Motion in one dimension

Scalar and Vector Quantities

Do you know what a physical quantity is? A physical quantity is any physical property that can be expressed in numbers. For example, time is a physical quantity as it can be expressed in numbers, but beauty is not as it cannot be expressed in numbers.

Scalar Quantities

- If a physical quantity can be completely described only by its magnitude, then it is a **scalar quantity**. To measure the mass of an object, we only have to know how much matter is present in the object. Therefore, mass of an object is a physical quantity that only requires magnitude to be expressed. Therefore, we say that **mass is a scalar quantity**.
- Some more examples of scalar quantities are time, area, volume, and energy.
- We can add scalar quantities by simple arithmetic means.
- It is difficult to plot scalar quantities on a graph.

Vector Quantities

- There are some physical quantities that cannot be completely described only by their magnitudes. These physical quantities require direction along with magnitude. For example, if we consider force, then along with the magnitude of the force, we also have to know the direction along which the force is applied. Therefore, to describe a force, we require both its magnitude and direction. This type of physical quantity is called a vector quantity.

  Therefore, we can define **vector quantity** as **the physical quantity that requires both magnitude and direction to be described**.
- Some examples of vector quantities are velocity, force, weight, and displacement.
- Vector quantities cannot be added or subtracted by simple arithmetic means.
- Vector quantities can easily be plotted on a graph.

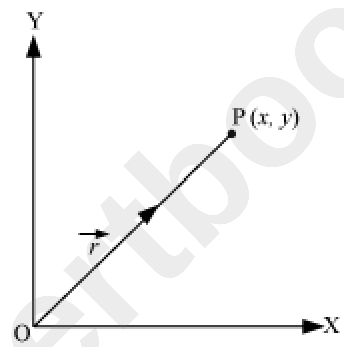
Scalars v/s Vectors

Scalars	Vectors

A scalar quantity has only magnitude.	A vector quantity has both magnitude and direction.
Scalars can be added, subtracted, multiplied, and divided just as ordinary numbers i.e., scalars are subjected to simple arithmetic operations.	Vectors cannot be added, subtracted, and multiplied following simple arithmetic laws. Arithmetic division of vectors is not possible at all.
Example: mass, volume, time, distance, speed, work, temperature	Example: displacement, velocity, acceleration, force

Position Vector

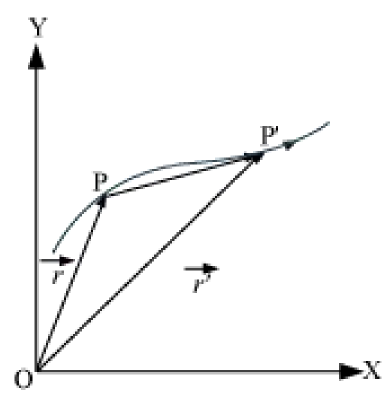
Position vector of a point in a coordinate system is the straight line that joins the origin and the point.



Magnitude of the vector is the length of the straight line and its direction is along the angle θ from the positive x-axis.

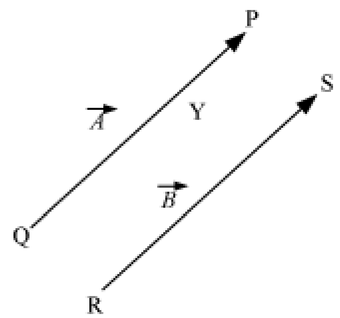
Displacement Vector

Displacement vector is the straight line joining the initial and final positions.

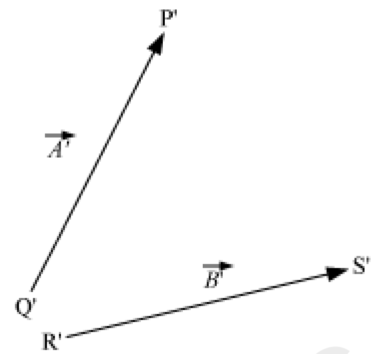


Equality of Vectors

Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal, if and only if they have the same magnitude and the same direction.



$\vec{A}$ and $\vec{B}$ are equal



$\vec{A'}$ and $\vec{B'}$ are unequal vectors; their magnitudes are same but directions are not same.

Rest and Motion

We use the word ‘rest’ very often. For example, when someone is doing no work or lying on the bed, we often say that the person is resting. This means that the person is not moving. Scientifically as well, the word ‘rest’ has a similar meaning.

Scientifically, we say an object is at **rest** when the position of the object **does not change** with **time**, with respect to its **surroundings.**

Similarly, **motion** is defined as the change of position of an object with **time**, with respect to its **surroundings**.

What do we mean by **with respect to the surroundings**?

We know that a moving train is in motion because its position changes with time. Now, consider a person sitting in the train. For someone standing on the platform, the person sitting in the train is in motion.

