

# Laws of Motion

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## Effects of Force

Force cannot be seen, heard, or tasted. Only its effects can be felt or seen. It is correctly defined as a push or pull upon an object resulting from the object's interaction with another object. The various effects of force are:

It can move a body initially at rest.

It can bring a moving body to rest.

It can change the direction of a moving body.

It can change the speed of a moving body.

It can change the shape of a body.

It can change the size of a body.

Let us take an example of a football lying in a field. When a player hits the ball, it starts moving, i.e., it starts moving only when we apply force. Thus, force can move a body initially at rest.



Now, if the goalkeeper catches the moving ball, then it comes to rest. The goalkeeper applies a force to stop the moving ball. Hence, we can say that force can bring a moving body to rest.



If another player kicks the moving ball in the opposite direction, then it starts moving in the direction towards which it is kicked i.e. the direction of the football changes. The

player applies force on the football to change its direction. Hence, force can change the direction of a moving body. Also, if the player hits the ball hard, then the net speed of the ball will also change. Hence, the speed of a moving body can be changed by applying force.



The shape of a deflated football can be changed by inflating it. When you inflate a football, you apply force on the pump. Hence, force can change the shape of an object. Also, if you keep inflating the football, then its size will keep on increasing. Hence, force can change the size of an object.



A deflated football



Force can change the shape of an object



Force can change the size of an object

## Contact Forces

Anuj is cycling on the road. He observes that as he stops pedalling, the cycle stops moving after travelling for some distance. **Let us see why this happens?**



Forces acting between two bodies can be classified into two broad categories: **Contact force** and **non-contact force**. Let us learn about contact forces in detail.

**Contact forces are those that act between two objects, which are in direct contact with each other.** The two common examples of contact forces are **muscular** and **frictional**.

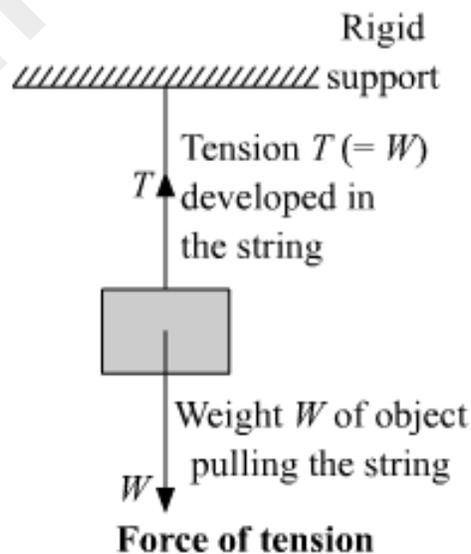
### Frictional Force

Earlier you had seen that as Anuj stops pedalling the bicycle, it stops after some time. This happens due to the external force acting between the road and tyres of the bicycle. This force is known as the **frictional force**. The force of friction acts between all moving bodies, which are in contact with one another. The force of friction always acts opposite to the direction of motion. The magnitude of this force depends on the nature of the surface in contact.

Frictional force is a contact force.
Frictional force always acts between two moving objects, which are in contact with one another.
Frictional force always acts opposite to the direction of motion.
Frictional force depends on the nature of the surface in contact.

### Tension Force

This force appears in a string, attached to a rigid support, when an object is suspended by it. In such case, the object pulls the string vertically downwards due to its weight and the string in its stretched condition pulls the object upwards by a force which balances the weight of the object. This force developed in the string is called tension  $T$ .

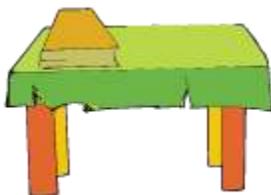


## Muscular Force

The force applied by the action of muscles in our body is termed as a **muscular force**.

For example, when you pick up a book placed on the table using your hands, you apply muscular force.

For lifting the book from the table using your hands, you had to touch the book. You cannot lift the book without making contact with it. Hence, **muscular force is a contact force**.



Like humans, animals also use muscular force to perform various activities. For example, birds fly in the air by flapping their wings.



## Mechanical Force

One more common example of contact forces is a **mechanical force** which is defined as the force generated by a machine. All the mechanical works are done by the mechanical force.

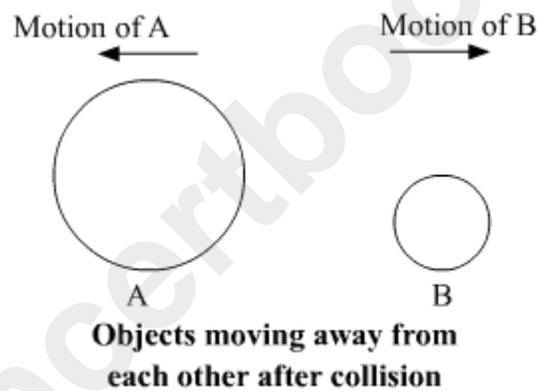
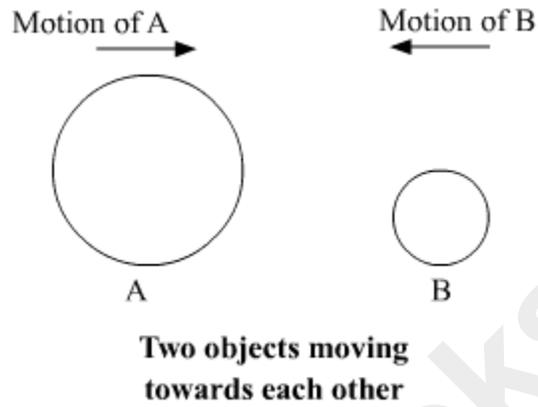


For example, when the car gets started the car's engine creates a mechanical force on the tyres that helps the car to accelerate. So here, the movement of car occurs due to the force generated by the machine on the tyres.

## Force Exerted During Collision

Two objects push each other with an equal but opposite forces if collision occurs between them. These forces are known as the force of action and force of reaction. Let

us consider a situation in which an object B in motion collides with a moving object A and applies a force  $F_{AB}$  on the object A i.e. an action force. At the same instant, the object A also applies an equal but opposite force of reaction  $F_{BA}$  on the object B. Because of this, they move apart from each other after collision.



Do you know the force(s) involved in the movement of trolley bags?



When we pull the trolley we are applying muscular force on it and at the same time, there is a frictional force acting between the tyres of the trolley and floor. These two forces are together responsible for the movement of the trolley.

So we can say that, **Combined forces** are nothing but varieties of forces acting on an object at the same time.

## Non-Contact Forces



### Does force act only when two objects are in contact?

To understand, let us perform a small activity. Take a bar magnet and an iron nail. Bring the magnet close to the iron nail, but do not bring them in contact.

### What do you observe?

The iron nail moves towards the magnet. This means that there must be a force that is acting between the magnet and the iron nail.

Since the iron piece moves towards the magnet (even when they are not in contact), we can say that the force exerted by the magnet on the iron piece is a **non-contact force**.

**Non-contact forces are those forces that act between two objects, but are not in direct contact with each another.** Examples of non-contact forces include magnetic force, electrostatic force, and gravitational force.

### Magnetic force



**What will happen if you bring the South Pole of a bar magnet close to the North Pole of another bar magnet?** The magnets will attract each other. They attract each other with **magnetic force**.

**What will happen if you bring the North Pole of both bar magnets close to each other?** The bar magnets will repel each other. The force with which they repel each other is known as **magnetic force**.



**Magnetic force can be attractive as well as repulsive.**

**Magnetic force is a non-contact force.**

**Magnetic force acts between two magnets, or between a magnet and a magnetic material (such as iron).**

**Magnetic force depends on the strength of the magnet used.**

**Magnetic force also depends on the distance between the interacting bodies.**

### **Electrostatic force**



Take a paper and tear it into pieces. Now, rub a plastic scale against dry hair and bring this scale close to the paper pieces. **What do you observe?**

You will observe that the pieces of paper are attracted towards the scale. This happens because rubbing of the scale against dry hair produces an electrostatic charge. Thus, the scale attracts the pieces of paper by a non-contact force known

as **electrostatic force**.

**Electrostatic force is a non-contact force.**

**Electrostatic force can be attractive as well as repulsive.**

**Electrostatic force is the force that exists either between two charged bodies, or between a charged and uncharged body.**

**Electrostatic force depends on the magnitude of charge present in the bodies.**

**Electrostatic force also depends on the distance between the interacting bodies.**



## Gravitational force

**Do you know why apples fall towards the ground from trees?  
Why does water from a tap flow down?**

The Earth attracts everything (that is near or on its surface) towards its centre by a non-contact force known as **gravitational force**. It is this force that makes an apple fall towards the ground from the tree and makes the water from a tap flow down.

