

Chapter 8

Ratio

POINTS TO REMEMBER

1. **Ratio** : If a and b are two non-zero quantities of the same kind and in the same units, then the fraction $\frac{a}{b}$ is called the ratio between a and b , and is written as $a : b$. a and b are called its terms, a is the first term or antecedent and b , the second term or consequent.

2. **An Important Property** : The value of a ratio remains unchanged, if both of its terms are multiplied or divided by the same non-zero number.

Proof : Let $a : b$ be given ratio and let m be a non-zero number.

Then, we may write, $a : b$ as $\frac{a}{b}$.

(i) Clearly, we have $\frac{a}{b} = \frac{ma}{mb}$.

This shows that $a : b$ is the same as $ma : mb$.

(ii) We may write, $\frac{a}{b} = \frac{(a/m)}{(b/m)}$.

This shows that $a : b$ is the same as $\frac{a}{m} : \frac{b}{m}$.

3. **A Ratio in its Lowest Terms** : When a and b have no common factor, other than 1, we say that $a : b$ is in its lowest terms.

4. **Composition of Ratios** :

(i) Duplicate ratio of $a : b$ is $a^2 : b^2$.

(ii) Triplicate ratio of $a : b$ is $a^3 : b^3$.

(iii) Sub-duplicate ratio of $a : b$ is $\sqrt{a} : \sqrt{b}$. or $a^{\frac{1}{2}} : b^{\frac{1}{2}}$

(iv) Sub-triplicate ratio of $a : b$ is $a^{1/3} : b^{1/3}$ or $\sqrt[3]{a} : \sqrt[3]{b}$

(v) Reciprocal ratio of $a : b$ is $\frac{1}{a} : \frac{1}{b}$, which is same as $b : a$.

(vi) Compounded ratio of $a : b$ and $c : d$ is $ac : bd$.

5. **Proportion** : Four non-zero quantities a, b, c, d are said to be in proportion, if $a : b = c : d$ and we write, $a : b :: c : d$. We read it as, ' a is to b as c is to d '.

If $a : b :: c : d$, then :

- (i) a, b, c, d are known as first, second, third and fourth term respectively.

(ii) d is called the **fourth proportional** to a, b, c .

(iii) a and d are called extremes, while b and c are called means.

An Important Property

If $a : b :: c : d$, then the product of extremes is equal to the product of means.

Proof : $a : b :: c : d \Leftrightarrow \frac{a}{b} = \frac{c}{d}$

$$\Leftrightarrow ad = bc$$

$\therefore$ Product of extremes = product of means.

6. Continued Proportion

We say that a, b, c, d, e, f etc. are in continued proportion, if

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \frac{e}{f}.$$

7. Mean Proportional (or Geometric Mean)

Let a, b, c be in continued proportion.

Then, $\frac{a}{b} = \frac{b}{c}$ or $b^2 = ac$ or $b = \sqrt{ac}$.

Here, b is called the **mean proportional** or **geometric mean** between a and c .

8. Third Proportional :

If $a : b = b : c$, then c is called the third proportional to a and b . Clearly, it is the fourth proportional to a, b, b .

9. An Important Property :

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, prove that each ratio in equality $\frac{a+c+e}{b+d+f}$

Proof : Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$,

then $a = bk, c = dk$ and $e = fk$.

$$\begin{aligned} \therefore \frac{a+c+e}{b+d+f} &= \frac{bk+dk+fk}{b+d+f} \\ &= \frac{k(b+d+f)}{(b+d+f)} = k. \end{aligned}$$

Hence, each ratio $= \frac{a+c+e}{b+d+f}$

$$= \frac{\text{Sum of antecedent}}{\text{Sum of consequents}}$$

10. Properties of Ratio and Proportion :

(1) **Invertendo** : If $a : b :: c : d$, then $b : a :: d : c$.

Proof : $a : b :: c : d \Rightarrow \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$

[$\therefore$ Reciprocals of equals are equal]

$$\Rightarrow b : a :: d : c.$$

$$\text{Hence, } a : b :: c : d$$

$$\Rightarrow b : a :: d : c$$

[Invertendo]

(2) Alternendo : If $a : b :: c : d$, then $a : c :: b : d$.

$$\text{Proof : } a : b :: c : d \Rightarrow \frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow ad = bc$$

[By cross multiplication]

$$\Rightarrow \frac{a}{c} = \frac{b}{d}$$

[On dividing both sides by cd]

$$\Rightarrow a : c :: b : d$$

$$\text{Hence, } a : b :: c : d$$

$$\Rightarrow a : c :: b : d$$

[Alternendo]

(3) Componendo : If $a : b :: c : d$, then $(a + b) : b :: (c + d) : d$.

$$\text{Proof : } a : b :: c : d \Rightarrow \frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow \frac{a}{b} + 1 = \frac{c}{d} + 1$$

[Adding 1 on both sides]

$$\Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$$

$$\Rightarrow (a + b) : b :: (c + d) : d.$$

$$\text{Hence, } a : b :: c : d$$

$$\Rightarrow (a + b) : b :: (c + d) : d$$

[componendo]

(4) Dividendo : If $a : b :: c : d$, then $(a - b) : b :: (c - d) : d$.

$$\text{Proof : } a : b :: c : d \Rightarrow \frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow \frac{a}{b} - 1 = \frac{c}{d} - 1$$

[On subtracting 1 from both sides]

$$\Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$$

$$\Rightarrow (a - b) : b :: (c - d) : d.$$

$$\text{Hence, } a : b :: c : d$$

$$\Rightarrow (a - b) : b :: (c - d) : d$$

[Dividendo]

(5) Componendo & Dividendo : If $a : b :: c : d$, then $(a + b) : (a - b) :: (c + d) : (c - d)$.

$$\text{Proof : } a : b :: c : d.$$

$$\Rightarrow \frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$$

...(i)

[By componendo]

$$\text{and } \frac{a-b}{b} = \frac{c-d}{d}$$

...(ii)

[By dividendo]

$$\Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d} \quad [\text{On dividing (i) by (ii)}]$$

$$\Rightarrow (a+b) : (a-b) :: (c+d) : (c-d)$$

$$\text{Hence, } a : b :: c : d \Rightarrow (a+b) : (a-b) :: (c+d) : (c-d) \quad [\text{Componendo and dividendo}]$$

In other words, we can say that

$$(i) \quad \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c} \quad [\text{Invertendo}]$$

$$(ii) \quad \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d} \quad [\text{Alternendo}]$$

$$(iii) \quad \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d} \quad [\text{Componendo}]$$

$$(iv) \quad \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{b} = \frac{c-d}{d} \quad [\text{Dividendo}]$$

$$(v) \quad \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d} \quad [\text{Componendo and dividendo}]$$

EXERCISE 8(A)

Q.1. Find the ratio between

- (i) 60 paise and Rs. 1.35
- (ii) 3.6m and 75 cm
- (iii) 35 min. and $1\frac{3}{4}$ hrs.
- (iv) 1.5 kg and 600 gm.

Sol. (i) 60 paise : Rs. 1.35
 $= 60 \text{ paise} : 135 \text{ paise}$
 $= 60 : 135 = 4 : 9 \quad [\text{Dividing by 15}]$
 $= 4 : 9$

(ii) 3.6 m : 75 cm
 $= 360 \text{ cm} : 75 \text{ cm} = 360 : 75$
 $= 24 : 5 \quad [\text{Dividing by 15}]$

(iii) 35 min : $1\frac{3}{4}$ hrs.
 $= 35 \text{ min} : \frac{7}{4} \times 60 \text{ min.}$
 $= 35 \text{ min} : 105 \text{ min}$
 $= 35 : 105 = 1 : 3 \quad [\text{Dividing by 35}]$

(iv) 1.5 kg : 600 gm
 $= 1500 \text{ gm} : 600 \text{ gm} = 1500 : 600$
 $= 5 : 2 \quad [\text{Dividing by 300}] \text{ Ans.}$

Q.2. Out of the monthly income of Rs. 15000, Rahul spends Rs. 10500 and the rest he saves. Find the ratio of his.

- (i) income to expenditure ;
- (ii) income to savings ;
- (iii) savings to expenditure.

