ICSE 2025 EXAMINATION

Sample Question Paper - 5

Mathematics

Time: 2 ½ Hours **Total Marks: 80**

General Instructions:

1. *Answers to this Paper must be written on the paper provided separately.*
2. *You will not be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.*
3. *The time given at the head of this Paper is the time allowed for writing the answers.*
4. *Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*
5. *The intended marks for questions or parts of questions are given in brackets [ ]*

Section A

(Attempt all questions from this section.)

Question 1

Choose the correct answers to the questions from the given options. [15]

i) Which of the following rational number is terminating?

(a) $\frac{13}{95}$

(b) $\frac{27}{405}$

(c) $\frac{11}{524}$

(d) $\frac{9}{1280}$

ii) Rates of interest for two consecutive years are 10% and 12% respectively. The percentage increase during these two years is

(a) 22%

(b) 23.2%

(c) 123.2%

(d) 122%

iii) If $x^2 - 4x + 1 = 0$, the value of $x^2 + \frac{1}{x^2} + 1$ is

(a) 8

(b) 10

(c) 15

(d) $\frac{2}{3}$

iv) The value of $\frac{0.8\times0.8\times0.8+0.5\times0.5\times0.5}{0.8\times0.8-0.8\times0.5+0.5\times0.5}$ is

(a) 0.13
(b) 0.3
(c) 0.25
(d) 1.3

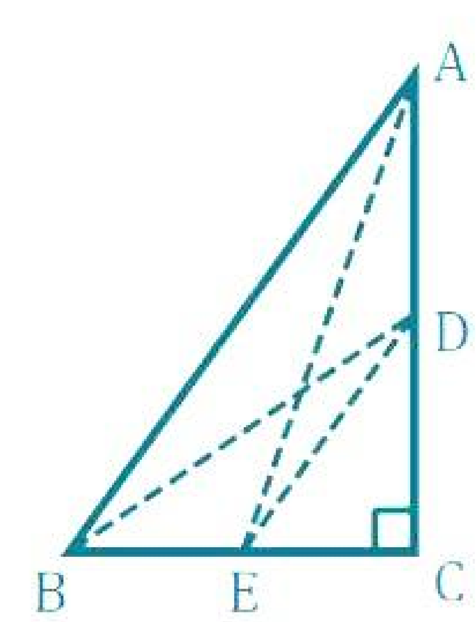
v) Solution of the equations x + y = 8 and 3x – 5y = 0 is
(a) x = 5, y = –3
(b) x = 5, y = 3
(c) x = –5, y = 3
(d) x = –5, y = –3

vi) If $4^{2x} = \frac{1}{32}$, the value of x is

(a) 1
(b) 1.25
(c) –1
(d) –1.25

vii) In quadrilateral PQRS, PQ = QR and PS = RS, then QS bisects
(a) ∠PQS
(b) ∠QPS
(c) ∠QRS
(d) ∠PQR

viii) In the figure given below, $AE^2 + BD^2$ is equal to
(a) $AB^2 - DE^2$
(b) $DE^2 - AB^2$
(c) $AB^2 + DE^2$
(d) DE × AB



ix) The line joining the mid-points of two chords of a circle passes through its centre, then the chords are
(a) parallel to each other
(b) not parallel
(c) equal to each other
(d) not equal

x) The class marks of a frequency distribution are 12, 16, 20, 24, 28, … The class corresponding to 20 is

(a) 10 – 14
(b) 12 – 16
(c) 18 – 22
(d) 22 – 26

xi) Mean of the data x – 2, x, x + 2, x + 4 is

(a) x + 1
(b) x
(c) x + 2
(d) 4x + 4

xii) How much water flows in 1 minute through the pipe of a uniform cross-section area of 12 cm^2 if water flows with the speed of 20 cm/s?

(a) 144 cm^3
(b) 1440 cm^3
(c) 0.144 m^3
(d) 0.0144 m^3

xiii) If A = 30°, then

Statement 1: $3\sin A - 4\sin^3 A = \sin 3A$

Statement 2: $3\sin A - 4\sin^3 A = 1$

Which of the following is valid?

(a) Both the statements are true.
(b) Both the statements are false.
(c) Statement 1 is true, and Statement 2 is false.
(d) Statement 1 is false, and Statement 2 is true.

xiv) Abscissa of a point is the solution of the equation 5x – 2 = 8 and its ordinate is the solution of the equation 10 – 3y = 1. The point is

(a) (2, 2)
(b) (4, –2)
(c) (–3, 4)
(d) (2, 3)

xv) **Assertion (A):** The point (x, y) is equidistant from the points (3, 6) and (−3, 4). Then the relation between x and y is $3x - y = 5$.

Reason (R): Distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(a) A is true, R is false

(b) A is false, R is true

(c) Both A and R are true, and R is the correct reason for A.

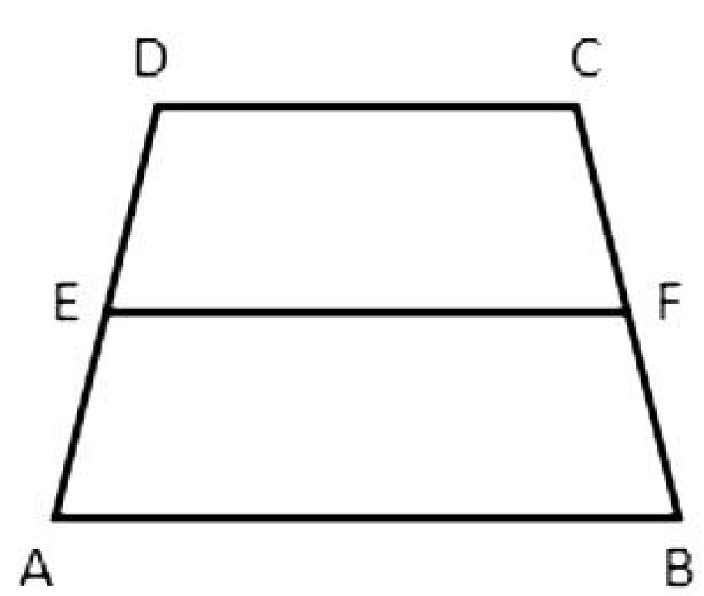
(d) Both A and R are true, and R is the incorrect reason for A.

Question 2

i) Two years ago, the population of a village was 4000. During the next year, it increased by 6%, but due to an epidemic, it decreased by 5% in the following year. What is its population now? [4]

ii) The sides of an equilateral triangle are $(6x + 5y)$ cm, $(7x + 3y + 1)$ cm and $2(x + 6y - 1)$ cm, respectively. Find the length of each side. [4]

iii) If E and F are the mid-points of non-parallel sides AD and BC, respectively, of a trapezium ABCD. [4]



Prove that:

A. EF is parallel to AB

B. EF = ½ (AB + CD)

Question 3

i) In a rhombus PQRS, side PQ = 17 cm and diagonal PR = 16 cm. Calculate the area of the rhombus. [4]

ii) In a pentagon ABCDE, AB is parallel to ED and angle B = 120°. Find the angles C and D if $\angle C : \angle D = 2 : 3$. [4]

iii) Factorise: [5]

A. $x^3 - 3x^2 + 3x + 7$

B. $3x^2 + 14x + 8$

Section B

(Attempt any four questions from this Section.)

Question 4

i) If $x = \frac{1}{5+2\sqrt{6}}$ and $y = \frac{1}{5-2\sqrt{6}}$, find the value of $x^2 + y^2$. [3]

ii) A woman saves Rs. 4,000 every year and invests it at the end of the year at 10% compound interest. Calculate the total amount of her savings at the end of the third year (Without using formula). [3]

iii) Two chords AB and CD of lengths 5 cm and 11 cm respectively, of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle. [4]

Question 5

i) If $(3a + 4b) = 16$ and $ab = 4$, find the value of $(9a^2 + 16b^2)$. [3]

ii) Factorise: $4(2a - 3)^2 - 3(2a - 3)(a - 1) - 7(a - 1)^2$. [3]

iii) The mean of 9 observations was found to be 35. Later on, it was detected that an observation 81 was misread as 18. Find the correct mean of the observations. [4]

Question 6

i) i) Solve using cross multiplication: $2x - 5y + 8 = 0$, $x - 4y + 7 = 0$. [3]

ii) ii) Simplify: $\frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 2^2 \times 5^n}$ [3]

iii) In a study of diabetic patients in a village, the following observations were noted.