But for the co-passengers, the person is at rest as the position of the person does not change with time. Therefore, we need to consider the surroundings or the point of observation while describing the state of motion of an object. The surroundings is called reference frame.

What the above discussion shows us is that rest and motion are relative. They can be different for different observers. If someone views the Earth from the universe, then all the things on the Earth (such as houses, trees, a moving train, etc.) are in motion for that person. But for a person on the Earth’s surface, things such as houses, trees, etc., are at rest. So, when we say that an object is at rest, what we really mean is that the object is at rest with respect to its surroundings.

Motion in a Straight Line

Rest and Motion

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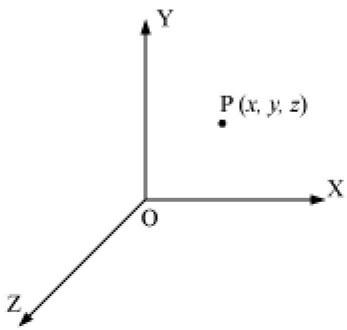
Point Mass Object

If the distance covered by an object is much greater than its size during its motion, then the object is considered as point mass object.

Rectilinear Motion − What it means?

Motion of a body that moves along a straight line such as the motion of a car moving on a straight road

Position − How to Describe it?



- Locating an object requires finding its position relative to a reference point.
- Reference point is often taken as the origin of a coordinate system.
- The coordinates (*x, y, z*) of the object describe the position of the object with respect to the coordinate axes.
- Coordinate system along with time constitutes a frame of reference.

Path Length

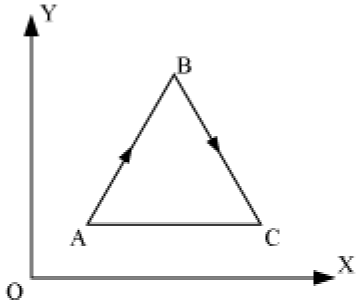
- Length of the actual path traversed by a body in a given time
- It is a **scalar quantity**. Therefore, only magnitude is important, not the direction of movement. (Implies that path length can never be negative)

Displacement

- A change of position ΔR from coordinate $R_1(x_1, y_1, z_1)$ to coordinate $R_2(x_2, y_2, z_2)$
- This is the shortest distance between the initial and final position of a body.
- It is a **vector quantity**. Therefore, both magnitude and direction are important to describe displacement. (Implies that displacement can be negative depending on the initial and final positions of a body in a coordinate system)

How Path Length and Displacement are Different?

Consider this example.



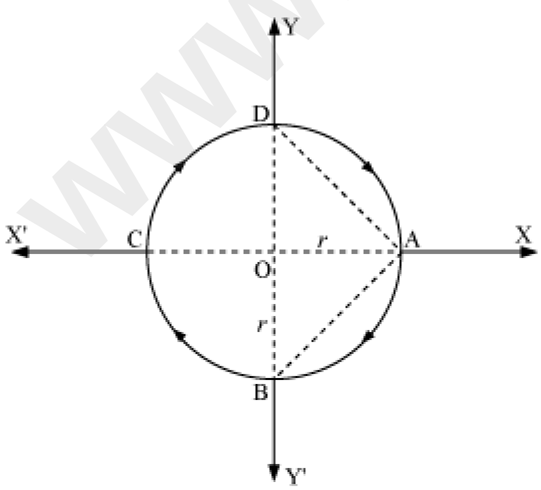
If an object goes from A to B and then B to C in time '*t*', then

- **path length = AB + BC** (arithmetic sum of the distances)
- displacement, $\Delta x = \overrightarrow{AC}$ (shortest distance between points A and C)

Problems based on Path Distance and Displacement

Example − A particle moves along a circle of radius '*r*'. It starts from A and moves clockwise. Calculate the distance travelled by the particle and its displacement in each case. Take centre of the circle as the origin.

1. From A to B
2. From A to C
3. From A to D
4. In one complete revolution



Solution

(i) Distance travelled by the particle from A to B $= \frac{2\pi r}{4} = \frac{\pi r}{2}$

Displacement = $\left|\overrightarrow{AB}\right| = \sqrt{OA^2 + OB^2}$

$= \sqrt{r^2 + r^2} = \sqrt{2}r$ direction is along negative X and negative Y axis.

(ii) Distance travelled by particle from A to $C = \frac{2\pi r}{2} = \pi r$

Displacement $= \overrightarrow{|AC|} = 2r$ direction is along negative X axis.

(iii) Distance travelled from A to D $= \frac{2\pi r}{4} \times 3 = \frac{3}{2}\pi r$

Displacement$= \left|\overrightarrow{AD}\right| = \sqrt{r^2 + r^2} = \sqrt{2}r$ Direction is along Positive X and Y axis.