Sol. Monthly income = Rs. 15000
 Expenditure = Rs. 10500
 $\therefore \text{Savings} = \text{Rs. } 15000 - \text{Rs. } 10500$
 $= \text{Rs. } 4500$

(i) Now ratio between
 Income and expenditure
 $= \text{Rs. } 15000 : \text{Rs. } 10500$
 $= 15000 : 10500$
 $= 10 : 7 \quad [\text{Dividing by 1500}]$

(ii) Ratio between
 Income and savings = Rs. 15000 : Rs. 4500
 $= 15000 : 4500$
 $= 10 : 3 \quad [\text{Dividing by 1500}]$

(iii) Ratio between
 Savings and expenditure = Rs. 4500 : Rs. 10500
 $= 4500 : 10500$
 $= 3 : 7 \quad [\text{Dividing by 1500}] \text{ Ans.}$

Q. 3. If $A : B = 5 : 6$ and $B : C = 9 : 11$,
find : (i) $A : C$ (ii) $A : B : C$.

Sol. (i) $A : B = 5 : 6$ and $B : C = 9 : 11$

$$\text{or } \frac{A}{B} = \frac{5}{6} \text{ and } \frac{B}{C} = \frac{9}{11}$$

$$\therefore \frac{A}{B} \times \frac{B}{C} = \frac{5}{6} \times \frac{9}{11} = \frac{45}{66} = \frac{15}{22}$$

$$\Rightarrow \frac{A}{C} = \frac{15}{22} \Rightarrow A : C = 15 : 22$$

(ii) Again $A : B = 5 : 6$ and $B : C = 9 : 11$

$$\Rightarrow A : B = 15 : 18 \text{ [Multiplying by 3]}$$

$$B : C = 18 : 22 \text{ [Multiplying by 2]}$$

$$\therefore A : B : C = 15 : 18 : 22 \text{ Ans.}$$

Q. 4. If $a : b = 3 : 5$ and $b : c = 2 : 9$,
find : (i) $a : c$ (ii) $a : b : c$.

$$\text{Sol. (i) } a : b = 3 : 5 \Rightarrow \frac{a}{b} = \frac{3}{5}$$

$$b : c = 2 : 9 \Rightarrow \frac{b}{c} = \frac{2}{9}$$

$$\therefore \frac{a}{b} \times \frac{b}{c} = \frac{3}{5} \times \frac{2}{9} \Rightarrow \frac{a}{c} = \frac{2}{15}$$

$$\therefore a : c = 2 : 15$$

(ii) $a : b = 3 : 5$ and $b : c = 2 : 9$

$$\therefore a : b = 6 : 10 \text{ [Multiplying by 2]}$$

$$\text{and } b : c = 10 : 45 \text{ [Multiplying by 5]}$$

$$\therefore a : b : c = 6 : 10 : 45 \text{ Ans.}$$

Q. 5. If $p : q = 7 : 4$ and $q : r = 5 : 14$,
find : (i) $r : p$ (ii) $p : q : r$.

$$\text{Sol. (i) } p : q = 7 : 4 \Rightarrow \frac{p}{q} = \frac{7}{4}$$

$$q : r = 5 : 14 \Rightarrow \frac{q}{r} = \frac{5}{14}$$

$$\therefore \frac{p}{q} \times \frac{q}{r} = \frac{7}{4} \times \frac{5}{14} = \frac{5}{8}$$

$$\therefore p : r = 5 : 8 \Rightarrow r : p = 8 : 5$$

(ii) $p : q = 7 : 4$ and $q : r = 5 : 14$

$$\therefore p : q = 35 : 20 \text{ [Multiplying by 5]}$$

$$\text{and } q : r = 20 : 56 \text{ [Multiplying by 4]}$$

$$\therefore p : q : r = 35 : 20 : 56 \text{ Ans.}$$

Q. 6. If $A : B = 2 : 3$, $B : C = 4 : 5$ and $C : D = 6 : 7$, then find $A : B : C : D$.

Sol. $A : B = 2 : 3$

$$B : C = 4 : 5$$

$$C : D = 6 : 7$$

$$A : B = 2 : 3 \quad \dots(i)$$

$$B : C = 4 : 5 \quad \dots(ii)$$

$$\therefore (\text{LCM of 3 and 4} = 12)$$

$\therefore$ Multiplying (i) by 4 and (ii) by 3,

$$A : B = 8 : 12$$

$$\text{and } B : C = 12 : 15$$

$$\therefore A : B : C = 8 : 12 : 15 \quad \dots(iii)$$

$$C : D = 6 : 7 \quad \dots(iv)$$

$$\text{LCM of 15 and 6} = 30$$

Now, multiplying (iii) by 2 and (iv) by 5.

$$A : B : C = 16 : 24 : 30 \text{ and}$$

$$C : D = 30 : 35$$

$$\therefore A : B : C : D = 16 : 24 : 30 : 35 \text{ Ans.}$$

Q. 7. If $A : B = \frac{1}{2} : \frac{3}{8}$, $B : C = \frac{1}{3} : \frac{5}{9}$ and $C : D = \frac{5}{6} : \frac{3}{4}$, find $A : B : C : D$.

$$\text{Sol. } A : B = \frac{1}{2} : \frac{3}{8}$$

$$\frac{4 : 3}{8} = 4 : 3$$

$$B : C = \frac{1}{3} : \frac{5}{9}$$

$$\frac{3 : 5}{9} = 3 : 5$$

$$C : D = \frac{5}{6} : \frac{3}{4}$$

$$\frac{10 : 9}{12} = 10 : 9$$

$$\text{Now, } A : B = 4 : 3 \quad \dots(i)$$

$$B : C = 3 : 5 \quad \dots(ii)$$

From (i) and (ii),

$$A : B : C = 4 : 3 : 5 \quad \dots(iii)$$

$$C : D = 10 : 9 \quad \dots(iv)$$

LCM of 5, 10 = 10

Multiplying (iii) by 2 and (iv) by 1

$$\therefore A : B : C = 8 : 6 : 10$$

$$C : D = 10 : 9$$

$$\therefore A : B : C : D = 8 : 6 : 10 : 9 \text{ Ans.}$$

Q. 8. If $3A = 5B = 6C$, find $A : B : C$.

Ans. Let $3A = 5B = 6C = x$, then

$$A = \frac{x}{3}, B = \frac{x}{5}, C = \frac{x}{6}$$

$$\therefore A : B : C = \frac{x}{3} : \frac{x}{5} : \frac{x}{6} = \frac{1}{3} : \frac{1}{5} : \frac{1}{6}$$

(Convert ratio in whole numbers)

(LCM of 3, 5, 6 = 30)

$$= \frac{1}{3} \times 30 : \frac{1}{5} \times 30 : \frac{1}{6} \times 30$$

$$= 10 : 6 : 5 \text{ Ans.}$$

Q. 9. If $a : b = 8 : 5$, find $(7a + 5b) : (8a - 9b)$.

Sol. $a : b = 8 : 5 \Rightarrow \frac{a}{b} = \frac{8}{5}$

$$\text{Now, } \frac{7a+5b}{8a-9b} = \frac{7\frac{a}{b}+5\frac{b}{b}}{8\frac{a}{b}-9\frac{b}{b}}$$

[Dividing each term by b]

$$= \frac{7\frac{a}{b}+5}{8\frac{a}{b}-9} = \frac{7 \times \frac{8}{5} + 5}{8 \times \frac{8}{5} - 9}$$

[Substituting the value of $\frac{a}{b}$]

$$= \frac{\frac{56}{5} + 5}{\frac{64}{5} - 9} = \frac{\frac{56+25}{5}}{\frac{64-45}{5}}$$

$$= \frac{\frac{81}{5}}{\frac{19}{5}} = \frac{81}{5} \times \frac{5}{19} = \frac{81}{19}$$

$$\therefore (7a + 5b) : (8a - 9b) = 81 : 19 \text{ Ans.}$$

Q. 10. If $x : y = 3 : 2$, find $(5x - 3y) : (7x + 2y)$.

Sol. $x : y = 3 : 2 \Rightarrow \frac{x}{y} = \frac{3}{2}$

$$\text{Now, } \frac{5x-3y}{7x+2y} = \frac{5\frac{x}{y}-3\frac{y}{y}}{7\frac{x}{y}+2\frac{y}{y}}$$

[Dividing by y]

$$= \frac{5\frac{x}{y}-3}{7\frac{x}{y}+2} = \frac{5 \times \frac{3}{2} - 3}{7 \times \frac{3}{2} + 2}$$

[Substituting the value of $\frac{x}{y}$]

$$= \frac{\frac{15}{2} - 3}{\frac{21}{2} + 2} = \frac{\frac{15-6}{2}}{\frac{21+4}{2}} = \frac{\frac{9}{2}}{\frac{25}{2}} = \frac{9}{2} \times \frac{2}{25} = \frac{9}{25}$$

$$\therefore (5x - 3y) : (7x + 2y) = 9 : 25 \text{ Ans.}$$

Q. 11. If $x : y = 10 : 3$, find $(3x^2 + 2y^2) : (3x^2 - 2y^2)$.

Sol. $x : y = 10 : 3 \Rightarrow \frac{x}{y} = \frac{10}{3}$

$$\text{Now } \frac{3x^2+2y^2}{3x^2-2y^2} = \frac{3\frac{x^2}{y^2}+2\frac{y^2}{y^2}}{3\frac{x^2}{y^2}-2\frac{y^2}{y^2}}$$

[Dividing by y^2]

$$= \frac{3\left(\frac{x}{y}\right)^2 + 2}{3\left(\frac{x}{y}\right)^2 - 2} = \frac{3 \times \left(\frac{10}{3}\right)^2 + 2}{3 \times \left(\frac{10}{3}\right)^2 - 2}$$

