Expansions

An equation which is true for all values of its variables is called an identity

Important Identities

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $(a+b)^2 + (a-b)^2 = a^2 + 2ab + b^2 + a^2 - 2ab + b^2 = 2(a^2 + b^2)$
4. $(a+b)^2 - (a-b)^2 = a^2 + 2ab + b^2 - (a^2 - 2ab + b^2) = 4ab$
5. $\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2 \Rightarrow a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$
6. $\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2 \Rightarrow a^2 + \frac{1}{a^2} = \left(a - \frac{1}{a}\right)^2 + 2$
7. $\left(a + \frac{1}{a}\right)^2 + \left(a - \frac{1}{a}\right)^2 = 2\left(a^2 + \frac{1}{a^2}\right)$
8. $\left(a + \frac{1}{a}\right)^2 - \left(a - \frac{1}{a}\right)^2 = 4$

Expansions of $(a \pm b)^3$

1. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \Rightarrow a^3 + b^3 = (a+b)^3 - 3ab(a+b)$
2. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 \Rightarrow a^3 - b^3 = (a-b)^3 + 3ab(a-b)$
3. $\left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right)$

   $\Rightarrow a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right)$
4. $\left(a - \frac{1}{a}\right)^3 = a^3 - \frac{1}{a^3} + 3\left(a - \frac{1}{a}\right)$

   $\Rightarrow a^3 - \frac{1}{a^3} = \left(a - \frac{1}{a}\right)^3 + 3\left(a - \frac{1}{a}\right)$

Expansion of $(x \pm a)(x \pm b)$

1. $(x + a)(x + b) = x^2 + (a + b)x + ab$
2. $(x + a)(x - b) = x^2 + (a - b)x - ab$
3. $(x - a)(x + b) = x^2 - (a - b)x - ab$
4. $(x - a)(x - b) = x^2 - (a + b)x + ab$

Expansion of $(a \pm b \pm c)^2$

$(a \pm b \pm c)^2 = a^2 + b^2 + c^2 \pm 2ab \pm 2bc \pm 2ca$

1. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
2. $(a + b - c)^2 = a^2 + b^2 + c^2 + 2(ab - bc - ca)$
3. $(a - b + c)^2 = a^2 + b^2 + c^2 - 2(ab + bc - ca)$
4. $(a - b - c)^2 = a^2 + b^2 + c^2 - 2(ab - bc + ca)$

Special Products

1. $(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$
2. $(a + b)(a^2 - ab + b^2) = a^3 + b^3$
3. $(a - b)(a^2 + ab + b^2) = a^3 - b^3$
4. $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc$