

PROPERTIES OF ANGLES AND LINES

(Including Parallel Lines)

18.1 PROPERTIES OF ADJACENT AND VERTICALLY OPPOSITE ANGLES

Property 1 :

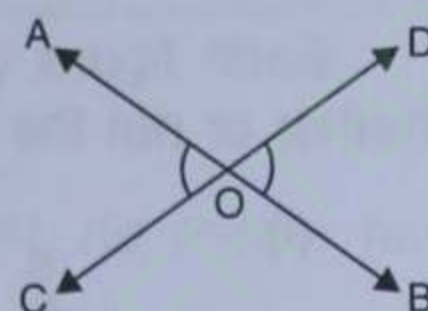
When two straight lines intersect :

- (i) *the sum of each pair of adjacent angles is always 180° .*
- (ii) *the vertically opposite angles are always equal.*

For example :

In the adjoining figure, the straight lines AB and CD intersect each other at point O, and so, we have :

- (i) the sum of adjacent angles = 180° ,
i.e. $\angle AOD + \angle DOB = 180^\circ$,
 $\angle BOD + \angle BOC = 180^\circ$,
 $\angle BOC + \angle COA = 180^\circ$,
 and $\angle COA + \angle AOD = 180^\circ$.



- (ii) the vertically opposite angles are equal, *i.e.* $\angle AOC = \angle BOD$ and $\angle BOC = \angle AOD$

Property 2 :

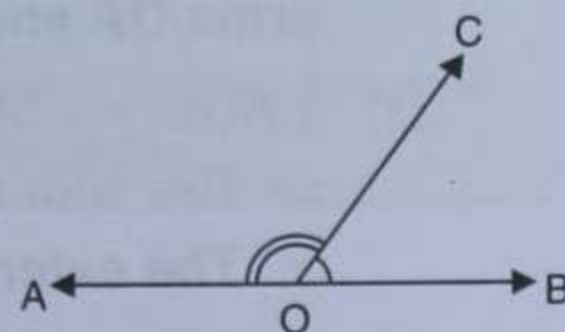
If the sum of two adjacent angles is 180° , their exterior arms are always in the same straight line.

Conversely, if the exterior arms of two adjacent angles are in the same straight line, the sum of the angles is always 180° .

For example :

Considering the two adjacent angles given alongside :

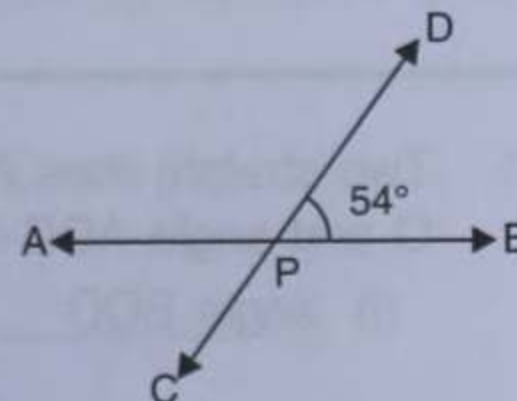
- (i) if $\angle AOC + \angle BOC = 180^\circ$, the exterior arms OA and OB are in the same straight line, *i.e.* AOB is a straight line.
- (ii) if exterior arms OA and OB are in the same straight line, *i.e.* if AOB is a straight line, $\angle AOC + \angle BOC = 180^\circ$.



Example 1 :

Two straight lines AB and CD intersect at point P. If angle BPD = 54° , find, giving reason :

- (i) $\angle APD$ (ii) $\angle APC$.



Solution :

- (i) When two straight lines intersect each other, the adjacent angles are supplementary.

$$\begin{aligned} \Rightarrow \angle APD + \angle BPD &= 180^\circ & \Rightarrow \angle APD + 54^\circ &= 180^\circ \\ & & \Rightarrow \angle APD &= 180^\circ - 54^\circ \\ & & &= 126^\circ \quad \text{(Ans.)} \end{aligned}$$

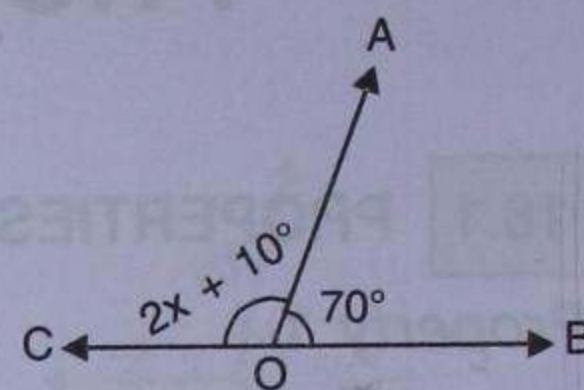
- (ii) When two straight lines intersect each other, the vertically opposite angles are equal.

$$\Rightarrow \angle APC = \angle BPD = 54^\circ$$

(Ans.)

Example 2 :

The figure given alongside, shows two adjacent angles AOB and AOC whose exterior arms OB and OC are along the same straight line. Find the value of x .



