ICSE 2025 EXAMINATION

Sample Question Paper - 3

Mathematics

Time: 2 ½ Hours **Total Marks: 80**

General Instructions:

1. *Answers to this Paper must be written on the paper provided separately.*
2. *You will not be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.*
3. *The time given at the head of this Paper is the time allowed for writing the answers.*
4. *Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*
5. *The intended marks for questions or parts of questions are given in brackets [ ].*

Section A

(Attempt all questions from this section.)

Question 1

Choose the correct answers to the questions from the given options. [15]

i. Which of the following is not an irrational number?

(a) $5+2\sqrt{3}$

(b) $\sqrt{23-7}+\sqrt{34+2}$

(c) $\sqrt{9}+\sqrt{7}$

(d) $\sqrt{51-2}-\sqrt{8}$

ii. A person invests Rs. 20,000 for the year at a certain rate of interest compounded annually. If the amount he receives after 1 year is Rs. 22,400, find the rate of interest per annum.

(a) 10%

(b) 11%

(c) 12%

(d) 13%

iii. Expand $(x + 9)(x + 11)$.

(a) $x^2 + 20x + 99$

(b) $x^2 + 2x + 99$

(c) $12x + 99$

(d) $x^2 - 2x + 2$

iv. The factors of $2a^2 + bc - 2ab - ac$ are
(a) $(a + b)$ and $(2a - c)$
(b) $(2a - b)$ and $(c + 2a)$
(c) $(a - c)$ and $(2a - b)$
(d) $(2a - c)$ and $(a - b)$

v. Which of the following ordered pair satisfies the two equations $2x + y = 35$, $3x + 4y = 65$?
(a) (12, 11)
(b) (14, 7)
(c) (15, 5)
(d) (16, 3)

vi. The value of $a^m \times a^n$ is equal to
(a) a^{m-n}
(b) a^{mn}
(c) a^{m+n}
(d) $a^{m/n}$

vii. Which of the following in not one of the congruency criteria?
(a) SSS
(b) AAA
(c) RHS
(d) SAS

viii. In ΔABC, $\angle C = 90^{o}$. If BC = 8 cm and AC = 6 cm, then AB =
(a) 2 cm
(b) 14 cm
(c) 10 cm
(d) 12 cm

ix. A chord of length 8 cm is drawn in a circle of diameter 20 cm, its distance from the centre of the circle is
(a) 4 cm
(b) 10 cm
(c) 6 cm
(d) 8 cm

x. Which term will be the median if there are 9 numbers arranged in ascending order?

(a) 2^{nd} & 3^{rd}
(b) 3^{rd} & 4^{th}
(c) 4^{th}
(d) 5^{th}

xi. For n = 2 years,

Statement 1: When interest is compounded yearly, $A = P\left(1 + \frac{r}{100}\right)^2$.

Statement 2: When interest is compounded half-yearly, $A = P\left(1 + \frac{r}{2 \times 100}\right)^2$.

Which of the following is valid?
(a) Both the statements are true.
(b) Both the statements are false.
(c) Statement 1 is true, and Statement 2 is false.
(d) Statement 1 is false, and Statement 2 is true.

xii. The dimensional ratio of a cuboid is 1 : 2 : 3. If its volume is 1296 cm^3, the actual dimensions of the cuboid are:
(a) 8 cm, 12 cm and 12 cm
(b) 12 cm, 8 cm and 8 cm
(c) 12 cm, 12 cm and 12 cm
(d) 6 cm, 12 cm and 18 cm

xiii. Given $\sec\theta = \frac{13}{12}$, then the value of sin θ is
(a) $\frac{12}{13}$
(b) $\frac{5}{13}$
(c) $\frac{1}{13}$
(d) 0

xiv. **Assertion (A):** Trinomial $6x^2 + 17x + 5$ is factorisable.
Reason (R): A trinomial $ax^2 + bx + c$ is factorisable if the value of $b^2 - 4ac$ is a perfect square.
(a) A is true, R is false.
(b) A is false, R is true.
(c) Both A and R are true, and R is the correct reason for A.
(d) Both A and R are true, and R is the incorrect reason for A.

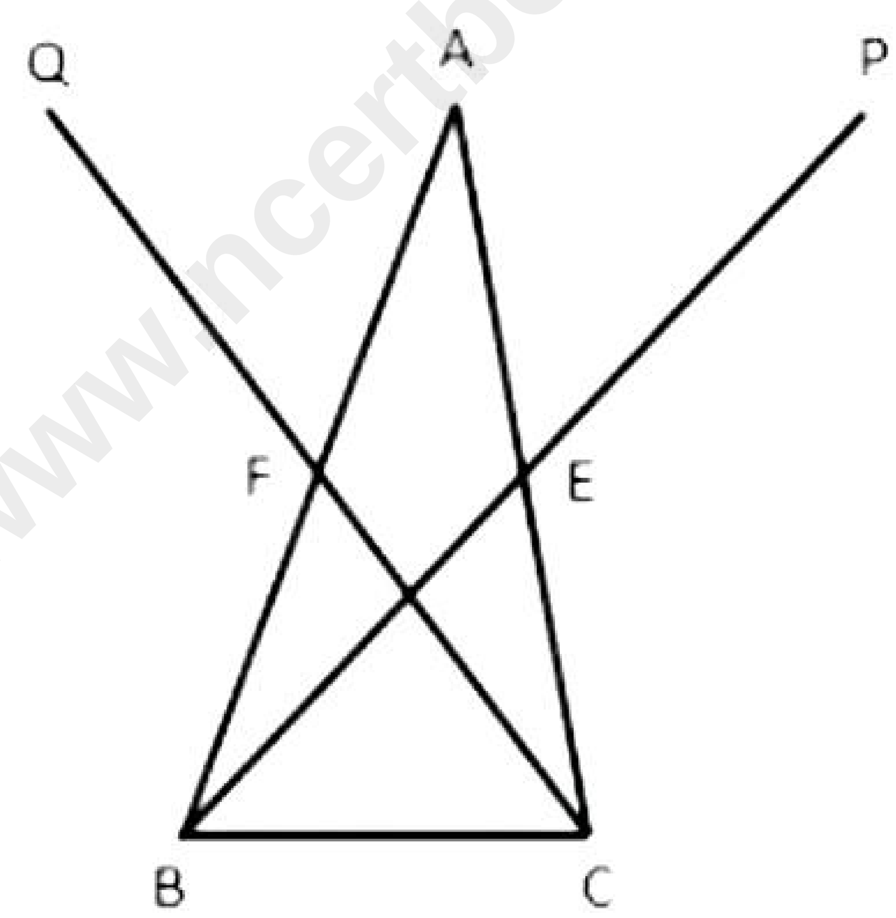
xv. Distance between the points P(5, 2) and Q(9, 5) is equal to
(a) 1 unit
(b) 3 units
(c) 5 units
(d) 6 units

Question 2

i. Find the amount and the compounded interest on Rs. 64000 for $1\frac{1}{2}$ years at 15% per annum, compounded half-yearly (Without using formula). [4]

ii. Solve using the method of elimination by equating coefficients:
$4x + \frac{6}{y} = 15, \; 3x - \frac{4}{y} = 7$ [4]

iii. In triangle ABC, the medians BE and CF are produced to points P and Q, respectively, such that EP = BE and FQ = CF. Prove that [4]
(a) Q, A and P are collinear.
(b) A is the mid-point of QP.



Question 3

i. A ladder 15 m long reaches a window which is 9 m above the ground on one side of the street. Keeping its foot at the same point, the ladder is turned to the other side of the street to reach a window 12 m high. Find the width of the street. [5]

ii. The diagonals of a quadrilateral are of lengths 10 cm and 24 cm. If the diagonals bisect each other at right angles, find the length of each side of the quadrilateral. Name the type of the quadrilateral. [4]

iii. Factorise: [4]

A. $x^3 + 3x^2 + 3x - 7$

B. $[(674)^2 - (326)^2]$

Section B

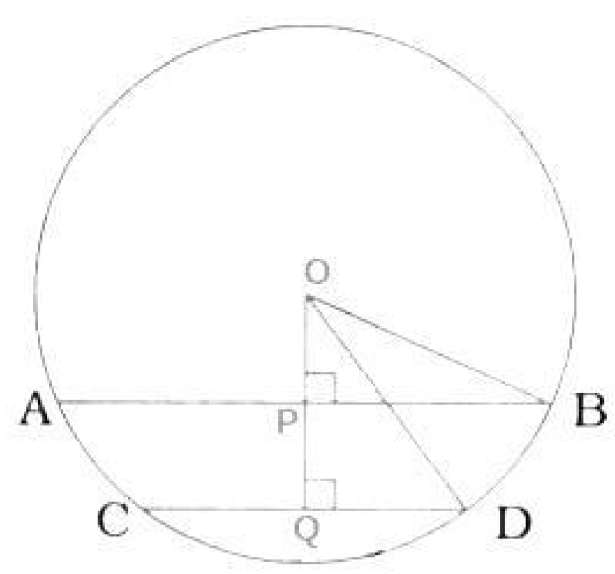
(Attempt any four questions from this Section.)