(iv) For one complete revolution i.e., motion from A to A,

Total distance travelled = $2\pi r$

Displacement = 0 [∵ the final position coincides with the initial position]

Speed, Velocity and Acceleration

Speed

- Speed = $\frac{\text{Distance travelled}}{\text{Time taken}} = \frac{\text{Path length}}{\text{Travelling time}}$
- It is a scalar quantity, which means that it requires no direction (it implies that speed cannot be negative).
- Instantaneous speed is the speed at a particular instant (when the interval of time is infinitely small).

i.e., instantaneous speed $= \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

Average speed:

- Average speed of a particle is defined as the total distance travelled by the particle divided by the total time taken during which the motion took place.

- Suppose that a car is covering a distance of 160 km from A to B and covers successive 40 km distances in time 1.2 h, 1.4 h, 1.6 h, and 0.9 h respectively. The speed of car is different at different at every successive interval. In such cases, we need to find the average speed.

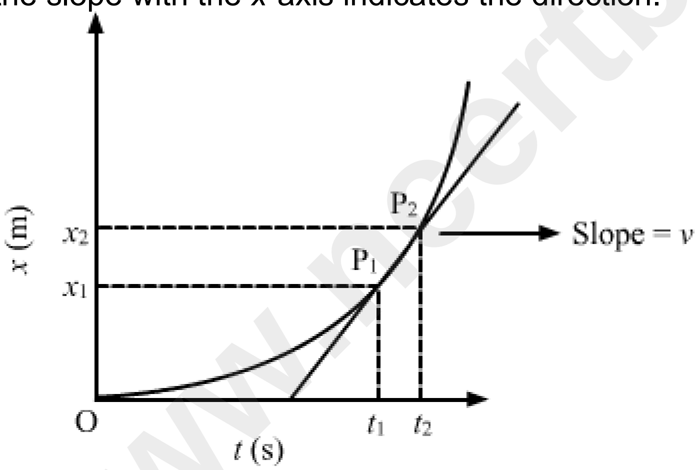
$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Velocity

- Velocity = $\dfrac{\text{Displacement}}{\text{Time interval}} = \dfrac{\text{Final position} - \text{Initial position}}{\text{Time interval}}$

- It is a vector quantity. Therefore, the direction of movement is taken into consideration (it implies that velocity contains algebraic sign).

- In a **position-time graph**, the slope of the curve indicates the velocity and the angle of the slope with the *x*-axis indicates the direction.



- Average velocity is the ratio of the change in displacement Δx to the time interval Δt in which the change in displacement occurred.

- If x_1 and x_2 are the positions of a particle at time t_1 and t_2, respectively, then the magnitude of displacement of the particle in time interval

interval $\Delta t = (t_2 - t_1)$ is $\Delta \vec{x} = \vec{x_2} - \vec{x_1}$.

Average velocity, $\vec{v}_{avg} = \dfrac{\text{Displacement}}{\text{Time taken}} = \dfrac{\vec{x_2} - \vec{x_1}}{t_2 - t_1} = \dfrac{\Delta \vec{x}}{\Delta t}$

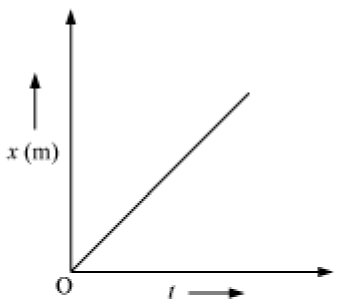
- Instantaneous velocity is the velocity at a particular instant (slope at a particular point on the *x-t* curve).

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

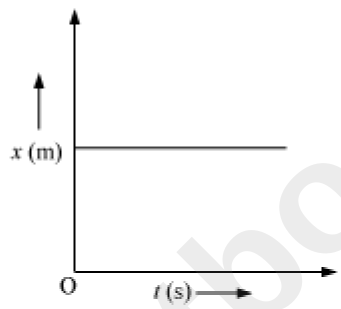
- When a motion is not uniform, sometimes instantaneous velocity has more importance than average velocity.

Uniform Motion: What it Means

In uniform motion, a body undergoes equal displacements in equal intervals of time.



Position – time graph of an object in uniform motion



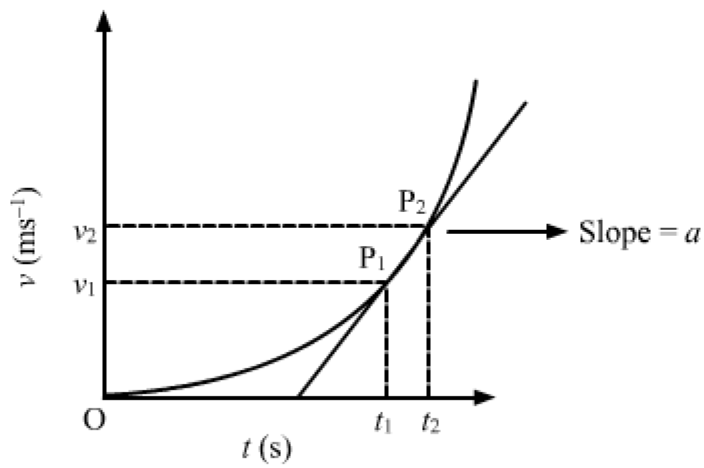
Position – time graph of a stationary object

Acceleration

- Acceleration is the rate of change of the velocity of an object.

