

# Similarity

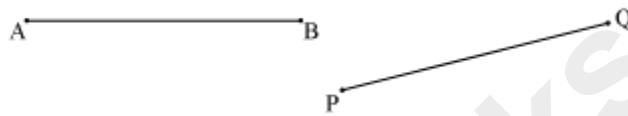
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## Difference Between Similarity and Congruence

### Congruency of line segments:

**“Two line segments are congruent to each other if their lengths are equal”.**

Consider the following line segments.



Here, the line segments AB and PQ will be congruent to each other, if they are of equal length.

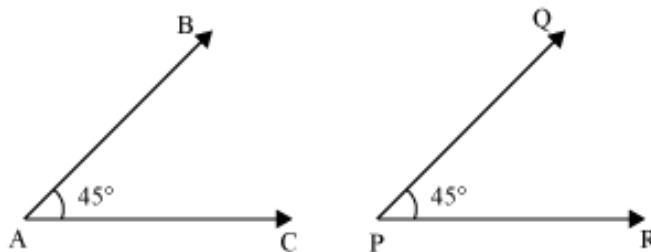
Conversely, we can say that, **“Two line segments are of equal length if they are congruent to each other”.**

i.e. if  $\overline{AB} \cong \overline{PQ}$ , then  $AB = PQ$ .

### Congruency of angles:

**“Two angles are said to be congruent to each other if they have the same measure”.**

The angles shown in the following figures are congruent to each other as both the angles are of the same measure  $45^\circ$ .



Thus, we can write  $\angle BAC \cong \angle QPR$ .

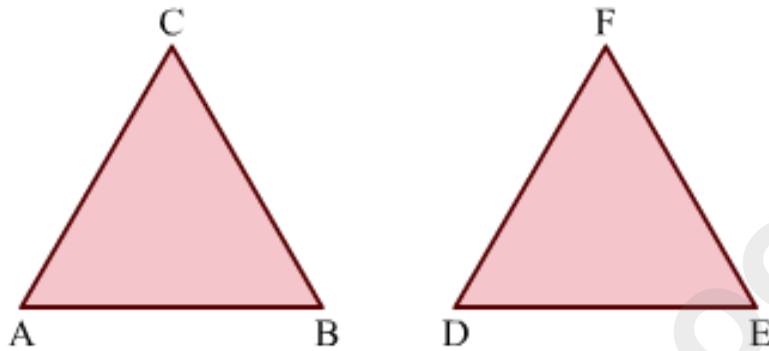
Its converse is also true.

**“If two angles are congruent to each other, then their measures are also equal”.**

There is one special thing about congruent figures that their corresponding parts are always equal.

For example, if two triangles are congruent then their corresponding sides will be equal. Also, their corresponding angles will be equal.

Look at the following triangles.



Here,  $\triangle ABC \cong \triangle DEF$  under the correspondence  $\triangle ABC \leftrightarrow \triangle DEF$ . This correspondence rule represents that in given triangles,  $AB \leftrightarrow DE$  (AB corresponds to DE),  $BC \leftrightarrow EF$ ,  $CA \leftrightarrow FD$ ,  $\angle A \leftrightarrow \angle D$ ,  $\angle B \leftrightarrow \angle E$ ,  $\angle C \leftrightarrow \angle F$ . These are **corresponding parts of congruent triangles** (CPCT),  $\triangle ABC$  and  $\triangle DEF$ .

Since  $\triangle ABC$  and  $\triangle DEF$  are congruent, their corresponding parts are equal. Therefore,  $AB = DE$ ,  $BC = EF$ ,  $CA = FD$

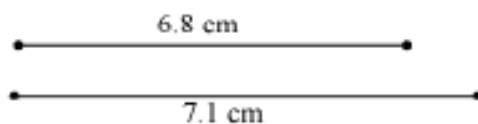
And,  $\angle A = \angle D$ ,  $\angle B = \angle E$ ,  $\angle C = \angle F$

Similarly, we can apply the method of CPCT on other congruent triangles also.

