line intersecting a circle at two points is called a **secant.***

*A straight line touching a circle at one point only is called a **tangent.***



In the figure given alongside, line AB intersects the given circle at points M and N. Therefore, AB is a *secant.* And line CD, which touches the circle at point P only is a *tangent* to the circle. The point P, at which the tangent touches the circle, is the **point of contact.**

21.6 ARC :

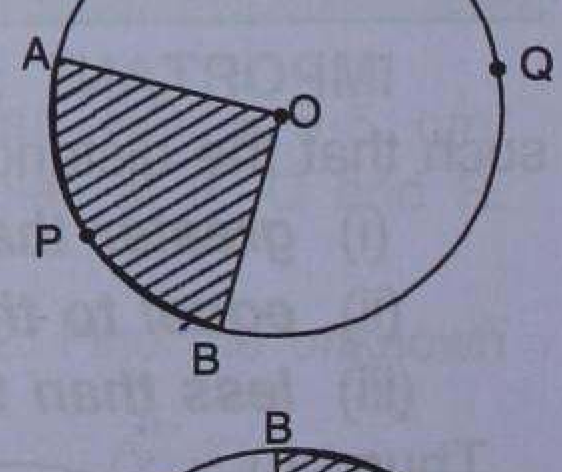


An arc is a part of the circumference of a circle.

Let A and B be two points on the circumference of a circle, as shown alongside. On joining AB, the circumference of the circle is divided into two parts, namely, APB and AQB. These two parts are known as the arcs of a circle.

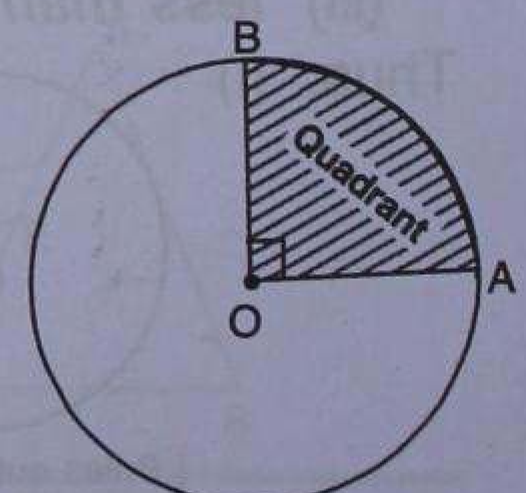
It is clear from the figure that *the length of the arc APB is smaller* than the *length of the arc AQB.* Therefore, **arc APB** is the ***minor arc*** and **arc AQB** is the ***major arc***.

21.7 SECTOR :



The part of a circle enclosed by any two radii (plural of radius) and an arc *is called a **sector**.*

In the figure given alongside, the arc APB and the radii OA and OB enclose the shaded portion OAPB, which is thus a sector of the circle.



It is clear from the figure that the radii OA and OB divide the circle into two sectors; the *minor sector is the shaded portion* OAPB and *the major sector is the unshaded portion* OAQB.

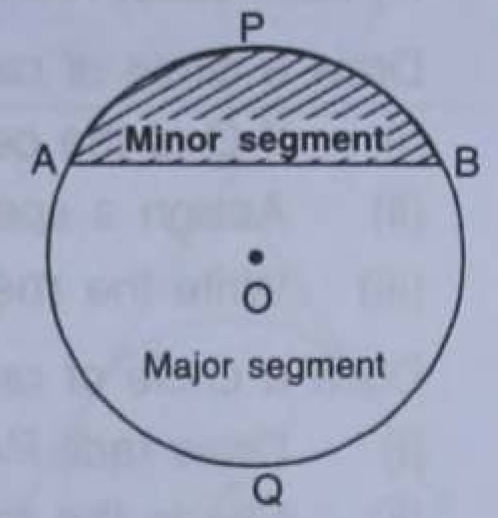
Note : The part of a circle enclosed by two mutually perpendicular radii, as shown alongside, is called a ***quadrant***.