[Substituting the value of $\frac{x}{y}$]

$$= \frac{3 \times \frac{100}{9} + 2}{3 \times \frac{100}{9} - 2} = \frac{\frac{100}{3} + 2}{\frac{100}{3} - 2}$$

$$= \frac{\frac{100+6}{3}}{\frac{100-6}{3}} = \frac{\frac{106}{3}}{\frac{94}{3}} = \frac{106}{3} \times \frac{3}{94} = \frac{53}{47}$$

$$\therefore (3x^2 + 2y^2) : (3x^2 - 2y^2) = 53 : 47 \text{ Ans.}$$

Q. 12. If $a : b = 2 : 5$, find $(3a^2 - 2ab + 5b^2) : (a^2 + 7ab - 2b^2)$.

Ans. Given $a : b = 2 : 5 \Rightarrow \frac{a}{b} = \frac{2}{5} \dots(i)$

$$\therefore \frac{3a^2 - 2ab + 5b^2}{a^2 + 7ab - 2b^2} = \frac{3\left(\frac{a}{b}\right)^2 - 2\left(\frac{a}{b}\right) + 5}{\left(\frac{a}{b}\right)^2 + 7\left(\frac{a}{b}\right) - 2}$$

(Dividing num. and Denom. by b^2)

$$= \frac{3 \cdot \frac{4}{25} - 2 \cdot \frac{2}{5} + 5}{\frac{4}{25} + 7 \cdot \frac{2}{5} - 2} \quad [\text{Using (i)}]$$

$$= \frac{12 - 20 + 125}{4 + 70 - 50} = \frac{117}{24} = \frac{39}{8}$$

Hence, $(3a^2 - 2ab + 5b^2) : (a^2 + 7ab - 2b^2) = 39 : 8 \text{ Ans.}$

Q. 13. If $(5x + 2y) : (7x + 4y) = 13 : 20$, find $x : y$.

Sol. $(5x + 2y) : (7x + 4y) = 13 : 20$

$$\Rightarrow \frac{5x+2y}{7x+4y} = \frac{13}{20}$$

$$\Rightarrow \frac{5\frac{x}{y} + 2\frac{y}{y}}{7\frac{x}{y} + 4\frac{y}{y}} = \frac{13}{20} \quad [\text{Dividing by } y]$$

$$\Rightarrow \frac{5\frac{x}{y} + 2}{7\frac{x}{y} + 4} = \frac{13}{20}$$

Let $\frac{x}{y} = a$, then $\frac{5a+2}{7a+4} = \frac{13}{20}$

$$\Rightarrow 20(5a+2) = 13(7a+4)$$

$$\Rightarrow 100a + 40 = 91a + 52$$

$$\Rightarrow 100a - 91a = 52 - 40 \Rightarrow 9a = 12$$

$$\therefore a = \frac{12}{9} = \frac{4}{3}$$

Hence, $\frac{x}{y} = \frac{4}{3} \Rightarrow x : y = 4 : 3 \text{ Ans.}$

Q.14. If $\frac{3x+5y}{3x-5y} = \frac{7}{3}$, find $x : y$

Sol. $\frac{3x+5y}{3x-5y} = \frac{7}{3}$

$$\Rightarrow 9x + 15y = 21x - 35y$$

[By cross multiplication]

$$\Rightarrow 21x - 9x = 15y + 35y$$

$$\Rightarrow 12x = 50y \Rightarrow \frac{x}{y} = \frac{50}{12} = \frac{25}{6}$$

Hence, $x : y = 25 : 6 \text{ Ans.}$

Q.15. If $(6x^2 - xy) : (2xy - y^2) = 6 : 1$, find $x : y$.

Sol. $(6x^2 - xy) : (2xy - y^2) = 6 : 1$

$$\Rightarrow \frac{6x^2 - xy}{2xy - y^2} = \frac{6}{1}$$

$$\Rightarrow \frac{6\frac{x^2}{y^2} - \frac{xy}{y^2}}{2\frac{xy}{y^2} - \frac{y^2}{y^2}} = \frac{6}{1} \quad [\text{Dividing by } y^2]$$

$$\Rightarrow \frac{6\left(\frac{x}{y}\right)^2 - \frac{x}{y}}{2\frac{x}{y} - 1} = \frac{6}{1}$$

Let $\frac{x}{y} = a$, then $\frac{6a^2 - a}{2a - 1} = \frac{6}{1}$

$$\Rightarrow (6a^2 - a) \times 1 = 6(2a - 1)$$

$$\Rightarrow 6a^2 - a = 12a - 6$$

$$\Rightarrow 6a^2 - a - 12a + 6 = 0$$

$$\Rightarrow 6a^2 - 13a + 6 = 0 \left\{ \begin{array}{l} \because 6 \times 6 = 36 \\ -13 = -9 - 4 \\ 36 = (-9) \times (-4) \end{array} \right\}$$

$$\Rightarrow 6a^2 - 9a - 4a + 6 = 0$$

$$\Rightarrow 3a(2a - 3) - 2(2a - 3) = 0$$

$$\Rightarrow (2a - 3)(3a - 2) = 0$$

[Zero Product Rule]

Either $2a - 3 = 0$, then $2a = 3 \Rightarrow a = \frac{3}{2}$

or $3a - 2 = 0$, then $3a = 2 \Rightarrow a = \frac{2}{3}$

(i) If $a = \frac{3}{2}$, then $\frac{x}{y} = \frac{3}{2} \Rightarrow x : y = 3 : 2$

(ii) If $a = \frac{2}{3}$, then $\frac{x}{y} = \frac{2}{3} \Rightarrow x : y = 2 : 3$

Hence, $x : y = 3 : 2$ or $2 : 3$ Ans.

Q. 16. If $(4x^2 - 3y^2) : (2x^2 + 5y^2) = 12 : 19$, find $x : y$.

Sol. $\frac{4x^2 - 3y^2}{2x^2 + 5y^2} = \frac{12}{19}$

Dividing by y^2

$$\Rightarrow \frac{\frac{4x^2}{y^2} - \frac{3y^2}{y^2}}{\frac{2x^2}{y^2} + \frac{5y^2}{y^2}} = \frac{12}{19}$$

$$\Rightarrow \frac{4\left(\frac{x}{y}\right)^2 - 3}{2\left(\frac{x}{y}\right)^2 + 5} = \frac{12}{19}$$

By cross multiplication

$$19 \left[4\left(\frac{x}{y}\right)^2 - 3 \right] = 12 \left[2\left(\frac{x}{y}\right)^2 + 5 \right]$$

$$\Rightarrow 76\left(\frac{x}{y}\right)^2 - 57 = 24\left(\frac{x}{y}\right)^2 + 60$$

$$\Rightarrow 76\left(\frac{x}{y}\right)^2 - 24\left(\frac{x}{y}\right)^2 = 57 + 60$$

$$\Rightarrow 52\left(\frac{x}{y}\right)^2 = 117$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{117}{52} = \frac{9}{4}$$

$$\Rightarrow \frac{x}{y} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$$

$$\Rightarrow x : y = 3 : 2 \text{ Ans.}$$

Q. 17. If $x^2 + 4y^2 = 4xy$, find $x : y$.

Sol. $x^2 + 4y^2 = 4xy$

Dividing by y^2

$$\frac{x^2}{y^2} + \frac{4y^2}{y^2} = \frac{4xy}{y^2}$$

$$\Rightarrow \left(\frac{x}{y}\right)^2 + 4 = 4\left(\frac{x}{y}\right)$$

$$\Rightarrow \left(\frac{x}{y}\right)^2 - 4\left(\frac{x}{y}\right) + 4 = 0$$

$$\Rightarrow \left[\frac{x}{y} - 2\right]^2 = 0$$

$$\Rightarrow \frac{x}{y} - 2 = 0 \Rightarrow \frac{x}{y} = \frac{2}{1}$$

$$\therefore x : y = 2 : 1 \text{ Ans.}$$

Q. 18. If $10x^2 - 23xy + 9y^2 = 0$, find $x : y$.

Sol. $10x^2 - 23xy + 9y^2 = 0$

$$\Rightarrow 10x^2 - 18xy - 5xy + 9y^2 = 0$$

$$\Rightarrow 2x(5x - 9y) - y(5x - 9y) = 0$$

$$\Rightarrow (5x - 9y)(2x - y) = 0$$

$$\Rightarrow 5x = 9y \text{ or } 2x = y$$

$$\Rightarrow \frac{x}{y} = \frac{9}{5} \text{ or } \frac{x}{y} = \frac{1}{2}$$

$$\text{Hence, } x : y = (9 : 5) \text{ or } (1 : 2).$$

Q. 19. A ratio is equal to $3 : 4$. If its consequent is 144, what is its antecedent?

Sol. The terms of the given ratio are 3, 4

Let antecedent = x , and 4 consequent

$$= 144, \text{ then } \frac{\text{antecedent}}{\text{consequent}} = \frac{x}{144} = \frac{3}{4}$$

$$\Rightarrow x = \frac{3}{4} \times 144 = 108$$

$$\therefore \text{Antecedent} = 108 \text{ Ans.}$$

Q. 20. Two numbers are in the ratio $5 : 7$. If 8 is subtracted from each, the ratio becomes $3 : 5$. Find the numbers.

Sol. Let the numbers be $5x$ and $7x$.

$$\text{Then } \frac{5x-8}{7x-8} = \frac{3}{5}$$

$$\Rightarrow 5(5x-8) = 3(7x-8)$$

[By cross multiplication]

$$\Rightarrow 25x - 40 = 21x - 24$$

$$\Rightarrow 25x - 21x = 40 - 24 \Rightarrow 4x = 16$$

$$\therefore x = \frac{16}{4} = 4$$

$$\therefore \text{First number} = 5x = 5 \times 4 = 20$$

$$\text{and second number} = 7x = 7 \times 4 = 28$$

$$\text{Hence, numbers are 20 and 28 Ans.}$$

Q. 21. Two numbers are in the ratio $8 : 13$. If 14 is added to each of the numbers, the ratio becomes $2 : 3$. Find the numbers.

