DESIGN OF THE QUESTION PAPER

MATHEMATICS - CLASS XI

Time : 3 Hours
Max. Marks : 100

The weightage of marks over different dimensions of the question paper shall be as follows:

1. Weigtage of Type of Questions **Marks**

(i) Objective Type Questions	:	(10)	$10 \times 1 =$	10
(ii) Short Answer Type questions	:	(12)	$12 \times 4 =$	48
(viii) Long Answer Type Questions	:	(7)	$7 \times 6 = 42$	
Total Questions	:	(29)		100

2. Weightage to Different Topics

S.No.	Topic	Objective Type Questions	S.A. Type Questions	L.A. Type Questions	Total
1.	Sets	-	1(4)	-	4(1)
2.	Relations and Functions	-	-	1(6)	6(1)
3.	Trigonometric Functions	2(2)	1(4)	1(6)	12(4)
4.	Principle of Mathematical Induction	-	1(4)	-	4(1)
5.	Complex Numbers and Quadratic Equations	2(2)	1(4)	- -	6(3)
6.	Linear Inequalities	1(1)	1(4)	-	5(2)
7.	Permutations and Combinations	-	1(4)	-	4(1)
8.	Binomial Theorem	-	-	1(6)	6(1)
9.	Sequences and Series	-	1(4)	-	4(1)
10.	Straight Lines	2(2)	1(4)	1(6)	12(4)
11.	Conic Section	-	-	1(6)	6(1)
12.	Introduction to three dimensional geometry	-	1(4)	-	4(1)
13.	Limits and Derivatives	1(1)	1(4)	-	5(2)
14.	Mathematical Reasoning	1(1)	1(4)	-	5(2)
15.	Statistics	-	1(4)	1(6)	10(2)
16.	Probability	1(1)	-	1(6)	7(2)
	Total	**10(10)**	**48(12)**	**42(7)**	**100(29)**

SAMPLE QUESTION PAPER
Mathematics Class XI

General Instructions

(i) The question paper consists of three parts A, B and C. Each question of each part is compulsory.

(ii) Part A (Objective Type) consists of 10 questions of 1 mark each.

(iii) Part B (Short Answer Type) consists of 12 questions of 4 marks each.

(iv) Part C (Long Answer Type) consists of 7 questions of 6 marks each.

PART - A

1. If $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$, then what is the value of $(\theta + \phi)$?

2. For a complex number z, what is the value of arg. z + arg. $\bar{z}$, $z \neq 0$?

3. Three identical dice are rolled. What is the probability that the same number will appear an each of them?

Fill in the blanks in questions number 4 and 5.

4. The intercept of the line $2x + 3y - 6 = 0$ on the x-axis is ....

5. $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$ is equal to ....

In Questions 6 and 7, state whether the given statements are True or False:

6. $x + \frac{1}{x} \geq 2, \quad \forall\, x > 0$

7. The lines $3x + 4y + 7 = 0$ and $4x + 3y + 5 = 0$ are perpendicular to each other.

In Question 8 to 9, choose the correct option from the given 4 options, out of which only one is correct.

8. The solution of the equation $\cos^2\theta + \sin\theta + 1 = 0$, lies in the interval

(A) $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ (B) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (C) $\left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ (D) $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$

9. If $z = 2 + \sqrt{3}i$, the value of $z \cdot \bar{z}$ is

(A) 7 (B) 8 (C) $2 - \sqrt{3}i$ (D) 1

10. What is the contrapositive of the statement? "If a number is divisible by 6, then it is divisible by 3.

PART - B

11. If A′ ∪ B = U, show by using laws of algebra of sets that A ⊂ B, where A′ denotes the complement of A and U is the universal set.

12. If $\cos x = \frac{1}{7}$ and $\cos y = \frac{13}{14}$, x, y being acute angles, prove that $x - y = 60°$.

13. Using the principle of mathematical induction, show that $2^{3n} - 1$ is divisible by 7 for all $n \in \mathbf{N}$.

14. Write $z = -4 + i\, 4\sqrt{3}$ in the polar form.

15. Solve the system of linear inequations and represent the solution on the number line:

$$3x - 7 > 2\,(x - 6) \quad \text{and} \quad 6 - x > 11 - 2x$$

16. If $a + b + c \neq 0$ and $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P., prove that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P.