    - Average acceleration, $\vec{a} = \frac{\text{Change in velocity}}{\text{Time taken}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$

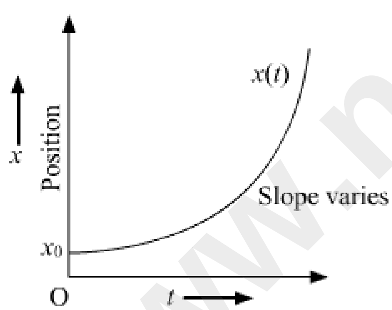
- In a velocity-time graph, the slope of the curve indicates the average acceleration and the angle of the slope indicates the direction of change of the velocity.



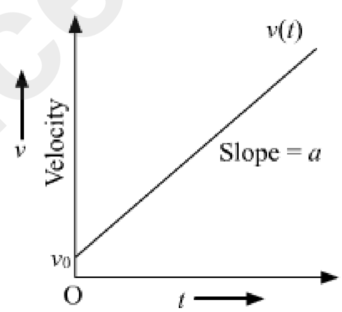
- Instantaneous acceleration is the acceleration at a particular instant (slope at a particular point on the *v-t* curve).

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

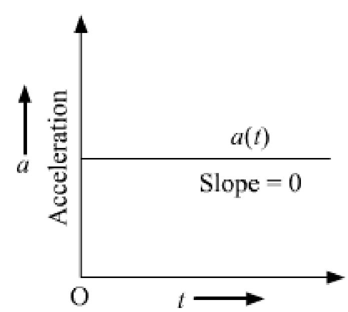
In this case, the rate of change of velocity with time remains constant. Graphically, such motion can be represented as



Position $x(t)$ of a particle moving with constant acceleration

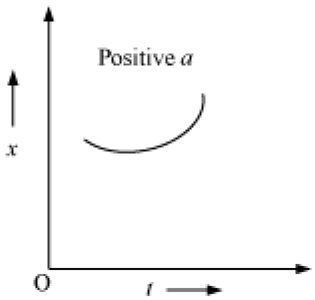


Its velocity $v(t)$, given at each point by the slope of the curve in (a).

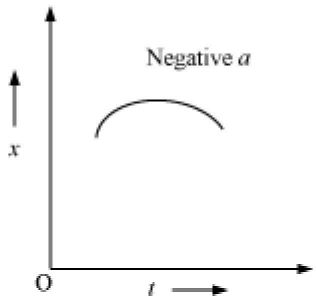


Its (constant) acceleration, equal to the (constant) slope of the curve of $v(t)$.

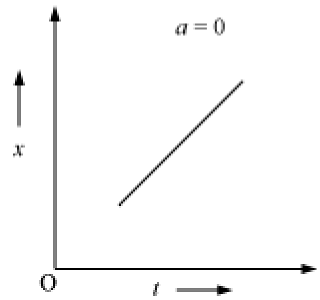
Graphical Representation of Accelerating Bodies



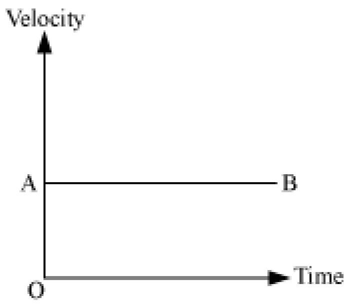
Position – time graph for motion with positive acceleration



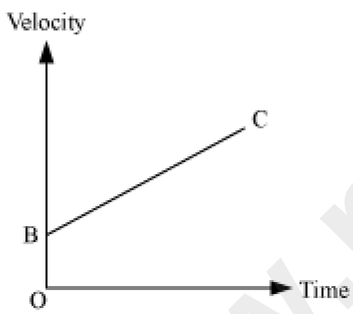
Position – time graph for motion with negative acceleration



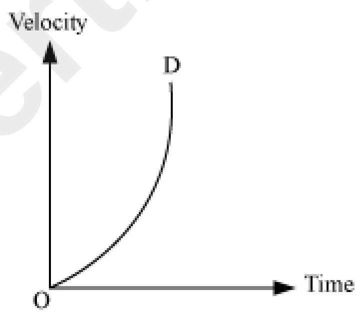
Position – time graph for motion with zero acceleration