**Gravitational force is a non-contact force.**

**Gravitational force is an attractive force.**

**Gravitational force is the force that is exerted by the earth on every object, which is near or on its surface.**

**Gravitational force depends on the mass of the body.**

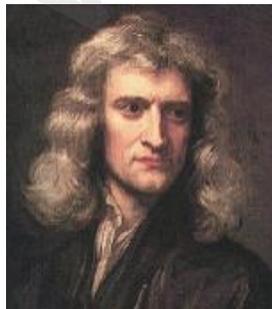
**Gravitational force also depends on the distance between the Earth and body.**

## Newton's First Law of Motion

What you have seen is not possible in real life. In reality, it is very difficult to achieve the condition of zero net **force** on the ball. This is because of the presence of the force of friction which acts opposite to the direction of motion of the ball. Thus, in reality, the ball will stop after travelling some distance.

This experiment was first conducted by Galileo Galilei, but the results of his experiment were not widely accepted by the people at that time.

### Know Your Scientist



Sir Isaac Newton (1642–1727), the English mathematician, astronomer and physicist,

was born at Woolsthorpe. He joined Cambridge University in 1661. He became a fellow of Trinity College in 1667 and Lucasian Professor of Mathematics in 1669.

He was at the University till 1696. His famous treatise *Philosophiae Naturalis Principia Mathematica* (Mathematical Principles of Natural Philosophy) was prepared during the years 1665–1666. *The Principia*, as it is commonly known, was not published until 1687.

For nearly 300 years, Newton has been considered as the exemplar of modern physical science. His accomplishments in mathematical research are as innovative as those in experimental investigations.

He is also known for his works on chemistry, the early history of Western civilisation and theology. Notable among his studies is the investigation of the form and dimensions of the biblical Solomon's Temple.

Isaac Newton used the results of Galileo's experiment to propound the first law of **motion**. It is stated as follows:

***A body at rest will remain at rest and a body in uniform motion will continue its uniform motion unless a unbalanced external force acts on it to change its state of rest or uniform motion.***

Let us understand Newton's first law of motion with the help of the following examples.



### **Stationary table**

A stationary table remains stationary as long as no one pushes or pulls it. The net force on the stationary table is zero. **An external force is required to change its state of rest.** When someone pushes or pulls the table, it moves along the direction of the applied force.

### Pen stand on a table



Suppose a pen is lying on a table. The pen stand cannot move by itself, i.e., it cannot change its state of rest by itself. **Its state of rest can be changed only when an external force is applied on it.** For example, if you lift the pen stand from the table, then its state of rest will change. This change is the result of the application of an external force on the pen stand by your hand.

### Newton's First Law of Motion

#### Space walk

An astronaut on a space station goes into outer space for a space walk without any harness. She leaps out from the space station and moves away from it. She has to apply a force toward the space station in order to stop from going too far from it.



#### What makes the astronaut move away from the space station?

The answer to this question is very simple. There are no forces acting on the astronaut in space. Hence, she keeps moving in a straight line away from the space station. She has to apply a force toward the space station to change her state of uniform motion.

In the context of the preceding examples, you will notice that every body resists a change in its state of motion or rest. If a body is at rest, then it tends to remain at rest; if a body is in a state of motion, then it continues to be in that state of motion. This property of a body is known as **inertia**.

### Inertia

**Mass is a measure of the amount of inertia**

Every object resists changes in its states of motion and rest. This implies that every object has inertia. However, all objects do not have the same tendency to resist changes. This tendency depends upon the mass of an object. Mass is a quantity that is dependent upon the inertia of an object. An object having greater inertia has a greater mass. Hence, a massive object has a greater tendency to resist changes in its states of motion and rest.

### **Examples of inertia**

- When a horse starts running suddenly, the rider falls backward due to the inertia of rest of the upper part of his body.
- The dust particles on a carpet fall off when beaten with a stick. The beating sets the carpet in motion, but the dust particles tend to remain at rest.

### **Types of inertia**

- Inertia of rest
- An object at rest, will remain at rest unless an external force is applied to change its state of rest. For example: on giving a jerk to the branches of a tree, the fruits fall down. This is because on shaking the branches of a tree the fruits attached to it comes in motion. But due to the inertia of rest of the fruits, they tend to remain in this state. Hence, they fall on the ground and attain the state of rest.
- Inertia of motion
- An object in a state of motion will continue to be in the state of motion with the same speed until an external force is applied on it to change its state of motion. For example: When a moving car stops abruptly, the passenger sitting inside it tends to lean forward. This is because when the car is in motion, the whole body of the passenger sitting inside the car is also in motion. On sudden application of brakes, the car and the lower half of the passenger's body (in contact with the car) comes to rest while the upper half remains in motion due to inertia. Thus, the passenger leans forward.
- Inertia of direction
- An object moving in a particular direction will continue to move in that direction until an external force is applied. For example: When a car running on a straight road suddenly takes a right turn, the person inside the car tends to lean leftwards. This is because when the car was moving in a straight road, the whole body of the passenger gained the inertia of moving in straight line. As soon as the car took a right turn, the car and lower half of the person's body changed their direction towards right but the upper half of the person's body still continued to move in a straight line due to inertia of direction. Thus, the person leans leftwards. In the similar way, when the car takes left turn, the person inside the car leans rightwards.

## Did You Know?

Inertia always resists a change in the state of motion or rest of a body. Thus, Newton's first law of motion is also known as the law of inertia.

### Do all bodies possess inertia?

Yes, all bodies, whether moving or at rest, possess inertia.

### Do they possess inertia in the same amount?

We know that pushing a wooden block is easier than pushing an iron block of the same size. We can easily move a football, but it takes a lot of effort to move a large rock. Hence, it can be said that heavier or massive objects possess greater inertia. Quantitatively, the inertia of an object is measured by its mass.

## Momentum

It is a common observation that more force is required to stop a heavier body than what is required for stopping a lighter body. Suppose a cricket ball and table-tennis ball are thrown towards you one after the other. To catch which of the two balls will you need to apply the greater force?

The cricket ball, of course! And the reason for this is that it has the greater mass of the two balls. So, we can conclude that the force required to stop the motion of a body is directly proportional to its mass. The same logic is at work when you have to throw the two balls. Since the mass of the cricket ball is greater than that of the table-tennis ball, the force required to throw the former will be greater than that required to throw the latter.

Now suppose you have two cricket balls of the same mass. You throw both the balls, but one with a lesser force than the other. What do you expect will happen? The ball thrown with the greater force will move with a greater velocity as compared to that thrown with the lesser force.

Hence, we can conclude that the effect of force on a body can be described with the help of its mass and velocity. Isaac Newton used the term 'momentum' to describe this effect. He defined momentum as the product of the mass and velocity of a body, i.e.,

### **Momentum = Mass × Velocity**

$$\text{Or, } p = m \times v$$

Where,  $p$  = Momentum

$m = \text{Mass}$

$v = \text{Velocity}$

This momentum is also known as linear momentum. You will learn another type of momentum called angular momentum in higher class.

### **Did You Know?**

Force can change the velocity of an object. Thus, force can change the momentum of an object.

### **Solved Examples**

#### **Easy**

#### **Example 1:**

**Find the momentum of a cricket ball weighing 150 g and moving at a velocity of 50 m/s.**

#### **Solution:**

It is given that:

Mass of the ball = 150 g = 0.15 kg

Its velocity = 50 m/s

We know that:

Momentum = Mass  $\times$  Velocity

$\therefore$  Momentum of the ball = 0.15 kg  $\times$  50 m/s

= 7.5 kg-m/s

#### **Medium**

#### **Example 2:**

**A bike weighing 200 kg accelerates from rest at the rate of 5 m/s<sup>2</sup>. Find its momentum after 10 s.**

#### **Solution:**

It is given that:

Initial speed ( $u$ ) of the car = 0

Its acceleration,  $a = 5 \text{ m/s}^2$

Time,  $t = 10 \text{ s}$

Let the speed of the car after 10 s be  $v$ .

Using the first equation of motion, we can compute the value of  $v$ .

$$v = u + at$$

$$\Rightarrow v = 0 + (5 \times 10)$$

$$\Rightarrow \therefore v = 50 \text{ m/s}$$

Now, the mass of the car is given as 200 kg.

So, the momentum of the car after 10 s = Mass  $\times$  Velocity

$$= 200 \times 50 = 10000 \text{ kg-m/s}$$

**Hard**

**Example 3:**

**The kinetic energy of a block of mass 3 kg is 150 J. Find its momentum.**

**Solution:**

It is given that:

Kinetic energy ( $k$ ) of the block = 150 J

Its mass,  $m = 3 \text{ kg}$

Let the velocity and momentum of the block be  $v$  and  $p$  respectively.

