

SUBSTITUTION

(Including Use of Brackets as Grouping Symbols)

14.1 BASIC CONCEPT

The value of an expression depends on the value(s) of its variable(s).

Consider the algebraic expression : $3x + 2$

In the expression $3x + 2$, the variable used is x , and so the value of the expression $3x + 2$ depends on the value of its variable x . That is :

- (i) if $x = 2$, the value of the expression $3x + 2 = 3 \times 2 + 2 = 6 + 2 = 8$
- (ii) if $x = 0$, the value of the expression $3x + 2 = 3 \times 0 + 2 = 0 + 2 = 2$
- (iii) if $x = -2$, the value of the expression $3x + 2 = 3 \times -2 + 2 = -6 + 2 = -4$ and so on.

Now consider the algebraic expression $5x - 2y$.

This expression consists of variables x and y .

Now, if :

- (i) $x = 3$ and $y = 2$, $5x - 2y = 5 \times 3 - 2 \times 2 = 15 - 4 = 11$
- (ii) $x = 8$ and $y = 5$, $5x - 2y = 5 \times 8 - 2 \times 5 = 40 - 10 = 30$.

The process of finding the value of an algebraic expression by substituting the given value (values) of its variable (variables) is called **substitution**.

Example 1 :

If $x = 6$ and $y = 3$, find the value of :

- (i) $4x + y$
- (ii) $3x - 4y$
- (iii) $3xy$
- (iv) $\frac{5x}{4y}$

Solution :

- (i) $4x + y = 4 \times 6 + 3 = 24 + 3 = 27$ (Ans.)
- (ii) $3x - 4y = 3 \times 6 - 4 \times 3 = 18 - 12 = 6$ (Ans.)
- (iii) $3xy = 3 \times 6 \times 3 = 54$ (Ans.)
- (iv) $\frac{5x}{4y} = \frac{5 \times 6}{4 \times 3} = \frac{30}{12} = \frac{5}{2} = 2\frac{1}{2}$ (Ans.)

Example 2 :

If $a = 2$, $b = 5$ and $c = 8$, find the value of : $3ab + 10bc - 2abc$.

Solution :

$$\begin{aligned}
 3ab + 10bc - 2abc &= 3 \times 2 \times 5 + 10 \times 5 \times 8 - 2 \times 2 \times 5 \times 8 \\
 &= 30 + 400 - 160 \\
 &= 430 - 160 = 270 \quad \text{(Ans.)}
 \end{aligned}$$

Example 3 :

If $p = 8$, $q = 1$ and $r = 2$, find the value of : $\frac{10pq - 3qr}{4pqr - 2p}$

Solution :

$$\frac{10pq - 3qr}{4pqr - 2p} = \frac{10 \times 8 \times 1 - 3 \times 1 \times 2}{4 \times 8 \times 1 \times 2 - 2 \times 8} = \frac{80 - 6}{64 - 16} = \frac{74}{48} = \frac{37}{24} = 1\frac{13}{24} \quad (\text{Ans.})$$

Example 4 :

If $x = 2$; find the value of $3x^2 + x$.

Solution :

$$\begin{aligned} 3x^2 + x &= 3(2)^2 + 2 \\ &= 3 \times 2 \times 2 + 2 = 12 + 2 = 14 \end{aligned} \quad (\text{Ans.})$$

Example 5 :

If $x = 5$, $y = 6$ and $z = 10$, find the value of :

(i) $\frac{3x^2}{x}$ (ii) $\frac{xy}{xz}$ (iii) $\frac{x^2y}{z}$

Solution :

$$(i) \quad \frac{3x^2}{x} = \frac{3 \times 5^2}{5} = \frac{3 \times 5 \times 5}{5} = 3 \times 5 = 15 \quad (\text{Ans.})$$

Alternative method :

$$\frac{3x^2}{x} = \frac{3 \times x \times x}{x} = 3 \times x = 3 \times 5 = 15 \quad (\text{Ans.})$$

$$(ii) \quad \frac{xy}{xz} = \frac{5 \times 6}{5 \times 10} = \frac{30}{50} = \frac{3}{5} \quad (\text{Ans.})$$