Question 4

i) Prove that $\sqrt{5}+\sqrt{11}$ is an irrational number. [3]

ii) Calculate the amount and the compound interest (Without using formula) on Rs. 25000 for 2 years at 8% per annum compounded annually. [3]

iii) If parallel chords AB and CD are in the same half plane of a line containing diameter parallel to them and AB = 8, CD = 6 and the perpendicular distance between them is 1, then find the length of the diameter of the circle. [4]



Question 5

i) Find $\frac{a^2}{bc}+\frac{b^2}{ac}+\frac{c^2}{ab}$, given a + b + c = 0. [3]

ii) Factorise: $a^3-\frac{1}{a^3}-2a+\frac{2}{a}$ [3]

iii) The mean of the ages of three students Vijay, Rahul and Rakhi is 15 years. If their ages are in the ratio 4:5:6, respectively, then find their ages. [4]

Question 6

i) Find the two numbers such that the sum of twice the first and thrice the second is 103, and four times the first exceeds seven times the second by 11. [3]

ii) Solve for x: $5^{x-3}\times 3^{2x-8}=225$ [3]

iii) The following table gives the distribution of two sections according to marks obtained:

Section A		Section B	
Marks	Frequency	Marks	Frequency
0–10	3	0–10	5
10–20	9	10–20	19
20–30	17	20–30	15
30–40	12	30–40	10
40–50	9	40–50	1

Represent the marks of students of both sections on the same graph by two frequency polygons.

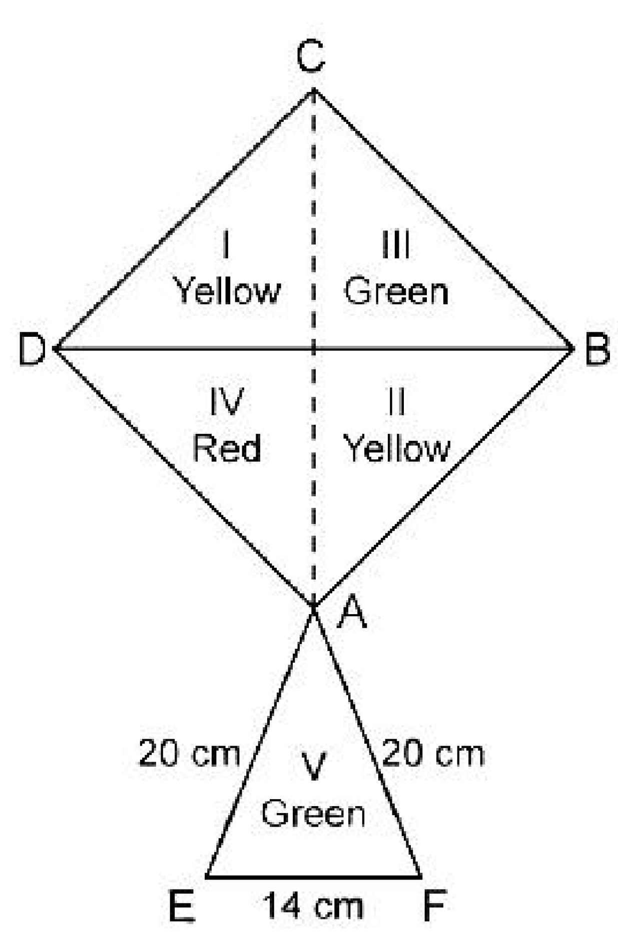
[4]

Question 7

i) If $\cot\theta = \dfrac{7}{8}$, evaluate [5]

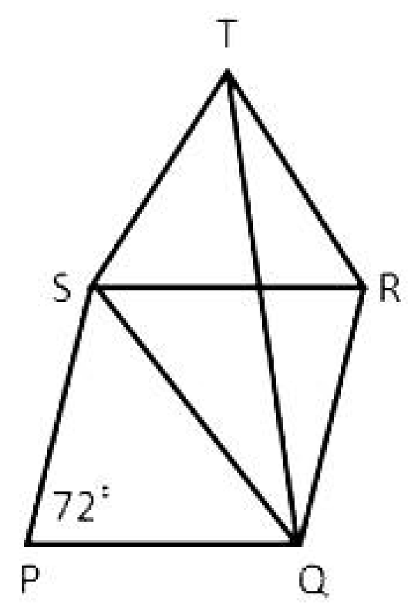
(a) $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ (b) $\tan^2\theta$

ii) In the given figure, ABCD is a square with diagonal 44 cm. How much paper of each shade is needed to make a kite given in the figure? $\left(\sqrt{39} = 6.25\right)$ [5]

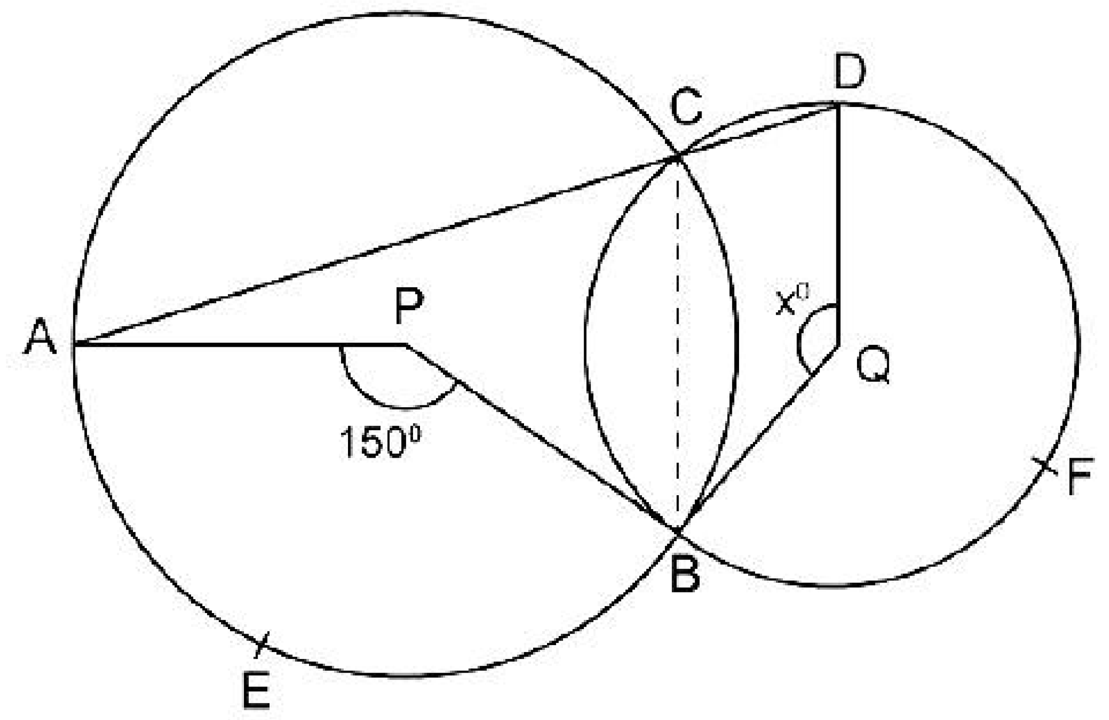


Question 8

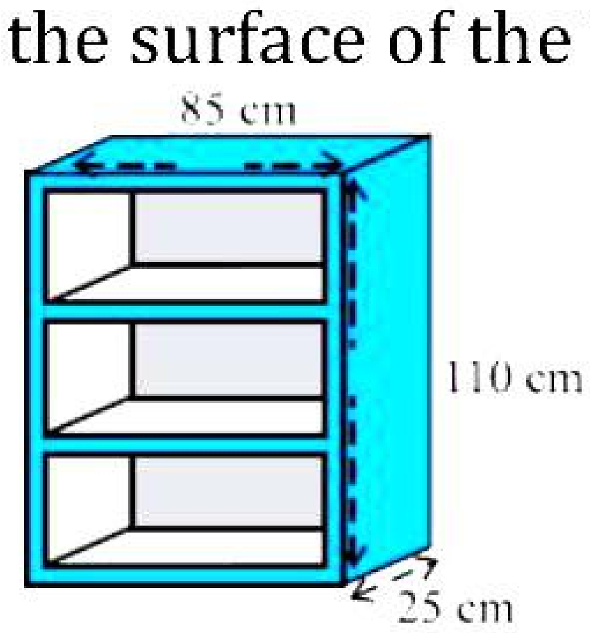
i) In the figure, PQRS is a rhombus and RST is an equilateral triangle. T and Q lie on opposite sides of RS. If ∠SPQ = 72°, then calculate the measures of ∠RQT and ∠SQT. [3]



ii) In the figure given below, P and Q are centres of the two circles intersecting at B and C, and ACD is a straight line. Find the value of x. [3]

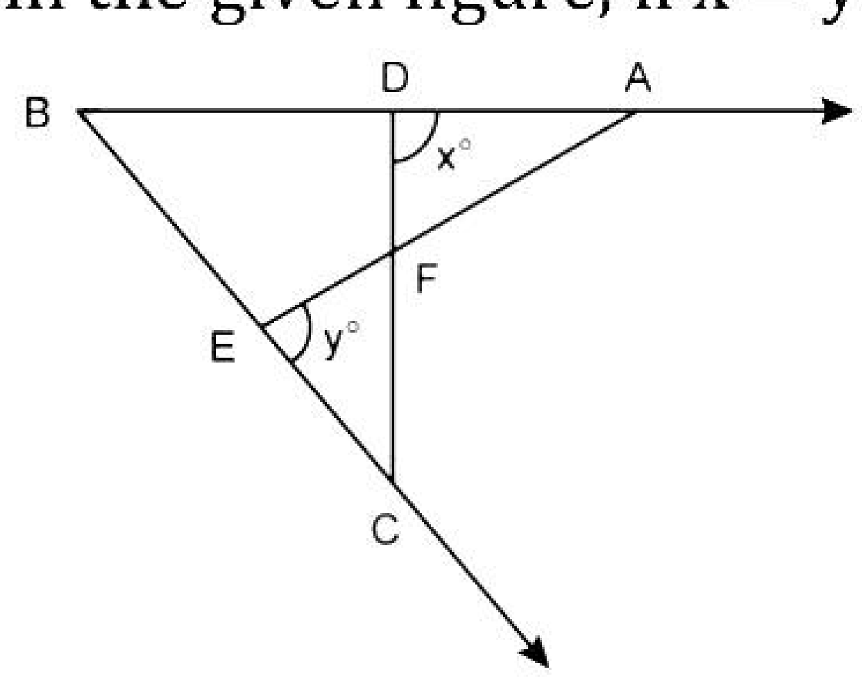


iii) A wooden bookshelf has external dimensions as follows:
Height = 110 cm, breadth = 25 cm, length = 85 cm.
The thickness of the plank is 5 cm everywhere. The external faces are to be polished, and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2, find the total expenses required for polishing and painting the surface of the bookshelf. [4]