Sol. Let the numbers be $8x$ and $13x$.

$$\text{Then } \frac{8x+14}{13x+14} = \frac{2}{3}$$

$$\Rightarrow (8x+14) \times 3 = (13x+14) \times 2$$

[By cross multiplication]

$$\Rightarrow 24x + 42 = 26x + 28$$

$$\Rightarrow 24x - 26x = 28 - 42$$

$$\Rightarrow -2x = -14$$

$$x = \frac{-14}{-2} = 7$$

$$\therefore \text{First number} = 8x = 8 \times 7 = 56$$

$$\text{and second number} = 13x = 13 \times 7 = 91$$

$$\text{Hence, numbers are 56, 91 Ans.}$$

Q. 22. What number must be added to each term of the ratio $5 : 7$ to make it $8 : 9$?

Sol. Let the number to be added be x . Then,

$$(5+x) : (7+x) = 8 : 9$$

$$\Rightarrow \frac{5+x}{7+x} = \frac{8}{9}$$

$$\Rightarrow 9 \times (5+x) = 8 \times (7+x)$$

[By cross multiplication]

$$\Rightarrow 45 + 9x = 56 + 8x \Rightarrow x = 11.$$

$$\text{Hence, the required number is 11 Ans.}$$

Q. 23. Find the number which bears the same ratio to $\frac{7}{33}$ that $\frac{8}{21}$ does to $\frac{4}{9}$.

Sol. Let the required number be x . Then,

$$x = \frac{7}{33} = \frac{8}{21} : \frac{4}{9}$$

$$\Rightarrow 33x : 7 = 72 : 84 \Rightarrow \frac{33x}{7} = \frac{72}{84}$$

$$\Rightarrow x = \frac{72}{84} \times \frac{7}{33} \Rightarrow x = \frac{2}{11}$$

$$\text{Hence, the required number is } \frac{2}{11}.$$

Q. 24. The ages of Shruti and Divya are in the ratio 5 : 7. Five years hence, their ages will be in the ratio 3 : 4. Find their present ages.

Sol. Let the age of Shruti = $5x$ years.

and age of Divya = $7x$ years.

5 years hence,

Age of Shruti will be = $(5x + 5)$ years

and age of Divya will be = $(7x + 5)$ years

According to the condition,

$$\frac{5x+5}{7x+5} = \frac{3}{4}$$

$$\Rightarrow 3(7x+5) = 4(5x+5)$$

[By cross multiplication]

$$\Rightarrow 21x + 15 = 20x + 20$$

$$\Rightarrow 21x - 20x = 20 - 15$$

$$\Rightarrow x = 5$$

$\therefore$ Present age of Shruti = $5 \times 5 = 25$ years.

and present age of Divya = 7×5

= 35 years **Ans.**

Q. 25. One year ago, the ratio of Gaurav's and Sachin's age was 6 : 7 respectively. Four years hence, this ratio would become 7 : 8. How old is Sachin?

Sol. 1 year ago, let age of Gaurav = $6x$ years

and age of Sachin = $7x$ years

$\therefore$ Present age of Gaurav = $(6x + 1)$ years

and age of Sachin = $(7x + 1)$ years

4 years hence,

$$\text{Age of Gaurav} = 6x + 1 + 4$$

$$= (6x + 5) \text{ years}$$

$$\text{and age of Sachin} = 7x + 1 + 4$$

$$= (7x + 5) \text{ years}$$

According to the condition,

$$\frac{6x+5}{7x+5} = \frac{7}{8}$$

By cross multiplication,

$$8(6x+5) = 7(7x+5)$$

$$\Rightarrow 48x + 40 = 49x + 35$$

$$\Rightarrow 49x - 48x = 40 - 35$$

$$\Rightarrow x = 5$$

$\therefore$ Present age of Sachin = $7x + 1$

$$= 7 \times 5 + 1 = 35 + 1$$

= 36 years **Ans.**

Q. 26. Peter is 40 years old and Ronald is 60 years old. How many years ago was the ratio of their ages 3 : 5?

Sol. Age of Peter = 40 years

and age of Ronald = 60 years

Let x years ago, their ages were, as given below :

Age of Peter = $(40 - x)$ years

and age of Ronald = $(60 - x)$ years

According to the condition,

$$\frac{40-x}{60-x} = \frac{3}{5}$$

$$\Rightarrow 5(40-x) = 3(60-x)$$

(By cross multiplication)

$$\Rightarrow 200 - 5x = 180 - 3x$$

$$\Rightarrow 200 - 180 = 5x - 3x \Rightarrow 20 = 2x$$

$$\Rightarrow 2x = 20 \Rightarrow x = \frac{20}{2} = 10$$

Hence, 10 years ago. **Ans.**

Q. 27. Divide Rs. 672 in the ratio 5 : 3.

Ans. Sum of the ratio terms = $(5 + 3) = 8$

$$\text{A's part} = \text{Rs.} \left[672 \times \frac{5}{8} \right] = \text{Rs.} 420$$

$$\text{B's part} = \text{Rs.} \left[672 \times \frac{3}{8} \right] = \text{Rs.} 252 \text{ **Ans.**}$$

Q. 28. Divide Rs. 1162 among A, B, C in the ratio 35 : 28 : 20.

Ans. Given ratio = 35 : 28 : 20

Sum of ratio terms = $(35 + 28 + 20) = 83$

$$\begin{aligned}\therefore \text{A's share} &= \text{Rs.} \left[1162 \times \frac{35}{83} \right] \\ &= 14 \times 35 = \text{Rs. } 490\end{aligned}$$

$$\begin{aligned}\text{B's share} &= \text{Rs.} \left[1162 \times \frac{28}{83} \right] \\ &= 14 \times 28 = \text{Rs. } 392\end{aligned}$$

$$\begin{aligned}\text{C's share} &= \text{Rs.} \left[1162 \times \frac{20}{83} \right] \\ &= 14 \times 20 = \text{Rs. } 280 \text{ Ans.}\end{aligned}$$

Q. 29. Divide Rs. 782 among P, Q, R in the ratio

$$\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$$

Ans. Given ratio = $\frac{1}{2} : \frac{2}{3} : \frac{3}{4} = 6 : 8 : 9$

Sum of ratio terms = $(6 + 8 + 9) = 23$

$$\begin{aligned}\therefore \text{A's share} &= \text{Rs.} \left[782 \times \frac{6}{23} \right] \\ &= 34 \times 6 = \text{Rs. } 204.\end{aligned}$$

$$\begin{aligned}\text{B's share} &= \text{Rs.} \left[782 \times \frac{8}{23} \right] \\ &= 34 \times 8 = \text{Rs. } 272\end{aligned}$$

$$\begin{aligned}\text{C's share} &= \text{Rs.} \left[782 \times \frac{9}{23} \right] \\ &= 34 \times 9 = \text{Rs. } 306.\end{aligned}$$

Q. 30. If Rs. 510 be divided among A, B, C in such a way that A gets $\frac{2}{3}$ of what B gets and B gets $\frac{1}{4}$ of what C gets, find their respective shares.

Sol. Total amount = Rs. 510

Let C's part = x

$$\therefore \text{B's part} = \frac{1}{4} \times x = \frac{x}{4}$$

$$\text{A's part} = \frac{x}{4} \times \frac{2}{3} = \frac{x}{6}$$

$$\therefore x + \frac{x}{4} + \frac{x}{6} = 510$$

$$\frac{12x + 3x + 2x}{12} = 510 \Rightarrow \frac{17x}{12} = 510$$

$$\Rightarrow x = \frac{510 \times 12}{17} = 360$$

$$\therefore \text{A's share} = \frac{x}{6} = \frac{360 \times 1}{6} = \text{Rs. } 60$$

$$\text{B's share} = \frac{x}{4} = \frac{360}{4} = \text{Rs. } 90$$

$$\text{C's share} = x = 360 = \text{Rs. } 360 \text{ Ans.}$$

Q. 31. A sum of Rs. 1300 is divided among P, Q, R and S such that

$$\frac{\text{P's share}}{\text{Q's share}} = \frac{\text{Q's share}}{\text{R's share}} = \frac{\text{R's share}}{\text{S's share}} = \frac{2}{3}$$

Find their respective shares.