Let us now try and apply what we have just learnt in some examples.

**Example 1:**

**Find which of the pairs of line segments are congruent.**



(i)

(ii)

**Solution:**

(i) Lengths of the two line segments are not same. Therefore, they are not congruent.

(ii) Each of the line segments is of length 3.1 cm, i.e. they are equal. Therefore, they are congruent.

**Example 2:**

If  $\overline{AB} \cong \overline{PQ}$  and  $\overline{PQ} = 9$  cm, then find the length of  $\overline{AB}$ .

**Solution:**

Since  $\overline{AB} \cong \overline{PQ}$ , i.e. line segment AB is congruent to line segment PQ, therefore,  $\overline{AB}$  and  $\overline{PQ}$  are of equal length.

$$\therefore \overline{AB} = 9 \text{ cm}$$

**Example 3:**

If  $\angle ABC \cong \angle PQR$  and  $\angle PQR = 75^\circ$ , then find the measure of  $\angle ABC$ .

**Solution:**

If two angles are congruent, then their measures are equal.

Since  $\angle ABC \cong \angle PQR$ ,

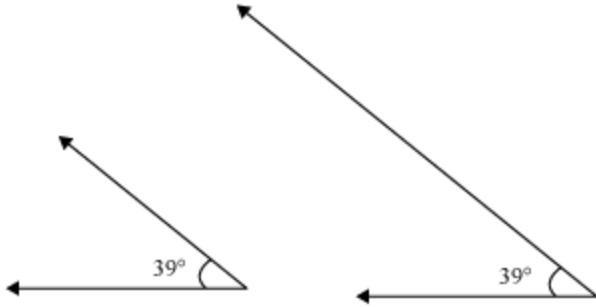
$$\therefore \angle ABC = \angle PQR$$

Therefore,  $\angle ABC = 75^\circ$

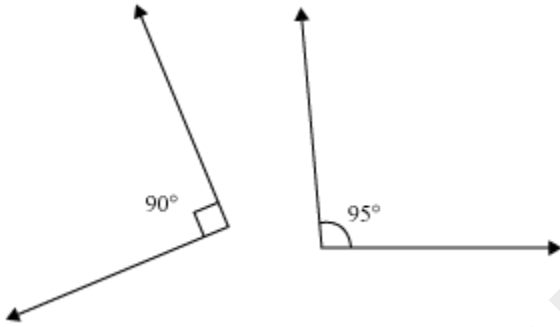
**Example 4:**

**Which of the following pairs of angles are congruent?**

(i)



(ii)



**Solution:**

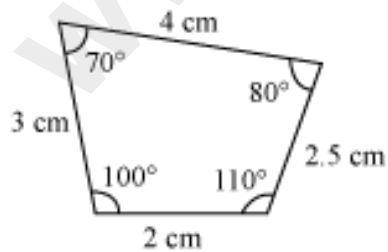
(i) The measure of both the angles is the same. Therefore, they are congruent.

(ii) The measures of the two angles are different. Therefore, they are not congruent.

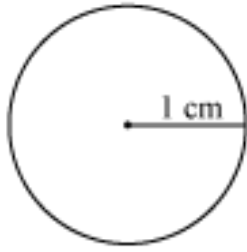
**Example 5:**

**Identify the pairs of similar and congruent figures from the following.**

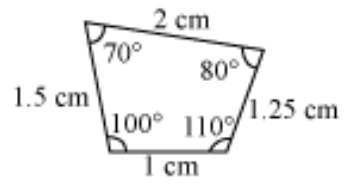
(i)



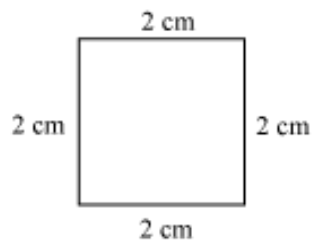
(ii)