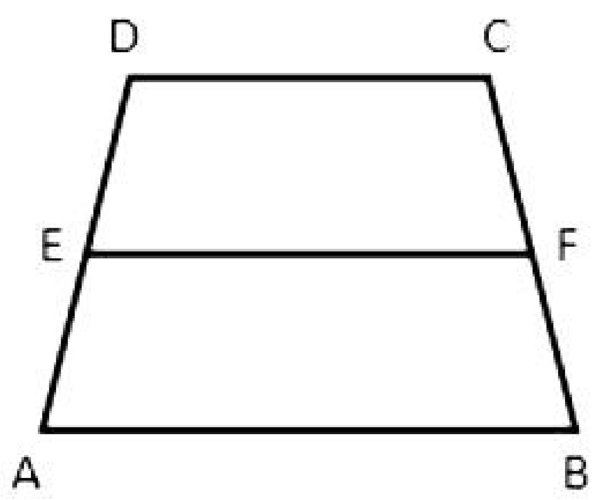


Question 9

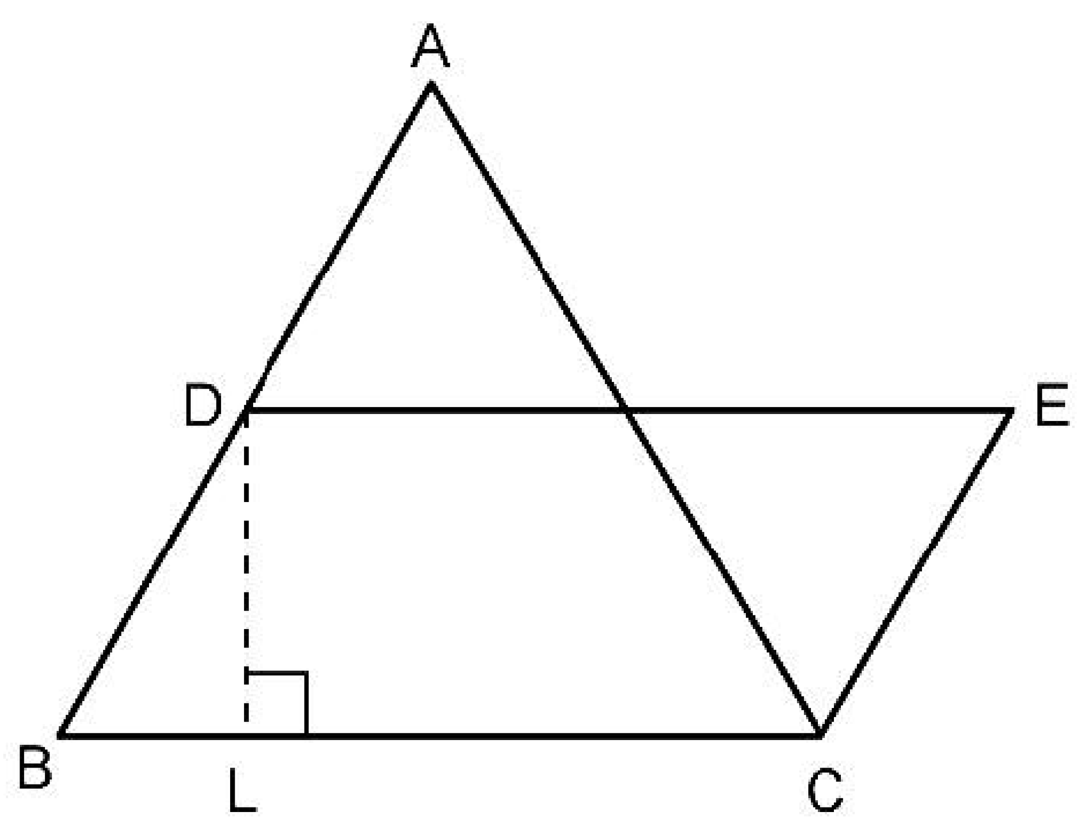
i) In the given figure, if x = y and AB = CB, then prove that AE = CD. [3]



ii) If E and F are the mid-points of non-parallel sides AD and BC, respectively, of a trapezium ABCD, then prove that EF is parallel to AB and EF = ½ (AB + CD). [3]



iii) In the given figure, a ΔABC has been given in which AB = 7.5 cm, AC = 6.5 cm and BC = 7 cm. On base BC, a parallelogram DBCE of the same area as that of ΔABC is constructed. Find the height DL of the parallelogram. [4]



Question 10

i) How many bricks will be required to construct a wall 8 m long, 6 m high and 22.5 cm thick if each brick measures 25 cm × 11.25 cm × 6 cm? [3]

ii) Find the area of a circle whose centre is (5, –3) and it passes through the point (–7, 2). [3]

iii) Solve the below pair of simultaneous equations graphically:
2x + 3y = 2 and x – 2y = 8 [4]

Solution

Section A

Solution 1

i) Correct option: (b)

Explanation:

$\sqrt{23-7}+\sqrt{34+2}=\sqrt{16}+\sqrt{36}=4+6=10$, which is a rational number.

ii) Correct option: (c)

Explanation:

For the 1st year:

P = Rs. 20,000 and A = Rs. 22,400

I = A – P = 22400 – 20000 = Rs. 2400

$\Rightarrow$ Rate of interest p. a. = $\frac{I \times 100}{P \times T}\% = \frac{2400 \times 100}{20000 \times 1}\% = 12\%$

iii) Correct option: (a)

Explanation:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

$(x + 9)(x + 11) = x^2 + (9 + 11)x + (9\times11) = x^2 + 20x + 99$

iv) Correct option: (d)

Explanation:

$2a^2 + bc - 2ab - ac$

$= 2a^2 - 2ab - ac + bc$

$= 2a(a - b) - c(a - b)$

$= (2a - c)(a - b)$

v) Correct option: (c)

Explanation:

At (15, 5)

$2x + y = 30 + 5 = 35$

$3x + 4y = 45 + 20 = 65$

$\therefore$ The ordered pair (15, 5) satisfies both the equations.

vi) Correct option: (c)

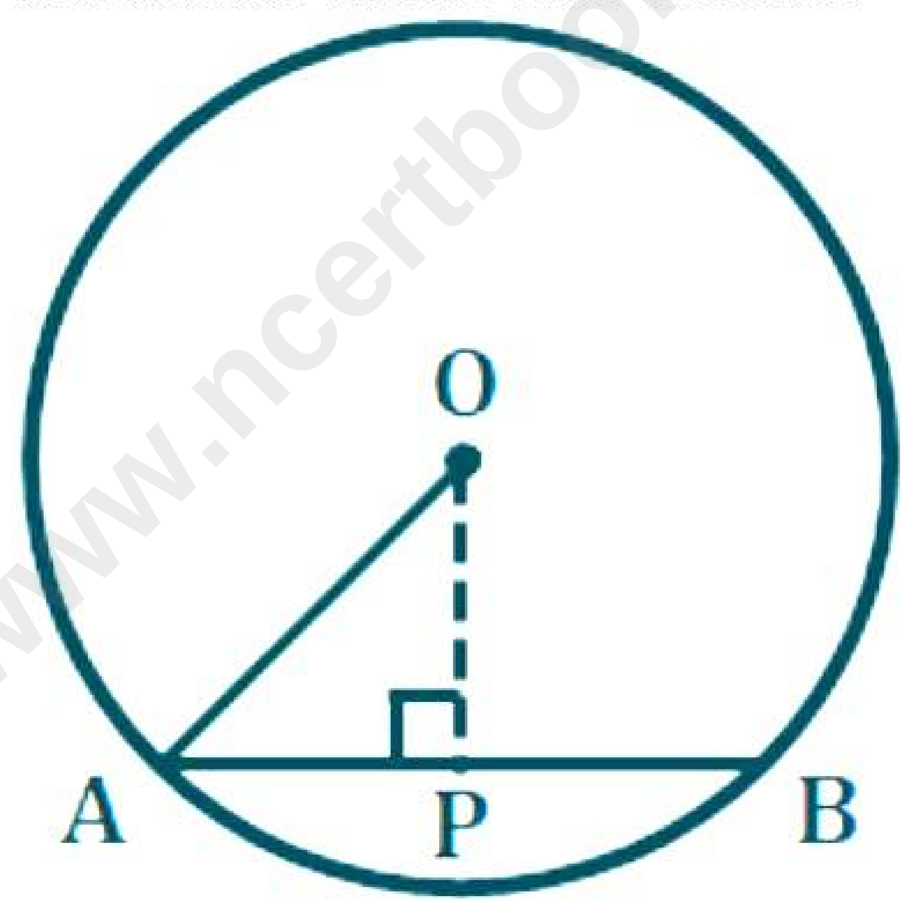
Explanation:

$a^m \times a^n = a^{m+n}$

vii) Correct option: (b)
Explanation:
AAA is not the congruency criteria because here all the corresponding angles are congruent but none of the corresponding sides are congruent.

viii) Correct option: (c)
Explanation:
In ΔABC, $\angle C = 90^0$.
$\therefore$ By Pythagoras' theorem,
$AB^2 = AC^2 + BC^2$
$\Rightarrow AB^2 = 6^2 + 8^2 = 36 + 64 = 100$
Taking the square root on both sides, we get
$AB = 10$ cm

ix) Correct option: (c)
Explanation:
Let AB be the chord of a circle with centre O.
Radius of this circle = 20/2 = 10 cm
Drop a perpendicular OP on the chord AB from the centre.