Age (in years)	10–20	20–30	30–40	40–50	50–60	60–70
Number of patients	2	5	12	19	9	4

Represent the above data by a frequency polygon (without using a histogram). [4]

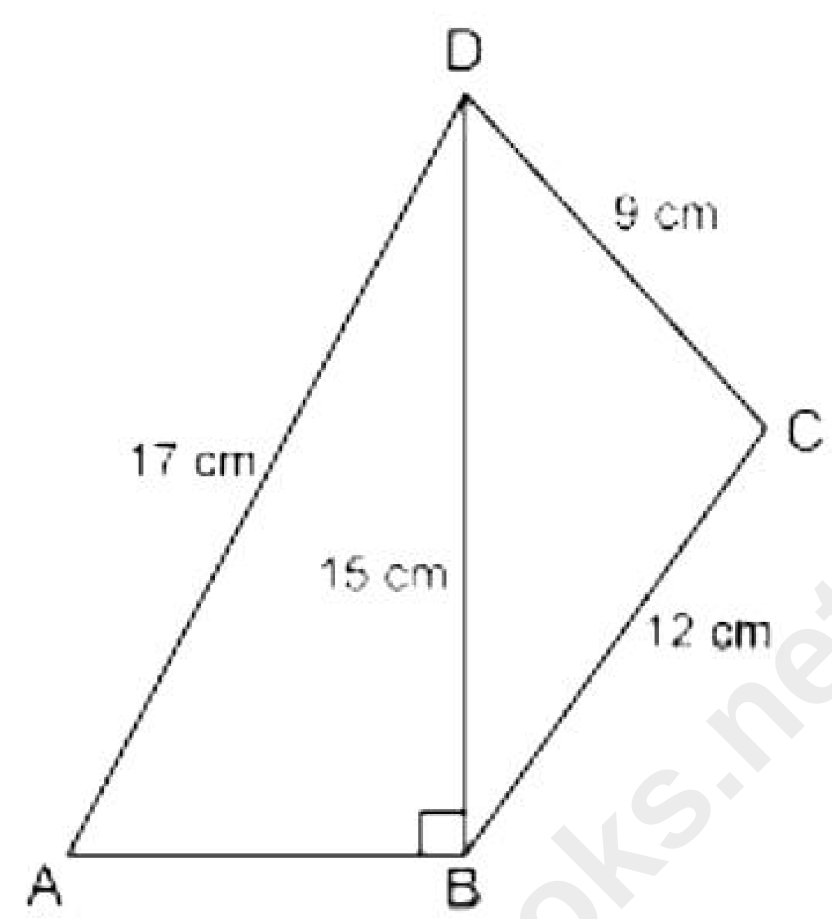
Question 7

i) If $\cot\theta = \frac{7}{8}$, evaluate [5]

(a) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(b) $\tan^2\theta$

ii) Find the perimeter and area of a quadrilateral ABCD in which BC = 12 cm, CD = 9 cm, BD = 15 cm, DA = 17 cm and ∠ABD = 90°. [5]



Question 8

i) Two opposite angles of a parallelogram are (3x – 2) and (50 – x). Find the measure of each angle of the parallelogram. [3]

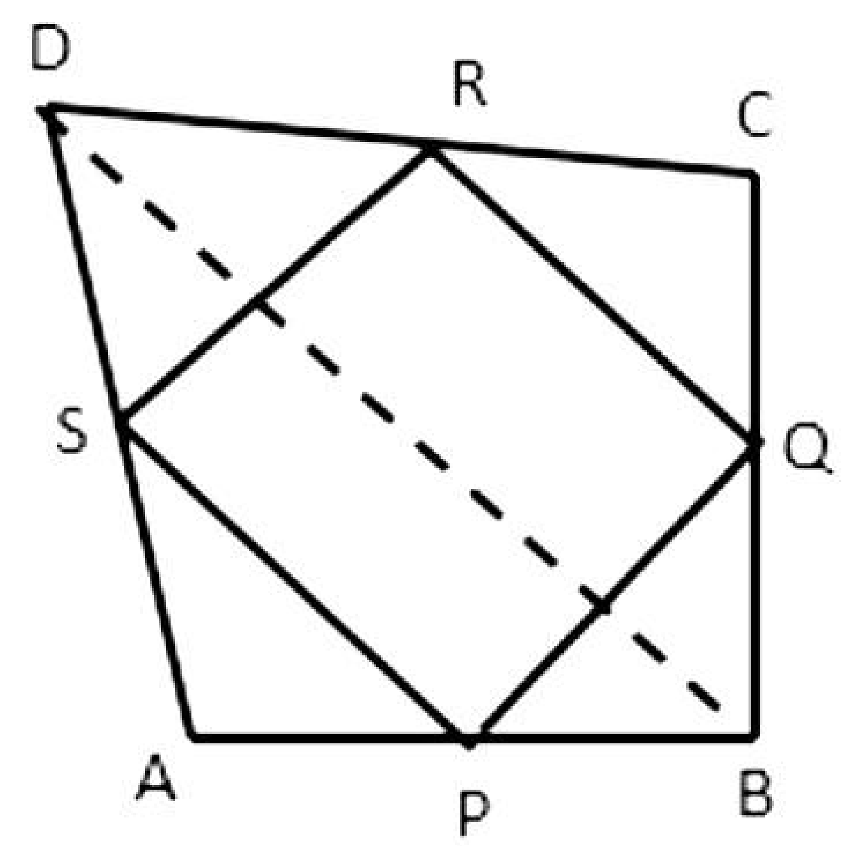
ii) Two circles with centres O and O' intersect at two points A and B. A line PQ is drawn parallel to OO' through A or B intersecting the circles at P and Q. Prove that PQ = 2OO'. [3]

iii) A godown measures 40 m × 25 m × 10 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m which can be stored in the godown. [4]

Question 9

i) The line segment joining the midpoints M and N of parallel sides AB and DC, respectively, of a trapezium ABCD is perpendicular to both sides AB and DC. Prove that AD = BC. [3]

ii) Prove that the figure obtained by joining the mid-points of the adjacent sides of a quadrilateral is a parallelogram. [3]



iii) Two sides of a triangular field are 85 m and 154 m in length and its perimeter is 324 m. Find [4]

a. the area of the field
b. the length of the perpendicular from the opposite vertex on the side measuring 154 m.

Question 10

i. An open rectangular cistern when measured from the outside is 1.35 m long, 1.08 m broad and 90 cm deep. It is made of iron which is 2.5 cm thick. Find the capacity of the cistern and the volume of the iron used. [3]

ii. Using the distance formula, show that the points (1, –1), (5, 2) and (9, 5) are collinear.[3]

iii. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region. [4]

Solution

Section A

Solution 1

i) Correct option: (d)

Explanation:

$\frac{9}{1280}=\frac{9}{2^8\times5^1}$ is terminating.

The denominators of other rational numbers are not of the form $2^n \times 5^m$ as shown below:

$\frac{13}{95}=\frac{13}{5\times19}$

$\frac{27}{405}=\frac{9}{135}=\frac{9}{5\times27}=\frac{1}{5\times3}$

$\frac{11}{524}=\frac{11}{2^2\times131}$

Hence, they are not terminating.

ii) Correct option: (b)

Explanation:

For the 1st year,

Interest = $\frac{x\times10\times1}{100}=\frac{x}{10}$

Amount $=x+\frac{x}{10}=\frac{11}{10}x$

For the 2nd year,

Interest $=\frac{\frac{11x}{10}\times12\times1}{100}=\frac{132x}{1000}$

Amount $=\frac{11x}{10}+\frac{132x}{1000}=\frac{1100+132}{1000}x=1.232x$

Increased amount = 1.232x – x = 0.232x

% increase = $\frac{0.232x}{x}\times100=23.2\%$

iii) Correct option: (c)

Explanation:

Given: $x^2 - 4x + 1 = 0$

$\Rightarrow x-4+\frac{1}{x}=0$

$\Rightarrow x+\frac{1}{x}=4$

Now,

$$x^2+\frac{1}{x^2}+1=x^2+\frac{1}{x^2}+2-2+1$$
$$=\left(x+\frac{1}{x}\right)^2-1$$
$$=15$$

iv) Correct option: (d)

Explanation:

$$\frac{0.8\times0.8\times0.8+0.5\times0.5\times0.5}{0.8\times0.8-0.8\times0.5+0.5\times0.5}=\frac{(0.8)^3+(0.5)^3}{(0.8)^2-0.8\times0.5+(0.5)^2}$$
$$=\frac{(0.8+0.5)\left[(0.8)^2-0.8\times0.5+(0.5)^2\right]}{(0.8)^2-0.8\times0.5+(0.5)^2}$$
$$=0.8+0.5$$
$$=1.3$$

v) Correct option: (b)

Explanation:

$x + y = 8$ ... (I)

$3x – 5y = 0$ ... (II)

$\Rightarrow 5x + 5y = 40$ ... (III)[Multiplying (i) by 5]

Adding (II) & (III), we get

$8x = 40$

$\Rightarrow x = 5$ and $y = 3$

vi) Correct option: (d)

Explanation:

$$4^{2x}=\frac{1}{32}$$
$$\Rightarrow\left(2^2\right)^{2x}=\frac{1}{2^5}$$
$$\Rightarrow 2^{4x}=2^{-5}$$
$$\Rightarrow 4x=-5$$
$$\Rightarrow x=-1.25$$

vii) Correct option: (d)

Explanation:

In ΔPQS and ΔRQS,

PQ = RQ ... (Given)

PS = RS ... (Given)

QS = QS ... (Common side)

⇒ ΔPQS ≅ ΔRQS ... By SSS congruency criteria

$\Rightarrow \angle PQS = \angle RQS$ ... C.P.C.T

Thus, QS bisects $\angle PQR$.

viii) Correct option: (c)

Explanation:

In ΔABC, we have

$AB^2 = BC^2 + AC^2$ ... (I)