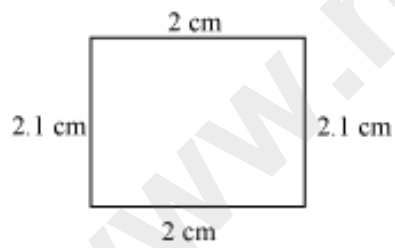
(iii)



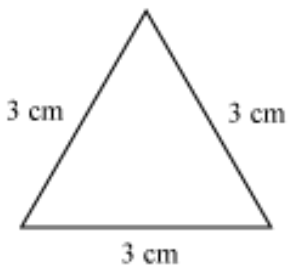
(iv)



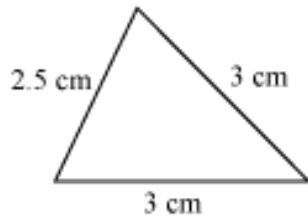
(v)



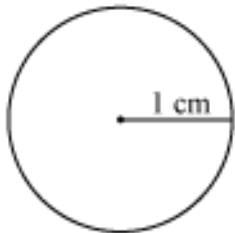
(vi)



(vii)



(viii)



**Solution:**

Figures (i) and (iii) are similar because their corresponding angles are equal and their corresponding sides are in the same ratio. However, these figures are not congruent as they are of different sizes.

Figures (ii) and (viii) are congruent as they are of the same shape and size (circles with radius 1 cm each).

**Example 6:**

**Are the following figures similar or congruent?**

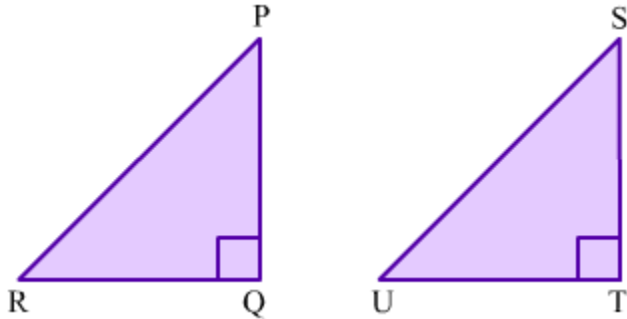


**Solution:**

The two given figures show two one-rupee coins. As both the figures represent the same coin in two different sizes, they are similar to each other. However, the pictures are not congruent because of their different sizes.

**Example 7:**

In the following figure,  $\Delta PQR$  and  $\Delta STU$  are congruent.



If  $PQ = 8$  cm,  $QR = 6$  cm then find the perimeter of  $\Delta STU$ .

**Solution:**

In  $\Delta PQR$ , we have

$PQ = 8$  cm,  $QR = 6$  cm and  $\angle Q = 90^\circ$

Applying Pythagoras theorem in  $\Delta PQR$ , we obtain

$$RP^2 = PQ^2 + QR^2$$

$$\Rightarrow RP^2 = 8^2 + 6^2$$

$$\Rightarrow RP^2 = 64 + 36$$

$$\Rightarrow RP^2 = 100$$

$$\Rightarrow RP = 10 \text{ cm}$$

Since  $\Delta PQR$  and  $\Delta STU$  are congruent, their corresponding parts will be equal.

Therefore,

$$PQ = 8 \text{ cm} = ST \quad (\text{CPCT})$$

$$QR = 6 \text{ cm} = TU \text{ and} \quad (\text{CPCT})$$

$$RP = 10 \text{ cm} = US \quad (\text{CPCT})$$

$$\therefore \text{Perimeter of } \Delta STU = ST + TU + US = 8 \text{ cm} + 6 \text{ cm} + 10 \text{ cm} = 24 \text{ cm}$$

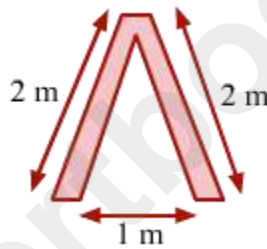
## Mid Point Theorem and Its Converse

### Midpoint Theorem

The given image is that of a food outlet named Adam's Diner. The logo of the outlet, as you can see on top of the building, is the letter 'A'.