We know that the perpendicular drawn to a chord from the centre of a circle, bisects the chord.
$\Rightarrow AP = PB = ½ AB = 4$ cm
By Pythagoras theorem, we have
$OA^2 = OP^2 + AP^2$
$\Rightarrow 100 = OP^2 + 64$
$\Rightarrow OP = 6$ cm

x) Correct option: (d)
Explanation:
When number of terms is odd, middle value becomes the median.
Since, number of terms = 9.
So, the median = $[(9+1)/2]^{th}$ term = 5^{th} term

xi) Correct option: (c)
Explanation:
For n = 2 years,

When interest is compounded yearly, $A = P\left(1 + \frac{r}{100}\right)^2$.

Hence, statement 1 is true.

When interest is compounded half-yearly, $A = P\left(1 + \frac{r}{2 \times 100}\right)^{2 \times 2}$.

Hence, statement 1 is false.

xii) Correct option: (d)
Explanation:
Let the dimensions be x, 2x and 3x.
Volume of a cuboid = $l \times b \times h = 6x^3$
$\Rightarrow 6x^3 = 1296\ cm^3$
$\Rightarrow x = 6$ cm
So, the dimensions are 6 cm, 12 cm and 18 cm.

xiii) Correct option: (b)
Explanation:
Since, $\cos\theta = \frac{1}{\sec\theta} = \frac{12}{13}$
Now, $\sin^2\theta + \cos^2\theta = 1$
$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$
$\Rightarrow \sin^2\theta = 1 - \frac{144}{169}$
$\Rightarrow \sin^2\theta = \frac{25}{169}$
Taking square root on both the sides,
$\Rightarrow \sin\theta = \frac{5}{13}$

xiv) Correct option: (c)
Explanation:
For trinomial $6x^2 + 17x + 5$,

$b^2 - 4ac = (17)^2 - 4(6)(5) = 289 - 120 = 169$, which is a perfect square.

$\Rightarrow 6x^2 + 17x + 5$ is factorisable.

Hence, both A and R are true, and R is the correct reason for A.

xv) Correct option: (c)
Explanation:
$$PQ = \sqrt{(5-9)^2 + (2-5)^2} = \sqrt{(-4)^2 + (-3)^2} = \sqrt{25} = 5 \text{ units}$$

Solution 2

i) Time = $1\frac{1}{2}$ years = 3 half years
For the first half year:
P = Rs. 64000, R = 15% and N = $\frac{1}{2}$ year
$$\text{Interest} = \frac{P \times R \times N}{100} = \frac{64000 \times 15 \times 1}{100 \times 2} = \text{Rs. } 4800$$
⇒ Amount at the end of the first half year = P + I = 64000 + 4800 = Rs. 68800
For the second half year:
P = Rs. 68800, R = 15% and N = $\frac{1}{2}$ year
$$\text{Interest} = \frac{P \times R \times N}{100} = \frac{68800 \times 15 \times 1}{100 \times 2} = \text{Rs. } 5160$$
Amount at the end of the second half year = P + I = 68800 + 5160 = Rs. 73960
For the third half year:
P = Rs. 73960, R = 15% and N = $\frac{1}{2}$ year
$$\text{Interest} = \frac{P \times R \times N}{100} = \frac{73960 \times 15 \times 1}{100 \times 2} = \text{Rs. } 5547$$
Amount at the end of the third half year = P + I = 73960 + 5547 = Rs. 79507
Compound interest for $1\frac{1}{2}$ years = Final amount – Initial principal
= 79507 – 64000
= Rs. 15507

ii)
$$4x + \frac{6}{y} = 15 \quad ...(i)$$
$$3x - \frac{4}{y} = 7 \quad ...(ii)$$
Multiplying (i) by 3 and (ii) by 4, we get
$$12x + \frac{18}{y} = 45 \quad ...(iii)$$
$$12x - \frac{16}{y} = 28 \quad ...(iv)$$
Subtracting (iv) from (iii), we get

$\frac{18}{y} + \frac{16}{y} = 45 - 28$

$\Rightarrow \frac{34}{y} = 17$

$\Rightarrow \frac{1}{y} = \frac{17}{34}$

$\Rightarrow y = 2$

Putting y = 2 in the equation (i), we get

$4x + \frac{6}{y} = 15$

$\Rightarrow 4x + \frac{6}{2} = 15$

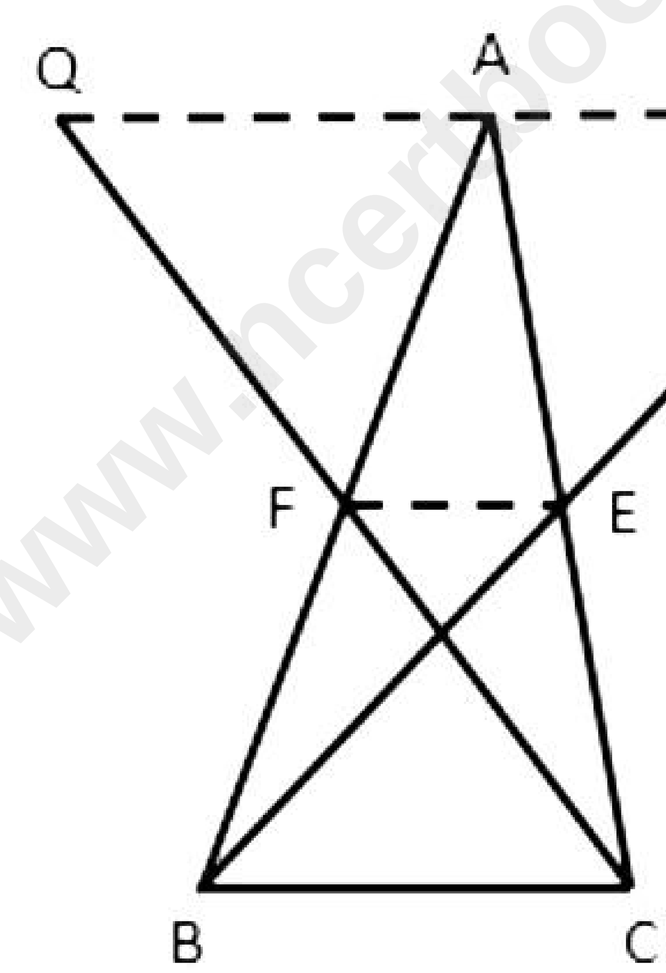
$\Rightarrow 4x + 3 = 15$

$\Rightarrow 4x = 12$

$\Rightarrow x = 3$

Hence, the solution is x = 3 and y = 2.

iii)



Given: EP = BE and FQ = CF, E and F are the mid-points of BP and CQ, respectively.
Construction: Join AP, AQ and FE.
In ΔABP, F is the mid-point of AB and E is the mid-point of BP; hence,
FE || AP and FE = ½ AP.
In ΔACQ, E is the mid-point of AC and F is the mid-point of CQ; hence,
FE || QA and FE = ½ QA.

(a) As FE || AP and FE || QA; hence, QA and AP lie along the same straight line.
Hence, Q, A and P are collinear.