Alternative method :

$$\frac{xy}{xz} = \frac{y}{z} = \frac{6}{10} = \frac{3}{5} \quad (\text{Ans.})$$

$$(iii) \quad \frac{x^2y}{z} = \frac{5^2 \times 6}{10} = \frac{5 \times 5 \times 6}{10} = \frac{150}{10} = 15 \quad (\text{Ans.})$$

Example 6 :

If $a = 2$, $b = 3$ and $c = 4$, find the value of :

(i) a^b (ii) b^a (iii) c^b (iv) $a^2 - b^2 + c^2$

Solution :

$$(i) \quad a^b = 2^3 = 2 \times 2 \times 2 = 8 \quad (\text{Ans.})$$

$$(ii) \quad b^a = 3^2 = 3 \times 3 = 9 \quad (\text{Ans.})$$

$$(iii) \quad c^b = 4^3 = 4 \times 4 \times 4 = 64 \quad (\text{Ans.})$$

$$\begin{aligned} (iv) \quad a^2 - b^2 + c^2 &= (2)^2 - (3)^2 + (4)^2 \\ &= 4 - 9 + 16 = 20 - 9 = 11 \end{aligned} \quad (\text{Ans.})$$

EXERCISE 14(A)

1. Fill in the following blanks, when : $x = 3$, $y = 6$, $z = 18$, $a = 2$, $b = 8$, $c = 32$ and $d = 0$.

(i) $x + y = 3 + 6 = 9$	(ix) $a + b + x = \dots\dots\dots$
(ii) $y - x = \dots\dots\dots$	(x) $b + z - d = \dots\dots\dots$
(iii) $\frac{y}{x} = \dots\dots\dots$	(xi) $a - b + y = \dots\dots\dots$
(iv) $c \div b = \dots\dots\dots$	(xii) $z - a - b = \dots\dots\dots$
(v) $z \div x = \dots\dots\dots$	(xiii) $d - a + x = \dots\dots\dots$
(vi) $y \times d = \dots\dots\dots$	(xiv) $xy - bd = \dots\dots\dots$
(vii) $d \div x = \dots\dots\dots$	(xv) $xz + cd = \dots\dots\dots$
(viii) $ab + y = \dots\dots\dots$	

2. Find the value of :

- $p + 2q + 3r$, when $p = 1$, $q = 5$ and $r = 2$
- $2a + 4b + 5c$, when $a = 5$, $b = 10$ and $c = 20$
- $3a - 2b$, when $a = 8$ and $b = 10$
- $5x + 3y - 6z$, when $x = 3$, $y = 5$ and $z = 4$
- $2p - 3q + 4r - 8s$, when $p = 10$, $q = 8$, $r = 6$, and $s = 2$
- $6m - 2n - 5p - 3q$, when $m = 20$, $n = 10$, $p = 2$ and $q = 9$

3. Find the value of :

- $4pq \times 2r$, when $p = 5$, $q = 3$ and $r = 1/2$
- $\frac{yx}{z}$, when $x = 8$, $y = 4$ and $z = 16$
- $\frac{a+b-c}{2a}$, when $a = 5$, $b = 7$ and $c = 2$

4. If $a = 3$, $b = 0$, $c = 2$ and $d = 1$, find the value of :

- $3a + 2b - 6c + 4d$
- $6a - 3b - 4c - 2d$
- $ab - bc + cd - da$
- $abc - bcd + cda$
- $a^2 + 2b^2 - 3c^2$
- $a^2 + b^2 - c^2 + d^2$
- $2a^2 - 3b^2 + 4c^2 - 5d^2$

5. Find the value of : $5x^2 - 3x + 2$, when $x = 2$.

6. Find the value of : $3x^3 - 4x^2 + 5x - 6$, when $x = -1$.

7. Show that the value of : $x^3 - 8x^2 + 12x - 5$ is zero, when $x = 1$.

8. State **true** and **false** :

- The value of $x + 5 = 6$, when $x = 1$
- The value of $2x - 3 = 1$, when $x = 0$
- $\frac{2x - 4}{x + 1} = -1$, when $x = 1$.