The carpenter who made the logo fixed the two legs of the logo, each of length 2 m, such that the distance between the two legs at the base was 1 m.

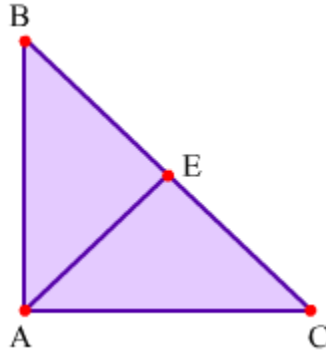


The carpenter then put the horizontal bar of the logo at the **mid-point** of the two legs so that it fitted exactly in the middle of the legs. The midpoint theorem came to his aid while doing so. In this lesson, we will study this theorem, its proof and its converse, and solve some related examples.

### Whiz Kid

#### Right triangle median theorem

The length of the median on the hypotenuse of a right triangle is half that of the hypotenuse.



Here,  $AE = \frac{1}{2} \times BC$

### Activity

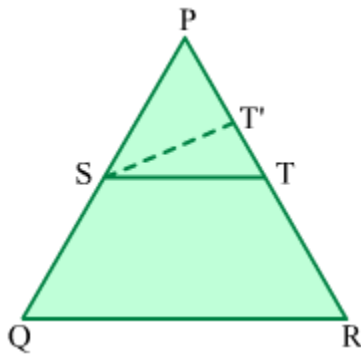
Perform the following activity to verify the midpoint theorem.

- Draw a large scalene triangle on a sheet of paper.
- Name its vertices A, B and C. Find the midpoints of sides AB and AC and name them D and E respectively. Draw a line joining the two midpoints.
- Cut out  $\triangle ABC$  and then cut it along line segment DE.
- Draw a quadrilateral BDEC. Place  $\triangle ADE$  on it such that vertices E and D of the triangle are respectively on vertex C and line segment BC of the quadrilateral. Mark the point where vertex D touches line segment BC.
- Shift  $\triangle ADE$  such that its vertices D and E are respectively on vertex B and line segment BC of the quadrilateral. Mark the point where vertex E touches line segment BC.
- What do you notice about the lengths of DE and BC? Do the marked points coincide?

### Proof of the Converse of the Midpoint Theorem

**Statement:** The line drawn through the midpoint of one side of a triangle and parallel to another side bisects the third side.

**Given:**  $\Delta PQR$  in which S is the midpoint of PQ and  $ST \parallel QR$



**To prove:** T is the midpoint of PR

**Construction:** Take a point T' on PR and join S and T'.

**Proof:** Let us say that T is not the midpoint of PR. Let T' be the midpoint of PR.

In  $\Delta PQR$ , S is the midpoint of PQ and T' is the midpoint of PR.

$\therefore$  By the midpoint theorem,  $ST' \parallel QR$  ... (1)

It is given that  $ST \parallel QR$ . ... (2)

From 1 and 2, we conclude that  $ST \parallel ST'$ , which is not possible.

So, our assumption was wrong.

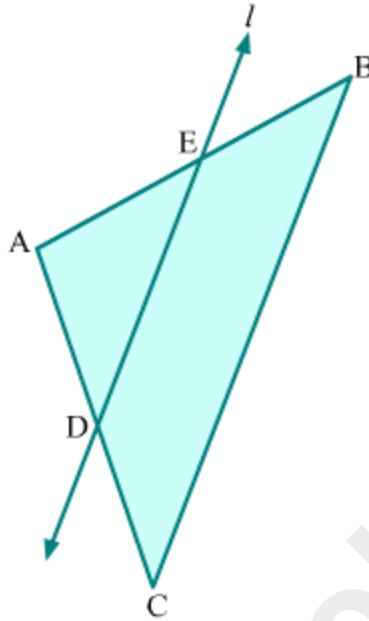
Hence, T is the midpoint of PR.