(b) As FE = ½ AP and FE = ½ QA

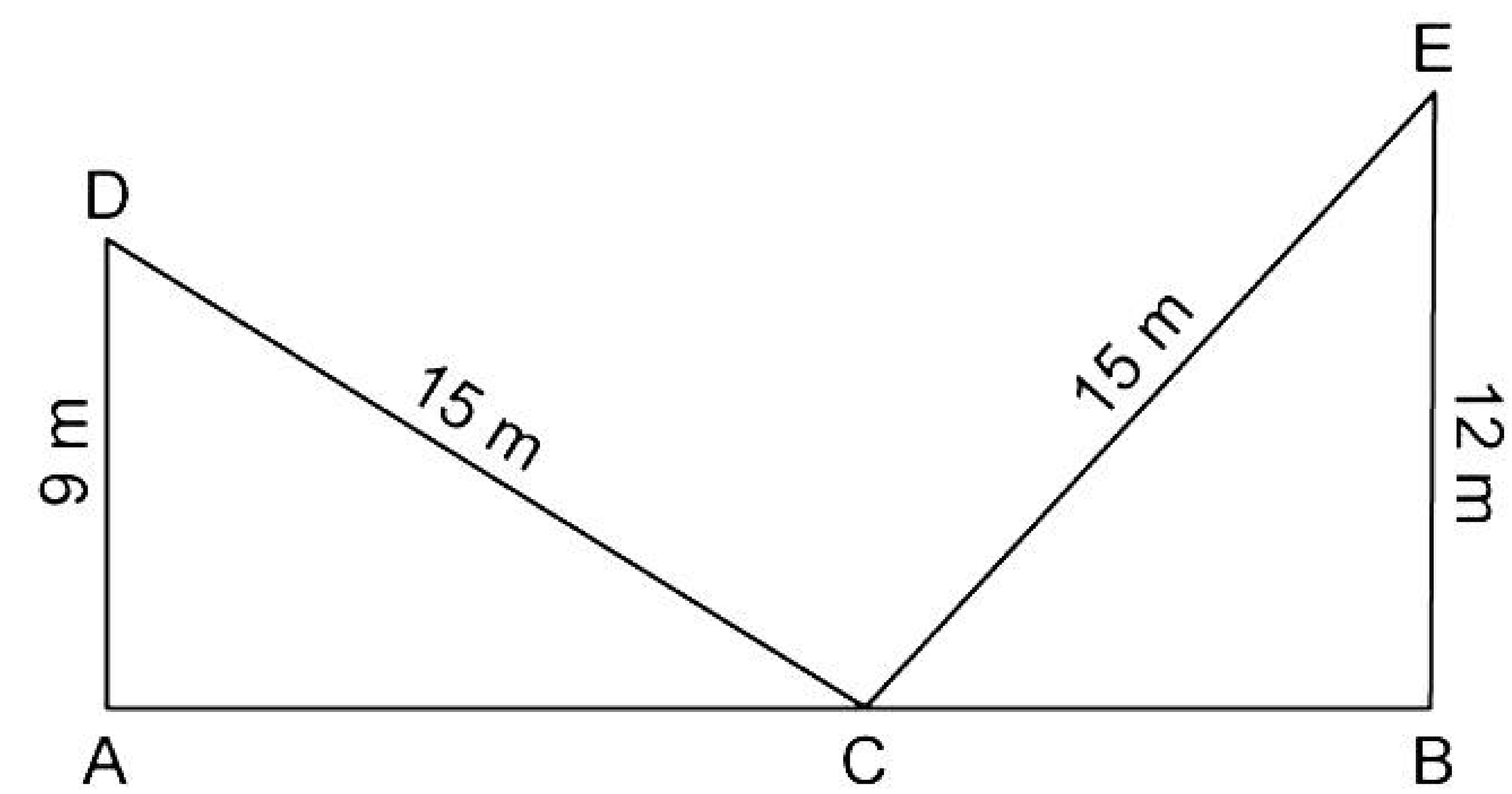
⇒ ½ AP = ½ QA

⇒ AP = QA

⇒ A is the mid-point of QP.

Solution 3

i)



In ΔACD, $\angle A = 90°$.

By Pythagoras' theorem, we get

$CD^2 = AD^2 + AC^2$

$\Rightarrow AC^2 = CD^2 - AD^2 = 15^2 - 9^2 = 225 - 81 = 144\ m^2$

$\Rightarrow AC^2 = 144\ m^2$

Taking the square root on both sides, we get

AC = 12 m

Similarly, in ΔBCE, $\angle B = 90^o$.

By Pythagoras' theorem, we get

$CE^2 = BC^2 + BE^2$

$\Rightarrow BC^2 = CE^2 - BE^2 = 15^2 - 12^2 = 225 - 144 = 81\ m^2$

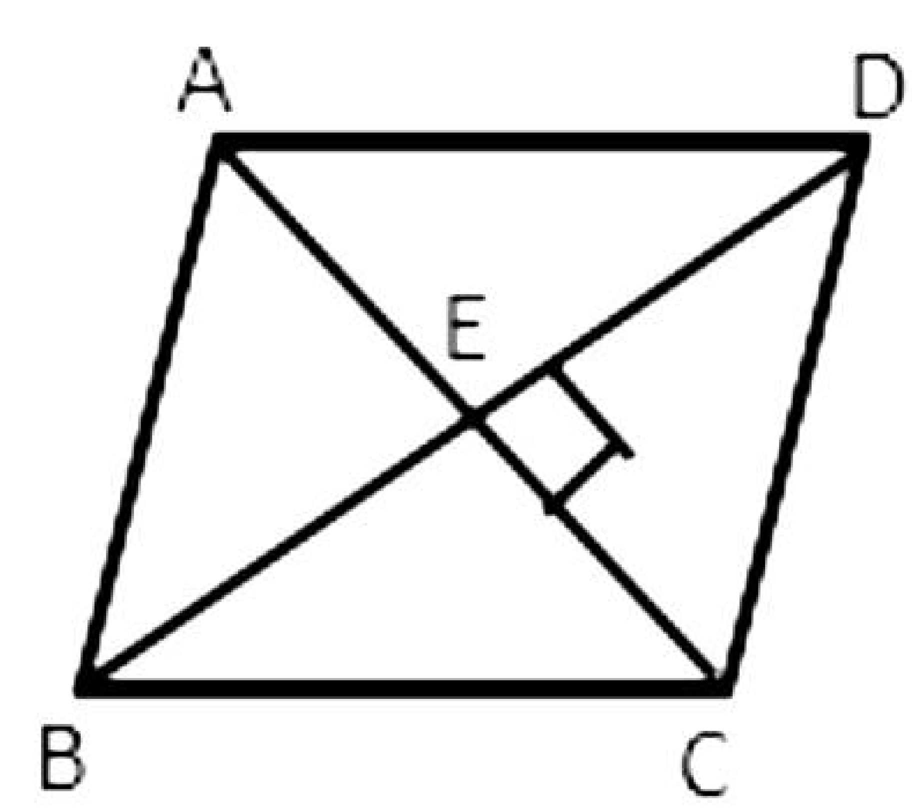
$\Rightarrow BC^2 = 81\ m^2$

Taking the square root on both sides, we get

BC = 9 m

Width of the street = AB = AC + BC = 12 + 9 = 21 m

ii)



ABCD is the given quadrilateral whose diagonal AC = 10 cm and diagonal BD = 24 cm.

Diagonals bisect each other at right angles at point E.

∴ BE = ½ BD = ½ (24) = 12 cm

∴ CE = ½ AC = ½ (10) = 5 cm

In right-angled triangle BEC,

$BC^2 = BE^2 + CE^2$

∴ $BC^2 = 12^2 + 5^2$

∴ $BC^2 = 144 + 25$

∴ $BC^2 = 169$

∴ BC = 13 cm

∴ Length of each side of the quadrilateral = 13 cm

The diagonals bisect each other at right angles and AB = BC = CD = AD = 13 cm.

Therefore, it is a rhombus.

iii)

A. $x^3 + 3x^2 + 3x - 7$
$= (x^3 + 3x^2 + 3x + 1) - 8$
$= (x + 1)^3 - (2)^3$
$= [(x + 1) - 2][(x + 1)^2 + 2(x + 1) + 2^2]$
$= (x - 1)(x^2 + 2x + 1 + 2x + 2 + 4)$
$= (x - 1)(x^2 + 4x + 7)$

B. $[(674)^2 - (326)^2]$
$= (674 + 326)(674 - 326)$ ... [Since $a^2 - b^2 = (a + b)(a - b)$]
$= 1000 \times 348$
$= 348000$

Section B

Solution 4

i)

Suppose that $\sqrt{5}+\sqrt{11}$ is a rational number.

$\Rightarrow \left(\sqrt{5}+\sqrt{11}\right)^2$ is a rational number.

$\Rightarrow 16+2\sqrt{55}$ is a rational number.

But we know that the sum of rational and irrational numbers is an irrational number.

Here, 16 is a rational number and $2\sqrt{55}$ is an irrational number.

This means that $16+2\sqrt{55}$ is an irrational number.

So, we arrive at a contradiction.

This contradiction arises by assuming that $\sqrt{5}+\sqrt{11}$ is a rational number.

Hence, $\sqrt{5}+\sqrt{11}$ is an irrational number.

ii) For the first year:

P = Rs. 25000, R = 8% and N = 1 year

$$\text{Interest} = \frac{P \times R \times N}{100} = \frac{25000 \times 8 \times 1}{100} = \text{Rs. } 2000$$

Amount at the end of the 1st year = P + I = 25000 + 2000 = Rs. 27000

For the second year:

P = Rs. 27000, R = 8% and N = 1 year

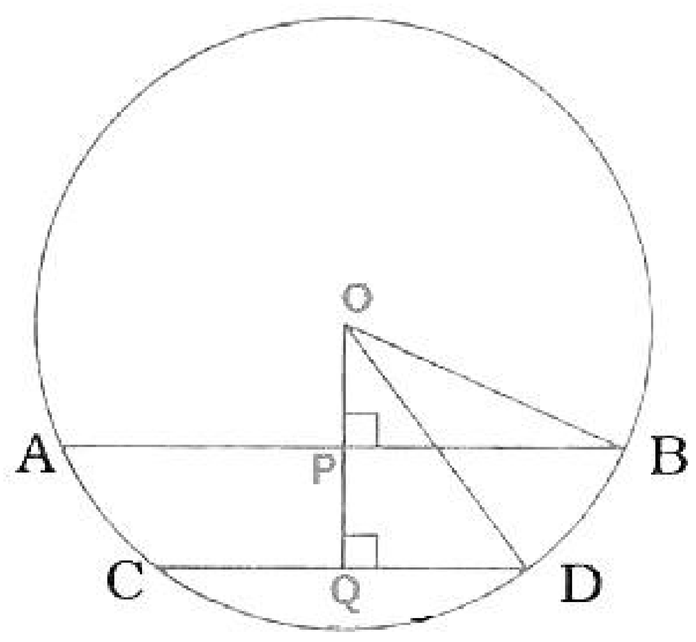
$$\text{Interest} = \frac{P \times R \times N}{100} = \frac{27000 \times 8 \times 1}{100} = \text{Rs. } 2160$$

Amount at the end of the 2nd year = P + I = 27000 + 2160 = Rs. 29160

Compound interest for 2 years = Final amount – Initial principal

= Rs. (29160 – 25000) = Rs. 4160

iii)



In a circle with centre O and radius r, AB and CD are two parallel chords lying on the same side of the centre.