21.8 SEGMENT :

Every chord divides a circle into two parts; these two parts of a circle are called its ***segments.***

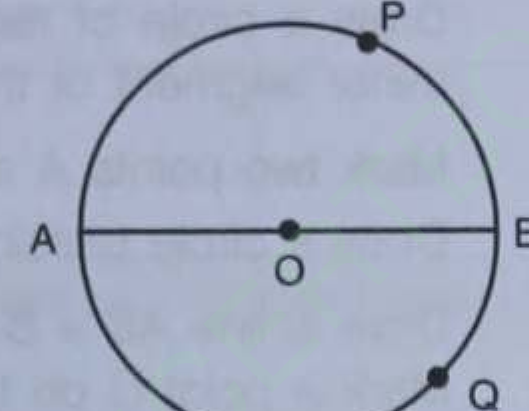
Therefore, *a segment is a part of a circle enclosed by a chord and an arc.*

In the figure given alongside , the chord AB divides the circle into two parts.



The ***smaller,*** *i.e.* the *shaded part of the circle, is called the* ***minor segment,*** and the ***larger,*** *i.e.* the *unshaded part of the circle, is called the* ***major segment****.*

1. The major segment contains the centre of the circle *i.e.* the minor segment does not contain the centre of the circle.
2. A chord through the centre, (*i.e.* a diameter) divides a circle into two equal parts, each equal part is called a **semi-circle.**

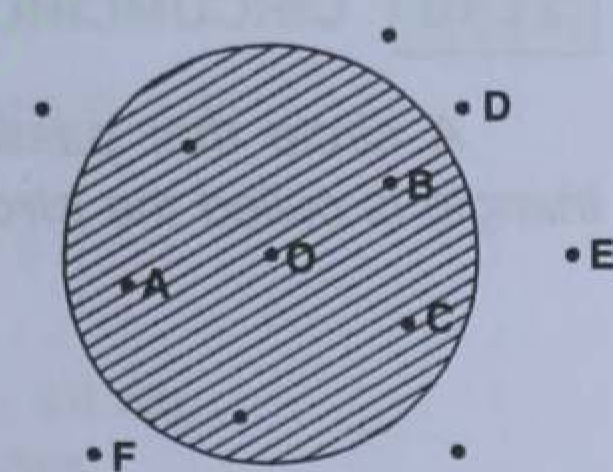


In the figure given alongside, AB is a diameter so APB and AQB are semi-circles.

21.9 INTERIOR AND EXTERIOR OF A CIRCLE :

The *part of the plane,* or the set of points in a plane *that lie inside a circle is called the interior of the circle.*

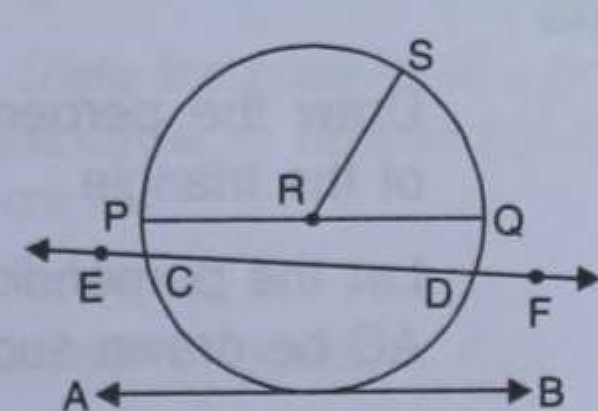
The part of the plane, or the set of points in a plane, *that lie outside a circle is called the exterior of the circle.*



The shaded part of the adjoining figure shows the interior of the circle. The points O, A, B, C, etc., lie inside the circle, whereas the points D, E, F, etc., lie outside the circle.

EXERCISE 21(A)

1. Use the figure given alongside to fill in the blanks :



(i) R is the .... of the circle.

(ii) Diameter of a circle is ....

(iii) Tangent to a circle is ....

(iv) EF is a .... of the circle.

(v) .... is a chord of the circle.

(vi) Diameter = 2 × ....

(vii) .... is a radius of the circle.

(viii) If the length of RS is 5 cm, the length of PQ = ...

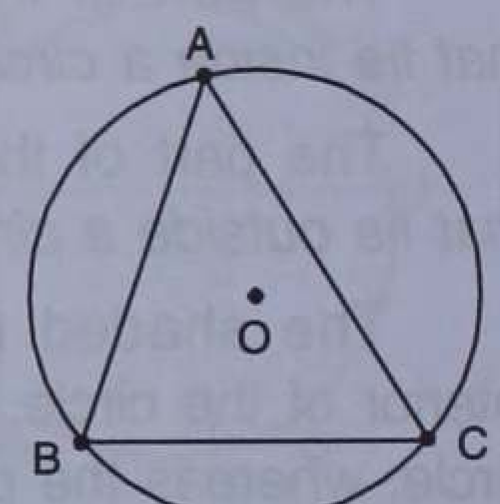
(ix) If PQ is 8 cm long, the length of RS = ...