We know that:

$$k = 0.5 mv^2 \dots (1)$$

$$p = mv \dots (2)$$

Using equation (1), we get:

$$k = \frac{1}{2}mv^2$$

$$\Rightarrow k = \frac{(mv)^2}{2m}$$

$$\Rightarrow k = \frac{p^2}{2m} \quad [\text{From equation (2): } p = mv]$$

$$\Rightarrow p = \sqrt{2mk}$$

$$\Rightarrow p = \sqrt{2 \times 3 \times 150}$$

$$\Rightarrow \therefore p = 30 \text{ kg-m/s}$$

## Newton's Second Law of Motion



Suppose a heavy wooden block is lying on a table. If we give it a gentle push, then it will move with a low velocity. In other words, if we apply a small **force** on the block, then its **momentum** will change slightly. Likewise, if we push the wooden block with a greater force, then the change in its momentum will be greater than before.

We can thus conclude that the change in the momentum of a body is directly proportional to the strength of the applied force. This brings us to Newton's second law of motion.

It is stated as follows:

***The rate of change of momentum of an object is directly proportional to the unbalanced external force acting on it. The direction of the unbalanced force is the same as the direction of the change of momentum.***

## Momentum and Newton's Second Law

Consider a body of mass  $m$ . It initially moves with velocity  $u$  and accelerates at a constant rate  $a$ . It attains a final velocity  $v$  after time  $t$ . This acceleration is induced by force  $F$ .

Now, Newton's second law of motion can be mathematically represented as follows:

The rate of change of momentum  $\propto \frac{mv - mu}{t}$

which is equal to unbalanced force,  $F$

$$F \propto \frac{mv - mu}{t}$$

$$F = k \frac{m(v - u)}{t}$$

Using the first equation of motion, we know that:

$$v = u + at$$

$$\Rightarrow \frac{v - u}{t} = a$$

Using this, we obtain:

$$F = ma = \text{Mass} \times \text{Acceleration}$$

Unit of force is taken Newton so the value of constant of proportionality ( $k$ ) becomes one.

$$1 \text{ Newton} = 1 \text{ kg } 1 \text{ ms}^{-2}$$

Thus, we can restate Newton's second law of motion as follows:

***Force acting on a body is equal to the product of its mass and acceleration.***

### **Impulse**

Impulse of a force is a measure of the total effect of the force

$$\text{Impulse} = \text{Force} \times \text{Time}$$

Forces which act on bodies for a short time are called impulsive forces.

Example: firing a gun, hitting a ball with a bat

It is a vector quantity.

### **Solved Examples**

## Easy

### Example 1:

A moving block of mass 2 kg changes its speed from 5 m/s to 15 m/s in 2 s. Find the net force acting on the block.

#### Solution:

It is given that:

Initial speed ( $u$ ) of the block = 5 m/s

Its final speed,  $v = 15$  m/s

Time taken,  $t = 2$  s

Let the acceleration due to gravity be  $a$ .

Using the first equation of motion, we know that:

$$v = u + at$$

$$\Rightarrow a = \frac{v - u}{t}$$

$$\Rightarrow \therefore a = \frac{15 - 5}{2} = 5 \text{ m/s}^2$$

It is given that the mass of the block is 2 kg.

From Newton's second law of motion, we know that:

$$F = ma = 2 \times 5 = 10 \text{ N}$$

Therefore, the net force acting on the block is 10 N.

## Medium

### Example 2:

A particle of mass 2 kg is subjected to a force  $F = kx$  with  $k = 20$  N/m and  $x$  being its distance from the origin. What is its initial acceleration if it is released from a point 30 cm away from the origin?

#### Solution:

It is given that:

Force ( $F$ ) applied on the particle =  $kx$

Where,  $k = 20 \text{ N/m}$

$x = 30 \text{ cm} = 0.3 \text{ m}$

$\therefore F = 20 \times 0.3 = 6 \text{ N}$

From Newton's second law of motion, we know that:

$$F = ma$$

$$\Rightarrow a = \frac{F}{m}$$

Here,  $F = 6 \text{ N}$  and  $m = 2 \text{ kg}$ .

$$\Rightarrow \therefore a = \frac{6}{2} = 3 \text{ m/s}^2$$

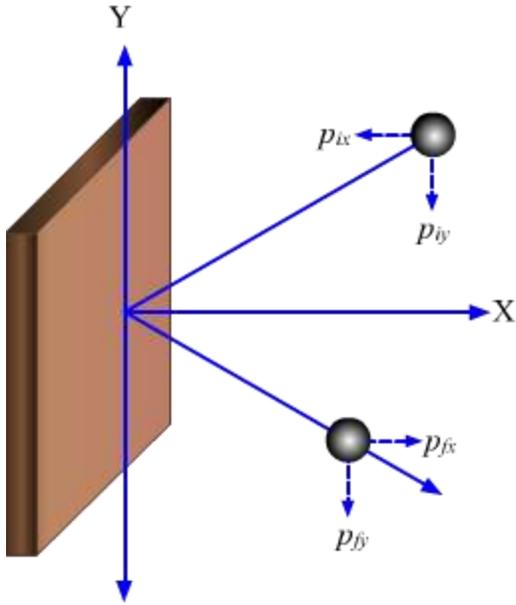
Therefore, the initial acceleration of the particle is  $3 \text{ m/s}^2$ .

**Hard**

**Example 3:**

**A ball of mass  $150 \text{ g}$  strikes a wall at a speed of  $10 \text{ m/s}$  and at an angle of  $30^\circ$ . The ball rebounds with the same speed. If the contact time is  $10^{-3} \text{ s}$ , then what is the force applied by the wall?**

**Solution:**



Mass ( $m$ ) of the ball = 150 g = 0.15 kg

Its initial velocity,  $u = 10$  m/s

Its initial momentum,  $p_i = mu = 0.15 \times 10 = 1.5$  kg-m/s

Initial momentum of the ball along the  $x$ -axis,  $p_{ix} = -1.5 \cos 30^\circ$

Initial momentum of the ball along the  $y$ -axis,  $p_{iy} = -1.5 \sin 30^\circ$

Final velocity ( $v$ ) of the ball = 10 m/s

Its final momentum,  $p_f = mv = 0.15 \times 10 = 1.5$  kg-m/s

Final momentum of the ball along the  $x$ -axis,  $p_{fx} = 1.5 \cos 30^\circ$

Final momentum of the ball along the  $y$ -axis,  $p_{fy} = -1.5 \sin 30^\circ$

Change in the momentum of the ball along the  $x$ -axis =  $p_{fx} - p_{ix} = 3 \cos 30^\circ = 3 \times 0.866 = 2.598$  kg-m/s

Change in the momentum of the ball along the  $y$ -axis =  $p_{fy} - p_{iy} = 0$

So, force acting along the  $x$ -axis =  $\frac{2.598}{10^{-3}} = 2598$  N

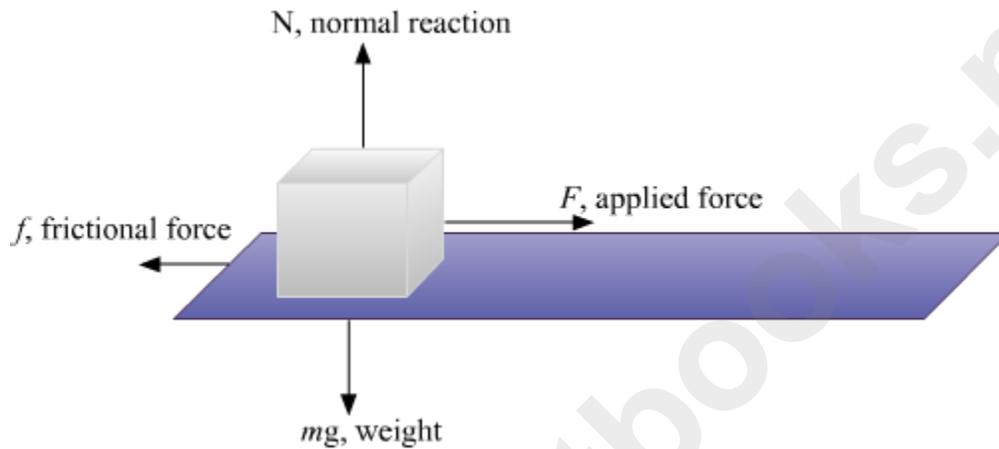
There is no change in the momentum along the  $y$ -axis; so, no force acts on the ball along it.

Thus, the force applied by the wall on the ball is 2598 N along the x-axis.

### Did You Know?

During the free fall of a ball, the earth pulls the ball toward itself. In turn, the ball also pulls the earth upward with an equal amount of force. However, the effect of this force on the earth is negligible.