### Solved Examples

#### Easy

#### Example 1:

Line  $l$  cuts the isosceles  $\Delta ABC$ , with  $AB = AC = 10$  cm and  $BC = 15$  cm, in such a way that  $\Delta ADE$  is also isosceles with  $AD = AE = 5$  cm. Find the length of DE.



**Solution:**

It is given that  $\triangle ABC$  is isosceles with  $AB = AC = 10$  cm,  $BC = 15$  cm and  $AD = AE = 5$  cm.

$$AC = AD + DC$$

$$\Rightarrow DC = AC - AD = (10 - 5) \text{ cm} = 5 \text{ cm}$$

Since  $AD = DC$ ,  $D$  is the midpoint of  $AC$ .

Similarly,  $E$  is the midpoint of  $AB$ .

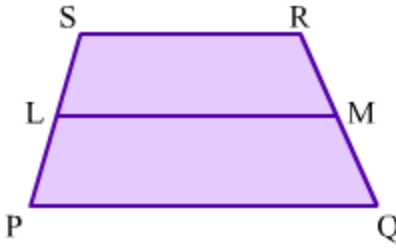
Using the midpoint theorem, we get:

$$DE = \frac{BC}{2} = \frac{15}{2} \text{ cm} = 7.5 \text{ cm}$$

**Medium**

**Example 1:**

In the given trapezium  $PQRS$ ,  $PQ \parallel SR$  and  $L$  is the midpoint of line segment  $PS$ . If  $LM \parallel PQ$ , then prove that  $RM = QM$ .



**Solution:**

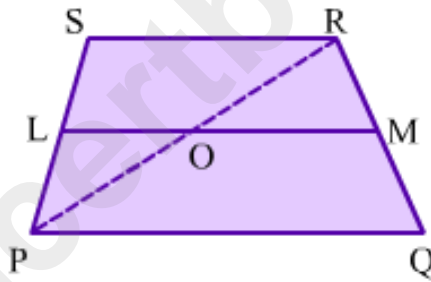
It is given that:

$$LM \parallel PQ$$

$$PQ \parallel SR$$

$\Rightarrow LM \parallel SR$  ( $\because$  Lines parallel to the same line are also parallel)

*Construction:* Join P to R such that PR intersects LM at point O.



In  $\triangle PSR$ , L is the midpoint of PS and  $LO \parallel RS$ .

So, by the converse of the midpoint theorem, we get O as the midpoint of PR.

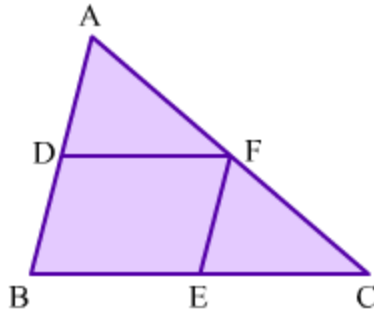
Now, in  $\triangle PQR$ , O is the midpoint of PR and  $OM \parallel PQ$ .

Again, by the converse of the midpoint theorem, we get M as the midpoint of QR.

$$\Rightarrow RM = QM$$

**Example 2:**

In the shown  $\triangle ABC$ , D, E and F are the midpoints of sides AB, BC and AC respectively. Prove that BEFD is a parallelogram.



**Solution:**

In  $\triangle ABC$ , D and F are the midpoints of AB and AC respectively.

So, by the midpoint theorem, we obtain  $DF \parallel BC$

$$\Rightarrow DF \parallel BE \dots (1)$$

Again, in  $\triangle ABC$ , E and F are the midpoints of BC and AC respectively.