Sol. $\frac{\text{P's share}}{\text{Q's share}} = \frac{2}{3} \Rightarrow P : Q = 2 : 3$

$$\frac{\text{Q's share}}{\text{R's share}} = \frac{2}{3} \Rightarrow Q : R = 2 : 3$$

$$\frac{\text{R's share}}{\text{S's share}} = \frac{2}{3} \Rightarrow R : S = 2 : 3$$

$$P : Q = 2 : 3 \quad \dots(i)$$

$$Q : R = 2 : 3 \quad \dots(ii)$$

Multiplying (i) by 2 and (ii) by 3,

$$P : Q = 4 : 6$$

$$Q : R = 6 : 9$$

$$\therefore P : Q : R = 4 : 6 : 9 \quad \dots(iii)$$

$$R : S = 2 : 3 \quad \dots(iv)$$

Multiplying (iii) by 2 and (iv) by 9

$$\therefore P : Q : R = 8 : 12 : 18$$

$$R : S = 18 : 27$$

$$\therefore P : Q : R : S = 8 : 12 : 18 : 27$$

$$\begin{aligned}\text{Sum of ratios} &= 8 + 12 + 18 + 27 \\ &= 65\end{aligned}$$

$$\text{Total Sum} = \text{Rs. } 1300$$

$$\therefore \text{P's share} = \text{Rs. } 1300 \times \frac{8}{65} = \text{Rs. } 160$$

$$\text{Q's share} = \text{Rs. } 1300 \times \frac{12}{65} = \text{Rs. } 240$$

$$\text{R's share} = \text{Rs. } 1300 \times \frac{18}{65} = \text{Rs. } 360$$

$$\text{S's share} = \text{Rs. } 1300 \times \frac{27}{65} = \text{Rs. } 540 \text{ Ans.}$$

Q. 32. Divide Rs. 8134 among A, B and C so that 4 times A's share, 5 times B's share and 7 times C's share may all be equal.

Sol. Total amount = Rs. 8134

We are given that

$$\begin{aligned} \text{A's share} \times 4 &= \text{B's share} \times 5 \\ &= \text{C's share} \times 7 = x \text{ (say)} \end{aligned}$$

$$\therefore \text{A's share} = \frac{x}{4}, \text{ B's share} = \frac{x}{5}$$

$$\text{C's share} = \frac{x}{7}$$

$$\frac{x}{4} + \frac{x}{5} + \frac{x}{7} = 8134$$

$$\Rightarrow \frac{35x + 28x + 20x}{140} = 8134$$

$$\Rightarrow 83x = 8134 \times 140$$

$$x = \frac{8134 \times 140}{83}$$

$$= 98 \times 140 = \text{Rs. } 13720$$

$$\therefore \text{A's share} = \frac{x}{4} = \frac{13720}{4} = \text{Rs. } 3430$$

$$\text{B's share} = \frac{x}{5} = \frac{13720}{5} = \text{Rs. } 2744$$

$$\text{C's share} = \frac{x}{7} = \frac{13720}{7} = \text{Rs. } 1960 \text{ Ans.}$$

Q. 33. In a school, the boys and girls are in the ratio 9 : 5. If there are 425 girls, what is the total number of students in the school?

Sol. Let the number of boys = x
and the number of girls = 425

$$\therefore \frac{x}{425} = \frac{9}{5}$$

$$\Rightarrow 5x = 9 \times 425$$

$$\begin{aligned} \Rightarrow x &= \frac{9 \times 425}{5} = 9 \times 85 \\ &= 765 \end{aligned}$$

$$\therefore \text{Number of boys} = 765$$

$$\begin{aligned} \text{and total number of students} &= 765 + 425 \\ &= 1190 \text{ Ans.} \end{aligned}$$

Q. 34. A sum of money is divided between A and B in the ratio 6 : 11. If B's share is Rs. 7315, find :

(i) A's share

(ii) the total amount of money.

Sol. Let A's share = Rs. x

and B's share = Rs. 7315

$$\therefore \frac{x}{7315} = \frac{6}{11}$$

$$\Rightarrow 11x = 6 \times 7315$$

$$\Rightarrow x = \frac{6 \times 7315}{11} = 6 \times 665 = 3990$$

(i) $\therefore$ A's share = Rs. 3990

(ii) Total amount = Rs. 3990 + Rs. 7315
= Rs. 11305 Ans.

Q. 35. Find the angles of a triangle which are in the ratio 4 : 3 : 2.

Sol. Sum of angles of a triangle = 180°

and ratio = 4 : 3 : 2

Let first angle = $4x$

Second angle = $3x$

and third angle = $2x$

$$\therefore 4x + 3x + 2x = 180$$

$$\Rightarrow 9x = 180 \Rightarrow x = \frac{180}{9} = 20$$

$$\therefore \text{First angle} = 4x = 4 \times 20 = 80^\circ$$

$$\text{Second angle} = 3x = 3 \times 20 = 60^\circ$$

$$\text{and third angle} = 2x = 2 \times 20 = 40^\circ \text{ Ans.}$$

Q. 36. The sides of a triangle are in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$ and its perimeter is 91 cm. Find the lengths of the sides of the triangle.

Sol. The ratio of sides are $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$

$$= \frac{1}{2} \times 12 : \frac{1}{3} \times 12 : \frac{1}{4} \times 12$$

[Multiplying by 12, the LCM of 2, 3 and 4]

$$= 6 : 4 : 3$$

Let first side = $6x$

second side = $4x$

and third side = $3x$

and perimeter = 91 cm

$$\therefore 6x + 4x + 3x = 91 \text{ cm}$$

$$\Rightarrow 13x = 91 \Rightarrow x = \frac{91}{13} = 7$$

$$\therefore \text{First side} = 6x = 6 \times 7 = 42 \text{ cm}$$

$$\text{Second side} = 4x = 4 \times 7 = 28 \text{ cm}$$

$$\text{and third side} = 3x = 3 \times 7$$

$$= 21 \text{ cm Ans.}$$

Q. 37. Reena reduces her weight in the ratio 3 : 2. What is her weight now if originally it was 81 kg ?

Sol. Let her reduced weight = x
and original weight = 81 kg.

$$\text{Ratio} = 3 : 2$$

According to the condition,

$$\therefore 81 : x = 3 : 2$$

$$\Rightarrow x \times 3 = 2 \times 81 \Rightarrow x = \frac{2 \times 81}{3}$$

$$\Rightarrow x = 54$$

Hence, reduced weight = 54 kg. **Ans.**

Q. 38. The cost of production of a radio set is Rs. 954. This cost was divided between material, labour and overheads in the ratio 3 : 4 : 2. Calculate the cost of material used in one radio set. (1994)

Sol. Cost of production = Rs. 954

Let cost of material = $3x$

Then, cost of labour = $4x$

and cost of overheads = $2x$

According to the condition,

$$3x + 4x + 2x = \text{Rs. } 954$$

$$\Rightarrow 9x = 954 \Rightarrow x = \frac{954}{9} = 106$$

$$\therefore \text{Cost of material} = 3x$$

$$= 3 \times 106 = \text{Rs. } 318 \text{ Ans.}$$

Q. 39. The cost of making an umbrella is divided between material, labour and overheads in the ratio of 6 : 4 : 1. If the material costs Rs. 75, find the cost of production of an umbrella.

Sol. Let cost of material = $6x$

then, cost of labour = $4x$

and cost of overheads = x

$$\therefore \text{Total cost of production} = 6x + 4x + x = 11x$$

Now, the cost of material = Rs. 75

$$\therefore 6x = 75 \Rightarrow x = \frac{75}{6} = \frac{25}{2}$$

Hence, total cost of production = $11x$

$$= 11 \times \frac{25}{2} = \frac{275}{2} = \text{Rs. } 137.5 \text{ Ans.}$$

Q. 40. What is the ratio whose terms differ by 40 and the measure of which is $\frac{2}{7}$?

Sol. Difference in terms = 40

$$\text{Measure of ratio} = \frac{2}{7}$$

$$\text{Let measure of ratio} = \frac{2x}{7x}$$

According to the condition,

$$7x - 2x = 40 \Rightarrow 5x = 40$$

$$\Rightarrow x = 8$$

$$\therefore \text{Ratio} = \frac{2x}{7x} = \frac{2 \times 8}{7 \times 8} = \frac{16}{56}$$

Hence, ratio = 16 : 56 **Ans.**

Q. 41. A purse contains one-rupee, 50-paise and 25-paise coins in the ratio 2 : 3 : 4. If the total amount in the purse is Rs. 180, find the number of coins of each kind.