Solution :

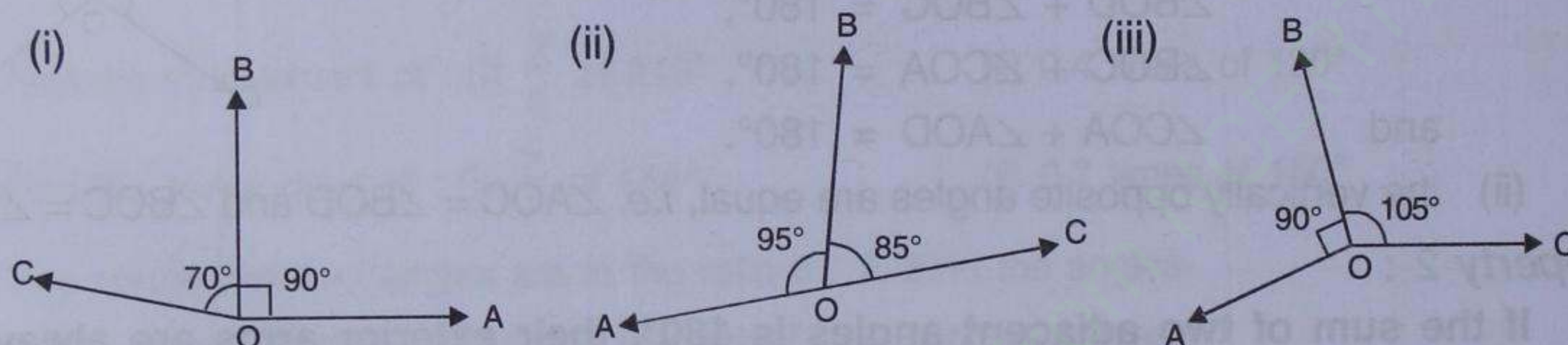
Since the exterior arms of the adjacent angles are in a straight line, the adjacent angles are supplementary.

$$\therefore \angle AOC + \angle AOB = 180^\circ \Rightarrow 2x + 10^\circ + 70^\circ = 180^\circ$$

$$\Rightarrow 2x = 180^\circ - 80^\circ \Rightarrow x = \frac{100^\circ}{2} = 50^\circ \quad (\text{Ans.})$$

Example 3 :

Each figure given below shows a pair of adjacent angles AOB and BOC. Find whether or not the exterior arms OA and OC are in the same straight line.



Solution :

(i) $\angle AOB + \angle BOC = 90^\circ + 70^\circ = 160^\circ$

Since the sum of adjacent angles AOB and BOC is not 180° , the exterior arms OA and OB are not in the same straight line. (Ans.)

(ii) $\angle AOB + \angle BOC = 95^\circ + 85^\circ = 180^\circ$

$\Rightarrow$ The sum of adjacent angles AOB and BOC is 180° .

$\therefore$ The exterior arms OA and OC are in the same straight line. (Ans.)

(iii) $\angle AOB + \angle BOC = 90^\circ + 105^\circ = 195^\circ$; which is not 180° .

$\Rightarrow$ The exterior arms OA and OC are not in the same straight line. (Ans.)

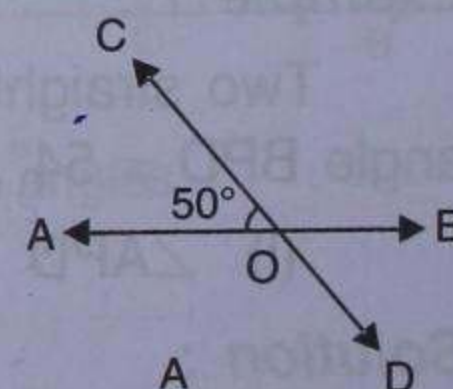
EXERCISE 18(A)

1. Two straight lines AB and CD intersect each other at a point O and angle AOC = 50° ; find :

(i) angle BOD

(ii) $\angle AOD$

(iii) $\angle BOC$



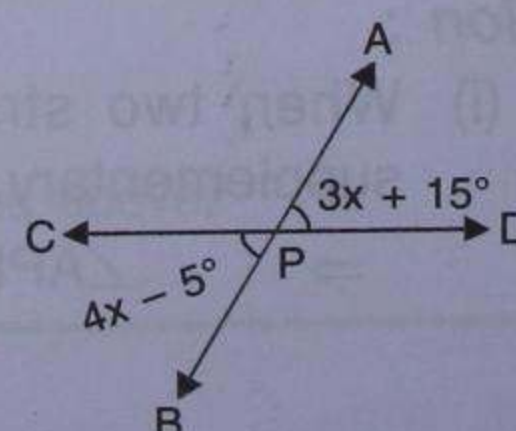
2. The adjoining figure shows two straight lines AB and CD intersecting at point P. If $\angle BPC = 4x - 5^\circ$ and $\angle APD = 3x + 15^\circ$, find :

(i) the value of x .