In ΔCDE, we have

$DE^2 = CD^2 + EC^2$ ... (II)

Now,

$AE^2 + BD^2 = AC^2 + EC^2 + BC^2 + CD^2$

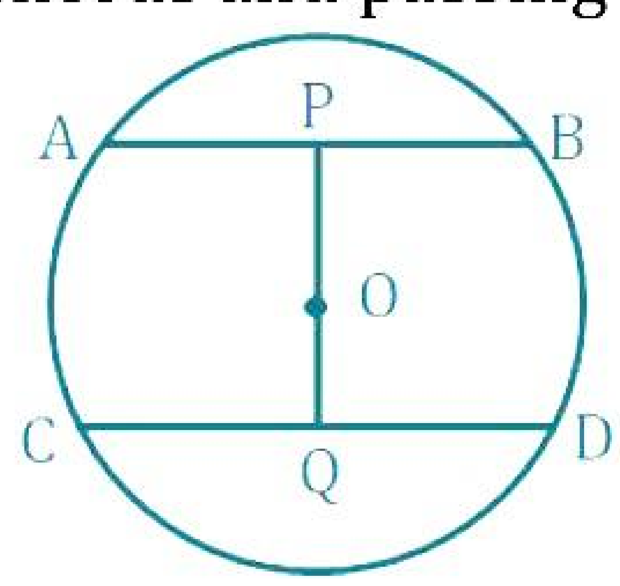
$= AC^2 + BC^2 + EC^2 + CD^2$

$= AB^2 + DE^2$

ix) Correct option: (a)

Explanation:

Let AB and CD be the two chords and PQ be the line joining the midpoints of the two chords and passing through the centre.



As OP bisects the chord and passes through the centre.

$\Rightarrow OP \perp AB$

Similarly, $OQ \perp CD$

$\Rightarrow \angle OPB = 90°$ & $\angle OQC = 90°$

Now, PQ is the transversal and the alternate angles are equal to each other.

Thus, AB || CD.

Hence, the chords are parallel to each other.

x) Correct option: (c)

Explanation:

The class marks are: 12, 16, 20, 24, 28, ...

Difference between every two consecutive class marks is 4.

So, the class width will be 4/2 = 2.

The classes are as follows:

10 – 14, 14 – 18, 18 – 22, 22 – 26, 26 – 30, ...

Thus, the class corresponding to 20 is 18 – 22.

xi) Correct option: (a)

Explanation:

Observations are: x – 2, x, x + 2, x + 4

$$\text{Mean} = \frac{x-2+x+x+2+x+4}{4} = \frac{4x+4}{4} = x+1$$

xii) Correct option: (d)

Explanation:

Volume of water flows every second = 12 × 20 cm^3 = 240 cm^3

Volume of water flows in 1 minute = 240 × 60 cm^3 = 14400 cm^3 = 0.0144 m^3

xiii) Correct option: (a)

Explanation:

3 sin A – 4 $\sin^3$A = 3 sin 30° – 4 $\sin^3$ (30°)

$$= 3\times\frac{1}{2} - 4\times\left(\frac{1}{2}\right)^3$$

$$= \frac{3}{2} - \frac{1}{2}$$

$$= 1$$

Now, sin 3A = sin 90° = 1

Hence, 3 sin A – 4 $\sin^3$A = sin 3A.

Thus, both the statements are true.

xiv) Correct option: (d)

Explanation:

Solution of the equation 5x – 2 = 8 is given by

5x = 8 + 2 = 10

⇒ x = 2

Solution of the equation 10 – 3y = 1 is given by

10 – 1 = 3y

⇒ y = 3

Abscissa = 2 and ordinate = 3.

Thus, the required point is (2, 3).

xv) Correct option: (b)

Explanation:

The statement given in reason is correct and hence, reason is true.

Let P(x, y), A(3, 6) and B(–3, 4).

PA = PB … (Given)

Distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$(3-x)^2+(6-y)^2=(-3-x)^2+(4-y)^2$$

$\Rightarrow 9 - 6x + x^2 + 36 - 12y + y^2 = 9 + 6x + x^2 + 16 - 8y + y^2$
$\Rightarrow -6x + 36 - 12y = 6x + 16 - 8y$
$\Rightarrow 6x + 6x - 8y + 12y = 36 - 16$
$\Rightarrow 12x + 4y = 20$
$\Rightarrow 3x + y = 5$
Thus, the assertion is false.

Solution 2

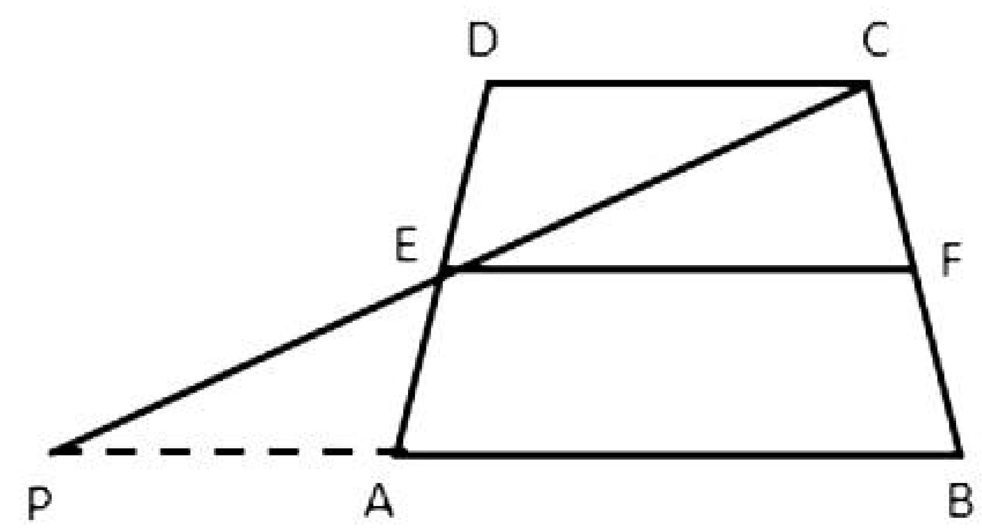
i) Two years ago, the population of a village was 4000.
So, P = 4000
During the next year, it increased by 6%.
$4000 + 6\% \text{ of } 4000 = 4000 + \frac{6}{100} \times 4000 = 4000 + 240 = 4240$
But due to an epidemic, it decreased by 5% in the following year.
$4240 - 5\% \text{ of } 4240 = 4240 - \frac{5}{100} \times 4240 = 4240 - 212 = 4028$
So, now the population is 4028.

ii) Since the triangle is equilateral and all sides of an equilateral triangle are equal,
$\Rightarrow 6x + 5y = 7x + 3y + 1 = 2(x + 6y - 1)$
Consider
$6x + 5y = 7x + 3y + 1$
$\Rightarrow -x + 2y = 1$
$\Rightarrow x = 2y - 1$ ....(i)
Now, consider
$6x + 5y = 2(x + 6y - 1)$
$\Rightarrow 6x + 5y = 2x + 12y - 2$
$\Rightarrow 4x - 7y = -2$ ... (ii)
Putting the value of x from (i) in equation (ii), we get
$4x - 7y = -2$
$\Rightarrow 4(2y - 1) - 7y = -2$
$\Rightarrow 8y - 4 - 7y = -2$
$\Rightarrow y = 2$
Putting y = 2 in equation (i), we get
$x = 2y - 1 = 2(2) - 1 = 4 - 1 = 3$
The sides of the triangle are (6x + 5y) cm, (7x + 3y + 1) cm and 2(x + 6y – 1) cm.
$6x + 5y = 6 \times 3 + 5 \times 2 = 18 + 10 = 28$ cm ... (since x = 3 and y = 2)
Therefore, the length of each side of an equilateral triangle is 28 cm.

iii) Given: ABCD is a trapezium in which AB ∥ DC. E and F are mid-points of non-parallel sides AD and BC, respectively.

To prove: EF ∥ AB and EF = ½ (AB + DC)

Construction: Join CE and produce it to meet BA at P.



Proof:

In ΔDEC and ΔEPA,

EA = ED (E is the mid-point of AD)

∠DEC = ∠AEP (vertically opposite angles)

∠DCE = ∠EPA (alternate angles, PA ∥ DC)

⇒ ΔDEC ≅ ΔEPA (By AAS congruence)

⇒ PE = EC (i) (c.p.c.t.)

⇒ PA = DC (ii) (c.p.c.t.)