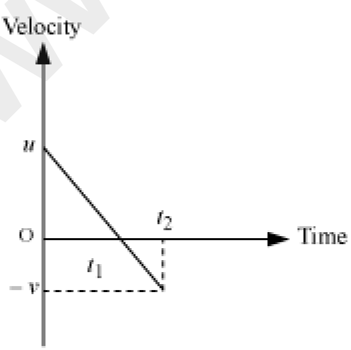
Object moving with constant velocity



Object moving with positive constant acceleration having some initial velocity



Object moving with increasing acceleration, having zero initial velocity



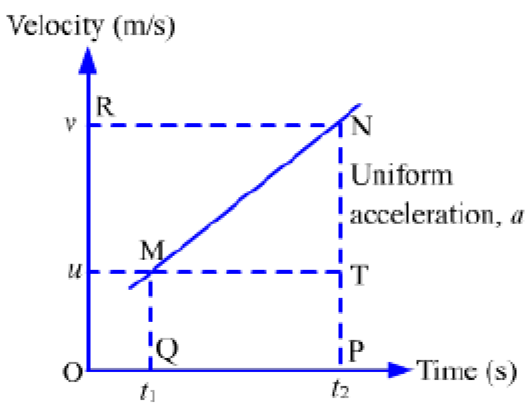
Object moving with uniform negative acceleration having positive initial velocity

First Equation of Motion

Velocity−Time Relation

Suppose a body is moving under a **uniform acceleration** in a given time interval. We can relate the change in the **velocity** of the moving body with the acceleration and time taken by using the one-dimensional velocity-time equation.

The velocity-time equation can be used for obtaining the final velocity, after time *t*, of a uniformly accelerating body.



Velocity−Time relation through the graphical method

Suppose a body is moving in a straight line, with an initial velocity *u* and under a uniform acceleration *a*. Its velocity becomes *v* after time *t*. The motion of this body is represented by the given velocity-time graph.

We can obtain the velocity-time equation if the velocities of a body (*u* and *v*) at times t_1 and t_2 are given, as shown in the velocity-time graph.

Initial velocity, *u* = MQ

Final velocity, *v* = NP

Time taken, $t = QP = (t_2 - t_1)$

Acceleration, *a* = Slope of line MN = NT/ MT = (NP - TP)/ (OP - OQ)

It is clear from the graph that TP = MQ

So, $a = (v - u)/ t_2 - t_1$

or, $a (t_2 - t_1) = v - u$ or, $v = u + a (t_2 - t_1)$

For initial time $t_1 = 0$, the equation reduces to: $v = u + a\,t_2$ or $v = u + at$ (as $t_2 = t$)

This is the **first equation of kinematics** and it is independent of the distance travelled. It is also known as the first equation of motion.

Solved Examples

Easy

Example 1:

On spotting a prey, a cheetah runs directly towards it with constant acceleration. The time taken by the cheetah is 50 s and its velocity, as it catches its prey, is 25 m/s. If we assume that the cheetah was initially at rest, then what is its acceleration?

Solution:

It is given that:

Initial velocity (u) of the cheetah = 0

Its final velocity, v = 25 m/s

Time taken (t) by it to catch its prey= 50 s

We can determine the acceleration (a) of the cheetah using the relation:

$$a = \frac{v-u}{t}$$
$$= \frac{25-0}{50} = \frac{25}{50} = 0.5 \text{ m/s}^2$$

Medium

Example 2:

A motorcyclist is travelling at a constant velocity of 10 m/s. In order to overtake a car, he accelerates at the rate of 0.2 m/s^2. If he overtakes the car in 60 seconds, then what is his velocity while overtaking?

Solution:

It is given that:

Initial velocity (u) of the motorcyclist = 10 m/s

His acceleration, a = 0.2 m/s^2

Time taken (t) by him to overtake the car = 60 s

Using the first equation of motion, we can compute the velocity (v) of the motorcyclist while overtaking the car.

$v = u + at$

$\Rightarrow v = 10 + 0.2 \times 60$

$\Rightarrow v = 10 + 12$

$\Rightarrow \therefore v = 22$ m/s

Hard

Example 3:

A train is moving under a constant acceleration of 150 km/h^2. It attains a velocity of
125 km/h in half-hour. What is the initial velocity of the train in SI unit?

Solution:

It is given that:

Final velocity (v) of the train= 125 km/h

Time taken (t) by it to attain the above velocity= 0.5 h

Its acceleration (a) = 150 km/h^2

Using the first equation of motion, we can compute the initial velocity (u) of the train.