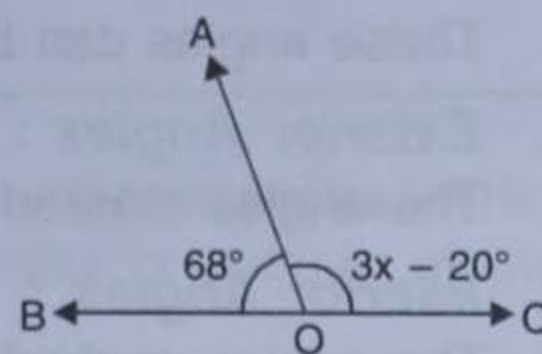
(ii) $\angle APD$

(iii) $\angle BPD$

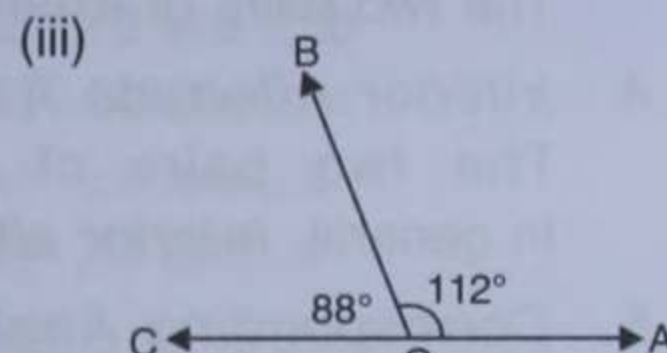
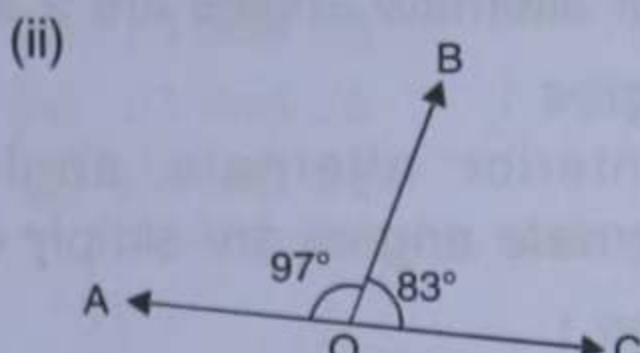
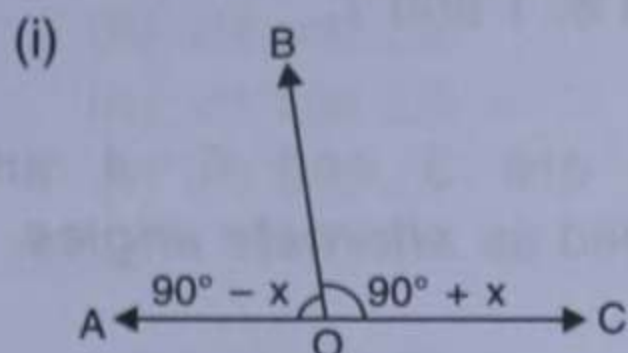
(iv) $\angle BPC$.



3. The figure given alongside, shows two adjacent angles AOB and AOC whose exterior sides are along the same straight line. Find the value of x .



4. Each figure given below shows a pair of adjacent angles AOB and BOC. Find whether or not the exterior arms OA and OC are in the same straight line.

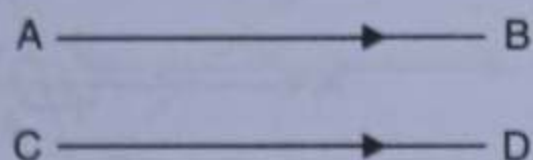


5. A line segment AP stands at point P of a straight line BC such that $\angle APB = 5x - 40^\circ$ and $\angle APC = x + 10^\circ$; find the values of x and angle APB.

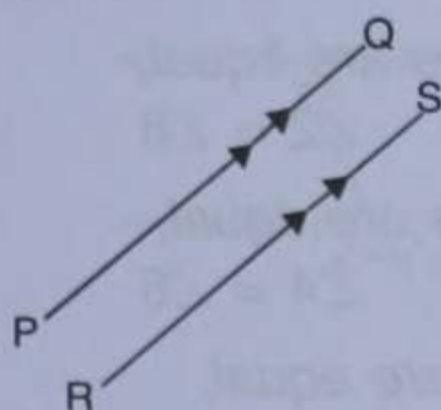
18.2 PARALLEL LINES

Two straight lines are said to be parallel if they do not meet, no matter how much they be extended in either direction.

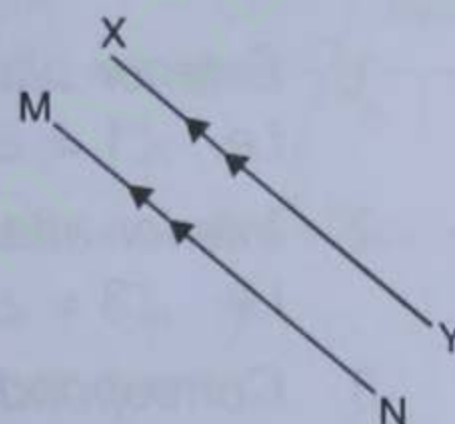
e.g. (i)



(ii)



(iii)

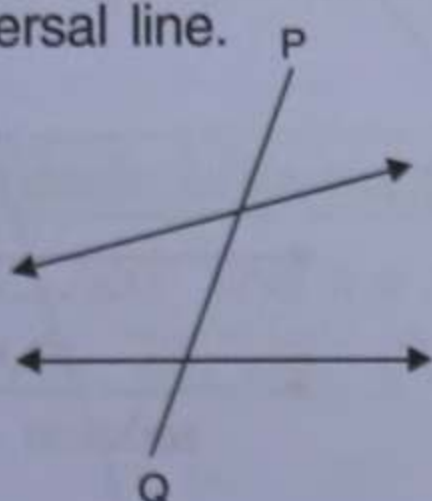


- Two parallel lines are represented by drawing arrows on both lines in the same direction. See the figures drawn above.
- The distance between two parallel lines does not change, i.e. neither does it increase nor does it decrease. That is why parallel lines never meet (intersect).

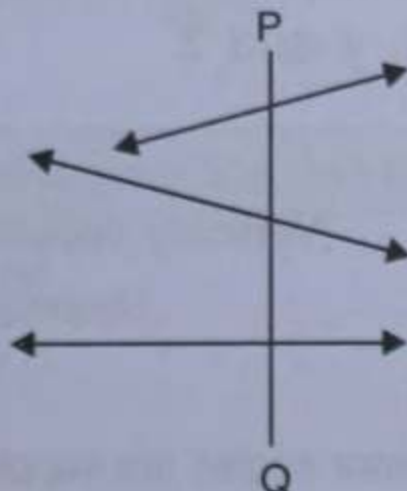
18.3 CONCEPT OF TRANSVERSAL LINES

When a line cuts two or more lines, whether (parallel or non-parallel), the line is called a transversal line, or simply, a transversal. In each of the following figures; PQ is a transversal line.