### Frictional Force



The normal reaction  $N$  is equal to the weight  $mg$ .

$$N = mg$$

The frictional force  $f$  is given by:

$$f = \mu N = \mu mg$$

Where,  $\mu$  is the coefficient of friction

If the applied force is greater than the frictional force, then the acceleration  $a$  of the block is found as:

$$ma = F - f$$

$$\Rightarrow ma = F - \mu mg$$

If the applied force just balances the frictional force, then there is no acceleration of the block and the block does not move.

$$\text{So, } F = f$$

Remember,  $a \neq \frac{f}{m}$  when  $f$  is the frictional force.

## Real-World Examples of Newton's Second Law of Motion

### High Jump

During an athletic event, the participants in the high jump event are provided with cushions to fall on after completing a jump. This is done to prevent any kind of injury to the athletes.



When an athlete falls on the cushion, it takes her a longer period of time to come to a stop. A small stopping force acts on her because her rate of change of velocity is low. As a result, she does not get hurt.

If the athlete were to fall on a hard surface, then her velocity would reduce to zero in a very short time. In this case, a large stopping force would act on her because her rate of change of velocity would be high. As a result, the athlete would get hurt.

### Seat belts



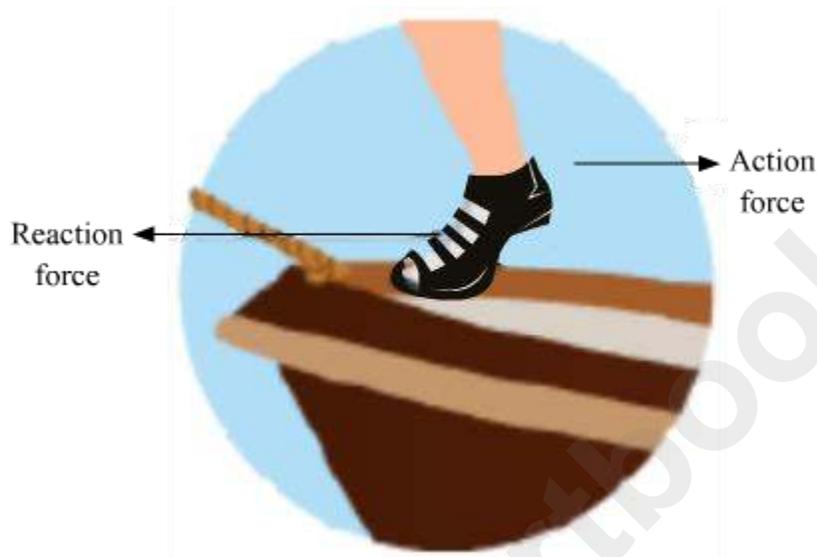
A very useful application of Newton's second law lies in the use of seat belts in cars. To prevent injuries to passengers in case of an accident, all cars are provided with seat belts. In the event of an accident, a fast-moving car stops suddenly, i.e., its high velocity is reduced to zero in a very short interval of time.

**The time taken by the passengers to fall gets increased because of the seat belts worn by them.** The rate of change of velocity of the passengers gets reduced because

of the increase in the time taken by them to fall. Hence, a lesser stopping force acts on them, as a result of which, injuries are reduced.

## Newton's Third Law of Motion

Newton's third law of motion states that for every action force there is always an equal and opposite reaction force, with the forces acting on different bodies.



Now, we are going to explain the example given in the overview.

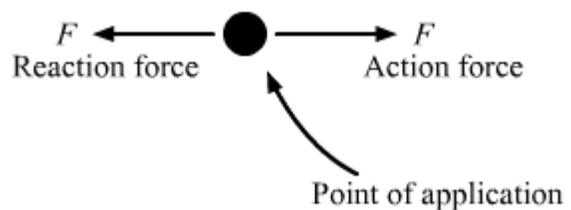
In order to jump from the boat, Payal applies a force on the boat with her leg. The direction of this force is opposite to that of her motion. As a result, the boat moves backward.

In this situation, the applied force is the **action force** and Payal's forward motion is the effect of the **reaction force** provided by the boat. Hence, the boat moves backward because of the action force exerted by Payal.

This situation can be summarized as follows:

**Action force** → Force exerted by Payal's leg on the boat

**Reaction force** → Force exerted by the boat on Payal's leg



It is clear from the above example that action and reaction forces are of equal magnitude and act in opposite directions. If Payal applies a force of magnitude  $F$  newton on the boat, then the boat in turn reacts with the same magnitude of force on her foot.

### Newton's Third Law of Motion

It is observed that both balances give the same reading. This implies that the force exerted by balance II on balance I is the same as the force exerted by balance I on balance II. Thus,

**Action force** → Force exerted by balance II on balance I,

**Reaction force** → Force exerted by balance I on balance II

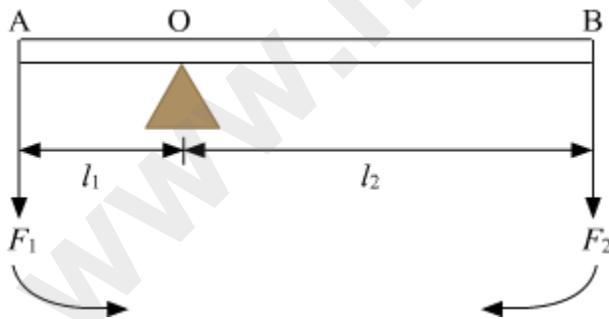
### Newton's Third Law of Motion

#### Rotational equilibrium

*Moment* is defined as the product of the force and the perpendicular length on a body or system.

A body is in rotational equilibrium when the algebraic sum of moments of all the forces acting on it about a fixed point is zero.

For example: In case of a beam balance or see-saw, the system will be in rotational equilibrium if



$$F_1 \times l_1 - F_2 \times l_2 = 0$$

Now,  $F_1 \times l_1 = \tau_1$  (anticlockwise moment)

And,  $F_2 \times l_2 = \tau_2$  (clockwise moment)

i.e., for rotational equilibrium, the total external force acting on the body must be zero

## Real-Life Applications of Newton's Third Law of Motion

### Flying of a bird



A bird can fly with the help of its wings. In this process, it pushes the air downward by flapping its wings. In turn, the air also exerts an equal force on the bottom of its wings. As a result, the bird gets a lift and can fly in the air.

The action–reaction forces in this case are described below.

**Action force:** Exerted by the wings on the air in the downward direction

**Reaction force:** Exerted by the air on the bottom of the wings in the upward direction

### Horse pulling a cart



The horse can pull and move a cart by exerting a force on the ground. In turn, the horse experiences a reaction force of equal magnitude in the opposite direction that causes the cart to move in that direction. In this case, the **action force** is the force applied by the horse on the ground and the **reaction force** is the force experienced by the horse from the **ground**.

## Firing of a bullet

The gun exerts a forward force on the bullet, the bullet in turn also exerts an equal and opposite reaction force on the gun.

**Action force** → Force exerted by the gun on the bullet

**Reaction force** → Force exerted by the bullet on the gun

## Rocket

Rockets work on the principle of Newton's third law of motion. In rockets, large amounts of hot gases are allowed to exit through a narrow opening. In turn, the fast-moving gases exert a force on the rocket which pushes the rocket upward.



In this case:

**Action force** → Exerted by the rocket on the exhaust gases

**Reaction force** → Exerted by the gases on the rocket

## Solved Examples

### Medium

**Example:**

**A 600 kg rocket is fired straight up from the earth, with the engines providing 9000 N of thrust. If  $g = 10 \text{ m/s}^2$ , then the acceleration of the rocket is**

1.  $5 \text{ m/s}^2$
2.  $10 \text{ m/s}^2$

3.  $15 \text{ m/s}^2$
4.  $50 \text{ m/s}^2$

**Solution:**

It is given that:

Upthrust,  $F = 9000 \text{ N}$

Mass ( $m$ ) of the rocket =  $600 \text{ kg}$

$g = 10 \text{ m/s}^2$

Let the acceleration of the rocket be  $a$ .

Net force in the upward direction is given as:

Upthrust – Weight of the rocket = Mass  $\times$  Acceleration

$$\Rightarrow F - mg = ma$$

$$\Rightarrow 9000 - (600 \times 10) = 600a$$

$$\Rightarrow 3000 = 600a$$

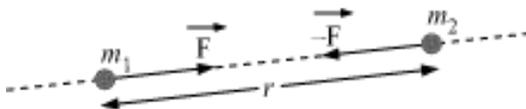
$$\therefore a = 5 \text{ m/s}^2$$

**Know More**

It is a common misconception that rockets are unable to accelerate in space. The fact is that rockets do accelerate in space. They are able to do so because they burn fuel and push the exhaust gases in the direction opposite to the direction in which they need to be accelerated.

**Universal Law of Gravitation and Gravitational Constant**

- Consider two bodies of masses  $m_1$  and  $m_2$  with their centres separated by a distance  $r$ .