9. If $x = 2$, $y = 5$ and $z = 4$, find the value of each of the following :

- $\frac{x}{2x^2}$
- $\frac{xz}{yz}$
- z^x
- y^x
- $\frac{x^2 y^2 z^2}{xz}$
- $\frac{5x^4 y^2 z^2}{2x^2}$
- $xy \div y^2 z$
- $\frac{x^2 y^x}{x}$

10. If $a = 3$, find the values of a^2 and 2^a .

11. If $m = 2$, find the difference between the values of $4m^3$ and $3m^4$.

14.2 BRACKETS

In general, the symbols (), { }, [], etc., are called **brackets**.

1. () is called a **small bracket** or **parenthesis**.
2. { } is called a **middle bracket** or **curly bracket**.
3. [] is called a **big bracket** or **square bracket**.

Note : If one more bracket is needed, we use the bar bracket, i.e. a line — is drawn over a group of terms.

Thus, in $2x + \overline{3y - 4z}$, the line over $3y - 4z$ serves as the **bar bracket** and is called **vinculum**.

If an expression is enclosed within a bracket, it is considered a single quantity even if it is made up of many terms.

For example :

Each of $(x - y)$ and $(2a + 3b - 2)$ will be treated as a single quantity.

While simplifying an expression containing a bracket, first of all, the terms inside brackets are operated (combined).

Example 7 :

Evaluate : (i) $15 - (8 - 6)$

(ii) $(15 - 8) - 6$

Solution :

$$\begin{aligned} \text{(i) } 15 - (8 - 6) &= 15 - 2 \\ &= 13 \end{aligned}$$

Simplifying the terms in the bracket first

(Ans.)

$$\text{(ii) } (15 - 8) - 6 = 7 - 6 = 1$$

(Ans.)

Similarly,

$$15 - (8 + 6) = 15 - 14 = 1; \quad (15 - 8) + 6 = 7 + 6 = 13 \quad \text{and so on.}$$

14.3 OPENING OR REMOVING A BRACKET

Removing a bracket

- (i) **When there is + ve (positive) sign before a bracket :**

The bracket is removed without changing the signs of the terms in the bracket.

For example :

$$\begin{aligned} \text{(i) } 10 + (7 - 3) \\ = 10 + 7 - 3 = 14 \end{aligned}$$

$$\begin{aligned} \text{(ii) } a + (b - c + d) \\ = a + b - c + d \end{aligned}$$

- (ii) **When there is - ve (negative) sign before a bracket :**

The bracket is removed and, at the same time, the sign of each term inside the bracket is also changed,

i.e. + sign will change to - sign, and - sign will change to + sign.

For example :

$$(i) \quad 12 - (8 - 5)$$

$$= 12 - 8 + 5 = 17 - 8 = 9$$

$$(ii) \quad a - (b - c + d)$$

$$= a - b + c - d$$

14.4 REMOVAL OF BRACKETS

In a combined operation, the brackets should be removed in the following order :

$$(i) \quad \text{—————}$$

$$(ii) \quad (\quad)$$

$$(iii) \quad \{ \quad \}$$

$$(iv) \quad [\quad]$$

For example :

$$(i) \quad 3x - (2y - \overline{x - y})$$

$$= 3x - (2y - x + y)$$

[On removing the bar bracket]

$$= 3x - (3y - x)$$

$$= 3x - 3y + x$$

[On removing the small bracket]

$$= 4x - 3y$$

$$(ii) \quad 6a - \{a + (2a - \overline{4 - a})\}$$

$$= 6a - \{a + (2a - 4 + a)\}$$

$$= 6a - \{a + (3a - 4)\}$$

$$= 6a - \{a + 3a - 4\}$$

$$= 6a - \{4a - 4\} = 6a - 4a + 4 = 2a + 4$$

First of all remove the bar bracket and simplify; then remove the small bracket () and simplify. Finally remove the curly bracket { } and simplify.