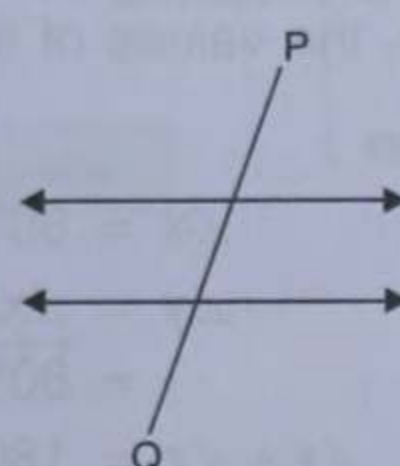
(i)



(ii)



(iii)



18.4 ANGLES FORMED BY TWO LINES AND THEIR TRANSVERSAL LINE

When a transversal cuts two parallel or non-parallel lines, eight (8) angles are formed : they are marked 1 to 8 in the following figure :

These angles can be distinguished as stated below :

1. **Exterior Angles :**

The angles marked 1, 2, 7 and 8 are *exterior angles*.

2. **Interior Angles :**

The angles marked 3, 4, 5 and 6 are interior angles.

3. **Exterior Alternate Angles :**

The two pairs of exterior alternate angles are 2 and 8; 1 and 7.

4. **Interior Alternate Angles :**

The two pairs of interior alternate angles are 3 and 5; 4 and 6.
In general, **interior alternate angles** are simply called as **alternate angles**.

5. **Corresponding Angles :**

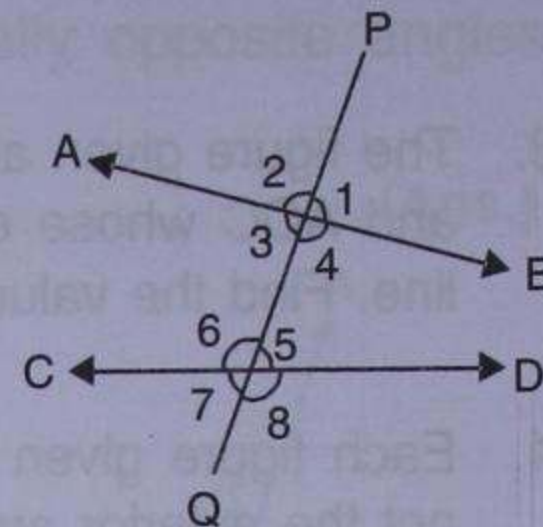
The four pairs of corresponding angles are 1 and 5; 2 and 6; 3 and 7; 4 and 8.

6. **Co-interior or Conjoined or Allied Angles :**

The two pairs of co-interior or allied angles are 3 and 6; 4 and 5.

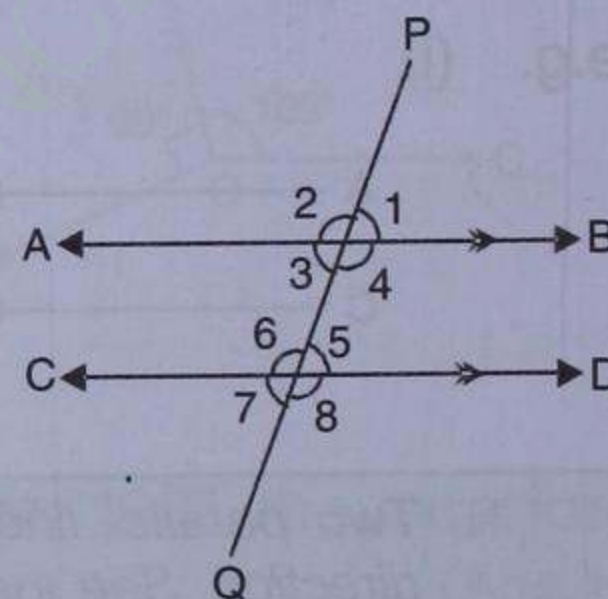
7. **Exterior Allied Angles :**

The two pairs of exterior allied angles are 2 and 7; 1 and 8.



18.5 WHEN TWO PARALLEL LINES ARE CUT BY A TRANSVERSAL

1. *Exterior alternate angles are equal,*
i.e. $\angle 1 = \angle 7$ and $\angle 2 = \angle 8$
2. *Interior alternate angles are equal,*
i.e. $\angle 3 = \angle 5$ and $\angle 4 = \angle 6$
3. *Corresponding angles are equal,*
i.e. $\angle 1 = \angle 5$, and $\angle 2 = \angle 6$;
 $\angle 3 = \angle 7$ and $\angle 4 = \angle 8$
4. *Co-interior (conjoined, allied) angles are supplementary,*
i.e. $\angle 4 + \angle 5 = 180^\circ$ and $\angle 3 + \angle 6 = 180^\circ$
5. *Exterior allied angles are supplementary,*
i.e. $\angle 2 + \angle 7 = 180^\circ$ and $\angle 1 + \angle 8 = 180^\circ$

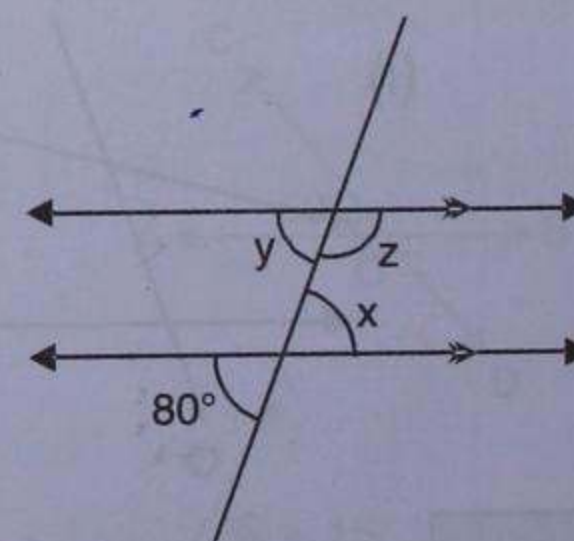


Example 4 :

In the figure given alongside, two parallel lines are cut by a transversal. Find, giving reasons, the values of the angles x, y and z.