$OP \perp AB,\ P \in AB$ and $OQ \perp CD,\ Q \in CD$

Then, O, P and Q are collinear.

Given: AB = 8 and CD = 6

AB > CD,

$\therefore OP < OQ$

$\therefore O - P - Q$

Let $OP = x$

$\Rightarrow OQ = x + PQ = x + 1$

Now, $OB = OD = r$

In ΔOPB, $m\angle P = 90°$

$PB = \frac{1}{2}(AB) = 4$

$\therefore OP^2 + PB^2 = OB^2$

$\therefore x^2 + 4^2 = r^2$

$\therefore x^2 + 16 = r^2 \qquad ...(1)$

In ΔOQD, $m\angle Q = 90°$

$QD = \frac{1}{2}CD = 3$

$\therefore OQ^2 + QD^2 = OD^2$

$\therefore (x+1)^2 + 3^2 = r^2$

$\therefore x^2 + 2x + 1 + 9 = r^2$

$\therefore x^2 + 2x + 10 = r^2 \qquad ...(2)$

From equation (1) and (2), we get

$x^2 + 2x + 10 = x^2 + 16$

$\therefore 2x = 6$

$\therefore x = 3$

$\text{Now, } r^2 = x^2 + 16 = 3^2 + 16 = 9 + 16 = 25 = (5)^2$

$\therefore r = 5$

Radius = 5

$\text{Diameter} = 2(\text{radius}) = 2 \times 5 = 10$

Hence, the diameter of the circle is 10.

Solution 5

i) We know that, if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

Here it is given that $a + b + c = 0$.

$$\frac{a^2}{bc} + \frac{b^2}{ac} + \frac{c^2}{ab} = \frac{a^3 + b^3 + c^3}{abc}$$

$$= \frac{3abc}{abc}$$

$$= 3$$

ii)

$$a^3 - \frac{1}{a^3} - 2a + \frac{2}{a} = \left[(a)^3 - \left(\frac{1}{a}\right)^3\right] - 2\left(a - \frac{1}{a}\right)$$

$$= \left[\left(a - \frac{1}{a}\right)\left(a^2 + a \times \frac{1}{a} + \frac{1}{a^2}\right)\right] - 2\left(a - \frac{1}{a}\right)$$

$$= \left(a - \frac{1}{a}\right)\left(a^2 + 1 + \frac{1}{a^2} - 2\right)$$

$$= \left(a - \frac{1}{a}\right)\left(a^2 + \frac{1}{a^2} - 1\right)$$

iii) The ages of Vijay, Rahul and Rakhi are in the ratio 4:5:6.

Let the age of Vijay = 4x, the age of Rahul = 5x and the age of Rakhi = 6x

Mean age = 15 years

And, number of students = 3

We know that,

$$\text{Mean} = \frac{\text{sum of all the observations}}{\text{number of observations}} a$$

$$\Rightarrow 15 = \frac{4x + 5x + 6x}{3}$$

$$\Rightarrow 15x = 15 \times 3$$

$$\Rightarrow x = 3$$

So, we have,

Age of Vijay = $4x = 4 \times 3 = 12$ years

Age of Rahul = $5x = 5 \times 3 = 15$ years

Age of Rakhi = $6x = 6 \times 3 = 18$ years

Solution 6

i) Let the first and second numbers be x and y respectively.

According to the first condition, we get

$2x + 3y = 103$ ... (i)

According to the second condition, we get

$4x - 7y = 11$ ... (ii)

Multiplying equation (i) by 2, we get

$4x + 6y = 206$ ... (iii)

Subtracting equation (ii) from (iii), we get

$$\begin{array}{rcrcr} 4x & + & 6y & = & 206 \\ 4x & - & 7y & = & 11 \\ - & & + & & - \\ \hline & & 13y & = & 195 \end{array}$$

$\Rightarrow y = 15$

Putting y = 15 in equation (ii), we get

$4x - 7 \times 15 = 11$

$\Rightarrow 4x = 11 + 105$

$\Rightarrow 4x = 116$

$\Rightarrow x = 29$

Hence, the two numbers are 29 and 15 respectively.

ii) $5^{x-3} \times 3^{2x-8} = 225$

$5^{x-3} \times 3^{2x-8} = 225$

$\Rightarrow 5^x \times \frac{1}{5^3} \times 3^{2x} \times \frac{1}{3^8} = 25 \times 9$

$\Rightarrow \frac{5^x \times 3^{2x}}{5^3 \times 3^8} = 5^2 \times 3^2$

$\Rightarrow 5^x \times 3^{2x} = 5^2 \times 3^2 \times 5^3 \times 3^8$

$\Rightarrow 5^x \times 3^{2x} = 5^5 \times 3^{10}$

$\Rightarrow x = 5$

iii)

1. The given data is in the exclusive form (class intervals are continuous).

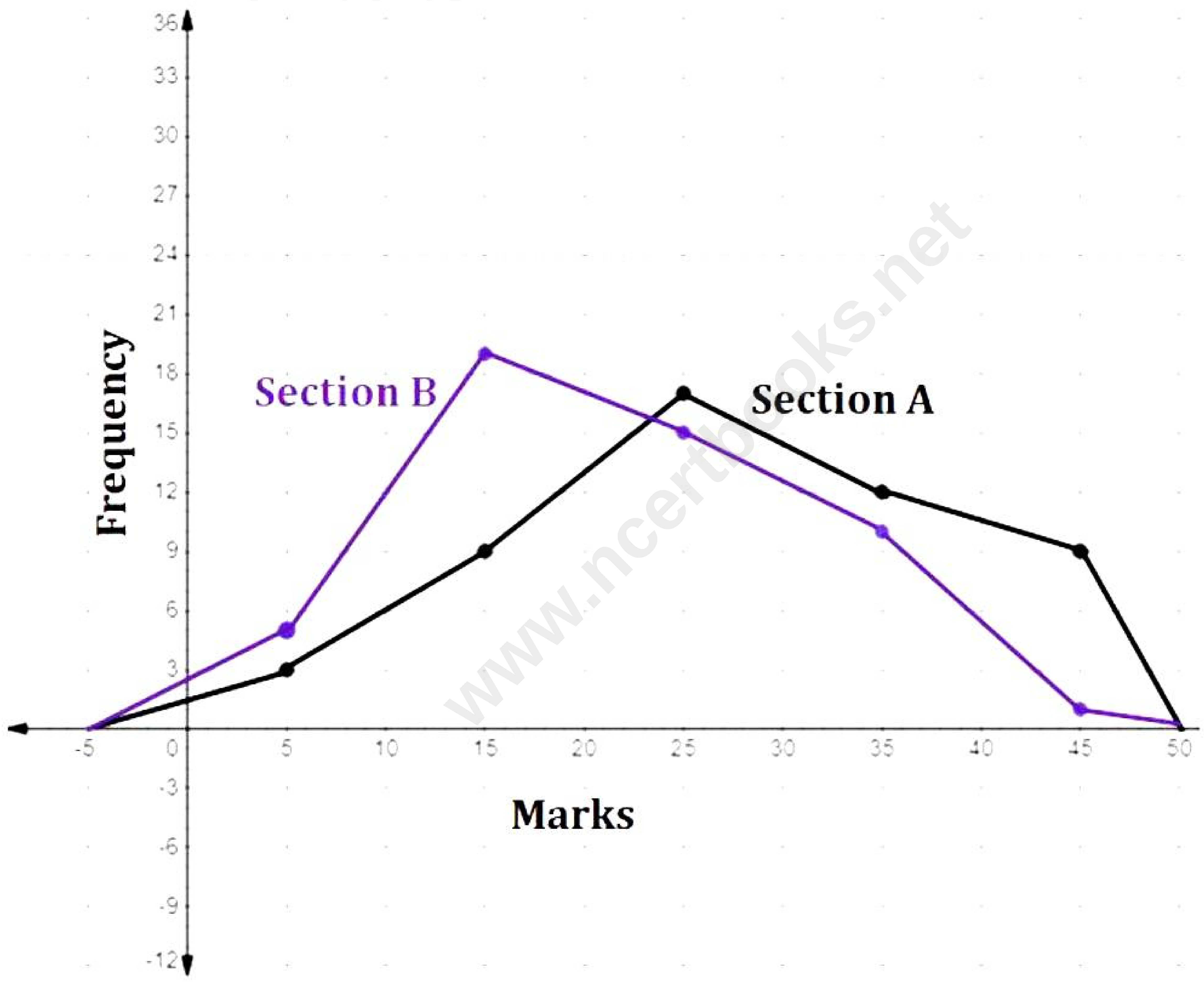
2. Find the class marks of the given class intervals.

$$\text{Class mark} = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Section A			Section B		
Marks	Class marks	Frequency	Marks	Class marks	Frequency
0–10	5	3	0–10	5	5
10–20	15	9	10–20	15	19
20–30	25	17	20–30	25	15
30–40	35	12	30–40	35	10
40–50	45	9	40–50	45	1

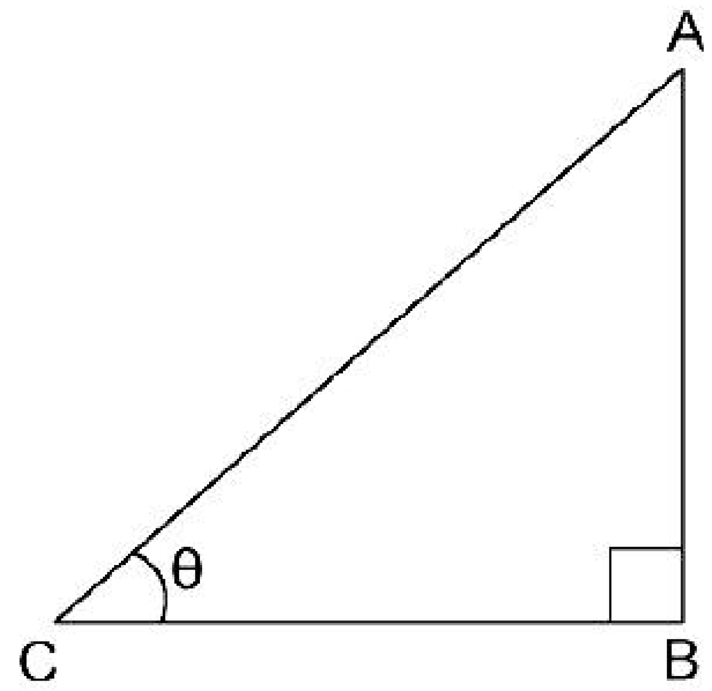
3. Now taking class marks on the x-axis and frequency on the y-axis, choose an appropriate scale (1 unit = 3 for the y-axis).