17. A mathematics question paper consists of 10 questions divided into two parts I and II, each containing 5 questions. A student is required to attempt 6 questions in all, taking at least 2 questions from each part. In how many ways can the student select the questions?

18. Find the equation of the line which passes through the point (–3, –2) and cuts off intercepts on x and y axes which are in the ratio 4 : 3.

19. Find the coordinates of the point R which divides the join of the points P(0, 0, 0) and Q(4, –1, –2) in the ratio 1 : 2 externally and verify that P is the mid point of RQ.

20. Differentiate $f(x) = \frac{3 - x}{3 + 4x}$ with respect to x, by first principle.

21. Verify by method of contradiction that $p = \sqrt{3}$ is irrational.

22. Find the mean deviation about the mean for the following data:

x_i	10	30	50	70	90
f_i	4	24	28	16	8

PART C

23. Let $f(x) = x^2$ and $g(x) = \sqrt{x}$ be two functions defined over the set of non-negative real numbers. Find:

(i) $(f+g)(4)$ (ii) $(f-g)(9)$ (iii) $(fg)(4)$ (iv) $\left(\frac{f}{g}\right)(9)$

24. Prove that: $\dfrac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$

25. Find the fourth term from the beginning and the 5th term from the end in the expansion of $\left(\dfrac{x^3}{3} - \dfrac{3}{x^2}\right)^{10}$.

26. A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point $(1, 5)$. Find the equation of this line.

27. Find the lengths of the major and minor axes, the coordinates of foci, the vertices, the ecentricity and the length of the latus rectum of the ellipse $\dfrac{x^2}{169} + \dfrac{y^2}{144} = 1$.

28. Find the mean, variance and standard deviation for the following data:

Class interval:	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
Frequency:	3	7	12	15	8	3	2

29. What is the probability that
(i) a non-leap year have 53 Sundays.
(ii) a leap year have 53 Fridays
(iii) a leap year have 53 Sundays and 53 Mondays.

MARKING SCHEME
MATHEMATICS CLASS XI

PART - A

Q. No.	Answer	Marks
1.	$\frac{\pi}{4}$	1
2.	Zero	1
3.	$\frac{1}{36}$	1
4.	3	1
5.	$\frac{1}{2}$	1
6.	True	1
7.	False	1
8.	D	1
9.	A	1
10.	If a number is not divisible by 3, then it is not divisible by 6.	1

PART - B

11. $B = B \cup \phi = B \cup (A \cap A')$ 1

$= (B \cup A) \cap (B \cup A')$ 1

$= (B \cup A) \cap (A' \cup B) = (B \cup A) \cap U$ (Given) 1

$= B \cup A$ $\frac{1}{2}$

$\Rightarrow A \subset B.$ $\frac{1}{2}$

12. $\cos x = \frac{1}{7} \Rightarrow \sin x = \sqrt{1-\cos^2 x} = \sqrt{1-\frac{1}{49}} = \frac{4\sqrt{3}}{7}$ 1

$\cos y = \frac{13}{14} \Rightarrow \sin y = \sqrt{1-\frac{169}{196}} = \frac{3\sqrt{3}}{14}$ 1

$\cos(x-y) = \cos x \cos y + \sin x \sin y$ $\frac{1}{2}$

$= \left(\frac{1}{7}\right)\left(\frac{13}{14}\right) + \frac{4\sqrt{3}}{7} \cdot \frac{3\sqrt{3}}{14} = \frac{1}{2}$ 1

$\Rightarrow x - y = \frac{\pi}{3}$ $\frac{1}{2}$

13. Let P(n) : "$2^{3n} - 1$ is divisble by 7" $\frac{1}{2}$

P(1) = $2^3 - 1 = 8 - 1 = 7$ is divisible by 7 $\Rightarrow$ P(1) is true. $\frac{1}{2}$

Let P(k) be true, i.e, "$2^{3k} - 1$ is divisible by 7", $\therefore$ $2^{3k} - 1 = 7a, a \in \mathbf{Z}$ 1

We have : $2^{3(k+1)} - 1 = 2^{3k} . 2^3 - 1$ 1

$= (2^{3k} - 1)\, 8 + 7 = 7a . 8 + 7 = 7(8a + 1)$ $\frac{1}{2}$

$\Rightarrow$ P(k + 1) is true, hence P(n) is true $\forall_n \in \mathbf{N}$ $\frac{1}{2}$