Sol. Let number of one-rupee coins = $2x$

then 50 - paise coins = $3x$

and 25 - paise coins = $4x$

Total money = Rs. 180

According to the condition,

$$2x + 3x \times \frac{50}{100} + \frac{4x \times 25}{100} = 180$$

$$\Rightarrow 2x + \frac{3x}{2} + x = 180$$

$$\Rightarrow 4x + 3x + 2x = 360$$

[Multiplying by 2]

$$\Rightarrow 9x = 360 \Rightarrow x = \frac{360}{9} = 40$$

$\therefore$ No. of one rupee-coins

$$= 2x = 40 \times 2 = 80$$

No. of 50 paise coins = $3x$

$$= 3 \times 40 = 120$$

No. of 25 paise coins = $4x$

$$= 4 \times 40 = 160 \text{ **Ans.**}$$

Q. 42. In a cash-bag, there are currency notes in denominations of Rs. 20, Rs. 10 and Rs. 5 in the ratio of 3 : 4 : 5. If the total amount in the bag is Rs. 2250, find the number of notes of each kind.

Sol. Let the no. of Rs. 20 notes = $3x$

No. of Rs. 10 notes = $4x$

and no. of Rs. 5 notes = $5x$

Total amount of the bag = Rs. 2250

According to the condition,

$$3x \times 20 + 4x \times 10 + 5x \times 5 = 2250$$

$$\Rightarrow 60x + 40x + 25x = 2250$$

$$\Rightarrow 125x = 2250 \Rightarrow x = \frac{2250}{125} = 18$$

$\therefore$ No. of 20 - rupee notes = $3x$

$$= 3 \times 18 = 54$$

No. of 10 - rupee notes = $4x$

$$= 4 \times 18 = 72$$

and no. of 5 - rupee notes = $5x$

$$= 5 \times 18 = 90 \text{ **Ans.**}$$

Q. 43. 85 kg. of a mixture contains milk and water in the ratio 27 : 7. How much more water is to be added to this mixture to get a new mixture containing milk and water in the ratio 3 : 1 ?

Sol. Total mixture of milk and water = 85 kg.

Ratio of milk and water = 27 : 7

$\therefore$ Quantity of milk

$$= \frac{85 \times 27}{27 + 7} = \frac{85 \times 27}{34} = \frac{5 \times 27}{2} \text{ kg.}$$

$$= \frac{135}{2} \text{ kg.}$$

and quantity of water

$$= \frac{85 \times 7}{34} = \frac{5 \times 7}{2} = \frac{35}{2} \text{ kg.}$$

Let more water to be added = x kg.

According to the condition,

$$\frac{135}{2} : \left(\frac{35}{2} + x \right) = 3 : 1$$

$$\Rightarrow 3 \left(\frac{35}{2} + x \right) = \frac{135}{2} \times 1$$

$$\Rightarrow \frac{35}{2} + x = \frac{135}{2 \times 3}$$

$$\Rightarrow \frac{35}{2} + x = \frac{45}{2}$$

$$\therefore x = \frac{45}{2} - \frac{35}{2} = \frac{10}{2} = 5$$

Hence, more water is to be added

$$= 5 \text{ kg. **Ans.**}$$

Q. 44. A mixture contains milk and water in the ratio 5 : 1. On adding 5 litres of water, the ratio of milk to water becomes 5 : 2. Find the quantity of milk in the original mixture.

Sol. Let in the original mixture of milk and water, milk be = $5x$ l and water = x l

By adding 5 l of water in the mixture, the ratio is 5 : 2

$$\therefore \frac{5x}{x+5} = \frac{5}{2}$$

$$\Rightarrow 10x = 5x + 25$$

$$\Rightarrow 10x - 5x = 25$$

$$\Rightarrow 5x = 25$$

$$\therefore x = \frac{25}{5} = 5$$

Hence, milk in the original mixture

$$= 5x = 5 \times 5$$

$$= 25 \text{ litres Ans.}$$

Q. 45. In a regiment, the ratio of number of officers to the number of soldiers was 3 : 31 before a battle. In the battle, 6 officers and 22 soldiers were killed. The ratio between the number of officers and the number of soldiers now is 1 : 13. Find the number of officers and soldiers in the regiment before the battle. (1992)

Sol. Before the battle, since the ratio of the number of officers to the number of soldiers was 3 : 31,

let the number of officers in the regiment before battle = $3x$,

then the number of soldiers in the regiment before battle = $31x$.

Number of officers killed = 6,

$\therefore$ The number of officers left after the battle = $3x - 6$.

Number of soldiers killed = 22,

$\therefore$ The number of soldiers left after the battle = $31x - 22$.

After battle, since the ratio of number of officers to the number of soldiers is 1 : 13,

$$\therefore \frac{3x - 6}{31x - 22} = \frac{1}{13}$$

$$\Rightarrow 13(3x - 6) = 1(31x - 22)$$

$$\Rightarrow 39x - 78 = 31x - 22$$

$$\Rightarrow 8x = 56$$

$$\Rightarrow x = 7.$$

$\therefore$ The number of officers in the regiment before battle = $3 \times 7 = 21$, and the number of soldiers in the regiment before battle = $31 \times 7 = 217$ **Ans.**

Q. 46. In an examination, the ratio of passes to failures was 4 : 1. Had 30 less appeared and 20 less passed, the ratio of passes to failures would have been 5 : 1. How many students appeared for the examination?

Sol. Let number of candidates appeared = x

Ratio of passes to failures = 4 : 1

$$\therefore \text{No. of passes} = \frac{x \times 4}{4+1} = \frac{4x}{5}$$

In second case,

No. of candidates appeared = $x - 30$

No. the ratio of passes to failures = 5 : 1

$\therefore$ No. of candidates passed

$$= \frac{(x-30) \times 5}{5+1} = \frac{5x-150}{6}$$

According to the condition,

$$\frac{4x}{5} - \frac{5x-150}{6} = 20$$

$$\Rightarrow 24x - 25x + 750 = 600$$

[Multiplying by 30, the LCM of 5 and 6]

$$\Rightarrow -x = 600 - 750$$

$$\Rightarrow -x = -150$$

$$\therefore x = 150$$

Hence, total no. of candidates appeared
= 150 **Ans.**

$$\Rightarrow \frac{19}{25} < \frac{17}{20}$$

$$\therefore (19 : 25) < (17 : 20) \text{ Ans.}$$

$$(iii) \frac{1}{2} : \frac{1}{5} \text{ and } 5 : 2$$

$$\Rightarrow \frac{1}{2} \times \frac{5}{1} \text{ and } \frac{5}{2}$$

$$\Rightarrow \frac{5}{2} \text{ and } \frac{5}{2}$$

$$\therefore \frac{5}{2} = \frac{5}{2}$$

$$\text{Hence, } \left(\frac{1}{2} : \frac{1}{5} \right) = (5 : 2) \text{ Ans.}$$

Q. 47. Compare the following ratios :

$$(i) (7 : 9) \text{ and } (11 : 16)$$

$$(ii) (19 : 25) \text{ and } (17 : 20)$$

$$(iii) \left(\frac{1}{2} : \frac{1}{5} \right) \text{ and } (5 : 2)$$

Sol. (i) 7 : 9 and 11 : 16

$$\Rightarrow \frac{7}{9} \text{ and } \frac{11}{16}$$

$$\text{Now } \frac{7}{9} \times \frac{16}{16} \text{ and } \frac{11}{16} \times \frac{9}{9}$$

[Making equal denominators]

$$\Rightarrow \frac{112}{144} \text{ and } \frac{99}{144}$$

$$\therefore \frac{112}{144} > \frac{99}{144}$$

$$\Rightarrow \frac{7}{9} > \frac{11}{16}$$

$$\therefore (7 : 9) > (11 : 16) \text{ Ans.}$$

$$(ii) 19 : 25 \text{ and } 17 : 20$$

$$\Rightarrow \frac{19}{25} \text{ and } \frac{17}{20}$$

Making equal denominators,

$$\frac{19}{25} \times \frac{4}{4} \text{ and } \frac{17}{20} \times \frac{5}{5}$$

$$\Rightarrow \frac{76}{100} \text{ and } \frac{85}{100}$$

$$\therefore \frac{76}{100} < \frac{85}{100}$$

Q. 48. Arrange the following ratios in descending order of magnitudes :

$$(i) (5 : 6), (8 : 9), (13 : 18) \text{ and } (19 : 24)$$

$$(ii) (6 : 7), (13 : 14), (19 : 21) \text{ and } (23 : 28)$$

$$(iii) (3 : 5), (4 : 9), (5 : 11) \text{ and } (10 : 17).$$

Sol. (i) (5 : 6), (8 : 9), (13 : 18) and (19 : 24)

$$\Rightarrow \frac{5}{6}, \frac{8}{9}, \frac{13}{18} \text{ and } \frac{19}{24}$$

$$\text{L.C.M. of } 6, 9, 18 \text{ and } 24 = 72$$

Making equal denominator i.e. each of 72

$$\frac{5}{6} = \frac{5}{6} \times \frac{12}{12} = \frac{60}{72}$$

$$\frac{8}{9} = \frac{8}{9} \times \frac{8}{8} = \frac{64}{72}$$

$$\frac{13}{18} = \frac{13 \times 4}{18 \times 4} = \frac{52}{72}$$

$$\text{and } \frac{19}{24} = \frac{19 \times 3}{24 \times 3} = \frac{57}{72}$$

It is clear from the above that

$$\frac{64}{72} > \frac{60}{72} > \frac{57}{72} > \frac{52}{72}$$

$$\therefore \frac{8}{9} > \frac{5}{6} > \frac{19}{24} > \frac{13}{18}$$

Hence $(8 : 9) > (5 : 6) > (19 : 24) > (13 : 18)$ **Ans.**

(ii) $(6 : 7), (13 : 14), (19 : 21)$ and $(23 : 28)$

$$\Rightarrow \frac{6}{7}, \frac{13}{14}, \frac{19}{21} \text{ and } \frac{23}{28}$$