Example 8 :

Simplify :

$$(i) \quad a - [b - \{c - (a - \overline{b - c})\}]$$

$$(ii) \quad 3a - [2a - \{a + (a - \overline{b - c} + c)\}]$$

Solution :

$$(i) \quad a - [b - \{c - (a - \overline{b - c})\}]$$

$$= a - [b - \{c - (a - b + c)\}]$$

[On removing the bar bracket]

$$= a - [b - \{c - a + b - c\}]$$

[On removing the small bracket]

$$= a - [b - c + a - b + c]$$

[On removing the middle bracket]

$$= a - b + c - a + b - c$$

[On removing the square bracket]

$$= 0$$

(Ans.)

$$(ii) \quad 3a - [2a - \{a + (a - \overline{b - c} + c)\}]$$

$$= 3a - [2a - \{a + (a - b + c + c)\}]$$

$$= 3a - [2a - \{a + a - b + c + c\}]$$

$$= 3a - [2a - a - a + b - c - c]$$

$$= 3a - 2a + a + a - b + c + c = 3a - b + 2c$$

(Ans.)

Order of removing the brackets is : $\overline{\quad}$, (), { } and finally [].

A number placed before a bracket indicates that each term inside the bracket is to be multiplied by that number.

Thus : (i) $5(x + y) = 5x + 5y$

(ii) $2(2a - b) = 4a - 2b$

(iii) $3(m - 2n + 4a) = 3m - 6n + 12a$

and so on.

Example 9 :

Simplify : (i) $2x + 3(x - y)$ (ii) $5a - 2\{2b - 3(a - b)\}$

Solution :

$$\begin{aligned} \text{(i)} \quad 2x + 3(x - y) &= 2x + 3x - 3y \\ &= 5x - 3y \quad (\text{Ans.}) \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad 5a - 2\{2b - 3(a - b)\} &= 5a - 2\{2b - 3a + 3b\} \\ &= 5a - 4b + 6a - 6b \\ &= 5a + 6a - 4b - 6b = 11a - 10b \quad (\text{Ans.}) \end{aligned}$$

EXERCISE 14(B)

1. Evaluate :

- | | | |
|-----------------------|------------------------|-----------------------|
| (i) $(23 - 15) + 4$ | (ii) $5x + (3x + 7x)$ | (iii) $6m - (4m - m)$ |
| (iv) $(9a - 3a) + 4a$ | (v) $35b - (16b + 9b)$ | (vi) $(3y + 8y) - 5y$ |

2. Simplify :

- | | |
|---|--|
| (i) $12x - (5x + 2x)$ | (ii) $10m + (4n - 3n) - 5n$ |
| (iii) $(15b - 6b) - (8b + 4b)$ | (iv) $-(-4a - 8a)$ |
| (v) $x - (x - y) - (-x + y)$ | (vi) $p + (-q - r - s) - (p - q - r)$ |
| (vii) $(a + b) - (c + d) - (e - f)$ | (viii) $3x + (8x - 5x) - (7x - x)$ |
| (ix) $a - (a - b - c)$ | (x) $6a^2 + (2a^2 - a^2) - (a^2 - b^2)$ |
| (xi) $2m - (3m + 2n - 6n)$ | (xii) $-m - n - (-m) - m$ |
| (xiii) $x + y - (x + \overline{y - x})$ | (xiv) $25y - (5x - 10y + 6x - 3y)$ |
| (xv) $3x + (2x - \overline{x + 2})$ | (xvi) $a - (2a - \overline{4a + 3a})$ |
| (xvii) $5x^2 - (3x - \overline{x^2 - 4})$ | (xviii) $-(y - x) - (x + y - \overline{2x + y})$ |

3. Simplify :

- | | |
|---|---|
| (i) $x - (y - z) + x + (y - z) + y - (z + x)$ | (ii) $x - [y + \{x - (y + x)\}]$ |
| (iii) $4x + 3(2x - 5y)$ | (iv) $2(3a - b) - 5(a - 3b)$ |
| (v) $p + 2(q - \overline{r + p})$ | (vi) $a - [-\{-\overline{(a - b - c)}\}]$ |
| (vii) $3x - [5y - \{6y + 2(10y - x)\}]$ | (viii) $5\{a^2 - a(\overline{a - a - 2})\}$ |

14.5 INSERTING A BRACKET

1. When any part of an expression is inserted within a bracket with a *positive* sign before it, the sign of each term kept inside the bracket remains unchanged.