(x) AB is a .... of the circle.

2. Draw a circle of radius 4·2 cm. Mark its centre as O. Take a point A on the circumference of the circle. Join AO and extend it till it meets point B on the circumference of the circle.
   (i) Measure the length of AB. (ii) Assign a special name to AB.
3. Draw circles with diameter :
   (i) 6 cm (ii) 8·4 cm
   In each case, measure the length of the radius of the circle drawn.
4. Draw a circle of radius 6 cm. In the circle, draw a chord AB = 6 cm.
   (i) If O is the centre of the circle, join OA and OB.
   (ii) Assign a special name to Δ AOB.
   (iii) Write the measure of angle AOB.
5. Draw a circle of radius 4·8 cm and mark its centre as P.
   (i) Draw radii PA and PB such that $\angle APB = 45°$.
   (ii) Shade the major sector of the circle.
6. Draw a circle of radius 3·6 cm. In the circle, draw a chord AB = 5 cm. Now shade the minor segment of the circle.
7. Mark two points A and B, 4 cm apart.
   Draw a circle passing through B and with A as centre.
8. Draw a line AB = 8·4 cm. Now draw a circle with AB as diameter.
   Mark a point C on the circumference of the circle. Measure angle ACB.

21.10 CIRCUMCIRCLE OF A TRIANGLE :

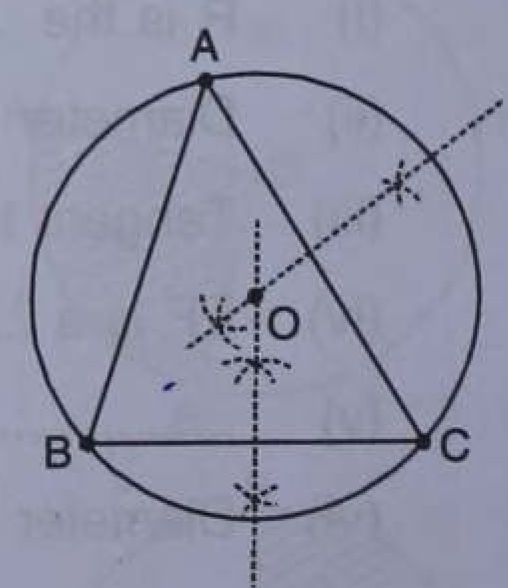
*A circle that passes through all the three vertices of a triangle is called the **circumcircle of the triangle**.*



To construct the circumcircle of a given triangle ABC.

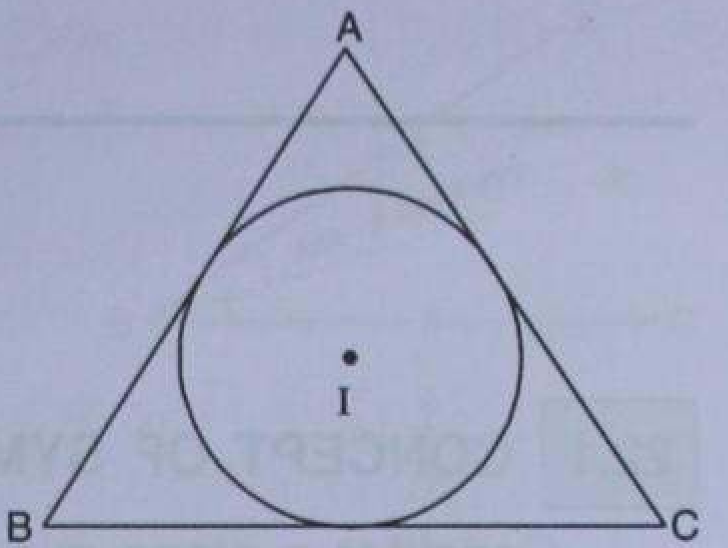
Steps :

1. Draw the perpendicular bisectors of any two sides of the triangle.
   Let the perpendicular bisectors of the sides BC and AC be drawn such that they intersect at point O.
2. Draw a circle taking O as centre and OA or OB or OC as radius. This circle will pass through all the three vertices, A, B and C.