In ΔCPB,

E is the mid-point of PC. [From (i)]

F is the mid-point of BC. (given)

⇒ EF ∥ PB

⇒ EF ∥ AB

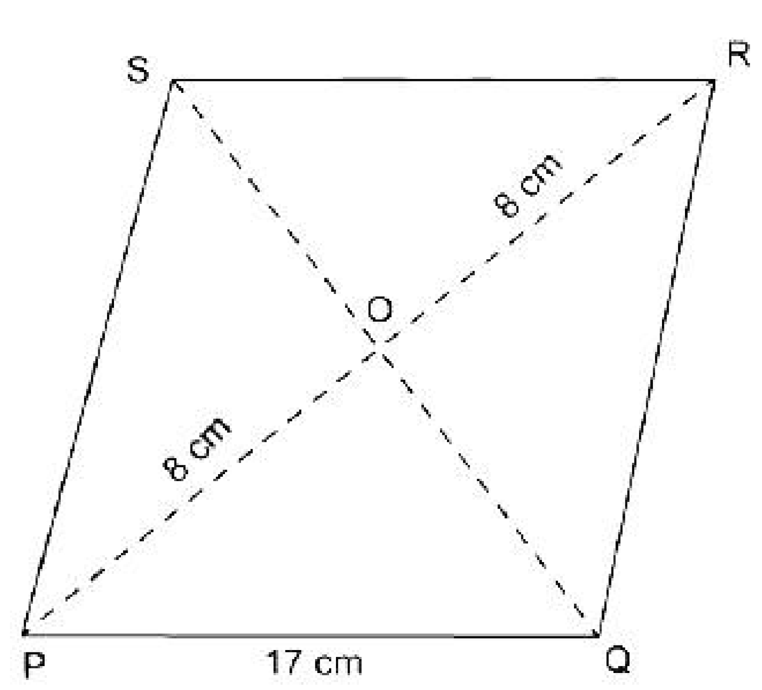
And, EF = ½ PB

⇒ EF = ½ (PA + AB)

⇒ EF = ½ (AB + DC) [from (ii)]

Solution 3

i)



Construction: Draw PR and QS such that they intersect each other at point O.

In ▱PQRS, PR and QS are the diagonals of a rhombus intersecting each other at point O.

Diagonals of a rhombus bisect each other at right angles.

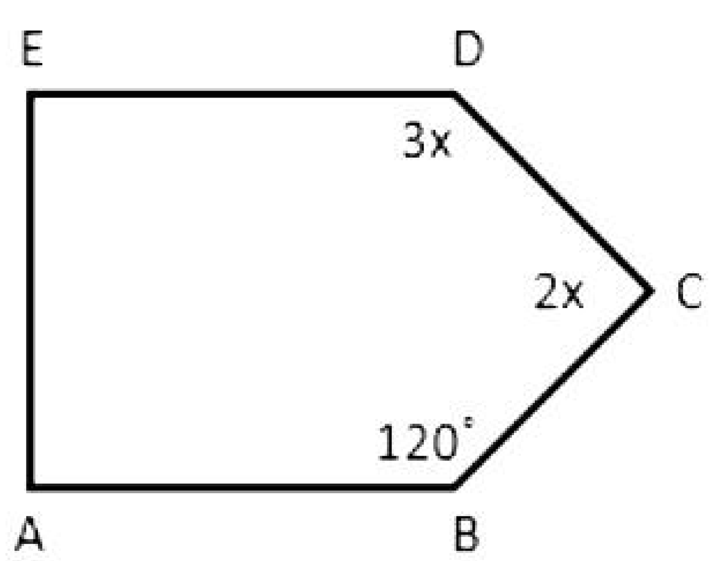
⇒ PR ⊥ QS

In ΔPOQ, ∠POQ = 90° (since PR ⊥ QS)

Then, by Pythagoras' theorem, we get

$PQ^2 = OP^2 + OQ^2$
$\Rightarrow OQ^2 = PQ^2 - OP^2$
$= 17^2 - 8^2 \quad (OP = ½ PR)$
$= 289 - 64$
$= 225$
$\Rightarrow OQ = 15$ cm
Diagonals of a rhombus bisect each other.
$\Rightarrow QS = 2OQ = 2 \times 15 = 30$ cm
Area of rhombus PQRS = $\frac{1}{2} \times PQ \times QS = \frac{1}{2} \times 16 \times 30 = 240 \text{ cm}^2$
Therefore, the area of the rhombus is 240 cm^2.

ii)



AB || ED
$\Rightarrow \angle A + \angle E = 180°$
$\angle C : \angle D = 2 : 3$
$\Rightarrow \angle C = 2x$ and $\angle D = 3x$
$\angle A + \angle B + \angle C + \angle D + \angle E = (n - 2) \times 180° = (5 - 2) \times 180°$
$\Rightarrow \angle A + \angle B + \angle C + \angle D + \angle E = 540°$
$\Rightarrow (\angle A + \angle E) + \angle B + \angle C + \angle D = 540°$
$\Rightarrow 180° + 120° + 2x + 3x = 540°$
$\Rightarrow 5x = 240°$
$\Rightarrow x = 48°$
$\angle C = 2x$ and $\angle D = 3x$
$\Rightarrow \angle C = 2 \times 48°$ and $\angle D = 3 \times 48°$
$\Rightarrow \angle C = 96°$ and $\angle D = 144°$

iii)

A. $x^3 - 3x^2 + 3x + 7 = (x^3 - 3x^2 + 3x - 1) + 8$
$= (x - 1)^3 + (2)^3$
$= [(x - 1) + 2][(x - 1)^2 - 2(x - 1) + 2^2]$
$= (x + 1)(x^2 - 2x + 1 - 2x + 2 + 4)$
$= (x + 1)(x^2 - 4x + 7)$

B. Since $12 + 2 = 14$ and $12 \times 2 = 24$
$\therefore 3x^2 + 14x + 8 = 3x^2 + 12x + 2x + 8$
$= 3x(x + 4) + 2(x + 4)$
$= (x + 4)(3x + 2)$

Section B

Solution 4

i)

We know that $x^2+y^2 = (x+y)^2 - 2xy$ ... (i)

$$x+y=\frac{1}{5+2\sqrt{6}}+\frac{1}{5-2\sqrt{6}}$$

$$=\frac{(5-2\sqrt{6})+(5+2\sqrt{6})}{(5+2\sqrt{6})(5-2\sqrt{6})}$$

$$=\frac{10}{25-24}$$

$$=10$$

$$xy=\frac{1}{5+2\sqrt{6}}\times\frac{1}{5-2\sqrt{6}}$$

$$=\frac{1}{25-24}$$

$$=1$$

Sustituting the values in (i), we get

$$x^2+y^2 = (x+y)^2-2xy=100-2=98$$

ii) For the 1st year:

P = 4000, R = 10% and N = 1

$$I=\frac{P\times R\times N}{100}=\frac{4000\times 10\times 1}{100}=400$$

Amount = P + I = 4000 + 400 = Rs. 4400

For the 2nd year:

P = 4000 + 4400 = 8400, R = 10% and N = 1

$$I=\frac{P\times R\times N}{100}=\frac{8400\times 10\times 1}{100}=840$$

Amount = P + I = 8400 + 840 = Rs. 9240

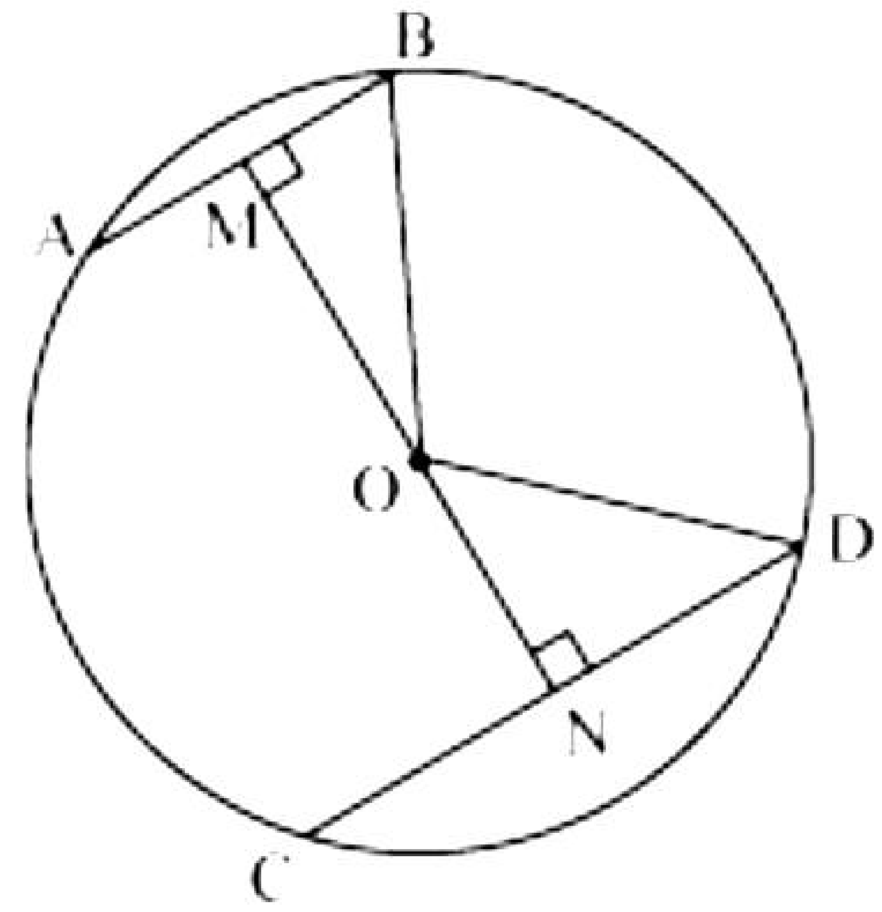
For the 3rd year:

P = 4000 + 9240 = 13240, R = 10% and N = 1

$$I=\frac{P\times R\times N}{100}=\frac{13240\times 10\times 1}{100}=1324$$

Amount = P + I = 13240 + 1324 = Rs. 14564

iii) Draw OM ⊥ AB and ON ⊥ CD. Join OB and OD



$BM = \frac{AB}{2} = \frac{5}{2}$ ... (Perpendicular from the centre bisects the chord)

$ND = \frac{CD}{2} = \frac{11}{2}$ ... (Perpendicular from the centre bisects the chord)

Let ON be x, so OM will be 6 – x.