Making equal denominator *i.e.* 84, the LCM of 7, 14, 21 and 28.

$$\frac{6}{7} = \frac{6 \times 12}{7 \times 12} = \frac{72}{84}$$

$$\frac{13}{14} = \frac{13 \times 6}{14 \times 6} = \frac{78}{84}$$

$$\frac{19}{21} = \frac{19 \times 4}{21 \times 4} = \frac{76}{84}$$

$$\text{and } \frac{23}{28} = \frac{23 \times 3}{28 \times 3} = \frac{69}{84}$$

It is clear from the above that

$$\frac{78}{84} > \frac{76}{84} > \frac{72}{84} > \frac{69}{84}$$

$$\Rightarrow \frac{13}{14} > \frac{19}{21} > \frac{6}{7} > \frac{23}{28}$$

Hence $(13 : 14) > (19 : 21) > (6 : 7) > (23 : 28)$ **Ans.**

(iii) $(3 : 5), (4 : 9), (5 : 11)$ and $(10 : 17)$

$$\Rightarrow \frac{3}{5}, \frac{4}{9}, \frac{5}{11}, \frac{10}{17}$$

LCM of 5, 9, 11 and 17

$$= 5 \times 9 \times 11 \times 17 = 8415$$

Now, making equal denominators *i.e.* each of 8415.

$$\frac{3}{5} = \frac{3 \times 1683}{5 \times 1683} = \frac{5049}{8415}$$

$$\frac{4}{9} = \frac{4 \times 935}{9 \times 935} = \frac{3740}{8415}$$

$$\frac{5}{11} = \frac{5 \times 765}{11 \times 765} = \frac{3825}{8415}$$

$$\text{and } \frac{10}{17} = \frac{10 \times 495}{17 \times 495} = \frac{4950}{8415}$$

It is clear from the above that

$$\frac{5049}{8415} > \frac{4950}{8415} > \frac{3825}{8415} > \frac{3740}{8415}$$

$$\Rightarrow \frac{3}{5} > \frac{10}{17} > \frac{5}{11} > \frac{4}{9}$$

$\therefore (3 : 5) > (10 : 17) > (5 : 11) > (4 : 9)$ **Ans.**

Q. 49. Arrange the following ratios in ascending order of magnitudes :

(i) $(7 : 12), (9 : 16), (13 : 20)$ and $(5 : 8)$

(ii) $(4 : 9), (6 : 11), (7 : 13)$ and $(27 : 50)$

(iii) $(2 : 3), (8 : 15), (11 : 12)$ and $(7 : 16)$

Sol. (i) Given ratios are $\frac{7}{12}, \frac{9}{16}, \frac{13}{20}, \frac{5}{8}$

L.C.M of 12, 16, 20, 8 = 240

$$\frac{7}{12} = \frac{7 \times 20}{12 \times 20} = \frac{140}{240};$$

$$\frac{9}{16} = \frac{9 \times 15}{16 \times 15} = \frac{135}{240};$$

$$\frac{13}{20} = \frac{13 \times 12}{20 \times 12} = \frac{156}{240};$$

$$\frac{5}{8} = \frac{5 \times 30}{8 \times 30} = \frac{150}{240}.$$

Since $135 < 140 < 150 < 156$,

$$\text{so } \frac{135}{240} < \frac{140}{240} < \frac{150}{240} < \frac{156}{240}$$

$$\therefore \frac{9}{16} < \frac{7}{12} < \frac{5}{8} < \frac{13}{20}$$

Hence, the given ratios in ascending order are : (9 : 16), (7 : 12), (5 : 8) and (13 : 20).

Alternative Method :

$$7 : 12 = \frac{7}{12} = 0.58 ;$$

$$9 : 16 = \frac{9}{16} = 0.563 ;$$

$$13 : 20 = \frac{13}{20} = 0.65 ;$$

$$5 : 8 = \frac{5}{8} = 0.625.$$

Since, $0.56 < 0.58 < 0.62 < 0.65$

So $(9 : 16) < (7 : 12) < (5 : 8) < (13 : 20)$

Ans.

$$(ii) \text{ Given ratios are } \frac{4}{9}, \frac{6}{11}, \frac{7}{13}, \frac{27}{50}$$

L.C.M. of 9, 11, 13, 50 = 64350

$$\frac{4}{9} = \frac{4 \times 7150}{9 \times 7150} = \frac{28600}{64350} ;$$

$$\frac{6}{11} = \frac{6 \times 5850}{11 \times 5850} = \frac{35100}{64350} ;$$

$$\frac{7}{13} = \frac{7 \times 4950}{13 \times 4950} = \frac{34650}{64350} ;$$

$$\frac{27}{50} = \frac{27 \times 1287}{50 \times 1287} = \frac{34749}{64350}.$$

Since, $28600 < 34650 < 34749 < 35100$

$$\text{So, } \frac{28600}{64350} < \frac{34650}{64350} < \frac{34749}{64350} < \frac{35100}{64350}$$

$$\therefore \frac{4}{9} < \frac{7}{13} < \frac{27}{50} < \frac{6}{11}.$$

Hence, the given ratios in ascending order are :

(4 : 9) (7 : 13) (27 : 50) (6 : 11) **Ans.**

Alternative method :

$$4 : 9 = \frac{4}{9} = 0.44 ;$$

$$6 : 11 = \frac{6}{11} = 0.545$$

$$7 : 13 = \frac{7}{13} = 0.538 ;$$

$$27 : 50 = \frac{27}{50} = 0.54$$

Since, $0.44 < 0.538 < 0.54 < 0.545$

So, $(4 : 9) < (7 : 13) < (27 : 50) < (6 : 11)$

Ans.

$$(iii) \text{ Given ratios are } \frac{2}{3}, \frac{8}{15}, \frac{11}{12}, \frac{7}{16}$$

L.C.M of 3, 15, 12, 16 = 240

$$\frac{2}{3} = \frac{2 \times 80}{3 \times 80} = \frac{160}{240} ;$$

$$\frac{8}{15} = \frac{8 \times 16}{15 \times 16} = \frac{128}{240}$$

$$\frac{11}{12} = \frac{11 \times 20}{12 \times 20} = \frac{220}{240} ;$$

$$\frac{7}{16} = \frac{7 \times 15}{16 \times 15} = \frac{105}{240}$$

Since, $105 < 128 < 160 < 220$

$$\text{So, } \frac{105}{240} < \frac{128}{240} < \frac{160}{240} < \frac{220}{240}$$

$$\therefore \frac{7}{16} < \frac{8}{15} < \frac{2}{3} < \frac{11}{12}$$

Hence, the given ratios in ascending order are :

(7 : 16) (8 : 15) (2 : 3) (11 : 12) **Ans.**

EXERCISE 8 (B)

Q. 1. Find the duplicate ratio of :

- (i) $9 : 4$ (ii) $\frac{1}{5} : \frac{1}{6}$
(iii) $3\sqrt{3} : 2\sqrt{5}$ (iv) $2\sqrt{a} : b$

Sol. (i) Duplicate ratio of $9 : 4$ will be
 $= (9)^2 : (4)^2 = 81 : 16$

(ii) Duplicate ratio of $\frac{1}{5} : \frac{1}{6}$ will be

$$= \left(\frac{1}{5}\right)^2 : \left(\frac{1}{6}\right)^2 = \frac{1}{25} : \frac{1}{36} = 36 : 25$$

(iii) Duplicate ratio of $3\sqrt{3} : 2\sqrt{5}$ will be
 $= (3\sqrt{3})^2 : (2\sqrt{5})^2 = 9 \times 3 : 4 \times 5$
 $= 27 : 20$

(iv) Duplicate ratio of $2\sqrt{a} : b$ will be
 $= (2\sqrt{a})^2 : (b)^2$
 $= 4a : b^2$ **Ans.**

Q.2. Find the triplicate ratio of :

- (i) $4 : 7$ (ii) $\frac{1}{2} : \frac{1}{3}$
(iii) $\sqrt{2} : \sqrt{3}$ (iv) $2\sqrt{2} : 5$

Sol. (i) Triplicate ratio of $4 : 7$ will be
 $= (4)^3 : (7)^3 = 64 : 343$

(ii) Triplicate ratio of $\frac{1}{2} : \frac{1}{3}$ will be

$$= \left(\frac{1}{2}\right)^3 : \left(\frac{1}{3}\right)^3 = \frac{1}{8} : \frac{1}{27} = 27 : 8$$

(iii) Triplicate ratio of $\sqrt{2} : \sqrt{3}$ will be

$$= (\sqrt{2})^3 : (\sqrt{3})^3 = 2\sqrt{2} : 3\sqrt{3}$$

(iv) Triplicate ratio of $2\sqrt{2} : 5$ will be

$$= (2\sqrt{2})^3 : (5)^3$$

$$= 8 \times 2\sqrt{2} : 125 = 16\sqrt{2} : 125$$
 Ans.