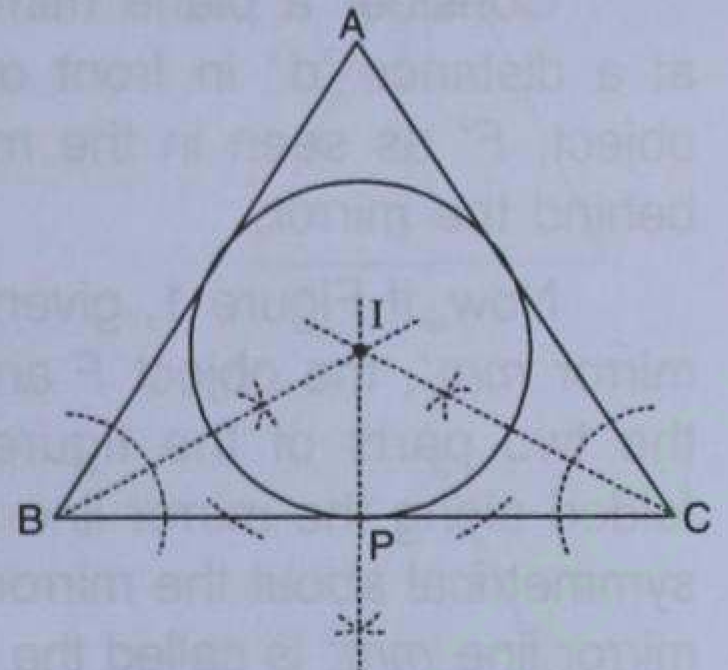
1. *The point where the perpendicular bisectors of the sides of a triangle meet, shown here as O, is called **circumcentre.***
2. *OA = OB = OC = radius of the circle = **circumradius.***

21.11 IN-CIRCLE OF A TRIANGLE :



A circle drawn inside a triangle *such that it touches all the three sides of the triangle is called the* **in-circle** *of the triangle.*

To construct the in-circle of a given triangle ABC.



Steps :

1. Draw the bisectors of any two angles of the triangle. Let these bisectors meet at point I.
2. From the point I, drop perpendicular IP on to BC.
3. With I as centre and IP as radius, draw a circle; this circle will touch all the three sides of the triangle.

1. *The point where the bisectors of the angles of a triangle meet, shown here as I, is called* **incentre**.
2. *The length of the perpendicular, here IP, is called* **inradius.**

EXERCISE 21(B)

1. Construct a triangle ABC with AB = 4·2 cm, BC = 6 cm and AC = 5 cm. Construct the circumcircle of the triangle drawn.
2. Construct a triangle PQR with QR = 5·5 cm, $\angle Q = 60°$ and angle R = 45°. Construct the circumcircle of the triangle PQR.
3. Construct a triangle ABC with AB = 5 cm, $\angle B = 60°$ and BC = 6·4 cm. Draw the incircle of the triangle ABC.
4. Construct a triangle XYZ in which XY = YZ = 4·5 cm and ZX = 5·4 cm. Draw the circumcircle of the triangle and measure its circumradius.
5. Construct a triangle PQR in which PQ = QR = RP = 5·7 cm. Draw the incircle of the triangle and measure its radius.

Revision Exercise (Chapter 21)

1. The centre of a circle is at point O and its radius is 8 cm. State the position of a point P (point P may lie inside the circle, on the circumference of the circle, or outside the circle), when : (a) OP = 10·6 cm (b) OP = 6·8 cm (c) OP = 8 cm
2. The diameter of a circle is 12·6 cm. State the length of its radius.
3. Can the length of a chord of a circle be greater than its diameter ? Explain
4. Draw a circle of diameter 7 cm. Draw two radii of this circle such that the angle between these radii is 90°. Shade the minor sector obtained. Write a special name for this sector.
5. State which of following statements are true and which are false :
   (i) If the end points A and B of a line segment lie on the circumference of a circle, AB is a diametre.
   (ii) The longest chord of a circle is its diametre.
   (iii) Every diametre bisects a circle and each part of the circle so obtained is a semi-circle.
   (iv) The diametres of a circle always pass through the same point in the circle.