In ΔMOB,

$OM^2 + MB^2 = OB^2$

$(6-x)^2 + \left(\frac{5}{2}\right)^2 = OB^2$

$36 + x^2 - 12x + \frac{25}{4} = OB^2$ .... (1)

In ΔNOD,

$ON^2 + ND^2 = OD^2$

$x^2 + \left(\frac{11}{2}\right)^2 = OD^2$

$x^2 + \frac{121}{4} = OD^2$ .... (2)

We have OB = OD ... (Radii of the same circle)

So, from equations (1) and (2),

$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$

$12x = 36 + \frac{25}{4} - \frac{121}{4}$

$= \frac{144 + 25 - 121}{4}$

$= \frac{48}{4} = 12$

$\Rightarrow x = 1$

From equation (2),

$$(1)^2 + \left(\frac{121}{4}\right) = OD^2$$

$$OD^2 = 1 + \frac{121}{4} = \frac{125}{4}$$

$$OD = \frac{5}{2}\sqrt{5}$$

So, the radius of the circle is $\frac{5}{2}\sqrt{5}$ cm.

Question 5

i) We know that $(x + y)^2 = x^2 + y^2 + 2xy$

Here, $x = 3a$ and $y = 4b$.

$$\therefore (3a + 4b)^2 = (3a)^2 + (4b)^2 + 2 \times 3a \times 4b$$
$$= 9a^2 + 16b^2 + 24ab$$

$$\Rightarrow 9a^2 + 16b^2 + 24ab = (3a + 4b)^2$$
$$\Rightarrow 9a^2 + 16b^2 = (3a + 4b)^2 - 24ab$$
$$= (16)^2 - 24 \times 4$$
$$= 256 - 96 = 160$$

ii) $4(2a - 3)^2 - 3(2a - 3)(a - 1) - 7(a - 1)^2$

Putting $(2a - 3) = x$ and $(a - 1) = y$,

$4x^2 - 3xy - 7y^2$

$= 4x^2 + 4xy - 7xy - 7y^2$

$= 4x(x + y) - 7y(x + y)$

$= (x + y)(4x - 7y)$

$= [(2a - 3) + (a - 1)][4(2a - 3) - 7(a - 1)]$

$= (3a - 4)(8a - 12 - 7a + 7)$

$= (3a - 4)(a - 5)$

iii) Here, mean of 9 observations = 35

We know that, $\text{Mean} = \frac{\text{sum of all the observations}}{\text{number of observations}}$

$\Rightarrow$ Sum of all the observations = Mean × number of observations

$= 35 \times 9$

$= 315$

It was detected that an observation 81 was misread as 18.

$\Rightarrow$ Sum of all the observations (correct value) $= 315 - 18 + 81 = 378$

$\therefore$ Correct mean $= \dfrac{\text{sum of all the observations (Correct value)}}{\text{number of observations}}$

$= \dfrac{378}{9}$

$= 42$

Therefore, the correct mean is 42.

Solution 6

i) Comparing $a_1x + b_1y + c_1 = 0$ with $2x - 5y + 8 = 0$
and $a_2x + b_2y + c_2 = 0$ with $x - 4y + 7 = 0$, we get
$a_1 = 2$, $b_1 = -5$, $c_1 = 8$ and $a_2 = 1$, $b_2 = -4$, $c_2 = 7$

We know that,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \text{ and } y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Substituting the values, we get

$$x = \frac{(-5)\times 7 - (-4)\times 8}{2\times(-4) - 1\times(-5)} \text{ and } y = \frac{8\times 1 - 7\times 2}{2\times(-4) - 1\times(-5)}$$

$$\Rightarrow x = \frac{-35 + 32}{-8 + 5} \text{ and } y = \frac{8 - 14}{-8 + 5}$$

$$\Rightarrow x = \frac{-3}{-3} \text{ and } y = \frac{-6}{-3}$$

$$\Rightarrow x = 1 \text{ and } y = 2$$

Hence, the solution is $x = 1$ and $y = 2$.

ii)

$$\frac{5^{n+3} - 6\times 5^{n+1}}{9\times 5^n - 2^2\times 5^n}$$

$$= \frac{5^n\times 5^3 - 6\times 5^n\times 5^1}{9\times 5^n - 2^2\times 5^n}$$

$$= \frac{5^n \times 5\left(5^2 - 6\right)}{5^n\left(9 - 2^2\right)}$$

$$= \frac{5\left(25 - 6\right)}{9 - 4}$$

$$= \frac{5 \times 19}{5} = 19$$

iii) The given frequency distribution is as below:

Age (in years)	10–20	20–30	30–40	40–50	50–60	60–70
Number of patients	2	5	12	19	9	4

STEPS:

The class mark of a class interval $= \frac{\text{lower limit+ upper limit}}{2}$

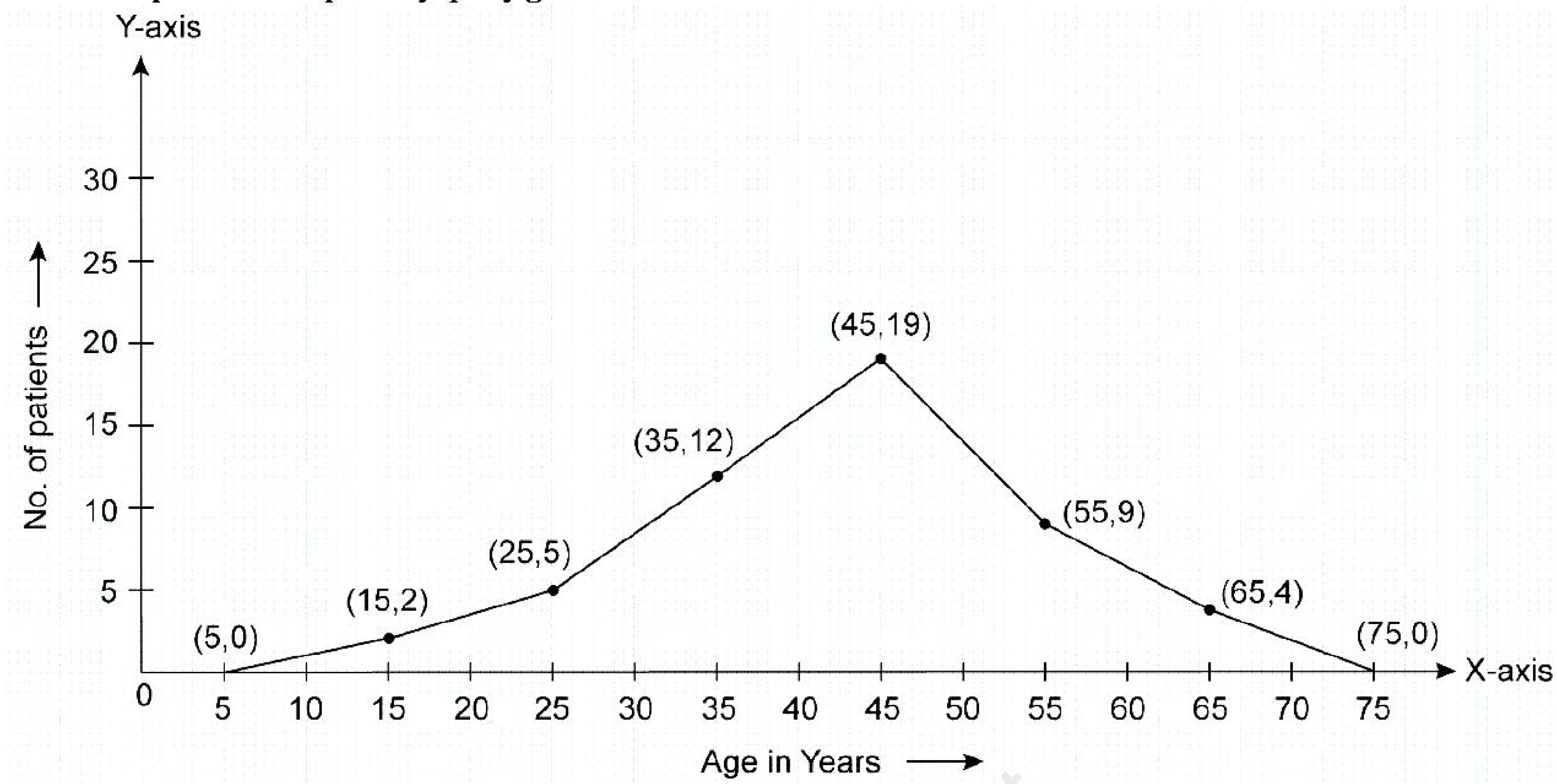
The frequency distribution table with class marks is given below:

Class Intervals	Class Marks	Frequency
0–10	5	0
10–20	15	2
20–30	25	5
30–40	35	12
40–50	45	19
50–60	55	9
60–70	65	4
70–80	75	0

In the above table, we have taken imaginary class intervals 0–10 at beginning and 70–80 at the end, each with frequency zero.
On a graph paper, take class marks along the x-axis and the corresponding frequencies along the y-axis.
On this graph paper, plot the points (5, 0), (15, 2) … (65, 4) and (75, 0).
Draw line segments joining the consecutive points marked in step (5).