Solution :

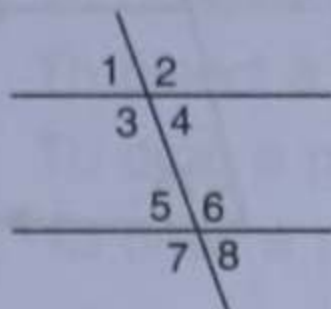
$$\begin{aligned}\angle x &= 80^\circ && \text{[Vertically opposite angles]} \\ \angle y &= \angle x && \text{[Alternate angles]} \\ &= 80^\circ \\ \angle x + \angle z &= 180^\circ && \text{[Co-interior angles are supplementary]} \\ 80^\circ + \angle z &= 180^\circ \\ \Rightarrow \angle z &= 180^\circ - 80^\circ = 100^\circ \\ \therefore \angle x &= 80^\circ, \angle y = 80^\circ \text{ and } \angle z = 100^\circ && \text{(Ans.)}\end{aligned}$$



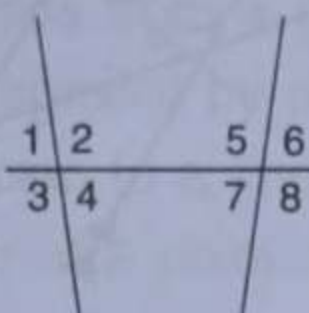
EXERCISE 18(B)

1. Identify the pair of angles in each of the figure given below : adjacent angles, vertically opposite angles, interior alternate angles, corresponding angles or exterior alternate angles.

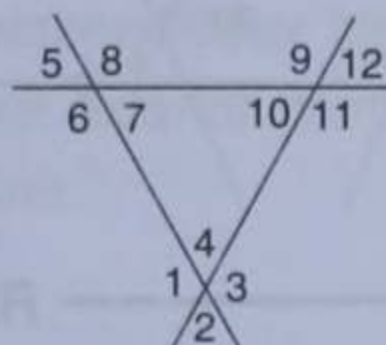
- (a) (i) $\angle 2$ and $\angle 4$
 (ii) $\angle 1$ and $\angle 8$
 (iii) $\angle 4$ and $\angle 5$
 (iv) $\angle 1$ and $\angle 5$
 (v) $\angle 3$ and $\angle 5$