$v = u + at$

$\Rightarrow u = v - at$

$\Rightarrow u = 125 - 150 \times 0.5$

$\Rightarrow u = 125 - 75$

$\Rightarrow \therefore u$ = 50 km/h

$$\text{Since } 1\,\text{km/h} = \left(\frac{5}{18}\right)\text{m/s}$$

$$50\,\text{km/h} = 50\left(\frac{5}{18}\right)\text{m/s}$$

$$= 13.89\ \text{m/s}$$

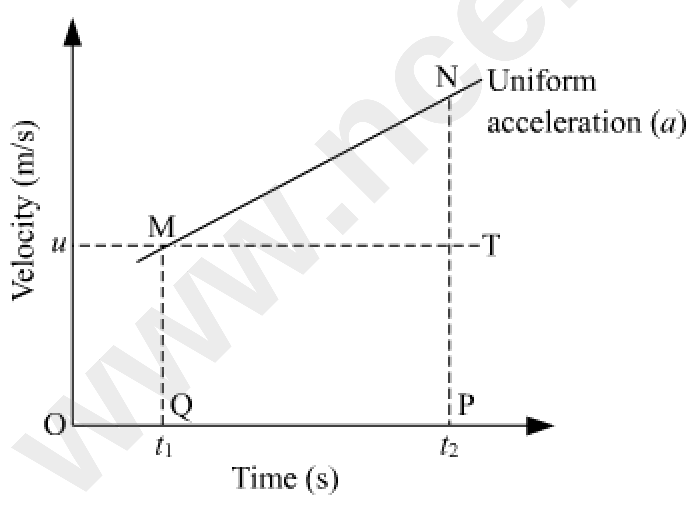
Therefore, the initial velocity of the train is 13.89 m/s.

Second Equation of Motion

Position−Time Relation

Suppose a body moving under a **uniform acceleration** covers a certain **distance** in a given time interval. We can relate the change in the **velocity** of the moving body with the acceleration, distance covered and time taken by using the one-dimensional position−time equation. The position−time equation is used to obtain the distance travelled by a uniformly accelerating body in a given interval of time.

Suppose a body is moving in a straight line, with an initial velocity u and under a uniform acceleration a. The distance covered by the moving body from time t_1 to time t_2 is represented in the given velocity−time graph.



It is clear from the graph that:

Initial velocity, u = MQ = TP

Time, t = PQ = $(t_2 - t_1)$ = MT

Change in velocity, NT = $a\,(t_2 - t_1)$

Distance, s = Area of trapezium QMNP

= Area of rectangle QMTP + Area of triangle MTN

= (MQ × QP) + ($\frac{1}{2}$ × NT × MT)

= $[u \times (t_2 - t_1)] + \frac{1}{2} \times a\,(t_2 - t_1) \times (t_2 - t_1)$

$\therefore s = u\,(t_2 - t_1) + \frac{1}{2}\,a\,(t_2 - t_1)^2$

For $t_1 = 0$ and $t_2 = t$, the equation reduces to:

$$s = ut + \frac{1}{2}\,at^2$$

This is the **second equation of kinematics** or the second equation of motion.

Average Velocity for Uniformly Accelerated Motion

We can obtain the relation for average velocity using the velocity−time and position−time equations.

The velocity−time equation is given as:

$v = u + at$

The position−time equation is given as:

$s = ut + \frac{1}{2}\,at^2$

$\Rightarrow \frac{s}{t} = u + \frac{1}{2}\,at$

$\Rightarrow \frac{s}{t} = \frac{2u + at}{2}$

$\Rightarrow \frac{s}{t} = \frac{u + (u + at)}{2}$

$\Rightarrow \frac{s}{t} = \frac{u + v}{2}$ (Using $v = u + at$)

$\therefore$Average velocity (v_{av}) = $\frac{s}{t} = \frac{u+v}{2}$

Did You Know?

If the acceleration is zero, then the second equation of motion denotes the distance travelled as the product of the initial velocity and time.

$$s = ut + \frac{1}{2}at^2$$
$$\Rightarrow s = ut + \frac{1}{2} \times 0 \times t^2$$
$$\Rightarrow s = ut$$

Solved Examples

Easy

Example 1:

A motorcyclist is travelling at a constant velocity of 10 m/s. He overtakes a car by accelerating at the rate of 0.2 m/s^2. If he overtakes the car in 60 s, then how much distance does he cover before overtaking the car?

Solution:

It is given that:

Initial velocity (u) of the motorcyclist = 10 m/s

His acceleration, a = 0.2 m/s^2

Time taken (t) by him to overtake the car = 60 s

Using the second equation of motion, we can compute the distance covered (s) by the motorcyclist before overtaking the car.

$$s = ut + \frac{1}{2}at^2$$
$$\Rightarrow s = 10 \times 60 + \frac{1}{2} \times 0.2 \times 60^2$$
$$\Rightarrow s = 600 + \frac{1}{2} \times 0.2 \times 3600$$
$$\Rightarrow s = 600 + \frac{1}{2} \times 720$$
$$\Rightarrow s = 600 + 360$$
$$\Rightarrow \therefore s = 960 \text{ m}$$

Medium

Example 2:

A train moving at a speed of 180 km/h comes to a stop at a constant acceleration in 15 min after covering a distance of 25 km. What is its acceleration?