Note: Join the class mark of the class interval just before the first class and the class mark of the class interval just after the last class. This completes the required **Frequency Polygon**.

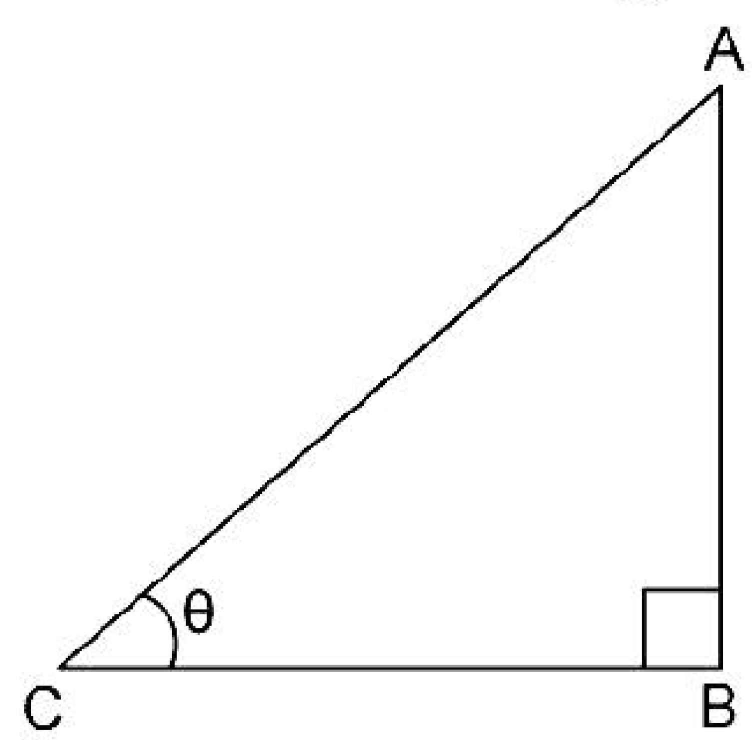
∴ The required frequency polygon will be



Solution 7

i)

Let us consider a right-angled ΔABC right-angled at point B.



$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{BC}{AB} = \frac{7}{8}$

If BC is 7K, AB will be 8K, where K is a positive integer.

Now applying Pythagoras' theorem in ΔABC,

$AC^2 = AB^2 + BC^2$

$= (8K)^2 + (7K)^2$

$= 64K^2 + 49K^2$

$= 113K^2$

$\therefore AC = \sqrt{113}\ K$

$$\sin\theta = \frac{\text{Side opposite to }\theta}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{8K}{\sqrt{113}\,K} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{\text{Side adjacent to }\theta}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7K}{\sqrt{113}\,K} = \frac{7}{\sqrt{113}}$$

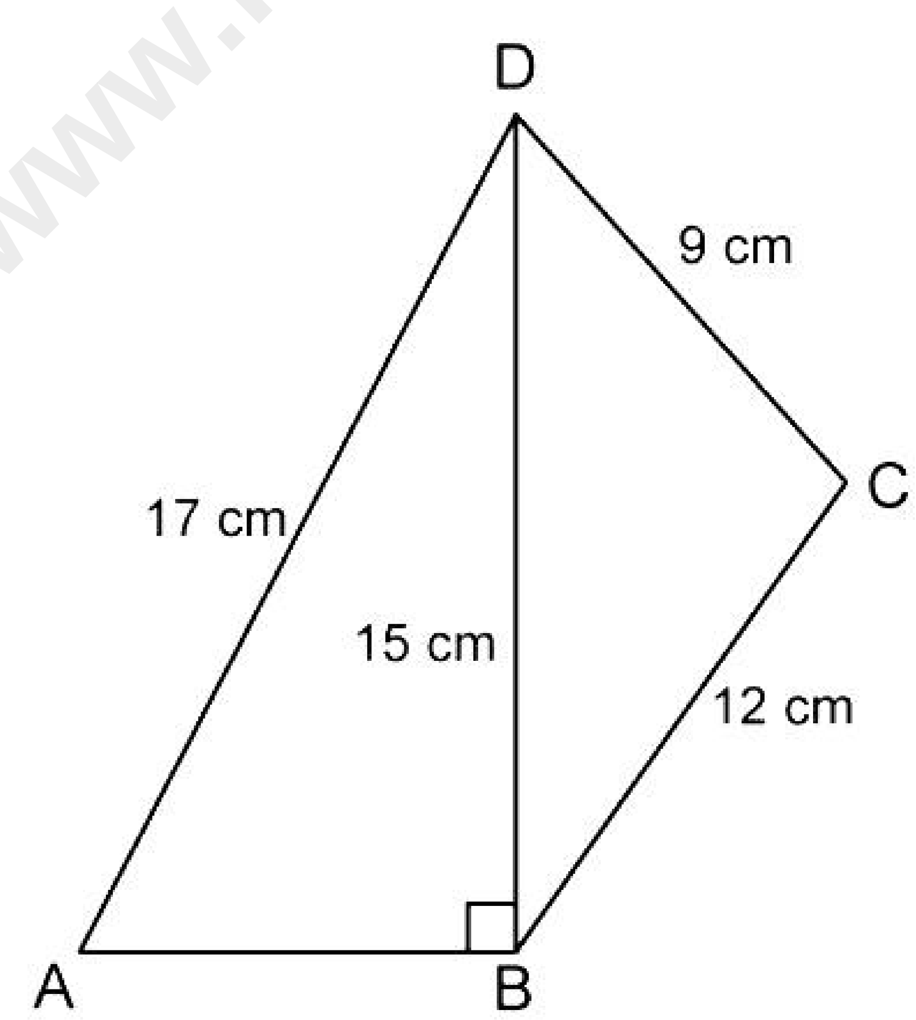
(a) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$$

(b) $\tan^2\theta = \left(\frac{1}{\cot\theta}\right)^2 = \left(\frac{8}{7}\right)^2 = \frac{64}{49}$

ii)



In ΔABD, by Pythagoras' theorem,

$AB^2 = AD^2 - BD^2 = 17^2 - 15^2 = 289 - 225 = 64\text{ cm}^2$

$\Rightarrow AB = 8$ cm

$\therefore$ Perimeter of quadrilateral ABCD = AB + BC + CD + AD

$= 8 + 12 + 9 + 17$

$= 46$ cm

Now,

$$A(\Delta ABD) = \frac{1}{2} \times AB \times BD = \frac{1}{2} \times 8 \times 15 = 60\ cm^2$$

In ΔBCD, BC = 12 cm, CD = 9 cm and BD = 15 cm

Let a = 12 cm, b = 9 cm and c = 15 cm

$$\text{Semi-perimeter, } s = \frac{a+b+c}{2} = \frac{12+9+15}{2}$$

$$= \frac{36}{2} = 18\ cm$$

$$\therefore \text{ Area of } \Delta BCD = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{18(18-12)(18-9)(18-15)}$$

$$= \sqrt{18 \times 6 \times 9 \times 3}$$

$$= \sqrt{6 \times 3 \times 6 \times 9 \times 3}$$

$$= 6 \times 3 \times 3$$

$$= 54\ cm^2$$

Thus, the area of quadrilateral ABCD = A(ΔABD) + A(ΔBCD)

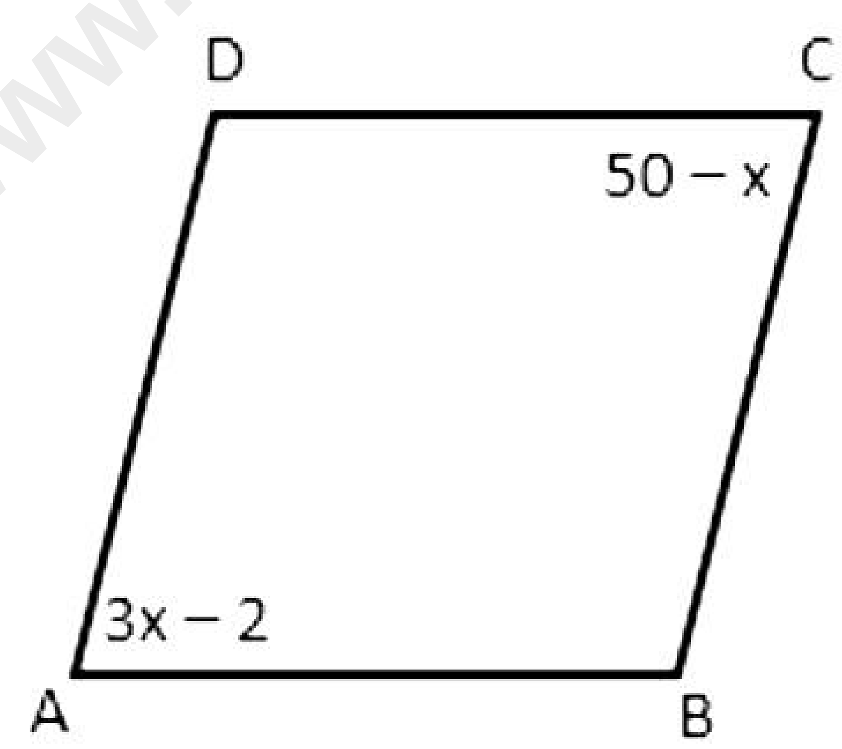
= (60 + 54) cm^2

= 114 cm^2

Solution 8

i)

ABCD is a parallelogram.