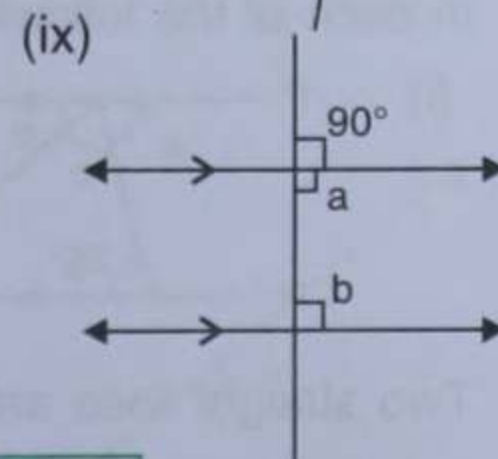
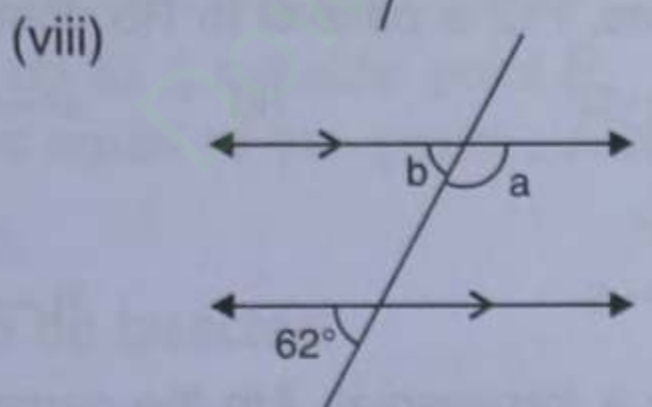
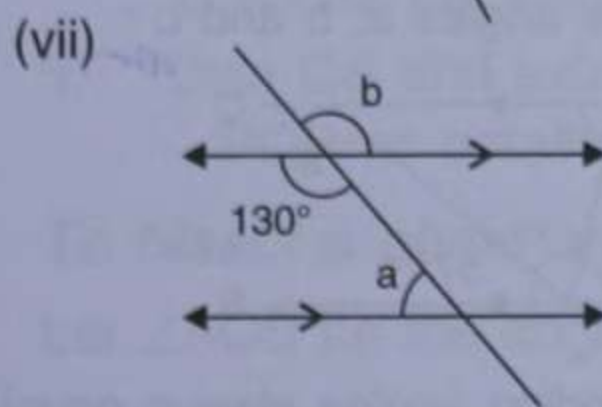
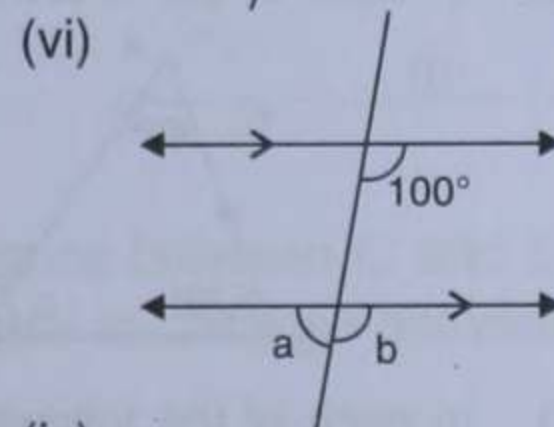
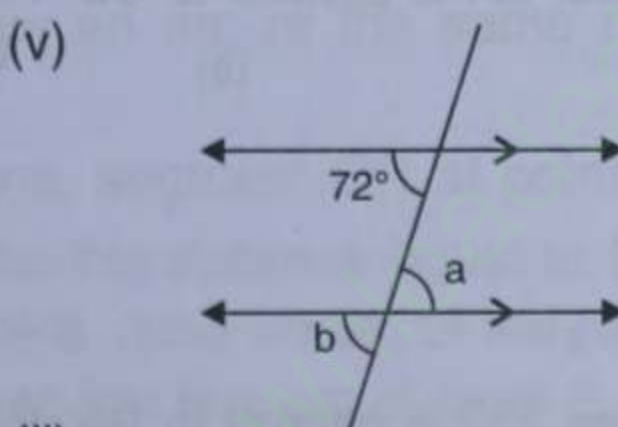
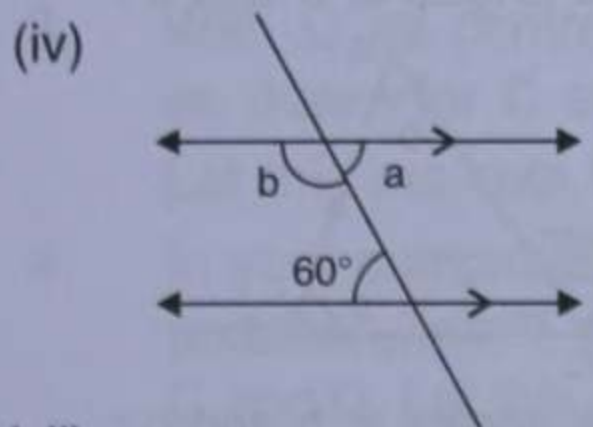
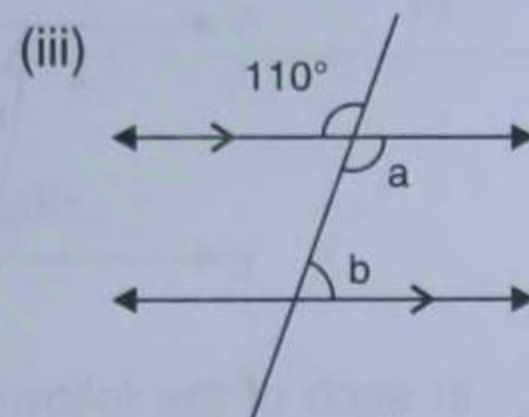
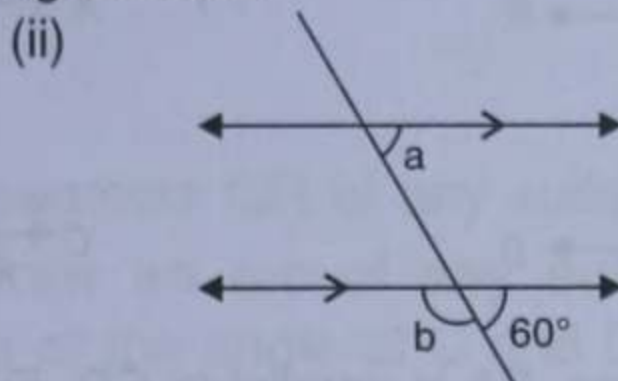
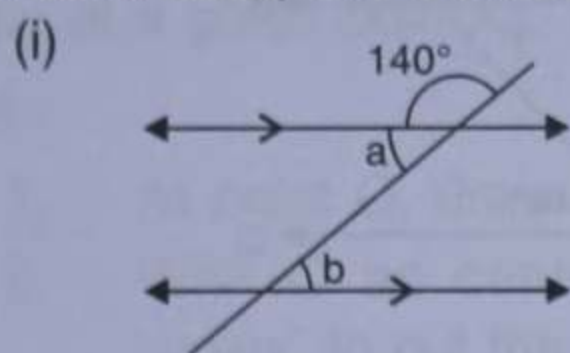
- (b) (i) $\angle 2$ and $\angle 7$
 (ii) $\angle 4$ and $\angle 8$
 (iii) $\angle 1$ and $\angle 8$
 (iv) $\angle 1$ and $\angle 5$
 (v) $\angle 4$ and $\angle 7$



- (c) (i) $\angle 1$ and $\angle 10$
 (ii) $\angle 6$ and $\angle 12$
 (iii) $\angle 8$ and $\angle 10$
 (iv) $\angle 4$ and $\angle 11$
 (v) $\angle 2$ and $\angle 8$
 (vi) $\angle 5$ and $\angle 7$

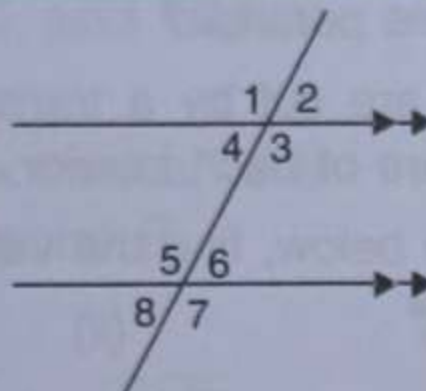


2. Each figure given below shows a pair of parallel lines cut by a transversal. For each case, find a and b , giving reasons.