Let  $F$  be the force of gravitational attraction between the two bodies. According to Newton's law of gravitation,

$$F \propto m_1 m_2$$

And,  $F \propto \frac{1}{r^2}$

Combining both the factors,

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\therefore F = G \frac{m_1 m_2}{r^2} \quad \dots(i)$$

Where,  $G$  is constant of proportionality known as gravitational constant

Its S.I value is  $6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$ .

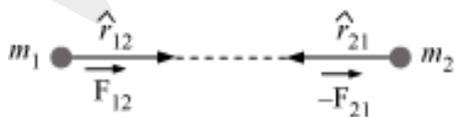
- If  $m_1 = m_2 = 1$  units and  $r = 1$  unit, then from equation (i),  $F = G$

Therefore, the universal gravitational constant ( $G$ ) is numerically equal to the force of attraction between two bodies for unit masses, separated by unit distance.

- **Some special features of gravitational force**

- It does not depend on the nature of the medium in which the masses are placed.
- It is extremely small in case of light bodies whereas it becomes appreciable in case of heavy bodies.
- It is a conservative force.
- It is always attractive.
- It is a central force.

- **Vector form of Newton's Law of Gravitation**



Consider two particles of masses  $m_1$  and  $m_2$ .

Let  $\vec{r}_{12}$  = Displacement vector from  $m_1$  to  $m_2$

$\vec{r}_{21}$  = Displacement vector from  $m_2$  to  $m_1$

$\vec{F}_{21}$  = Gravitational force exerted by  $m_1$  on  $m_2$

$\vec{F}_{12}$  = Gravitational force exerted by  $m_2$  on  $m_1$

$\hat{r}_{12}$  = Unit vector pointing towards  $m_2$

$\hat{r}_{21}$  = Unit vector pointing towards  $m_1$

In vector form, the Newton's law of gravitation is written as

$$\vec{F}_{12} = -G \frac{m_1 m_2}{r_{21}^2} \hat{r}_{21} \quad \dots(i)$$

Negative sign indicates that the direction of  $\vec{F}_{12}$  is opposite to that of  $\hat{r}_{21}$ .

$$\vec{F}_{21} = -G \frac{m_1 m_2}{r_{12}^2} \hat{r}_{12}$$

Similarly,

Where,  $\hat{r}_{21}$  is a unit vector pointing towards  $m_2$

Also,  $\hat{r}_{21} = -\hat{r}_{12}$  and  $r_{21}^2 = r_{12}^2$

$$\therefore \vec{F}_{21} = G \frac{m_1 m_2}{r_{21}^2} \hat{r}_{21} \quad \dots(2)$$

From equations (1) and (2),

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

Acceleration Due to Gravity

## Acceleration Due to Gravity - An Overview

The force of gravity bounds all objects on or near Earth. When there is force, there must be acceleration. The force of gravity on objects of different masses is different, but the

acceleration due to gravity remains the same. What can be the reason for this perplexing behaviour?

What are the parameters that determine the acceleration due to gravity of a planet? How does acceleration due to gravity vary with height? When you jump from a height, are you under a free fall?

Let us explore the answers to the what, how and when in the above questions.

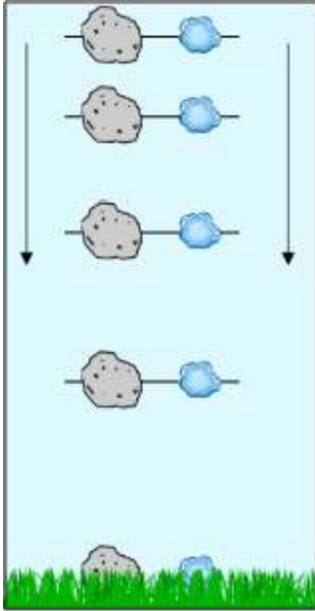
Whenever an object falls towards Earth, it experiences acceleration. This is called **acceleration due to gravity** and is denoted by the letter 'g'. It is a constant for every object falling on Earth's surface.

- **Acceleration due to gravity does not depend on the mass of the falling object. The value of 'g' changes slightly from place to place on Earth. The value of acceleration due to Earth's gravity is about  $9.8 \text{ m/s}^2$  near Earth's surface.**

**Differences between 'G' and 'g':**

<b>Universal gravitation constant (G)</b>	<b>Acceleration due to gravity (g)</b>
1. It is defined as the force of attraction acting between two bodies, each of unit mass, whose centres are placed at unit distance from each other.	1. It is defined as the constant acceleration produced in a body when it falls freely under the effect of gravity.
2. Its value is the same throughout the universe.	2. Its value changes from one place to another.
3. It is a scalar quantity.	3. It is a vector quantity.

**Free Fall and Acceleration Due to Gravity (g)**



A free-falling object is an object that falls under the sole influence of gravity. Any object that is being acted upon only by the force of gravity is said to be in a state of free fall.

Say, a cotton ball and a large rock are dropped from the same height at the same time. Assuming that air resistance can be eliminated such that neither object experiences any air drag during the course of its fall, **which object will hit the ground first?**

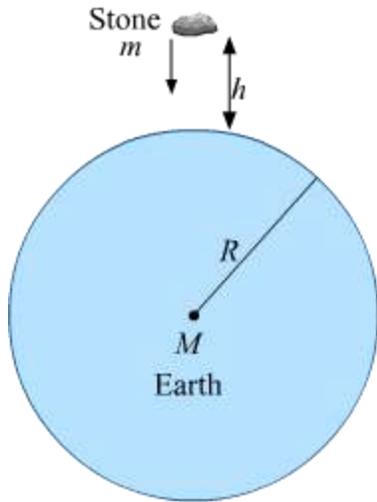
You might say that the rock will hit the ground first, but this is not true. It is generally thought that lighter objects fall slowly, while heavier objects fall rapidly when dropped from the same height. However, objects of different masses fall at the same rate.

There is a famous anecdote that Galileo dropped two rocks having different masses from the Leaning Tower of Pisa to show that different objects fall at the same rate, but the credibility of this anecdote is doubtful.

In actual conditions, if you drop a cotton ball and a rock from the same height, then the cotton ball will fall at a slower speed because of air resistance. In ideal conditions, when only gravitational force acts on the cotton ball and the rock (i.e., air resistance is not present), both the objects will fall at the same rate.

This was shown by Robert Boyle when he performed this experiment using a feather and a stone. The feather and stone were put in a tall glass jar and air was removed from the jar using a vacuum pump. When the jar was inverted, both objects fell to the bottom of the jar at the same time. This proved that in the absence of air resistance, all objects fall at the same rate.

**Equation for 'g'**



Let us consider a stone of mass  $m$ , dropped from a height  $h$ . The stone will fall towards Earth's surface having the mass  $M$  and radius  $R$ . This motion of the stone is called a **free fall** under the influence of Earth's gravity.

**Free fall is the motion of an object falling solely under the influence of Earth's gravity.**

Using Newton's second law of motion, the force on the stone can be given by the product of its mass and acceleration.

$$F = ma$$

Suppose the stone falls freely with an acceleration  $g$ .

$$F = mg \dots (i)$$

Force exerted by Earth on the stone is given by Newton's law of gravitation:

$$F = G \frac{Mm}{(R+h)^2} \dots (ii)$$

From equations (i) and (ii), we obtain:

$$mg = G \frac{Mm}{(R+h)^2}$$

$$\text{Or, } g = \frac{GM}{(R+h)^2} = \frac{GM}{R^2 \left(1 + \frac{h}{R}\right)^2}$$

The height  $h$  is very small compared to Earth's radius  $R$ . Hence, the term  $h/R$  will be very small and can be neglected. So, we get:

$$g = G \times \frac{M}{R^2}$$

Where,

$G$  = Universal gravitational constant =  $6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$

$M$  = Earth's mass

$R$  = Earth's radius

This equation expresses the value of the acceleration due to gravity of an object placed on Earth's surface. This value decreases as we move above from Earth's surface or go below it. Earth's radius  $R$  increases when we go from the poles to the equator. Consequently, the value of  $g$  decreases.

### Solved Examples

#### Easy

#### Example 1:

Calculate the value of the acceleration due to gravity on the surface of a planet X having a mass of  $5 \times 10^{20} \text{ kg}$  and the radius as 1800 km. ( $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$ )

**Solution:**

$$g = G \times \frac{M}{R^2}$$

It is given that:

$$G = 6.7 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2$$

$$M = 5 \times 10^{20} \text{ kg}$$

$$R = 1800 \text{ km} = 1800 \times 1000 \text{ m} = 1.8 \times 10^6 \text{ m}$$

On putting all these value in the above formula, we get:

$$g = \frac{6.7 \times 10^{-11} \times 5 \times 10^{20}}{(1.8 \times 10^6)^2} = 1.03 \times 10^{-2} \text{ m/s}^2$$

**Example 2:**

Calculate the value of acceleration due to gravity on the surface of the moon. The mass of the moon and its radius are  $7.4 \times 10^{22}$  kg and 1740 km respectively. The value of universal gravitational constant ( $G$ ) is  $6.7 \times 10^{-11}$  Nm<sup>2</sup>/kg<sup>2</sup>.