4. Draw the frequency polygon as below:



Solution 7

i) Let us consider a right-angled ΔABC.



$$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{BC}{AB} = \frac{7}{8}$$

If BC is 7K, AB will be 8K, where K is a positive integer.

Now applying Pythagoras' theorem in ΔABC,

$AC^2 = AB^2 + BC^2$

$= (8K)^2 + (7K)^2$

$= 64K^2 + 49K^2$

$= 113K^2$

$\therefore AC = \sqrt{113}\ K$

$$\sin\theta = \frac{\text{Side opposite to }\theta}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{8K}{\sqrt{113}\,K} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{\text{Side adjacent to }\theta}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7K}{\sqrt{113}\ K} = \frac{7}{\sqrt{113}}$$

(a) $$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$$

$$= \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{\left(\frac{7}{\sqrt{113}}\right)^2}{\left(\frac{8}{\sqrt{113}}\right)^2}$$

$$= \frac{49}{113} \times \frac{113}{64}$$

$$= \frac{49}{64}$$

(b) $$\tan^2\theta = \left(\frac{1}{\cot\theta}\right)^2 = \left(\frac{8}{7}\right)^2 = \frac{64}{49}$$

ii)

$$\text{Area of square sheet ABCD} = \frac{1}{2} \times (\text{Diagonal})^2$$

$$= \frac{1}{2} \times 44 \times 44$$

$$= 968\ \text{cm}^2$$

Area of yellow sheet

$$= \text{Area of Region I} + \text{Area of Region II}$$

$$= \frac{1}{2} \times \text{Area of square sheet ABCD}$$

$$= \frac{1}{2} \times 968$$

$$= 484 \text{ cm}^2$$

Area of Red sheet = Area of Region IV

$$= \frac{1}{4} \times \text{Area of square sheet ABCD}$$

$$= \frac{1}{4} \times 968$$

$$= 242 \text{ cm}^2$$

In ΔAEF, AE = 20 cm, EF = 14 cm and AF = 20 cm

Let a = 20 cm, b = 14 cm and c = 20 cm

Semi-perimeter, $s = \frac{a+b+c}{2}$

$$= \frac{20+14+20}{2}$$

$$= \frac{54}{2}$$

$$= 27 \text{ cm}$$

$\therefore$ Area of Region V = Area of ΔAEF

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{27(27-20)(27-14)(27-20)}$$

$$= \sqrt{27 \times 7 \times 13 \times 7}$$

$$= 21\sqrt{39}$$

$$= 21 \times 6.25$$

$$= 131.25 \text{ cm}^2$$

Area of Green sheet = Area of Region III + Area of Region V

$$= \frac{1}{4} \times \text{Area of square sheet ABCD} + 131.25$$

$$= \frac{1}{4} \times 968 + 131.25$$

$$= 242 + 131.25$$

$$= 373.25 \text{ cm}^2$$

Solution 8

i) PQRS is a rhombus.

$\Rightarrow \angle QRS = \angle SPQ = 72°$ ... (Opposite angles of a rhombus)

RST is an equilateral triangle.

$\therefore \angle SRT = 60°$

$\angle QRT = \angle QRS + \angle SRT = 72° + 60° = 132°$

Now, SR = RT ... (Equilateral triangle)

Also, SR = QR ... (sides of a rhombus)

$\therefore$ RT = QR

$\Rightarrow \angle RQT = \angle RTQ$ ...(i) (Opposite angles of equal sides)

In ΔRTQ,

$\angle RQT + \angle QRT + \angle RTQ = 180°$

$\therefore \angle RQT + 132° + \angle RTQ = 180°$

$\therefore 2\angle RQT = 48°$

$\therefore \angle RQT = \frac{1}{2} \times 48° = 24°$

In ΔSPQ,

SP = PQ ... (sides of a rhombus)

$\Rightarrow \angle PQS = \angle QSP$ ...(Opposite angles of equal sides)

$\Rightarrow \angle QSP = \frac{1}{2}(180° - 72°) = 54°$

Now, $\angle SQR = \angle PSQ = 54°$ ... ($\because$ Alternate interior angles)

$\therefore \angle SQT = \angle SQR - \angle RQT = 54° - 24° = 30°$

ii) We know that the angle subtended by an arc of a circle at its centre is twice the angle subtended by the same arc at a point on the circumference.

$\Rightarrow \angle APB = 2\angle ACB$

$\Rightarrow \angle ACB = \frac{1}{2}\angle APB = \frac{1}{2} \times 150° = 75°$

Now, ACD is a straight line.

$\Rightarrow \angle ACB + \angle DCB = 180°$

$\Rightarrow 75° + \angle DCB = 180°$

$\Rightarrow \angle DCB = 105°$

Again,

$\angle DCB = \frac{1}{2} \times \text{reflex } \angle BQD$

$\Rightarrow 105° = \frac{1}{2} \times (360° - x)$

$\Rightarrow 210° = 360° - x$

$\Rightarrow x = 150°$

iii) <u>Polishing expense</u>

External length (l) of the bookshelf = 85 cm

External breadth (b) of the bookshelf = 25 cm

External height (h) of the bookshelf = 110 cm

External surface area of the shelf while leaving the front face of the shelf

$= lh + 2(lb + bh)$
$= [85 \times 110 + 2(85 \times 25 + 25 \times 110)]\ cm^2$
$= [9350 + 2(2125 + 2750)]\ cm^2$
$= [9350 + 9750]\ cm^2$
$= 19100\ cm^2$

Area of the front face
$= [85 \times 110 - 75 \times 100 + 2(75 \times 5)]\ cm^2$
$= [9350 - 7500 + 750]\ cm^2$
$= 2600\ cm^2$

Area to be polished
$= (19100 + 2600)\ cm^2$
$= 21700\ cm^2$
Cost of polishing 1 cm^2 area = 20 paise = Rs. 0.20
Cost of polishing 21700 cm^2 area = Rs. (21700×0.20) = Rs. 4340

Painting expense
Height of the bookshelf = 3 × height of open part + 4 × thickness
$110 = 3h + 4 \times 5$
$3h = 90 \Rightarrow h = 30$ cm
Now, length (l), breadth (b) and height (h) of each row of the bookshelf are 75 cm, 20 cm and 30 cm, respectively.
Area to be painted in 1 row
$= 2(l + h)b + lh$
$= [2(75 + 30) \times 20 + 75 \times 30]\ cm^2$
$= (4200 + 2250)\ cm^2$
$= 6450\ cm^2$

Area to be painted in 3 rows = $(3 \times 6450)\ cm^2 = 19350\ cm^2$

Cost of painting 1 cm^2 area = Rs. 0.10

Cost of painting 19350 cm^2 area = Rs. (19350×0.10) = Rs. 1935

Total expense required for polishing and painting the surface of the bookshelf

= Rs. (4340 + 1935) = Rs. 6275

Solution 9

i) Given: AB = BC and x = y

To prove: AE = CD

Proof:

In ΔABE,

Ext. $\angle AEC = \angle EBA + \angle BAE$

$\Rightarrow y^\circ = \angle EBA + \angle BAE$

Also, in ΔBCD,

Ext. $\angle ADC = \angle CBA + \angle BCD$

$x^\circ = \angle CBA + \angle BCD$

Since, $x = y \quad$ (Given)

$\Rightarrow \angle EBA + \angle BAE = \angle CBA + \angle BCD$

$\Rightarrow \angle BAE = \angle BCD \quad [\because \angle EBA = \angle CBA]$

Now, in ΔBCD and ΔBAE,

$AB = BC \quad$ (Given)

$\angle B = \angle B \quad$ (Common)

$\angle BCD = \angle BAE \quad$ (Proved above)

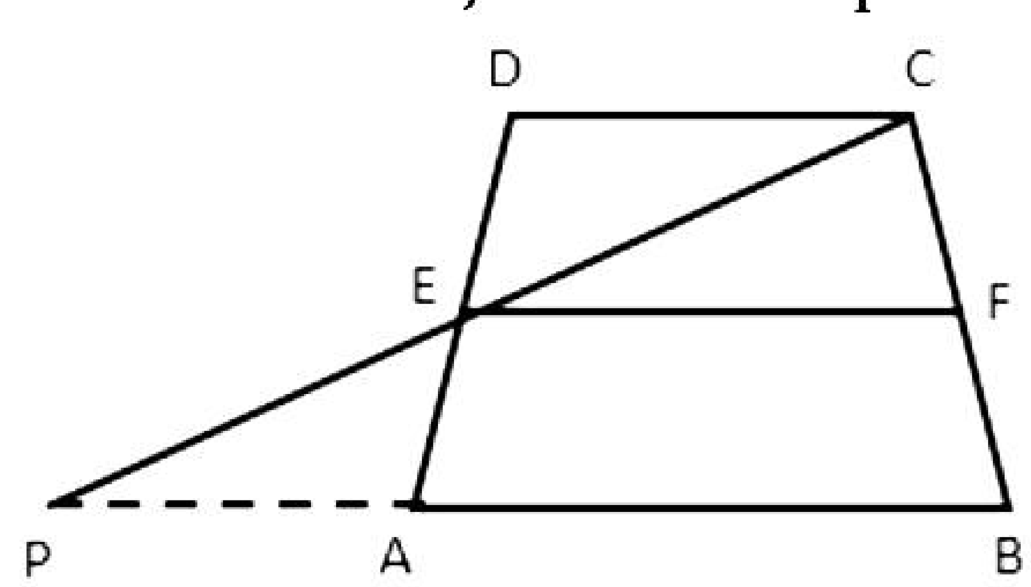
$\therefore \Delta BCD \cong \Delta BAE \quad$ (ASA congruence rule)

$\Rightarrow AE = CD \quad$ (C.P.C.T.)

ii) Given: ABCD is a trapezium in which AB || DC. E and F are the mid-points of non-parallel sides AD and BC, respectively.