14. Let $-4 + i\, 4\sqrt{3} = r(\cos\theta + i \sin\theta)$ $\frac{1}{2}$

$\Rightarrow r \cos\theta = -4,\ r \sin\theta = 4\sqrt{3} \Rightarrow r^2 = 16 + 48 = 64 \Rightarrow r = 8.$ $1\frac{1}{2}$

$$\tan\theta = -\sqrt{3} \Rightarrow \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$ $1\frac{1}{2}$

$$\therefore \quad z = -4 + i4\sqrt{3} = 8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$ $\frac{1}{2}$

15. The given in equations are :

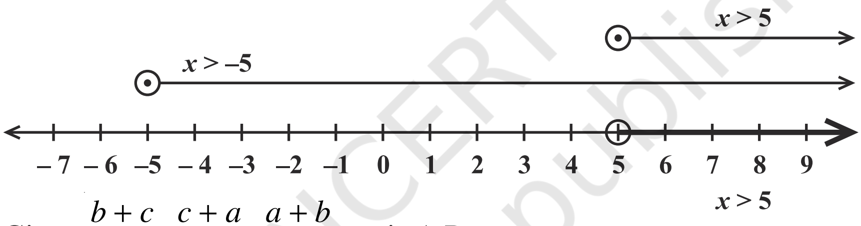
$3x - 7 > 2(x - 6)$ ... (i) and $6 - x > 11 - 2x$ ... (ii)

(i) $\Rightarrow 3x - 2x > -12 + 7$ or $x > -5$ ... (A) 1

(ii) $\Rightarrow -x + 2x > 11 - 6$ or $x > 5$ ... (B) 1

From A and B, the solutions of the given system are $x > 5$ 1

Graphical representation is as under:



1

16. Given $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P.

$\therefore 1 + \frac{b+c}{a}, 1 + \frac{c+a}{b}, 1 + \frac{a+b}{c}$ will also be in A.P. $1\frac{1}{2}$

$\Rightarrow \frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ will be in A.P.

1

Since, $a + b + c \neq 0$

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ will also be in A.P. $1\frac{1}{2}$

17. Following are possible choices:

Choice	Part I	Part II
(i)	2	4
(ii)	3	3
(iii)	4	2

1

$\therefore$ Total number of ways of selecting the questions are:

$= \left({}^5C_2 \times {}^5C_4 + {}^5C_3 \times {}^5C_3 + {}^5C_4 \times {}^5C_2\right)$ $1\frac{1}{2}$

$=10 \times 5 + 10 \times 10 + 5 \times 10 = 200$ $1\frac{1}{2}$

18. Let the intercepts on x-axis and y-axis be $4a$, $3a$ respectively $\frac{1}{2}$

$\therefore$Equation of line is : $\frac{x}{4a} + \frac{y}{3a} = 1$ $1\frac{1}{2}$

or $3x + 4y = 12a$

$(-3, -2)$ lies on it $\Rightarrow 12a = -17$ $1\frac{1}{2}$

Hence, the equation of the line is

$3x + 4y + 17 = 0$ $\frac{1}{2}$

19. Let the coordinates of R be (x, y, z)

$\therefore x = \frac{1(4) - 2(0)}{1-2} = -4$ 1

$y = \frac{1(-1) - 2(0)}{1-2} = 1$ 1

$z = \frac{1(-2) - 2(0)}{1-2} = 2$ $\therefore$ R is $(-4, 1, 2)$ 1

Mid point of QR is $\left(\frac{-4+4}{2}, \frac{1-1}{2}, \frac{2-2}{2}\right)$ i.e., $(0, 0, 0)$ 1

Hence verified.

20. $f(x) = \frac{3-x}{3+4x}$ $\therefore f(x + \Delta x) = \frac{3-(x+\Delta x)}{3+4(x+\Delta x)}$ $\frac{1}{2}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{\lim\limits_{\Delta x \to 0} \frac{3 - x - \Delta x}{3 + 4x + 4\Delta x} - \frac{3-x}{3+4x}}{\Delta x}$$ 1

$$= \lim_{\Delta x \to 0} \frac{(3 - x - \Delta x)(3 + 4x) - (3 + 4x + 4\Delta x)(3 - x)}{(\Delta x)(3 + 4x + 4\Delta x)(3 + 4x)}$$ $\frac{1}{2}$

$$= \lim_{\Delta x \to 0} = \frac{9 + 12x - 3x - 4x^2 - 3\Delta x - 4x\,\Delta x - 9 + 3x - 12x + 4x^2 - 12\Delta x + 4x\Delta x}{(\Delta x)(3 + 4x + 4\Delta x)(3 + 4x)}$$ 1

$$= \lim_{\Delta x \to 0} = \frac{-15\Delta x}{(\Delta x)(3 + 4x + 4\Delta x)(3 + 4x)} = \frac{-15}{(3 + 4x)^2}$$ 1