**Solution:**

The formula for acceleration due to gravity is:

$$g = \frac{GM}{R^2}$$

In the present case:

$$G = \text{Universal gravitational constant} = 6.7 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

$$M = \text{Mass of the moon} = 7.4 \times 10^{22} \text{ kg}$$

$$R = \text{Radius of the moon} = 1740 \text{ km} = 1.74 \times 10^6 \text{ m}$$

So, the acceleration due to gravity on the surface of the moon is given as:

$$g_{\text{moon}} = \frac{6.7 \times 10^{-11} \times 7.4 \times 10^{22}}{(1.74 \times 10^6)^2} = 1.63 \text{ m/s}^2$$

**Example 3:**

**A block of mass 15 kg falls with an acceleration of 4 m/s<sup>2</sup> on a distant planet. What will be the acceleration of a block of mass 5 kg on the same planet?**

**Solution:**

The acceleration produced by the gravitational force does not depend on the mass of an object. Therefore, the acceleration produced in a block of mass 15 kg will be the same as that produced in a block of mass 5 kg. Hence, both the blocks will fall with the same acceleration, i.e., 4 m/s<sup>2</sup>.

**Medium**

**Example 4:**

**Consider a planet whose mass and radius are each twice as those of Earth. Calculate the acceleration due to gravity on this planet.**

**Solution:**

Let  $M$  and  $M'$  be the masses of Earth and the planet respectively.

Let  $R$  and  $R'$  be the radii of Earth and the planet respectively.

The acceleration due to gravity on Earth's surface is:

$$g = \frac{GM}{R^2}$$

On Earth's surface, the value of  $g$  is  $9.8 \text{ m/s}^2$ .

It is given that:  $M' = 2M$  and  $R' = 2R$

$$g' = \frac{GM'}{R'^2}$$

$$\Rightarrow g' = \frac{G2M}{(2R)^2}$$

$$\Rightarrow g' = \frac{1}{2} \times \frac{GM}{(R)^2}$$

$$\Rightarrow g' = \frac{9.8}{2}$$

So,  $g' = 4.9 \text{ m/s}^2$ .

**Hard**

**Example 5:** The value of acceleration due to gravity at a place is 2% less than its value on Earth's surface. Find the height of that place above Earth's surface. (Given: Earth's radius = 6400 km)

**Solution:**

Consider the height of the place above Earth's surface as  $h$ .

The formula for the acceleration due to gravity at this place is:

$$g' = \frac{GM}{(R+h)^2} = \frac{GM}{\left(R\left(1+\frac{h}{R}\right)\right)^2}$$

$$g' = \frac{GM}{R^2} \times \left(\frac{R}{R+h}\right)^2$$

$$g' = g \left(\frac{R}{R+h}\right)^2 \quad \dots \quad \text{(i)}$$

According to the problem, we have:

$$g' = g - \frac{2}{100}g = \frac{98}{100}g \quad \dots \quad \text{(ii)}$$

On combining equations (i) and (ii), we obtain:

$$\frac{98g}{100} = g \left(\frac{R}{R+h}\right)^2$$

On further solving, we get:

$$h = R \left[ \frac{10}{\sqrt{98}} - 1 \right]$$

On putting the value of  $R$ , we get the height of the place as:

$$h = 65.299 \text{ km}$$

### Whiz Kid

**Why does a parachutist fall down slowly?**

**Solution:**

The surface area of a parachute is large. This increases the air resistance on the parachutist. This air resistance acts in the direction opposite to Earth's gravitational force. Consequently, the parachutist falls down slowly.

### SI Unit and Value of 'g'

Acceleration due to gravity is given as:  $g = \frac{GM}{R^2}$  On substituting the SI units of  $G$ ,  $M$  and  $R$  in this equation, we obtain the SI unit of acceleration due to gravity.

$$g = \frac{\text{Nm}^2}{\text{kg}^2} \times \frac{\text{kg}}{\text{m}^2} = \frac{\text{N}}{\text{kg}} = \frac{\text{kg.m/s}^2}{\text{kg}} = \text{m/s}^2$$

$$g = \frac{GM}{R^2}$$

We will use the following values of  $G$ ,  $M$  and  $R$  in the formula for acceleration due to gravity, in order to obtain the value of  $g$ .

$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

$$\text{For Earth, } M = 6 \times 10^{24} \text{ kg}$$

For Earth,  $R = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$  So, we have:

$$g = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^6)^2}$$

$$g = 9.8 \text{ m/s}^2$$

**Earth's acceleration due to the gravity is approximately  $9.8 \text{ m/s}^2$ . However, this value varies from place to place.**

### Know More

Almost all planets are flattened at the poles. This is because of their rotation. This flattening of planets is known as oblateness. The degree of flattening of a planet at its poles is directly proportional to the value of oblateness of the planet. The planets Mercury and Venus rotate extremely slowly. Hence, they are not at all oblate.

### Did You Know?

The value of acceleration due to gravity at Earth's centre is zero. This is because the net force of gravity at this place is zero.

### Equations of Motion for an Object under the Influence of Earth's Gravity

We have three equations of motion that relate the initial ( $u$ ) and final ( $v$ ) velocities of a moving object with its acceleration  $a$  along a straight distance  $s$  in time  $t$ . These equations are given as follows:

First equation of motion:  $v = u + at$

Second equation of motion:  $s = ut + \frac{1}{2} at^2$

Third equation of motion:  $v^2 = u^2 + 2as$

If an object moves only under the influence of gravity, then we can take its acceleration  $a$  as the acceleration due to gravity  $g$ . Hence, the three equations of motion for acceleration  $a$  will be valid for acceleration due to gravity  $g$ . These equations are given in the following table.

S. No.	Relation	Object falling downward ( $a = g$ )	Object moving upward ( $a = -g$ )
1.	Velocity-time	$v = u + gt$	$v = u - gt$
2.	Distance-time	$s = ut + \frac{1}{2}gt^2$	$s = ut - \frac{1}{2}gt^2$
3.	Velocity-distance	$v^2 = u^2 + 2gs$	$v^2 = u^2 - 2gs$

### Solved Examples

#### Medium

##### Example 1:

When a ball is thrown vertically upward, it rises to a distance of 20 m. Find the velocity with which the ball is thrown upward. (Take  $g = 9.8 \text{ m/s}^2$ )

**Solution:** We have:

Initial velocity of the ball =  $u$ .

Final velocity ( $v$ ) of the ball = 0

Acceleration due to gravity,  $g = -9.8 \text{ m/s}^2$

Height,  $h = 20 \text{ m}$

On substituting these values in the third equation of motion, we get:

$$v^2 = u^2 + 2gh$$

$$\Rightarrow (0)^2 = u^2 + 2 \times (-9.8) \times 20$$

$$\Rightarrow 0 = u^2 - 392$$

$$\text{So, } u = 19.8 \text{ m/s}$$

Therefore, the ball is thrown upward with a velocity of 19.8 m/s.

**Example 2: A coin is dropped from the top floor of a tall building. The coin takes ten seconds to reach the ground. What is the height of the building? (Take  $g = 9.8 \text{ m/s}^2$ )**

**Solution:**

We have:

Initial velocity ( $u$ ) of the coin = 0

Time taken ( $t$ ) by the coin to reach the ground = 10 s

Acceleration due to gravity,  $g = 9.8 \text{ m/s}^2$

Height of the building =  $h$

On substituting these values in the second equation of motion, we get:

$$h = ut + \frac{1}{2}gt^2$$

$$\Rightarrow h = 0 + \frac{1}{2} \times 9.8 \times 100$$

$$\text{So, } h = 490 \text{ m}$$

**Hard**

**Example 3: A cricket ball is thrown upward with some velocity. The ball goes up and comes down in four seconds. What is the velocity with which the ball was thrown? (Take  $g = 9.8 \text{ m/s}^2$ )**

**Solution:**

We have:

Initial velocity of the ball =  $u$

Final velocity,  $v = 0$  (Since at the highest point, the velocity of the ball is zero)

The total time taken by the ball to go up and come down is four seconds.

So, time taken ( $t$ ) by the ball to reach the highest point =  $\frac{4}{2} = 2\text{ s}$

Acceleration due to gravity,  $g = -9.8\text{ m/s}^2$

Using the first equation of motion, we get:

$$v = u + gt$$

$$\text{or } 0 = u - 9.8 \times 2$$

$$\text{or } u = 19.6\text{ m/s}$$

Hence, the ball was thrown upward with a velocity of 19.6 m/s.