To prove: EF || AB and EF = ½ (AB + DC)

Construction: Join CE and produce it to meet BA produced at P.



Proof:

In ΔDEC and ΔAEP,

EA = ED (E is the mid-point of AD)

$\angle DEC = \angle AEP$ (vertically opposite angles)

$\angle DCE = \angle EPA$ (alternate angles, PA || DC)

$\Rightarrow \Delta DEC \cong \Delta AEP$ (AAS congruence)

$\Rightarrow$ PE = EC (i) (c.p.c.t.)

$\Rightarrow$ PA = DC (ii) (c.p.c.t.)

In ΔCPB,

E is the mid-point of PC … From (i)
F is the mid-point of BC … (Given)
$\Rightarrow$ EF || PB
$\Rightarrow$ EF || AB
and EF = ½ PB … By Mid-point theorem
$\Rightarrow$ EF = ½ (AB + AP)
$\Rightarrow$ EF = ½ (AB + CD) … From (ii)

iii) In ΔABC, AB = 7.5 cm, BC = 7 cm and AC = 6.5 cm
Let a = 7.5 cm, b = 7 cm and c = 6.5 cm

$$\text{Semi-perimeter } (s) = \frac{a+b+c}{2} = \frac{7.5+7+6.5}{2} = \frac{21}{2} = 10.5 \text{ cm}$$

$$\therefore \text{ Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{10.5 \times (10.5-7.5)(10.5-7)(10.5-6.5)}$$
$$= \sqrt{10.5 \times 3 \times 3.5 \times 4}$$
$$= \sqrt{441}$$
$$= 21 \text{ cm}^2$$

Now, Area($\square$DBCE) = Area (ΔABC)
$\Rightarrow BC \times DL = 21$
$\Rightarrow 7 \times DL = 21$
$\Rightarrow DL = 3$ cm

Solution 10

i) Length of the wall = 8 m = 800 cm
Breadth of the wall = 22.5 cm
Height of the wall = 6 m = 600 cm
$\therefore$ Volume of the wall = length × breadth × height
= (800 × 600 ×22.5) cm^3
Length of the brick = 25 cm
Breadth of the brick = 6 cm
Height of the brick = 11.25 cm
$\therefore$ Volume of the brick = (25 × 6 × 11.25) cm^3

$$\therefore \text{ Number of bricks required} = \frac{\text{Volume of the wall}}{\text{Volume of brick}} = \frac{800 \times 600 \times 22.5}{25 \times 6 \times 11.25} = 6400$$

ii) According to the question,

Radius of a circle(r) = Distance between the centre (5, –3) and the point (–7, 2)

Distance between the given points $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$r=\sqrt{(-7-5)^2+(2+3)^2}$$
$$=\sqrt{144+25}$$
$$=\sqrt{169}$$
$$=13 \text{ units}$$

∴ Area of a circle = $\pi r^2 = 3.14 \times (13)^2 = 530.66$ sq. units

iii)

A. $2x + 3y = 2$

$\Rightarrow x = \dfrac{2-3y}{2}$

When $y = 2 \Rightarrow x = \dfrac{2-3\times 2}{2}=\dfrac{2-6}{2}=\dfrac{-4}{2}=-2$

When $y = 0 \Rightarrow x = \dfrac{2-3\times(0)}{2}=\dfrac{2}{2}=1$

When $y = -2 \Rightarrow x = \dfrac{2-3\times(-2)}{2}=\dfrac{2+6}{2}=\dfrac{8}{2}=4$

x	–2	1	4
y	2	0	–2

1. Plot the points (–2, 2), (1, 0), (4, –2) on the graph paper, taking 1 cm = 1 unit on both axes.
2. Draw a straight line AB passing through the points plotted.

B. $x - 2y = 8$

$\Rightarrow x = 8 + 2y$

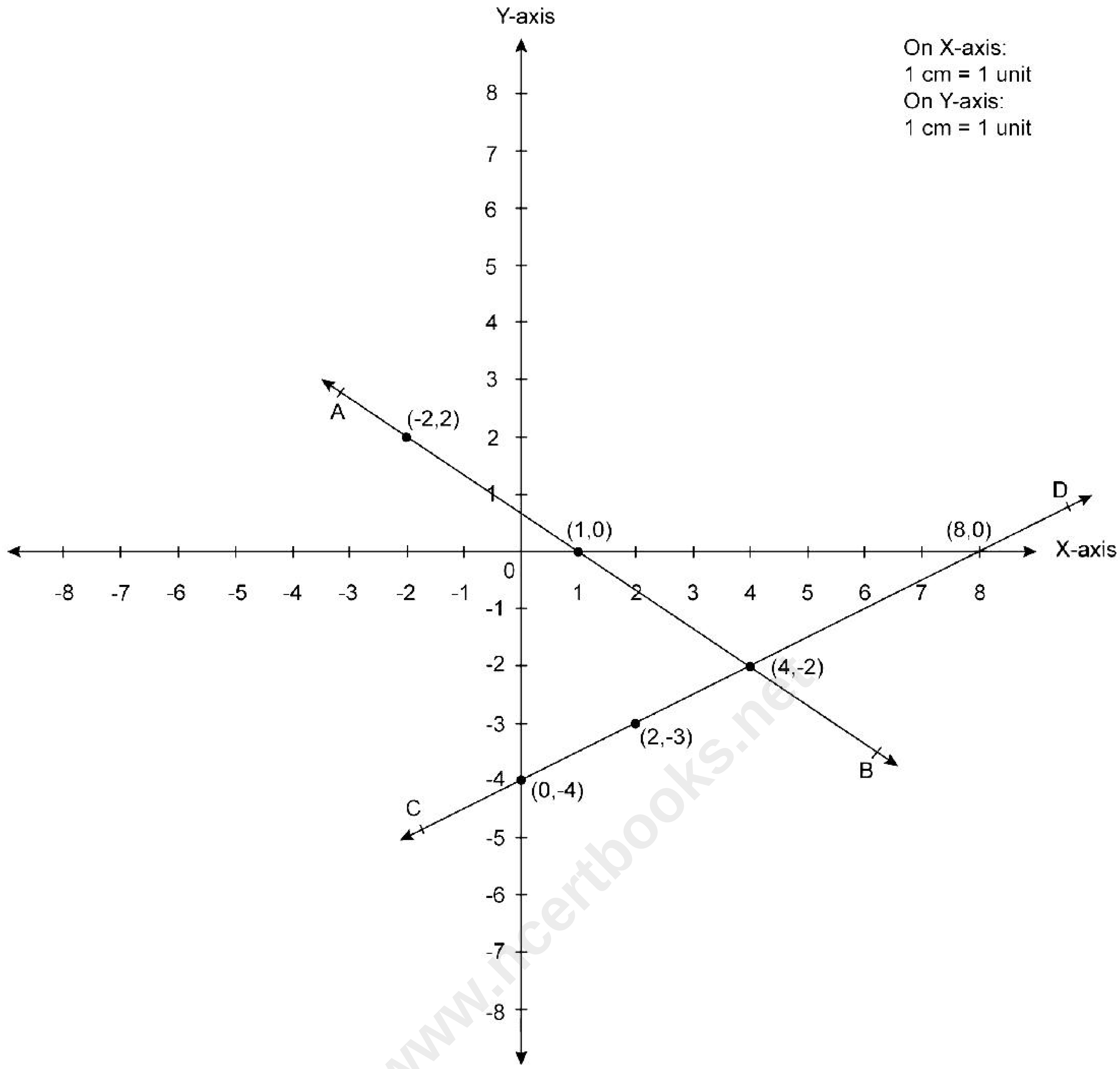
When $y = -3 \Rightarrow x = 8 + 2(-3) = 8 - 6 = 2$

When $y = -4 \Rightarrow x = 8 + 2(-4) = 8 - 8 = 0$

When $y = 0 \quad \Rightarrow x = 8 + 2(0) = 8$

x	2	0	8
y	–3	–4	0

1. Plot the points (2, –3), (0, –4), (8, 0) on the graph paper, taking 1 cm = 1 unit on both axes.
2. Draw a straight line CD passing through the points plotted.



From the graph, lines AB and CD intersect each other at point (4, –2).

$\Rightarrow$ The solution of the given simultaneous linear equations is x = 4 and y = –2.