# Linear Equations in One Variable

## Important concepts

1. An equation is a statement which relates two expressions with an equality sign.

2. The equation in which the variable is of the first order is called a simple or a linear equation.

3. An equation remains unaffected:

   (i) if we add the same quantity to both sides.

   (ii) if we subtract the same quantity from both sides.

   (iii) if we multiply both sides by the same quantity.

   (iv) if we divide both sides by a same non zero quantity.

4. To solve a problem based on equations, first translate the relation given in the problem into an algebraic equation and then solve the equation.

5. Steps in solving a linear equation:

   (1) Simplify all brackets, fractions, etc., if required.

   (2) Bring all the terms containing the variables on one side and all the constant terms on the other side.

   (3) Solve the equation, obtained in the previous step, to get the value of its variable.

6. Consecutive integers, natural numbers and whole numbers are taken as, n, n+1, n+2, n+3,….

7. Consecutive even integers, natural numbers and whole numbers differ by 2 and hence are taken as, n, n+2, n+4, n+6,….

8. Consecutive odd integers, natural numbers and whole numbers differ by 2 and hence are taken as, n, n+2, n+4, n+6,….

# Simultaneous Linear Equations

## Related Terms

1. If two linear equations in the same two variables x, y are satisfied by the same pair of values of x and y then such linear equations are called simultaneous linear equations.

## Methods to Solve Simultaneous Linear Equations

1. Elimination Method: In this method, we eliminate either of the unknown quantity by addition or subtraction of the equation. Then we get an equation in one variable only which can be easily solved.
2. Substitution Method: In this method, first, we solve one equation for one unknown in terms of the other unknown. Then we substitute the expression for this unknown in the second equation. After solving the resulting equation, we obtain the value of one unknown; substitute this value in one of the given equations to find the value of second unknown.
   The following example depicts the method of elimination as well as substitution.
   **Example:**
   Solve: $x + y = 7; 5x + 2y = 7$
   Solution:
   Step 1 : $x + y = 7 \Rightarrow y = 7 - x$
   Step 2: $5x + 12y = 7$
   $\Rightarrow 5x + 12(7 - x) = 7$
   $\Rightarrow 5x + 84 - 12x = 7$
   $\Rightarrow -7x = -77$
   $\Rightarrow x = 11$
   Step 3:
   $y = 7 - x \Rightarrow y = 7 - 11 = -4$
   $\therefore$ Solution is: $x=11$ and $y=-4$

3. Method of elimination by equating coefficients
   Step 1: Multiply one or both of the equations by a suitable number or numbers such that the coefficients of x or the coefficients of y in both the equations become numerically equal.
   Step 2: Add or subtract one equation from the other
   Step 3: Solve the resulting equation
   Step 4: Substitute this value in any of the two equations and find the value of the other unknown.

**Example:**

Solve: $7x + 6y = 61; 5x - 8y = -23$

Solution:

$7x + 6y = 71....(1)$

$5x - 8y = -23....(2)$

Step 1: Multiply equation (1) by 8 and equation (2) by 6.

The resulting equations are:

$56x + 48y = 568....(3)$

$30x - 48y = 138....(4)$

Step 2:

Adding equations (3) and (4):

$$\begin{array}{l} 56x + 48y = 568....(3) \\ \underline{30x - 48y = -138}....(4) \\ 86x \qquad\quad = 430 \end{array}$$

Step 3:

$86x = 430$

$\Rightarrow x = \frac{430}{86} = 5$

Step 4:

Substituting x=5 in equation (1), we have

$7 \times (5) + 6y = 71$

$\Rightarrow 35 + 6y = 71$

$\Rightarrow 6y = 71 - 35$

$\Rightarrow 6y = 36$

$\Rightarrow y = 6$

Hence x=5 and y=6

## Problems Based On Simultaneous Equations

1. To Solve a problem based on simultaneous equations:
   Step 1: Assume the two variables as x and y
   Step 2: Form two equations in x and y
   Step 3: Solve the equations using any method

# Graphical Solution

## Graphs of Linear Equations in Two Variables Related Terms

1. An equation of the form $ax + by + c = 0$ is called a linear equation in two variables.
2. In the equation, $ax + by + c = 0$; x and y are the variables and a, b and c are the constants.