∠A = ∠C ... (∵ Opposite angles of a parallelogram)

⇒ 3x – 2 = 50 – x

⇒ 4x = 52

⇒ x = 13

∴ The opposite angles are ∠A = 3x – 2 = 37° and ∠C = 50 – x = 37°

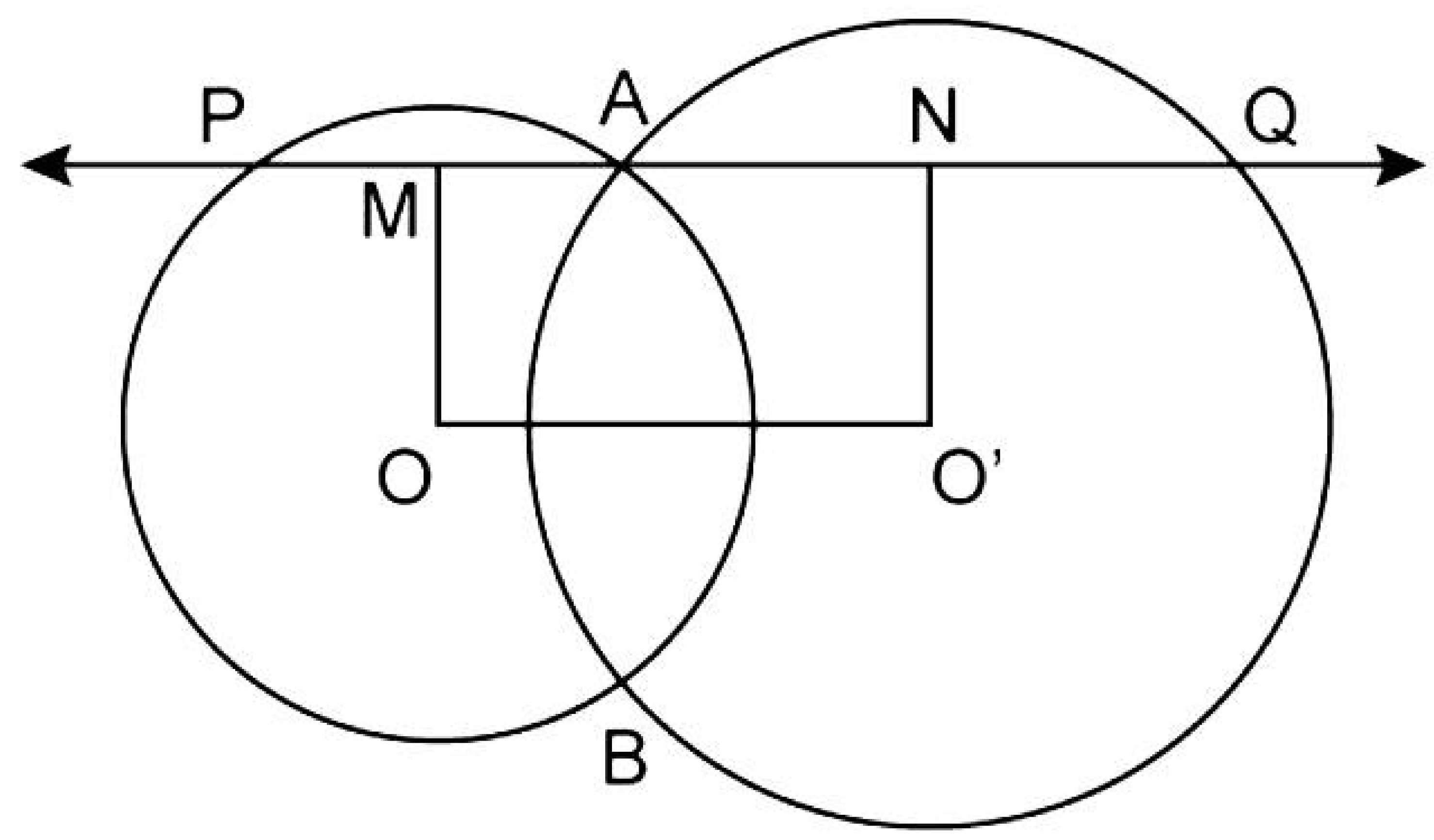
∠A + ∠B = 180° ... (∵ Adjacent angles of a parallelogram are supplementary)

⇒ 37° + ∠B = 180°

⇒ ∠B = 143° = ∠D

Hence, the angles of a parallelogram are 37°, 143°, 37°, 143°.

ii)



Draw OM ⊥ PQ and O'N ⊥ PQ.

⇒ OM ⊥ AP

⇒ AM = PM (perpendicular from the centre of a circle bisects the chord)

⇒ AP = 2AM ... (i)

And O'N ⊥ PQ

⇒ O'N ⊥ AQ

⇒ AN = QN (perpendicular from the centre of a circle bisects the chord)

⇒ AQ = 2AN ... (ii)

Now,

PQ = AP + AQ

⇒ PQ = 2AM + 2AN ... [From (i) and (ii)]

⇒ PQ = 2(AM + AN)

⇒ PQ = 2MN

⇒ PQ = 2OO' (since MNO'O is a rectangle)

iii)

Length (l_1) of the godown = 40 m

Breadth (b_1) of the godown = 25 m

Height (h_1) of the godown = 10 m

Volume of godown = $l_1 \times b_1 \times h_1 = (40 \times 25 \times 10)\ m^3 = 10000\ m^3$

Length (l_2) of a wooden crate = 1.5 m

Breadth (b_2) of a wooden crate = 1.25 m

Height (h_2) of a wooden crate = 0.5 m

Volume of a wooden crate = $l_2 \times b_2 \times h_2 = (1.5 \times 1.25 \times 0.5)\ m^3 = 0.9375\ m^3$

Let n wooden crates be stored in the godown.

Volume of n wooden crates = volume of the godown

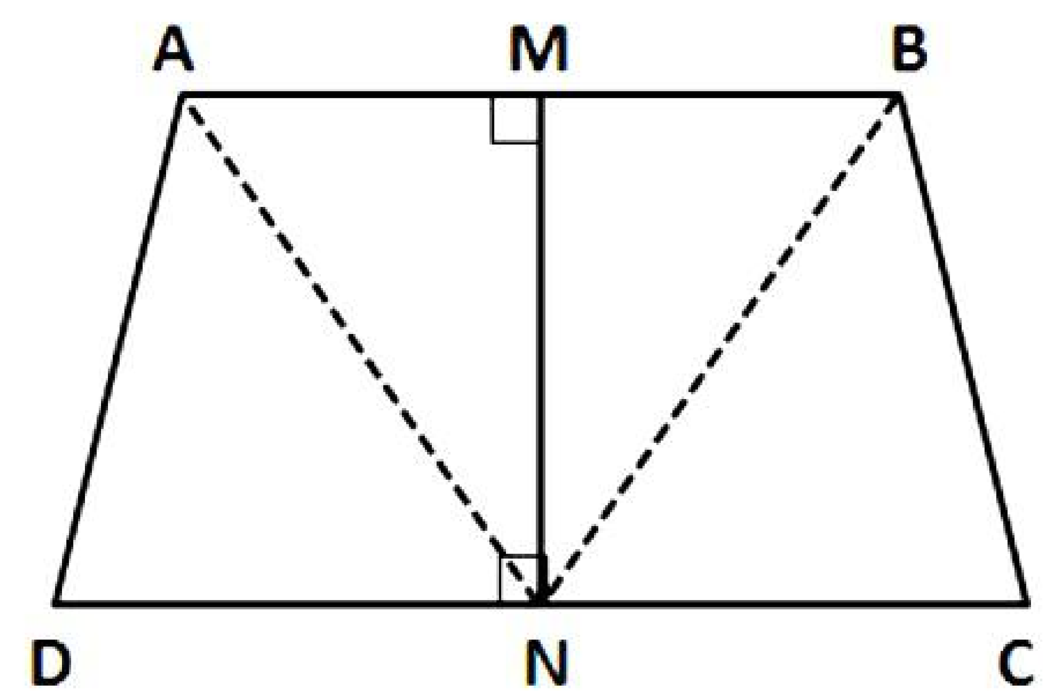
$0.9375 \times n = 10000$

$$n = \frac{10000}{0.9375} = 10666.66$$

Thus, 10666 wooden crates can be stored in the godown.