21. Assume that p is false, i.e., $\sim p$ is true

i.e., $\sqrt{3}$ is rational $\frac{1}{2}$

$\therefore$ There exist two positive integers a and b such that

$\sqrt{3} = \frac{a}{b}$, a and b are coprime $\frac{1}{2}$

$\Rightarrow a^2 = 3b^2 \Rightarrow 3$ divides $a^2 \Rightarrow 3$ divides a 1

$\therefore\ a = 3c$, c is a positive integer,

$\therefore\ 9c^2 = 3b^2 \Rightarrow b^2 = 3c^2 \Rightarrow 3$ divides b also 1

$\therefore$ 3 is a common factor of a and b which is a contradiction

as a, b are coprimes. 1

Hence p : $\sqrt{3}$ is irrational is true.

22.

x_i:	10	30	50	70	90		
f_i:	4	24	28	16	8	$\therefore \sum f_i = 80$	$\frac{1}{2}$
$f_i x_i$:	40	720	1400	1120	720	$\therefore \sum f_i x_i = 4000$	1
$\lvert d_i \rvert = \lvert x_i - \bar{x} \rvert$:	40	20	0	20	40	$\therefore$ Mean = 50	$\frac{1}{2}$
$f_i \lvert d_i \rvert$:	160	480	0	320	320	$\therefore \sum f_i \lvert d_i \rvert = 1280$	1

$\therefore$ Mean deviation $= \frac{1280}{80} = 16$ 1

PART C

23. $(f+g)(4) = f(4) + g(4) = (4)^2 + \sqrt{4} = 16 + 2 = 18$ $1\frac{1}{2}$

$(f-g)(9) = f(9) - g(9) = (9)^2 - \sqrt{9} = 81 - 3 = 78$ $1\frac{1}{2}$

$(f\,.\,g)(4) = f(4)\,.\,g(4) = (4)^2 \cdot \sqrt{(4)} = (16)(2) = 32$ $1\frac{1}{2}$

$\left(\frac{f}{g}\right)(9) = \frac{f(9)}{g(9)} = \frac{(9)^2}{\sqrt{9}} = \frac{81}{3} = 27$ $1\frac{1}{2}$

24. $\sin 7x + \sin 5x = 2\sin 6x \cos x$ 1

$\sin 9x + \sin 3x = 2\sin 6x \cos 3x$ 1

$\cos 7x + \cos 5x = 2\cos 6x \cos x$ 1

$\cos 9x + \cos 3x = 2\cos 6x \cos 3x$ 1

$$\therefore\ \text{L.H.S} = \frac{2\sin 6x\cos x + 2\sin 6x\cos 3x}{2\cos 6x\cos x + 2\cos 6x\cos 3x}$$ $\frac{1}{2}$

$$= \frac{\sin 6x\,(\cos 3x + \cos x)}{\cos 6x\,(\cos 3x + \cos x)} = \frac{\sin 6x}{\cos 6x}$$ 1

$$= \tan 6x$$ $\frac{1}{2}$

25. Using $T_{r+1} = {}^nC_r\, x^{n-r} \cdot y^r$ we have 1

$$T_4 = 10C_3 \left(\frac{x^3}{3}\right)^7 \cdot \left(\frac{-3}{x^2}\right)^3$$ 1

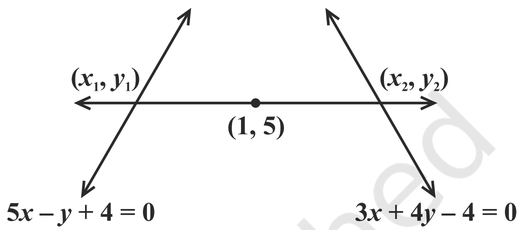
$$= -\frac{10.9.8}{3.2.1} \cdot \frac{1}{3^4} \cdot x^{15} = -\frac{40}{27}x^{15}$$ 1