**Example 4: A gun is fired such that the bullet moves vertically upward with a velocity of 300 m/s. What will be the maximum height attained by the bullet? (Take  $g = 9.8\text{ m/s}^2$ )**

**Solution:**

We have:

Initial velocity ( $u$ ) of the bullet = 300 m/s

Final velocity ( $v$ ) of the bullet = 0 m/s

Acceleration due to gravity,  $g = -9.8\text{ m/s}^2$

Using the third equation of motion, we get:

$$v^2 = u^2 + 2gh$$

$$\Rightarrow 0 = u^2 + 2gh$$

$$\Rightarrow u^2 = -2gh$$

$$\Rightarrow (300)^2 = -2(-9.8) \times h$$

$$\Rightarrow 90000 = 19.6h$$

So,  $h = 4591.84\text{ m} = 4.6\text{ km}$

Hence, the bullet will attain a maximum height of 4.6 km.

Do you know that the value of 'g' is not constant and depends on various other factors?

### Variations in Value of 'g'

**1. Change with depth:** The value of 'g' varies with the change in the depth. As we move deeper inside the earth, the value of 'g' decreases.

**2. Change with height:** The value of 'g' is inversely proportional to height. This means that with the increase in the height, there is a decrease in the value of g.

**3. Change along the surface of earth:** The value of 'g' is not same everywhere on the earth's surface. It is because the earth is not perfectly spherical. The earth is bulged at the equators and flattened at the poles. This means that the radius of earth is greater at the equator and less at the poles. From the following equation we can see that there is an inverse relationship between radius of earth and the value of 'g'.

$$g = \frac{GM}{R^2}$$

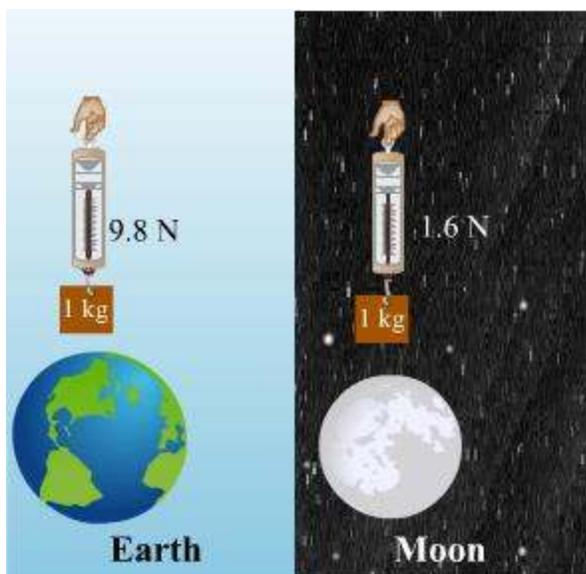
So, we can infer that the value of 'g' is highest at the poles and lowest at the equator.

Mass and Weight

### Mass and Weight: An Overview

Do you know that the mass of a body remains constant? And that the weight of the same body can vary from zero to any finite value, depending upon the celestial body on which it is kept? Have you heard about weightlessness?

A body has weight because of gravity; the same body can experience weightlessness under the same gravity. Strange, isn't it?



Suppose a body of mass 1000 kg is placed on Earth's surface and then on the surface of the moon. The mass of the body will remain the same at both places, but it will have different weights (9800 N on Earth and 1600 N on the moon).

**The mass of an object is defined as the amount of matter present in it.**

It is the measure of the inertia possessed by an object. It is one of the three fundamental physical quantities, the other two being length and time. The mass of an object is usually represented by the small letter ' $m$ '. Its SI unit is kilogram (kg).

The mass of an object is a conserved quantity. It can be neither created nor destroyed during physical or chemical changes. During a physical or chemical process, the total mass of the objects involved remains constant.

### Did You Know?

You might know the famous mass–energy equation given by Albert Einstein.

$$E = mc^2$$

This equation expresses the amount of energy created when mass ( $m$ ) is lost in a process. The letter ' $c$ ' represents the speed of light in vacuum and is numerically equal to  $3 \times 10^8$  m/s. Let us consider that some how we are able to completely convert 1 g (= 0.001 kg) of mass into energy. The resultant energy is given as:

$$E = 0.001 \times (3 \times 10^8)^2$$

$$\Rightarrow E = 10^{-3} \times 9 \times 10^{16}$$

$$\Rightarrow E = 9 \times 10^{13} \text{ J}$$

This energy is enough to meet the electricity needs of India for more than a year!!

A body contains the same quantity of matter whether it is on Earth, on Mars or in outer space. So, if the mass of an object is 10 kg on Earth, then it will have the same mass on Mars, on the moon and even in outer space. The mass of an object can never be zero.

## Weight

**The weight of an object is the force of gravity on the object and may be defined as the product of its mass and acceleration due to gravity.**

We know that:

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

The acceleration produced by Earth's force of attraction is known as acceleration due to gravity and is denoted by the letter ' $g$ '. Thus, the downward force acting on a body is given by:

$$\text{Force} = m \times g$$

Where,  $m$  is the mass of the body

By definition, Earth's force of attraction on a body is known as the weight of the body. Hence, on writing 'Weight' ( $W$ ) in place of 'Force' in the above equation, we get:

$$\text{Weight, } W = m \times g$$

Where,  $m$  = Mass of the body

$g$  = Acceleration due to gravity

Weight has the same SI unit as force, i.e., newton (N).

Now, let us calculate the weight of an object having a mass of 1 kg on Earth's surface.

We know that acceleration due to gravity on Earth's surface is  $9.8 \text{ m/s}^2$ .

$$\text{Therefore, weight of the object} = m \times g = 1 \text{ kg} \times 9.8 \text{ m/s}^2 = 9.8 \text{ N}$$

## Know More

Weight has magnitude as well as direction. The weight of a body acts in vertically downward direction and is given by  $mg$ . Since the value of 'g' (acceleration due to gravity) changes from place to place, the weight of a body also changes from place to place, i.e., the weight of a body is not constant.

In interplanetary space, acceleration due to gravity is negligible. Thus, the weight of a body is zero in interplanetary space, and because of this, one experiences weightlessness.

Acceleration due to gravity increases at the poles. As a result, the weight of an object increases at the poles. Acceleration due to gravity decreases at higher altitudes. As a result, the weight of an object decreases at higher altitudes. Acceleration due to gravity decreases under Earth's surface. As a result, the weight of an object decreases under Earth's surface and becomes zero at Earth's centre.

The weight of an object on Earth is the force with which Earth attracts the object toward itself. Similarly, the weight of an object on the moon is the force with which the moon attracts the object toward itself.

### Differences Between Mass and Weight

S. No.	Mass	Weight
1.	Mass is the amount of matter contained in a body.	Weight is the force exerted on a body due to the gravitational pull of another body such as Earth, the sun and the moon.
2.	Mass is an intrinsic property of a body.	Weight is an extrinsic property of a body.
3.	Mass is the measure of inertia.	Weight is the measure of force.
4.	The mass of a body remains the same everywhere in the universe.	The weight of a body depends on the local acceleration due to gravity where it is placed.
5.	The mass of a body cannot be zero.	The weight of a body can be zero.
6.	The SI unit of mass is kilogram (kg).	Since weight is a force, its SI unit is newton (N).
7.	The mass of a body can be measured using a beam balance and a pan balance.	The weight of a body can be measured using a spring balance and a weighing machine.

## Spring Balance and Beam Balance



While commonly used for measuring the mass of a body, what a spring balance actually measures is the weight of the body (or the force acting in the downward direction).

It can be used locally to measure mass when calibrated correctly according to the value of acceleration due to gravity at the given place.

A spring balance shows different readings on different planets because of the differing values of acceleration due to gravity.

In a spring balance,

$$mg = kx$$

Where,  $x$  is the extension produced in the spring and  $k$  is the spring constant.

$$\therefore x = \frac{mg}{k}$$

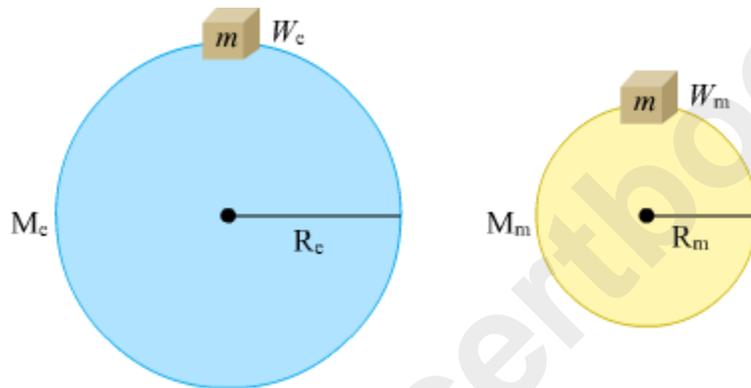
So, for differing values of  $g$ ,  $x$  also has different values.