## To Draw a Graph of a Linear Equation :

1. Make x or y, the subject of the equation.
2. Give at least three suitable values to the variable on the right-hand side and find the corresponding values of the variable on the left hand side.
3. Construct a table for the different pairs of values of x and y.
4. Plot at least three ordered pairs (points) from the table on a graph paper.
5. Draw a straight line passing through the points plotted on the graph.

Example: Draw the graph of $3x + 2y = 6$:

Step1:

$$3x + 2y = 6$$
$$\Rightarrow 2y = 6 - 3x$$
$$\Rightarrow y = \frac{6 - 3x}{2} \quad [\text{Making y, the subject}]$$

Step2:

Now give at least three different values to the variable x and find the corresponding values of y.

Let $x = 0$; then $y = \frac{6 - 3 \times 0}{2} = 3$

Let $x = 2$; then $y = \frac{6 - 3 \times 2}{2} = 0$

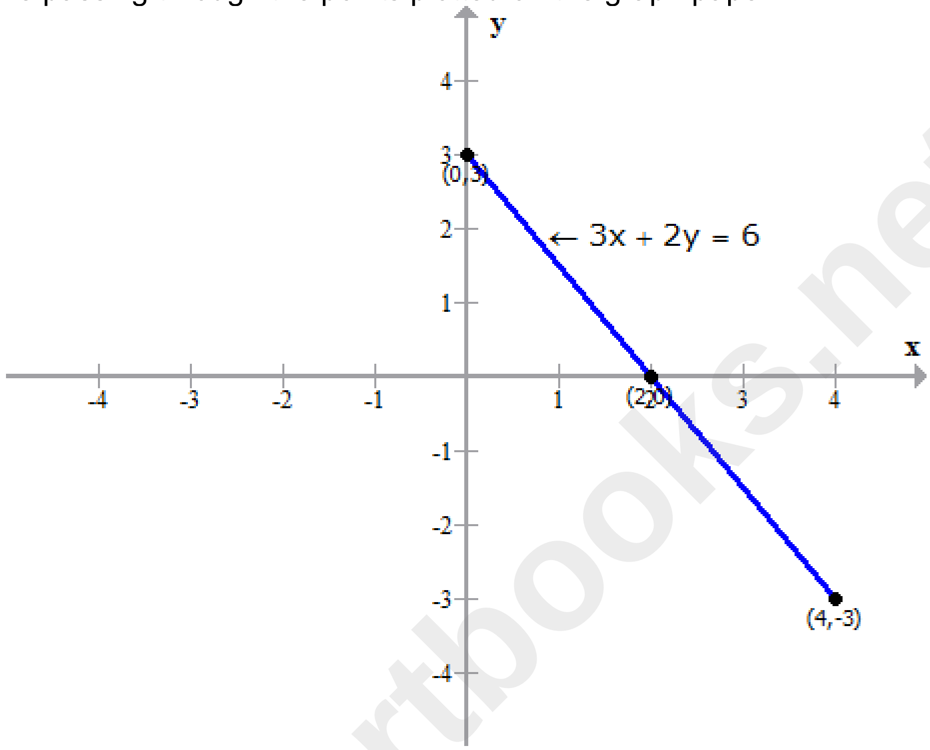
Let $x = 4$; then $y = \frac{6 - 3 \times 4}{2} = -3$

Step3:

Therefore, table for x and y is:

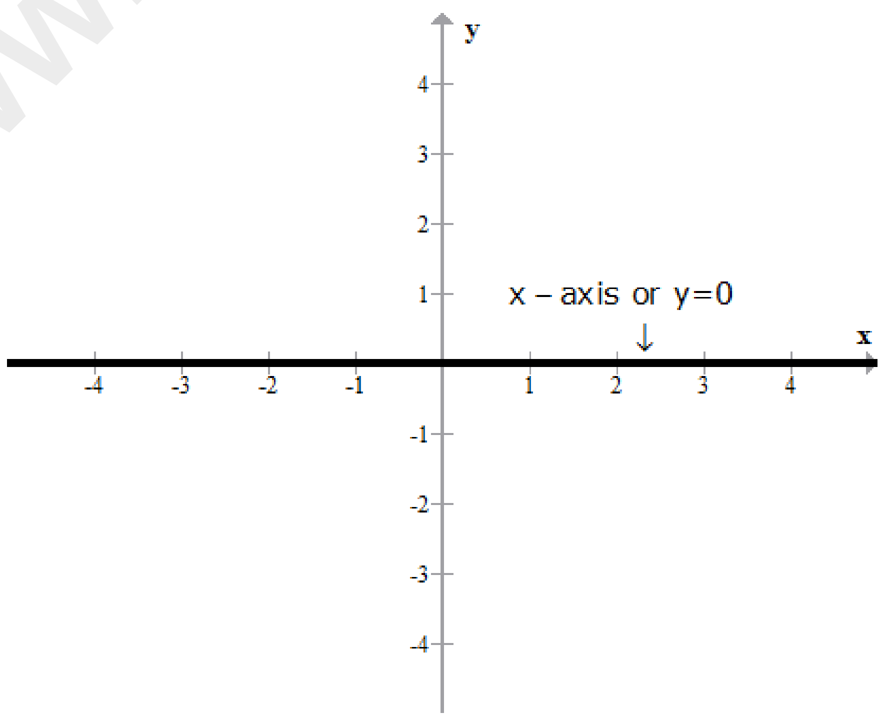
| | | | |
|---|---|---|---|
| x | 0 | 2 | 4 |
| y | 3 | 0 | –3 |

Step4: Plot the points $(0,3),(2,0)$ and $(4,-3)$ on a graph paper.
Draw a straight line passing through the points plotted on the graph paper.