5^{th} term from end $= (11 - 5 + 1) = 7^{th}$ term from beginning 1

$$\therefore \quad T_7 = 10C_6 \left(\frac{x^3}{3}\right)^4 \cdot \left(\frac{3}{x^2}\right)^6$$ 1

$$= \frac{10.9.8.7}{4.3.2.1} \cdot \frac{3^2}{1} = 1890$$ 1

26. Let the required line intersects the line $5x - y + 4 = 0$ at (x_1, y_1) and the line $3x + 4y - 4 = 0$ at (x_2, y_2).



$\therefore \quad 5x_1 - y_1 + 4 = 0 \Rightarrow$
$y_1 = 5x_1 + 4$

$$3x_2 + 4y_2 - 4 = 0 \Rightarrow y_2 = \frac{4 - 3x_2}{4}$$

$$\therefore \quad \text{Points of inter section are } (x_1, 5x_1 + 4), \left(x_2, \frac{4 - 3x_2}{4}\right)$$ $\frac{1}{2}$

$$\therefore \quad \frac{x_1 + x_2}{2} = 1 \text{ and } \frac{\frac{4 - 3x_2}{4} + 5x_1 + 4}{2} = 5$$ 1

$$\Rightarrow x_1 + x_2 = 2 \text{ and} 20x_1 - 3x_2 = 20$$ $\frac{1}{2}$

$$\text{Solving to get } x_1 = \frac{26}{23}, \quad x_2 = \frac{20}{23}$$ 1

$$\therefore \qquad y_1 = \frac{222}{23}, \quad y_2 = \frac{8}{23}$$ $\frac{1}{2}$

$$\therefore \quad \text{Equation of line is } y - 5 = \frac{\frac{222}{23} - 5}{\frac{26}{23} - 1}(x - 1)$$ 1

or $\qquad 107x - 3y - 92 = 0$ $\frac{1}{2}$

27. Here $a^2 = 169$ and $b^2 = 144 \Rightarrow a = 13, b = 12$ 1

$\therefore$ Length of major axis = 26

Length of minor axis = 24

Since $e^2 = 1 - \dfrac{b^2}{a^2} = 1 - \dfrac{144}{169} = \dfrac{25}{169} \therefore e = \dfrac{5}{13}$ 1

foci are $(\pm ae, 0) = \left(\pm 13 \cdot \dfrac{5}{13}, 0\right) = (\pm 5, 0)$ 1

vertices are $(\pm a, 0) = (\pm 13, 0)$ 1

latus rectum $= \dfrac{2b^2}{a} = \dfrac{2(144)}{13} = \dfrac{288}{13}$ 1

28.

Classes:	30-40	40-50	50-60	60-70	70-80	80-90	90-100		
f:	3	7	12	15	8	3	2	$\therefore \sum f = 50$	$\frac{1}{2}$
x_i:	35	45	55	65	75	85	95		
$d_i: = \dfrac{x_i - 65}{10}$	–3	–2	–1	0	1	2	3		
$f_i d_i$:	–9	–14	–12	0	8	6	6	$\sum f_i d_i = -15$	1
$f_i d_i^2$:	+27	28	12	0	8	12	18,	$\sum f_i d_i^2 = 105$	1

Mean $\bar{x} = 65 - \dfrac{15}{50} \times 10 = 65 - 3 = 62$ 1

Variance $\sigma^2 = \left[\dfrac{105}{50} - \left(\dfrac{-15}{50}\right)^2\right] \cdot 10^2 = 201$ $1\frac{1}{2}$

S.D. $\sigma = \sqrt{201} = 14.17$ 1

29. (i) Total number of days in a non leap year = 365

= 52 weeks + 1 day 1

$\therefore$ P(53 sun days) = $\frac{1}{7}$ 1

(ii) Total number of days in a leap year = 366

= 52 weeks + 2 days 1

$\therefore$ These two days can be Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday, Thursday and Friday, Friday and Saturday, Saturday and Sunday, Sunday and Monday

$\therefore$ P(53 Fridays) = $\frac{2}{7}$ $\frac{1}{2}$

(iii) P(53 Sunday and 53 Mondays) = $\frac{1}{7}$ (from ii) $1\frac{1}{2}$

Notes