Factorisation

Important concepts

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1. When a polynomial is the product of two or more polynomials, each of the later polynomials is called its factors.

2. The method of expressing a given polynomial as a product of two or more polynomials is called factorization.

3. If a given polynomial contains a common factor which may be either a constant or a variable, we divide each term separately by this factor.

   Example: Each term of the expression $3a^2 - 9ab$ has 3a as a common factor.

   Therefore, we have $3a^2 - 9ab = 3a(a - 3b)$

4. If the polynomial has even number of terms, then the terms are first arranged in groups such that each group has a common factor.

   Example: Factorise : $a^3 + a - 3a^2 - 3$

$$a^3 + a - 3a^2 - 3 = (a^3 - 3a^2) + (a - 3) \quad [\text{Forming groups}]$$
$$= a^2(a - 3) + 1(a - 3) \quad [\text{Taking out common factors from each group}]$$
$$= (a - 3)(a^2 + 1) \quad [\text{Taking } (a - 3) \text{ common}]$$

5. If the polynomial is trinomial in nature, i.e. , it has 3 terms ,we first arrange the terms in descending order.

   Then, split the middle term in such a way that the product is equal to the product of first and the last term.

   Example : Factorise $a^2 + 10a + 24$
   $$a^2 + 10a + 24 = a^2 + 6a + 4a + 24 \quad [\text{since } 6 + 4 = 10 \text{ and } 6 \times 4 = 24]$$
   $$= a(a + 6) + 4(a + 6) \quad [\text{Taking out common factors from each group}]$$
   $$= (a + 6)(a + 4) \quad [\text{Taking } (a + 6) \text{ common}]$$

6. Factorisation using difference of two squares: $x^2 - y^2 = (x + y)(x - y)$

Example : Factorise $9a^2 + 3a - 8b - 64b^2$

$$9a^2 + 3a - 8b - 64b^2 = 9a^2 - 64b^2 + 3a - 8b \quad [\text{Forming groups}]$$

$$= (3a)^2 - (8b)^2 + 3a - 8b \quad [\text{Rewriting}]$$

$$= (3a - 8b)(3a + 8b) + (3a - 8b) \quad \begin{bmatrix} \text{Writing as difference} \\ \text{of squares} \end{bmatrix}$$

$$= (3a - 8b)(3a + 8b + 1) \quad [\text{Taking } (3a - 8b) \text{ common}]$$

7. Sum of Difference of Two Cubes:

1. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

2. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$