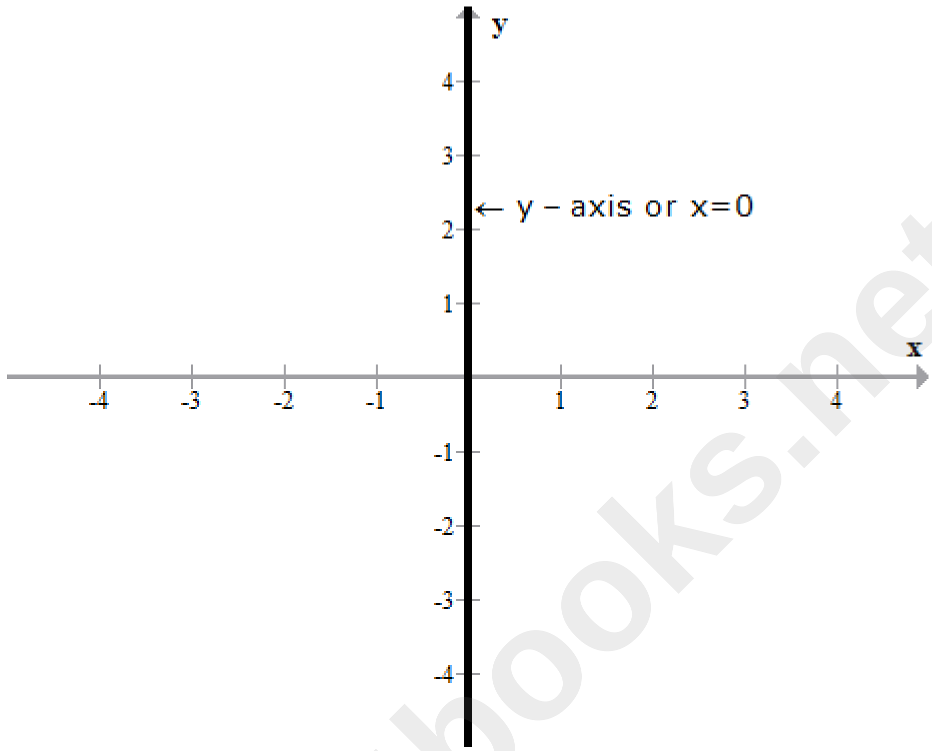



## Graphs of Coordinate Axes

1. The equation of x-axis is y=0.

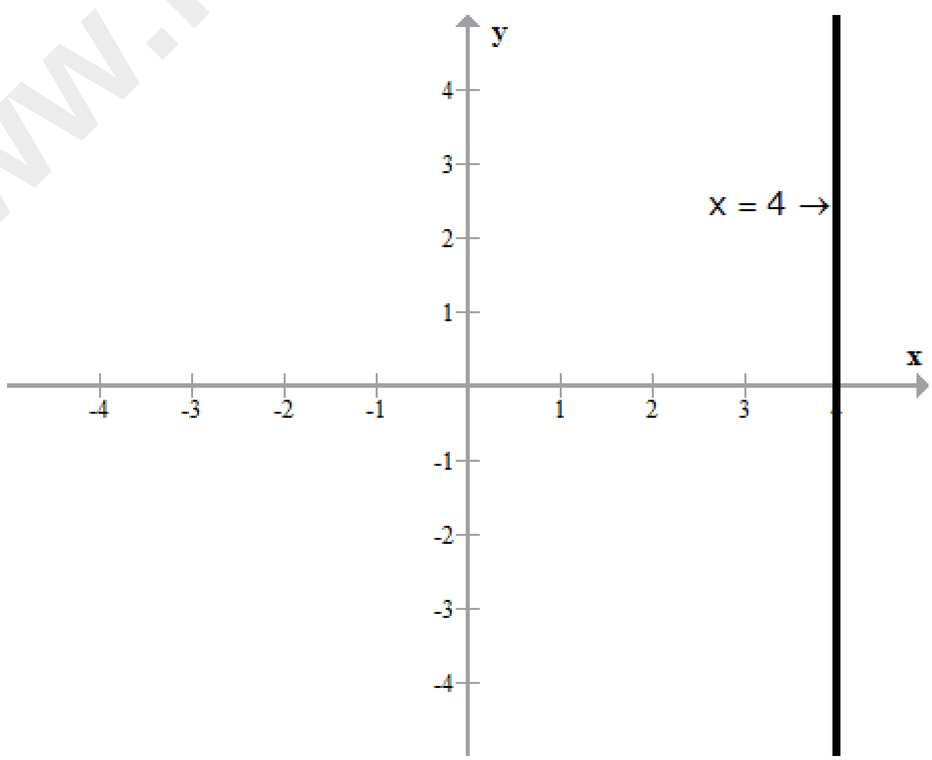


2. The equation of y-axis is x=0

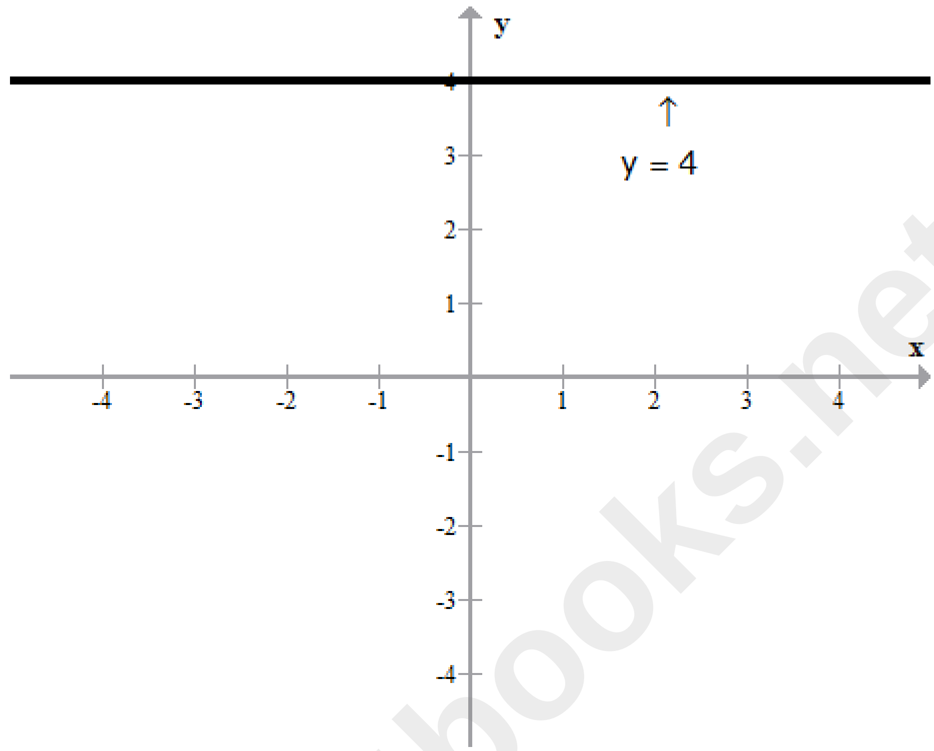



## Graphs of Some Special Functions

1. The graph of x=a (a is some constant) is a straight line parallel to y-axis and at a distance of 'a' units from the y-axis