A beam balance is also used for measuring the mass of a body. It does so by comparing the mass of the body with a given standard mass.

## Mass and Weight

### Weight of an Object on the Moon



Suppose an object having a mass  $m$  and weight  $W_e$  on Earth, is brought to the surface of the moon.

So, we have:

Mass of the object =  $m$

Weight of the object on Earth =  $W_e$

Let us take:

Mass of Earth =  $M_e$

Radius of Earth =  $R_e$

Weight of the object on the moon =  $W_m$

Mass of the moon =  $M_m$

Radius of the moon =  $R_m$

Since the mass of the object remains the same everywhere in the universe, it will be the same on both Earth and the moon.

Newton's law of gravitation gives the weight of the object on the moon as:

$$W_m = G \frac{M_m \times m}{R_m^2} \dots (i)$$

Its weight on Earth is given as:

$$W_e = G \frac{M_e \times m}{R_e^2} \dots (ii)$$

The values of the mass and radius of Earth and the moon are given in the following table.

	Mass	Radius
Earth	$5.98 \times 10^{24}$ kg	$6.37 \times 10^6$ m
Moon	$7.36 \times 10^{22}$ kg	$1.74 \times 10^6$ m

Hence, equation (ii) gives the weight of the object on Earth as:

$$W_e = G \times \frac{5.98 \times 10^{24} \times m}{(6.37 \times 10^6)^2}$$

$$W_e = 1.4737 \times 10^{11} m \text{ G... (iii)}$$

Equation (i) gives its weight on the moon as:

$$W_m = \frac{G \times 7.36 \times 10^{22} \times m}{(1.74 \times 10^6)^2}$$

$$W_m = 2.4309 \times 10^{10} m \text{ G... (iv)}$$

On dividing equation (iv) by equation (iii), we obtain:

$$\frac{W_m}{W_e} = \frac{2.4309 \times 10^{10} mG}{1.4737 \times 10^{11} mG}$$

$$\frac{W_m}{W_e} = \frac{1}{6}$$

$\frac{\text{Weight of the object on the moon}}{\text{Weight of the object on Earth}} = \frac{1}{6}$
------------------------------------------------------------------------------------------------------

From the above result, we can infer that:

- The weight of an object on the moon is one-sixth of its weight on Earth.
- The acceleration due to gravity on the moon is one-sixth of the acceleration due to gravity on Earth.

### Solved Examples

#### Easy

##### Example 1:

**A toy has a mass of 1 kg. Its weight is measured at the equator and at the North Pole using a spring balance. Where do you think the toy would weigh more?**

##### Solution:

Acceleration due to gravity is more at the North Pole than at the equator. Thus, an object weighs more at the North Pole than at the equator. Hence, the toy will weigh more at the North Pole.

##### Example 2:

**A block of mass 10 kg is taken to the moon. If the acceleration due to gravity on the moon is  $1.63 \text{ m/s}^2$ , then what is the weight of the block on the moon?**

##### Solution:

Weight,  $W = mg$

Where,  $m = \text{Mass of the block} = 10 \text{ kg}$

$g = \text{Acceleration due to gravity on the moon} = 1.63 \text{ m/s}^2$

$$\therefore W = 10 \times 1.63 = 16.3 \text{ N}$$

### Example 3:

**A horizontal force of 10 N acts on a block weighing 9.8 N. What is the acceleration produced in the block? (Take  $g = 9.8 \text{ m/s}^2$ )**

#### Solution:

Weight of the block = Mass of the block  $\times$  Acceleration due to gravity  
Let the mass of the block be  $m$ .

$$\Rightarrow 9.8 = m \times 9.8$$

$$\therefore m = 1 \text{ kg}$$

Acceleration produced in the block

$$= \frac{\text{Force acting on the block}}{\text{Mass of the block}} = \frac{10 \text{ N}}{1 \text{ kg}} = 10 \text{ m/s}^2$$

### Medium

#### Example 4:

**Why does the weight of an object change when we move from the poles to the equator?**

#### Solution:

Earth's radius increases when we move from the poles to the equator. The value of acceleration due to gravity is **inversely proportional** to Earth's radius ( $R$ ). So, as we move from the poles to the equator, the gravitational force decreases.

$$g = \frac{GM}{R^2}$$

The equation makes it clear that as  $R$  increases, the value of  $g$  decreases.

Now, the weight of an object is the product of its mass and the gravitational force. So, the weight of the object will decrease as we move from the poles to the equator.

### Hard

**Example 5:**

If a man's weight is 80 N on Earth's surface, then how far must he go from Earth's centre so as to weigh 40 N? (Take Earth's radius = 6400 km)

**Solution:**

Weight ( $W$ ) of the man on Earth's surface = 80 N

$$g = \frac{GM}{(R+h)^2}$$

Weight of the man at height  $h$  is:

$$W = mg = \frac{GMm}{(R+h)^2} = \frac{GMm}{\left(R\left(1+\frac{h}{R}\right)\right)^2}$$

$$\Rightarrow W = \frac{GMm}{R^2} \times \left(\frac{R}{R+h}\right)^2$$

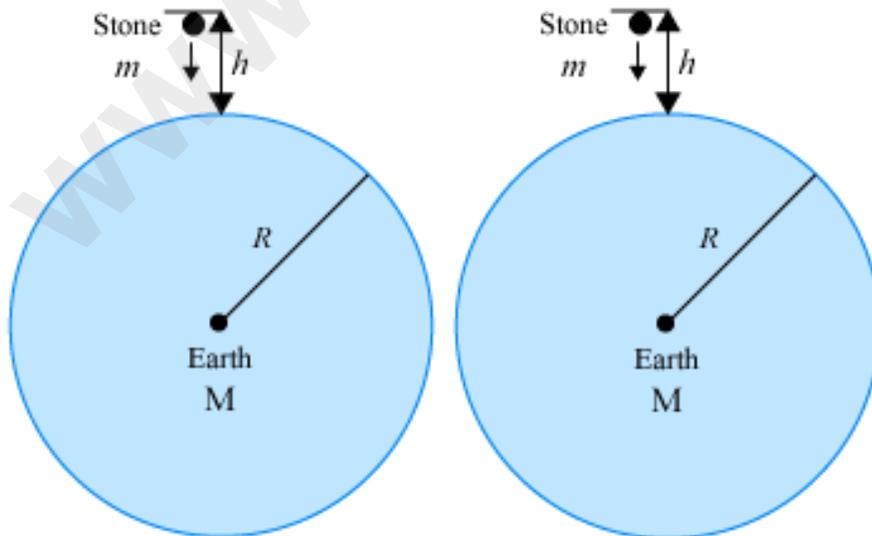
$$\Rightarrow W = mg \left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow 40 = 80 \times \left(\frac{R}{R+h}\right)^2$$

On solving, we get:

$$h = (\sqrt{2} - 1)R = 0.414R$$

$$\text{So, } h = 2.65 \times 10^6 \text{ m}$$



Therefore, the man must go  $(R+h) = (6.4 \times 10^6 + 2.65 \times 10^6 = 9.05 \times 10^6)$  m far from Earth's centre so as to weigh 40 N.

### **Weightlessness**

Weightlessness describes the situation wherein the weight of a body becomes zero.

The effective weight of the body at a place (or in a situation) is zero when the effective acceleration due to gravity at that point is zero.

Let us read about the situations wherein the weight of a body becomes zero.

#### **Case I: When the body is taken to Earth's centre**

The effective value of acceleration due to gravity at Earth's centre is zero.

Therefore, weight of the body at Earth's centre =  $mg' = m \times 0 = 0$

#### **Case II: When the body is revolving around Earth under the influence of the gravitational force**

Earth's gravitational pull on the body (acting towards Earth's centre) is balanced by the centrifugal force on the body (acting away from Earth's centre). In consequence, the effective weight of the body becomes zero.

#### **Case III: When the body is inside a lift falling freely under Earth's gravitational force**

Acceleration of the lift,  $a = g$

Effective acceleration due to gravity =  $g' = g - a = g - g = 0$

Hence, effective weight of the body = 0

### **Solved Examples**

#### **Easy**

**Example:**

**What is the weight of a body of mass  $m$  near Earth's surface during its free fall?**

**Solution:**

Weight is a physical quantity that can be experienced only when the body opposes the force of gravity. During free fall, the body does not oppose Earth's gravitational force; hence, its weight is zero.

**Did You Know?**

The motion of a satellite around Earth is an example of free fall. The satellite, at every point, is falling freely toward Earth.

A black hole is formed when a star completely collapses on its gravitational force. A black hole has an intense gravitational field around itself. Nothing can escape from this gravitational field, not even light!