Solution:

It is given that:

Initial velocity (u) of the train = 180 km/h

Distance covered (s) by it = 25 km

Time taken (t) by it to cover the above distance = 15 min = 0.25 h

Using the second equation of motion, we can compute the acceleration (a) of the train.

$$s = ut + \frac{1}{2}at^2$$
$$\Rightarrow 25 = 180 \times 0.25 + \frac{1}{2} \times a \times 0.25^2$$
$$\Rightarrow 25 = 45 + \frac{1}{2} \times a \times 0.0625$$
$$\Rightarrow 25 - 45 = 0.03125a$$
$$\Rightarrow a = \frac{-20}{0.03125}$$
$$\Rightarrow \therefore a = -640 \text{ km/h}^2$$

Hence, the train is retarding at a rate of 640 km/h^2. Note that since the speed of the train is decreasing, the acceleration comes out to be negative.

Hard

Example 3:

Brakes are applied on a car moving at a velocity of 72 km/h. It decelerates uniformly at the rate of 4 m/s^2 until it stops after 5 s. How far does the car go before it stops?

Solution:

It is given that:

Initial velocity (*u*) of the car = 72 km/h

$= 72 \times \left(\frac{5}{18}\right)$ m/s

= 20 m/s

Its acceleration, $a = -4$ m/s^2 (since the car decelerates)

Time taken (*t*) by it to stop = 5 s

Using the second equation of motion, we can compute the distance covered (*s*) by the car before stopping.

$s = ut + \frac{1}{2} at^2$

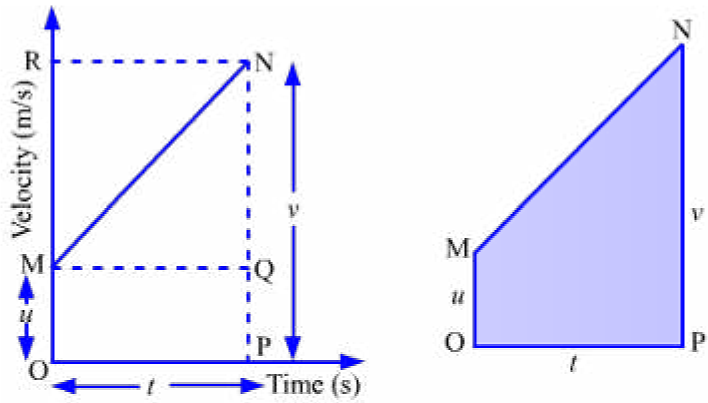
$\Rightarrow s = 20 \times 5 + \frac{1}{2} \times (-4) \times (5)^2$

$\Rightarrow s = 100 - 50$

$\therefore s = 50$ m

Third Equation of Motion

Position−Velocity Relation



Suppose a body is moving in a straight line, with an initial velocity *u* and under a uniform acceleration *a*. Its velocity becomes *v* after time *t* and it covers a distance *s* in the given time interval. The motion of this body is represented in the given velocity−time graph.

It is clear from the graph that:

Initial velocity, *u* = MO = QP

Final velocity, *v* = OR = NP

The straight line MN represents the velocity−time curve.

Distance (*s*) covered by the body = Area of trapezium OMNP

$$= \frac{1}{2} \times (\text{OM} + \text{PN}) \times \text{OP} = \frac{1}{2} \times (u + v) \times t$$

$$\therefore s = \frac{1}{2}(u + v)\, t \ldots \text{(i)}$$

Now, let us eliminate time *t* from this equation.

The velocity-time equation is given as:

v = *u* + *at*

$$\therefore t = \frac{v - u}{a} \quad \ldots \text{(ii)}$$

On substituting the value of *t* from equation (ii) in equation (i), we obtain:

$$s = \frac{1}{2} \times (u + v) \times \left(\frac{v-u}{a}\right)$$

$$\Rightarrow s = \frac{(u+v)(v-u)}{2a}$$

$$\Rightarrow s = \frac{v^2 - u^2}{2a}$$

$$\therefore \boxed{v^2 - u^2 = 2as}$$

This is the **third equation of kinematics**. It is independent of time. It is also known as the third equation of motion.

Deriving the Second Equation of Motion

The third equation of motion is given as:

$v^2 = u^2 + 2as$ … (i)

The first equation of motion is given as:

$v = u + at$ … (ii)

On eliminating velocity v from equation (i) with the help of equation (ii), we obtain:

$(u + at)^2 = u^2 + 2as$

$\Rightarrow u^2 + 2uat + a^2t^2 = u^2 + 2as$

$\Rightarrow 2uat + a^2t^2 = 2as$

$\Rightarrow s = \frac{1}{2a}(2uat + a^2t^2)$

$\Rightarrow s = ut + \frac{1}{2}at^2$ … (iii)

This is the second equation of motion.