2. The graph of y=b (b is some constant) is a straight line parallel to x-axis and at a distance of 'b' units from the x-axis






## Graphical Solution of Simultaneous Linear Equations

1. In order to solve simultaneous linear equations graphically:
   Step1: Draw a graph for each given equations
   Step2: Find the coordinates of the point of intersection of the two lines drawn.
   Step3: The coordinates of the point of intersection gives the solution of the given equation.
   Example:

   Solve the following system of equations graphically:
   $2x + y = 23....(1)$
   $4x - y = 19....(2)$
   Now consider equation (1).
   $2x + y = 23$
   $\Rightarrow y = 23 - 2x$
   Thus the table for equation (1) is:

| | | | |
|---|---|---|---|
| x | 0 | 7 | 9 |
| y | 23 | 9 | 5 |

   Plot the points $(0,23), (7,9)$ and $(9,5)$ on a graph paper.

Let us consider equation (2).

$4x - y = 19$

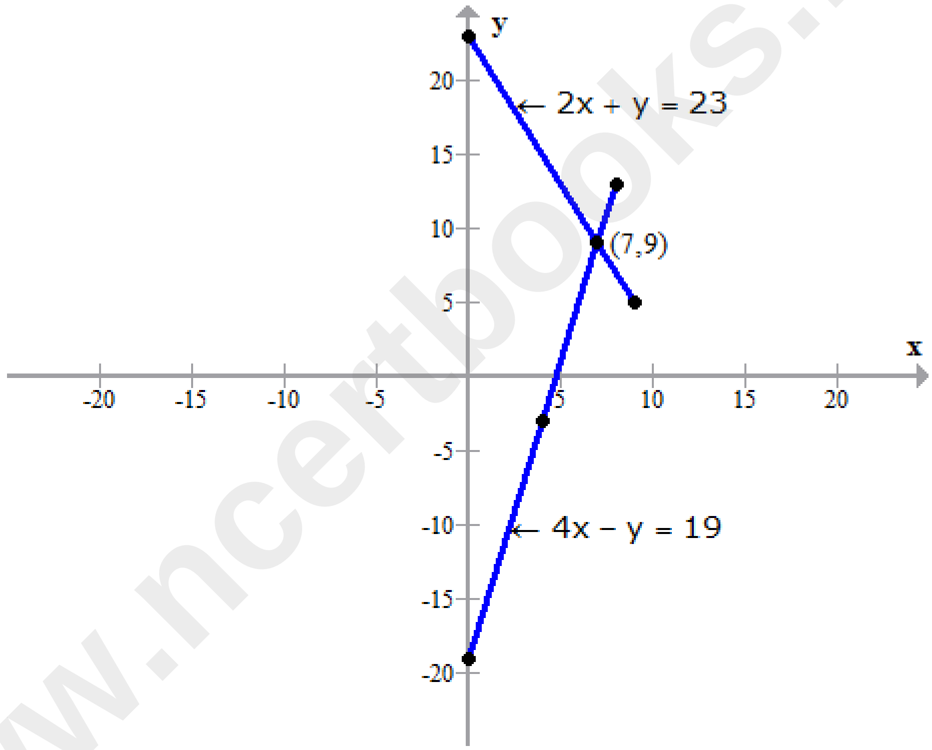
$\Rightarrow y = 4x - 19$

Thus the table for equation (1) is:

| | | | |
|---|---|---|---|
| x | 0 | 4 | 7 |
| y | –19 | –3 | 9 |

Plot the points $(0, -19), (4, -4)$ and $(7, 9)$ on the same graph paper.

Thus, we have




It is clear from the graph that the point of intersection of the given system of equations is (7, 9)