For example :

The expression $a - b + c - d$ may be written as :

$$a + (-b + c - d) \quad \text{or} \quad a - b + (c - d)$$

2. When any part of an expression is to be inserted within a bracket with a *negative* sign before it, the sign of each term kept inside the bracket gets changed.

For example :

The expression $a - b + c - d$ may be written as :

$$a - (b - c + d) \quad \text{or} \quad a - b - (-c + d)$$

EXERCISE 14(C)

1. Fill in the blanks :

- | | |
|---|--|
| (i) $2a + b - c = 2a + (\dots\dots\dots)$ | (ii) $3x - z + y = 3x - (\dots\dots\dots)$ |
| (iii) $6p - 5x + q = 6p - (\dots\dots\dots)$ | (iv) $a + b - c + d = a + (\dots\dots\dots)$ |
| (v) $5a + 4b + 4x - 2c$
$= 4x - (\dots\dots\dots)$ | (vi) $7x + 2z + 4y - 3 = -3 + 4y + (\dots\dots\dots)$ |
| (vii) $3m - 2n + 6 = 6 - (\dots\dots\dots)$ | (viii) $2t + r - p - q + s = 2t + r - (\dots\dots\dots)$ |

2. Insert the bracket as indicated :

- | | |
|---|---|
| (i) $x - 2y = - (\dots\dots\dots)$ | (ii) $m + n - p = - (\dots\dots\dots)$ |
| (iii) $a + 4b - 4c = a + (\dots\dots\dots)$ | (iv) $a - 3b + 5c = a - (\dots\dots\dots)$ |
| (v) $x^2 - y^2 + z^2 = x^2 - (\dots\dots\dots)$ | (vi) $m^2 + x^2 - p^2 = - (\dots\dots\dots)$ |
| (vii) $2x - y + 2z = 2z - (\dots\dots\dots)$ | (viii) $ab + 2bc - 3ac = 2bc - (\dots\dots\dots)$ |

REVISION EXERCISE (Chapter 14)

- Find the value of $3ab + 10bc - 2abc$ when $a = 2$, $b = 5$ and $c = 8$.
- If $x = 2$, $y = 3$ and $z = 4$, find the value of $3x^2 - 4y^2 + 2z^2$.
- If $x = 3$, $y = 2$ and $z = 1$, find the value of :

(i) x^y	(ii) y^x	(iii) $3x^2 - 5y^2$
(iv) $2x - 3y + 4z + 5$	(v) $y^2 - x^2 + 6z^2$	(vi) $xy + y^2z - 4zx$
- If $P = -12x^2 - 10xy + 5y^2$, $Q = 7x^2 + 6xy + 2y^2$ and $R = 5x^2 + 2xy + 4y^2$, find :

(i) $P - Q$	(ii) $Q + P$	(iii) $P - Q + R$	(iv) $P + Q + R$
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- If $x = a^2 - bc$, $y = b^2 - ca$ and $z = c^2 - ab$, find the value of : (i) $ax + by + cz$ (ii) $ay - bx + cz$
- Multiply and then evaluate :

(i) $(4x + y)$ and $(x - 2y)$, when $x = 2$ and $y = 1$.
(ii) $(x^2 - y)$ and $(xy - y^2)$, when $x = 1$ and $y = 2$.
(iii) $(x - 2y + z)$ and $(x - 3z)$, when $x = -2$, $y = -1$ and $z = 1$.
- Simplify :

(i) $5(x + 3y) - 2(3x - 4y)$	(ii) $3x - 8(5x - 10)$	(iii) $6\{3x - 8(5x - 10)\}$
(iv) $3x - 6\{3x - 8(5x - 10)\}$	(v) $2(3x^2 - 4x - 8) - (3 - 5x - 2x^2)$	
(vi) $8x - (3x - \overline{2x - 3})$	(vii) $12x^2 - (7x - \overline{3x^2 + 15})$	
- If $x = -3$, find the value of $2x^3 + 8x^2 - 15$.