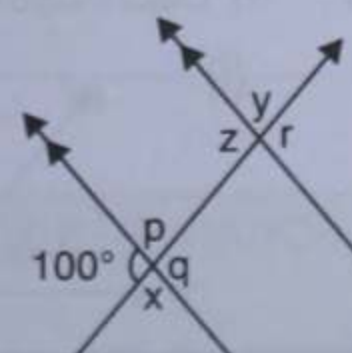


The arrows in the diagrams given above, indicate that the lines are parallel.

3. If $\angle 1 = 120^\circ$, find the measures of : $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$, $\angle 7$ and $\angle 8$. Give reasons.

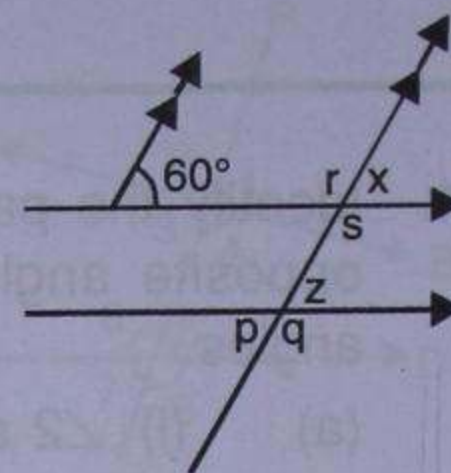


4. In the figure given alongside, find the measure of the angles denoted by x , y , z , p , q and r .

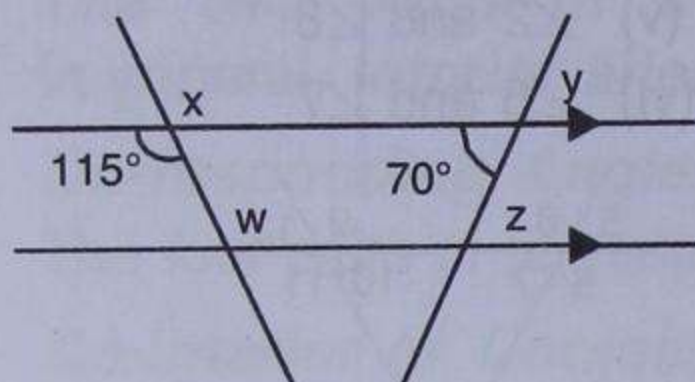


5. Using the figure given alongside, fill in the blanks :

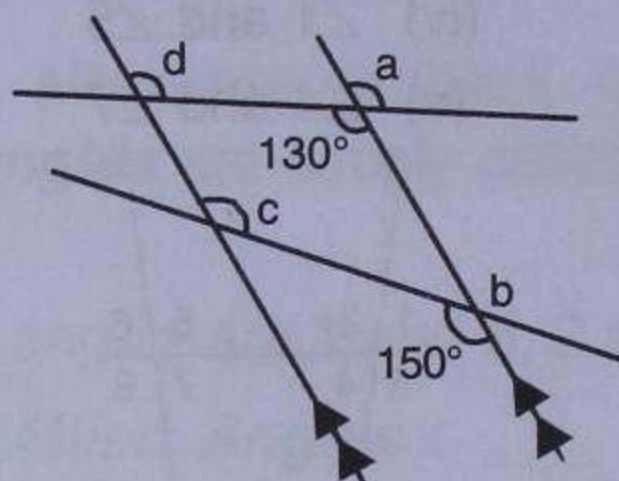
$\angle x = \dots\dots\dots$; $\angle z = \dots\dots\dots$;
 $\angle p = \dots\dots\dots$; $\angle q = \dots\dots\dots$;
 $\angle r = \dots\dots\dots$; $\angle s = \dots\dots\dots$;



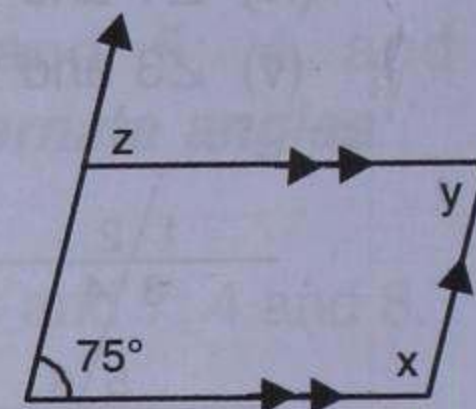
6. In the figure given alongside, find the angles shown by x , y , z and w . give reasons.