Solution 9

i) Construction: Join AN and BN.



In ΔANM and ΔBNM,

AM = BM ... (M is the mid-point of AB)

$\angle AMN = \angle BMN$ ... (Each 90°)

MN = MN ... (common)

$\therefore \Delta ANM \cong \Delta BNM$ (by SAS congruence)

$\Rightarrow$ AN = BN ... (c.p.c.t.) ... (i)

And $\angle ANM = \angle BNM$ ... (c.p.c.t.)

$\Rightarrow 90° - \angle ANM = 90° - \angle BNM$

$\Rightarrow \angle AND = \angle BNC$ ... (ii)

In ΔAND and ΔBNC,

AN = BN ... [From (i)]

$\angle AND = \angle BNC$ ... [From (ii)]

DN = CN ... (N is the mid-point of DC)

$\therefore \Delta AND \cong \Delta BNC$ ... (by SAS congruence)

$\Rightarrow$ AD = BC ... (c.p.c.t.)

ii) Given: P, Q, R and S are the mid-points of the sides AB, BC, CD and DA, respectively, of quadrilateral ABCD.

To prove: PQRS is a parallelogram.

Construction: Join BD

Proof:

In ΔABD,

PS || BD and PS = ½ BD ... (i) (mid-point theorem)

In ΔBCD,

QR || BD and QR = ½ BD ... (ii) (mid-point theorem)

$\Rightarrow$ PS || QR and PS = QR ... From (i) and (ii)

$\Rightarrow$ PQRS is a parallelogram.

iii)One side of a triangular field = 85 m
Second side of a triangular field = 154 m
Let the third side of a triangular field be x m.
Perimeter = 324 m
Then, 85 + 154 + x = 324
⇒ x = 85 m = third side of a triangle.
Let a = 85 m, b = 154 m and c = 85 m

$$\text{Now, semiperimeter}(S) = \frac{1}{2}(a+b+c)$$
$$= \left(\frac{85+154+85}{2}\right)$$
$$= \frac{324}{2} = 162 \text{ m}$$

$$\therefore \text{ Area of the triangle} = \sqrt{S(S-a)(S-b)(S-c)}$$
$$= \sqrt{162(162-85)(162-154)(162-85)}$$
$$= \sqrt{162\times77\times8\times77}$$
$$= \sqrt{2\times9\times9\times7\times11\times2\times2\times2\times7\times11}$$
$$= \sqrt{11\times11\times9\times9\times7\times7\times2\times2\times2\times2}$$
$$= 11\times9\times7\times2\times2$$

$$\therefore \text{ Area of triangle} = 2772 \text{ m}^2$$

$$\text{Also, area of triangle} = \frac{1}{2}\times\text{base}\times\text{height}$$

$$\Rightarrow 2772 = \frac{1}{2}\times154\times h$$
$$\Rightarrow 77h = 2772$$
$$\therefore h = \frac{2772}{77} = 36 \text{ m}$$

Therefore, the length of the perpendicular from the opposite vertex to the side measuring 154 m is 36 m.

Solution 10

i) External length of the cistern = 1.35 m = 135 cm
External breadth of the cistern = 1.08 m =108 cm
External height of the cistern = 90 cm
∴ External volume of the cistern = (135 × 108 × 90) cm^3

= 1312200 cm^3

Internal length of the cistern = (135 – 2 × 2.5) cm

= (135 – 5) cm = 130 cm

Internal breadth of the cistern = (108 – 2 × 2.5) cm

= (108 – 5) cm = 103 cm

Internal height of the cistern = (90 – 2.5) cm = 87.5 cm

Therefore,

Capacity of the cistern = Internal volume of the cistern

= (130 × 103 × 87.5) cm^3

= 1171625 cm^3

Volume of the iron used = External volume of the cistern – Internal volume of the cistern

= (1312200 – 1171625) cm^3

= 140575 cm^3

ii) Let the points be A(1, –1), B(5, 2) and C(9, 5).

Distance between the given points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(5-1)^2 + (2+1)^2} = \sqrt{16+9} = \sqrt{25} = 5$ units

$BC = \sqrt{(9-5)^2 + (5-2)^2} = \sqrt{16+9} = \sqrt{25} = 5$ units

$AC = \sqrt{(9-1)^2 + (5+1)^2} = \sqrt{64+36} = \sqrt{100} = 10$ units

Here, AB + BC = 5 + 5 = 10 = AC

⇒ The points A(1, –1), B(5, 2) and C(9, 5) are collinear.

iii)

A. x – y + 1 = 0

⇒ y = x + 1

When x = 0, y = 1

When x = 1, y = 2

When x = 2, y = 3

x	0	1	2
y	1	2	3

i. Plot the points (0, 1), (1, 2) and (2, 3).

ii. Draw a straight line AB passing through the plotted points.

B. $3x + 2y - 12 = 0 \Rightarrow y = \frac{12-3x}{2}$

When $x = 0$, $y = 6$

When $y = 2$, $y = 3$

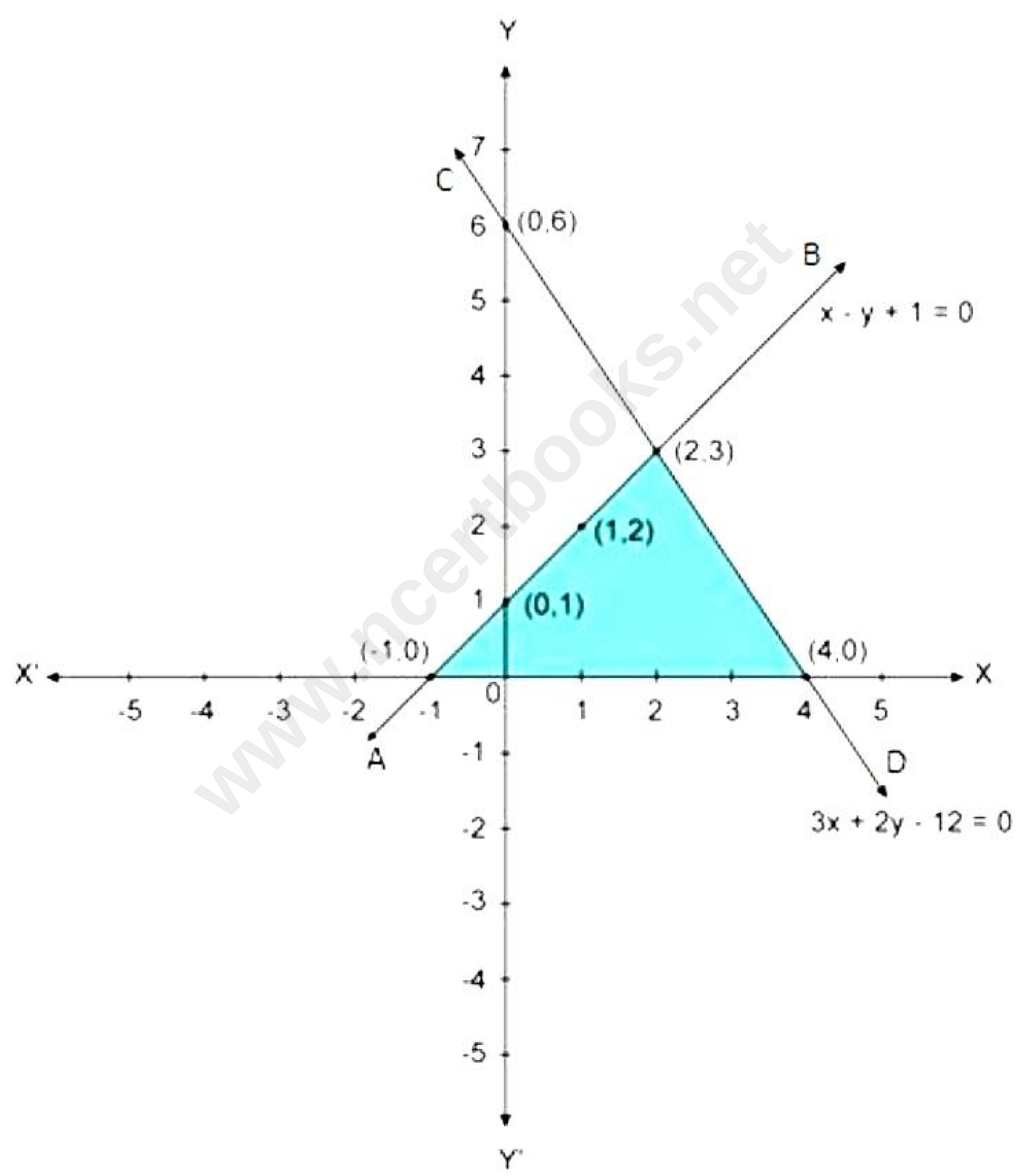
When $x = 4$, $y = 0$

x	0	2	4
y	6	3	0

i. Plot the points (0, 6), (2, 3) and (4, 0).

ii. Draw a straight line CD passing through the plotted points.

The triangle formed by the two lines and the x-axis can be shown by the shaded part as



From the graph, it can be observed that the co-ordinates of the vertices of the triangle so formed are (2, 3), (−1, 0), and (4, 0).