Q.3. Find the sub-duplicate ratio of :

- (i) $16 : 9$ (ii) $\frac{1}{9} : \frac{1}{4}$
(iii) $9a^2 : 25b^2$ (iv) $9 : 8$

Sol. (i) Sub-duplicate ratio of $16 : 9$ will be
 $= \sqrt{16} : \sqrt{9} = 4 : 3$

(ii) Sub-duplicate ratio of $\frac{1}{9} : \frac{1}{4}$ will be

$$= \sqrt{\frac{1}{9}} : \sqrt{\frac{1}{4}} = \frac{1}{3} : \frac{1}{2} = 2 : 3$$

(iii) Sub-duplicate ratio of $9a^2 : 25b^2$ will be
 $= \sqrt{9a^2} : \sqrt{25b^2} = 3a : 5b$

(iv) Sub-duplicate ratio of $9 : 8$ will be
 $= \sqrt{9} : \sqrt{8} = 3 : \sqrt{4 \times 2}$
 $= 3 : 2\sqrt{2}$ **Ans.**

Q.4. Find the sub-triplicate ratio of :

- (i) $125 : 216$ (ii) $\frac{1}{8} : \frac{1}{27}$
(iii) $x^3 : 64y^3$ (iv) $2\sqrt{2} : 3\sqrt{3}$

Sol. (i) Sub-triplicate ratio of $125 : 216$ will be
 $= \sqrt[3]{125} : \sqrt[3]{216} = \sqrt[3]{5^3} : \sqrt[3]{6^3} = 5 : 6$

(ii) Sub-triplicate ratio of $\frac{1}{8} : \frac{1}{27}$ will be

$$= \sqrt[3]{\frac{1}{8}} : \sqrt[3]{\frac{1}{27}} = \sqrt[3]{\left(\frac{1}{2}\right)^3} : \sqrt[3]{\left(\frac{1}{3}\right)^3}$$
$$= \frac{1}{2} : \frac{1}{3} = 3 : 2$$

(iii) Sub-triplicate ratio of $x^3 : 64y^3$ will be
 $= \sqrt[3]{x^3} : \sqrt[3]{64y^3} = \sqrt[3]{(x)^3} : \sqrt[3]{(4y)^3}$
 $= x : 4y$

(iv) Sub-triplicate ratio of $2\sqrt{2} : 3\sqrt{3}$ will be

$$\begin{aligned} &= \sqrt[3]{2\sqrt{2}} : \sqrt[3]{3\sqrt{3}} \\ &= \sqrt[3]{\sqrt{2} \times \sqrt{2} \times \sqrt{2}} : \sqrt[3]{\sqrt{3} \times \sqrt{3} \times \sqrt{3}} \\ &= \sqrt[3]{(\sqrt{2})^3} : \sqrt[3]{(\sqrt{3})^3} \\ &= \sqrt{2} : \sqrt{3} \text{ Ans.} \end{aligned}$$

Q. 5. Find the reciprocal ratio of :

(i) $16 : 25$ (ii) $\frac{1}{5} : \frac{1}{7}$

(iii) $4 : \frac{1}{9}$ (iv) $\frac{x}{3} : \frac{y}{5}$

Sol. (i) Reciprocal ratio of $16 : 25$ will be

$$= \frac{1}{16} : \frac{1}{25} = 25 : 16$$

(ii) Reciprocal ratio of $\frac{1}{5} : \frac{1}{7}$ will be $= 5 : 7$

(iii) Reciprocal ratio of $4 : \frac{1}{9}$ will be

$$= \frac{1}{4} : \frac{9}{1} = 1 : 36$$

(iv) Reciprocal ratio of $\frac{x}{3} : \frac{y}{5}$ will be

$$= \frac{3}{x} : \frac{5}{y} = 3y : 5x \text{ Ans.}$$

Q. 6. Find the compounded ratio of :

(i) $(3 : 4)$ and $(8 : 9)$

(ii) $(2 : 3)$, $(9 : 16)$ and $(4 : 7)$

(iii) $(a - b) : (a + b)$, $(a + b)^2 : (a^2 + b^2)$
and $(a^4 - b^4) : (a^2 - b^2)^2$

Sol. (i) Compounded ratio of

$$(3 : 4) \text{ and } (8 : 9) = \frac{3}{4} \times \frac{8}{9} = \frac{2}{3} = 2 : 3$$

(ii) Compounded ratio of $(2 : 3)$, $(9 : 16)$ and $(4 : 7)$

$$= \frac{2}{3} \times \frac{9}{16} \times \frac{4}{7} = \frac{3}{14} = 3 : 14$$

(iii) Compounded ratio of

$$(a - b) : (a + b), (a + b)^2 : (a^2 + b^2)$$

$$\text{and } (a^4 - b^4) : (a^2 - b^2)^2$$

$$= \frac{a - b}{a + b} \times \frac{(a + b)^2}{a^2 + b^2} \times \frac{a^4 - b^4}{(a^2 - b^2)^2}$$

$$= \frac{(a - b)}{a + b} \times \frac{(a + b)(a + b)}{a^2 + b^2}$$

$$\times \frac{(a - b)(a + b)(a^2 + b^2)}{(a + b)^2 (a - b)^2}$$

$$= \frac{1}{1} = 1 : 1 \text{ Ans.}$$

Q. 7. Show that the ratio compounded of the duplicate of $7 : 6$, the sub-duplicate of $25 : 49$ and the reciprocal of $35 : 36$ is $1 : 1$.

Sol. Duplicate of $7 : 6 = (7)^2 : (6)^2 = 49 : 36$

Sub-duplicate of $25 : 49$

$$= \sqrt{25} : \sqrt{49} = 5 : 7$$

$$\text{Reciprocal of } 35 : 36 = \frac{1}{35} : \frac{1}{36} = 36 : 35$$

Now, L.H.S. = compounded ratio of $(49 : 36)$, $(5 : 7)$ and $(36 : 35)$

$$= \frac{49}{36} \times \frac{5}{7} \times \frac{36}{35} = \frac{1}{1} = 1 : 1$$

= R.H.S.

Hence proved.

Q. 8. Find the ratio compounded of the reciprocal of $15 : 28$, the sub-duplicate of $36 : 49$ and the triplicate of $5 : 4$.

Sol. Reciprocal of $15 : 28$

$$= \frac{1}{15} : \frac{1}{28} = 28 : 15$$

Sub-duplicate of $36 : 49$

$$= \sqrt{36} : \sqrt{49} = 6 : 7$$

$$\text{Triplicate of } 5 : 4 = (5)^3 : (4)^3 = 125 : 64$$

Now, compounded ratio of $(28 : 15)$, $(6 : 7)$ and $(125 : 64)$

$$= \frac{28}{15} \times \frac{6}{7} \times \frac{125}{64}$$

$$= \frac{25}{8} = 25 : 8 \text{ Ans.}$$

Q. 9. If $(2x + 5) : (8x + 9)$ is the duplicate ratio of $3 : 5$, find the value of x .

Sol. Duplicate ratio of $3 : 5 = (3)^2 : (5)^2$

$$= 9 : 25$$

$$\therefore (2x + 5) : (8x + 9) = 9 : 25$$

$$\Rightarrow \frac{2x+5}{8x+9} = \frac{9}{25}$$

$$\Rightarrow 25(2x + 5) = 9(8x + 9)$$

[By cross multiplication]

$$\Rightarrow 50x + 125 = 72x + 81$$

$$\Rightarrow 50x - 72x = 81 - 125$$

$$\Rightarrow -22x = -44$$

$$\Rightarrow x = \frac{-44}{-22} = 2$$

Hence $x = 2$ Ans.

Q. 10. If $(4x + 3) : (9x + 10)$ is the triplicate ratio of $3 : 4$, find the value of x .

Sol. Triplicate ratio of $3 : 4 = (3)^3 : (4)^3$

$$= 27 : 64$$

$$\therefore (4x + 3) : (9x + 10) = 27 : 64$$

$$\Rightarrow \frac{4x+3}{9x+10} = \frac{27}{64}$$

$$\Rightarrow (4x + 3) \times 64 = 27 \times (9x + 10)$$

[By cross multiplication]

$$\Rightarrow 256x + 192 = 243x + 270$$

$$\Rightarrow 256x - 243x = 270 - 192$$

$$\Rightarrow 13x = 78$$

$$\Rightarrow x = \frac{78}{13} = 6$$

Hence $x = 6$ Ans.