Solved Examples

Easy

Example 1:

On applying the brakes, a cyclist travelling initially at 2 m/s comes to a halt at a constant retardation of 2 m/s^2. How much distance does the cyclist cover before coming to rest?

Solution:

It is given that:

Initial velocity (*u*) of the cyclist = 2 m/s

His final velocity, $v = 0$

His acceleration, $a = -2$ m/s^2 (since he is decelerating)

Using the third equation of motion, we can compute the distance covered (*s*) by the cyclist before stopping.

$$v^2 - u^2 = 2as$$
$$\Rightarrow v^2 = u^2 + 2as$$
$$\Rightarrow 0^2 = 2^2 - 2 \times 2 \times s$$
$$\Rightarrow 4 = 4s$$
$$\Rightarrow s = \frac{4}{4}$$
$$\Rightarrow \therefore s = 1 \text{ m}$$

Medium

Example 2:

A car covers 40 m in 8.5 s while applying brakes to a final speed of 2.8 m/s.

(i) What is the initial speed of the car?

(ii) What is its acceleration?

Solution:

It is given that:
Final velocity (v) of the car = 2.8 m/s
Distance covered, $s = 40$ m
Time taken (t) to cover the above distance = 8.5 s

Let us take:
Initial velocity of the car = u
Acceleration of the car = a

We have the relation:

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow 40 = 8.5u + \frac{1}{2}a \times 8.5^2 \qquad ...(1)$$

We know that:

$$v = u + at$$

$$\Rightarrow u = v - at$$

$$\Rightarrow u = 2.8 - a \times 8.5 \qquad ...(2)$$

From (1) and (2) we get:

$$40 = 8.5(2.8 - a \times 8.5) + \frac{1}{2}a \times 8.5^2$$

$$\Rightarrow \therefore a = -0.45 \text{ m/s}^2$$

On substituting the value of a in (2), we get:

$$u = 2.8 - (-0.45) \times 8.5$$

$$\Rightarrow \therefore u = 6.63 \text{ m/s}$$

Thus,
(i) The initial velocity of the car is 6.63 m/s.
(ii) The acceleration of the car is -0.45 m/s^2.

Hard

Example 3:

When the brakes are applied, a racing car stops within 0.0229 of a mile from a speed of
60 mi/h and within 0.0399 of a mile from a speed of 80 mi/h.

(i) What is the braking acceleration of the car for 60 mi/h to rest?

(ii) What is its braking acceleration for 80 mi/h to rest?

(iii) What is its braking acceleration for 80 mi/h to 60 mi/h?

Solution:

(i) In the first case:
Inital velocity (u) of the car = 60 mi/h = 26.82 m/s
Its final velocity, $v = 0$
Distance covered, s = 0.0229 mile = 36.88 m
Let the acceleration of the car be a.

We know that:

$v^2 - u^2 = 2as$

$\Rightarrow v^2 = u^2 + 2as$

$\Rightarrow 0 = 26.82^2 + 2 \times a \times 36.88$

$\Rightarrow \therefore a = -9.75 \text{ m/s}^2$

Thus, the braking acceleration of the car for 60 mi/h to rest is -9.75 m/s^2.

(ii) In the second case:
Initial velocity (u) of the car = 80 mi/h = 35.76 m/s
Its final velocity, $v = 0$
Distance covered, s = 0.0399 mile = 64.31 m
Again, let the acceleration of the car be a.

We know that:

$v^2 - u^2 = 2as$

$\Rightarrow v^2 = u^2 + 2as$

$\Rightarrow 0 = 35.76^2 + 2 \times a \times 64.31$

$\Rightarrow \therefore a = -9.94 \text{ m/s}^2$

Thus, the braking acceleration of the car for 80 mi/h to rest is -9.94 m/s^2.

(iii) In the third case:

Initial velocity (u) of the car $= 80$ mi/h $= 35.76$ m/s

Its final velocity, $v = 60$ mi/h $= 26.82$ m/s

Distance covered, $s =$ Distance covered in the second case $-$ Distance covered in the first case

$= 64.31 - 36.88 = 27.43$ m

Again, the acceleration is taken as a.

We know that:

$v^2 - u^2 = 2as$

$\Rightarrow v^2 = u^2 + 2as$

$\Rightarrow 26.82^2 = 35.76^2 + 2 \times a \times 27.43$

$\Rightarrow \therefore a = -10.19 \text{ m/s}^2$

Thus, the braking acceleration of the car for 80 mi/h to 60 mi/h is -10.19 m/s^2.