7. Find a , b , c and d in the figure given below :

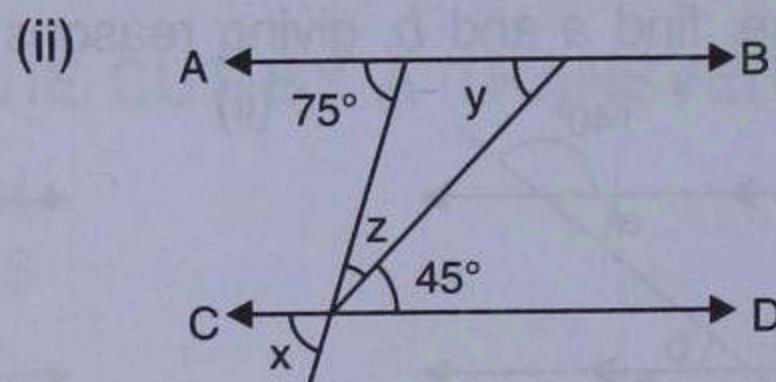
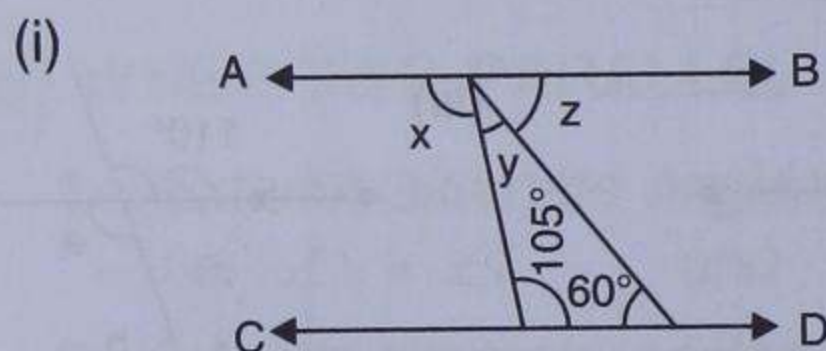


8. Find x , y and z in the figure given below :

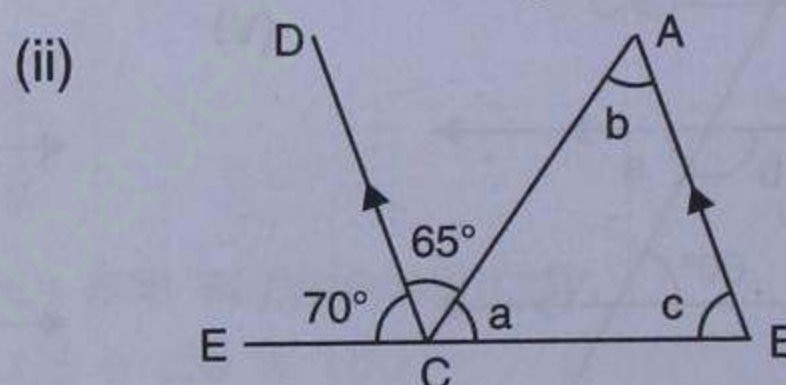
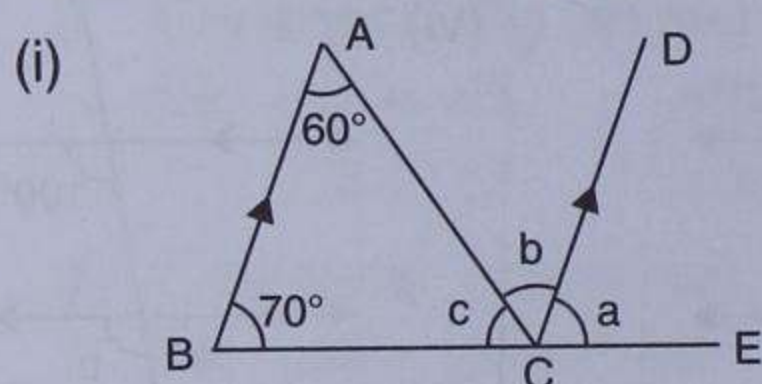


Revision Exercise (Chapter 18)

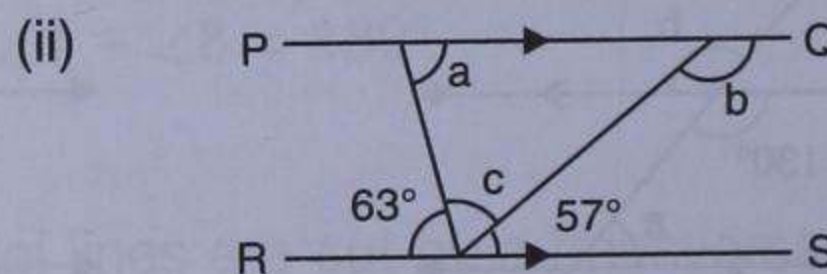
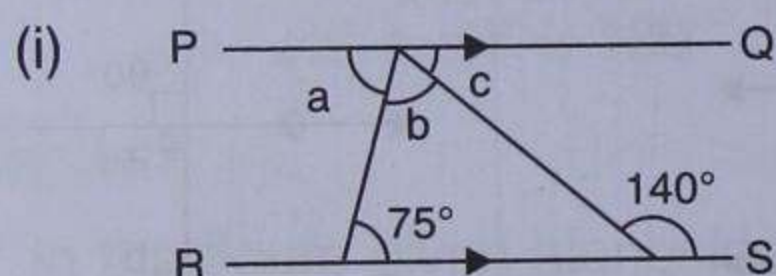
1. In both the following figures, AB is parallel to CD ; find the values of angles x , y and z :



2. In each of the following figures, BA is parallel to CD . Find the angles a , b and c :



3. In each of the following figures, PQ is parallel to RS . Find the angles a , b and c :



4. Two straight lines are cut by a transversal. Are the corresponding angles always equal ?

5. Two straight lines are cut by a transversal so that the co-interior angles are supplementary. Are the straight lines parallel ?

6. Two straight lines are cut by a transversal so that the co-interior angles are equal. What must be the measure of each interior angle to make the straight lines parallel to each other ?

7. In each case given below, find the value of x , if POQ is a straight line