So, by the midpoint theorem, we obtain  $EF \parallel BA$

$$\Rightarrow EF \parallel BD \dots (2)$$

Now in quadrilateral BEFD, we have:

$DF \parallel BE$  and  $EF \parallel BD$  (From 1 and 2)

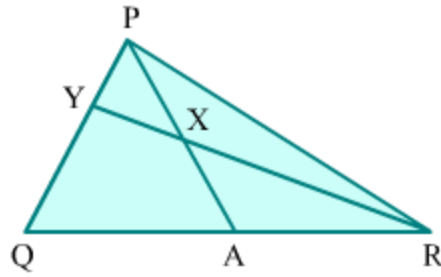
$\Rightarrow$  BEFD is a parallelogram. ( $\because$  There are two pairs of parallel opposite sides)

**Hard**

**Example 1:**

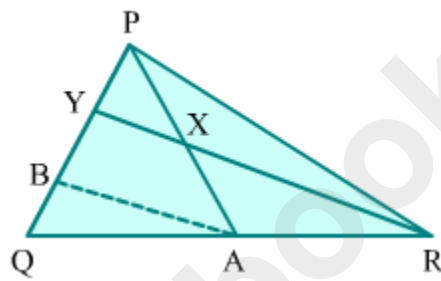
In the given  $\triangle PQR$ , PA is the median to side QR and X is the midpoint of PA.

When produced, RX meets PQ at Y. Prove that  $PY = \frac{1}{3} PQ$ .



**Solution:**

*Construction:* Draw a line AB such that it is parallel to RY.



In  $\triangle PAB$ , X is the midpoint of AP and  $XY \parallel AB$ .

So, by the converse of the midpoint theorem, we obtain Y as the midpoint of PB.

$$\Rightarrow PY = YB \dots (1)$$

In  $\triangle QYR$ , A is the midpoint of QR. ( $\because$  AP is the median to side QR)

Also,  $RY \parallel AB$

So, by the converse of the midpoint theorem, we obtain B as the midpoint of YQ.

$$\Rightarrow YB = BQ \dots (2)$$

From equations 1 and 2, we obtain:

$$PY = YB = BQ \dots (3)$$

From the figure, we have:

$$PQ = PY + YB + BQ$$

$$\Rightarrow PQ = PY + PY + PY$$

$$\Rightarrow PQ = 3PY$$

$$\Rightarrow \therefore PY = \frac{1}{3}PQ$$

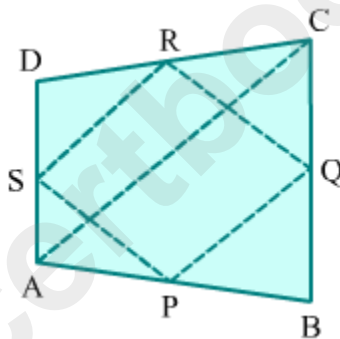
**Example 2:**

**Prove that the quadrilateral formed by joining the midpoints of all sides of a quadrilateral is a parallelogram.**

**Solution:**

Let ABCD be a quadrilateral and P, Q, R and S the respective midpoints of sides AB, BC, CD and DA.

*Construction:* Draw diagonal AC of quadrilateral ABCD.



To prove that PQRS is a parallelogram, we have to prove that it has one pair of parallel and equal opposite sides.

In  $\triangle ACD$ , R and S are the midpoints of sides CD and DA respectively.

So, by the midpoint theorem, we obtain  $SR \parallel AC$  and  $SR = \frac{1}{2}AC$  ... (1)

Now, in  $\triangle ABC$ , P and Q are the midpoints of sides AB and BC respectively.

So, by midpoint theorem, we obtain  $PQ \parallel AC$  and  $PQ = \frac{1}{2}AC$  ... (2)

From 1 and 2, we obtain:

$PQ \parallel RS$  ( $\because$  Lines parallel to the same line are also parallel)

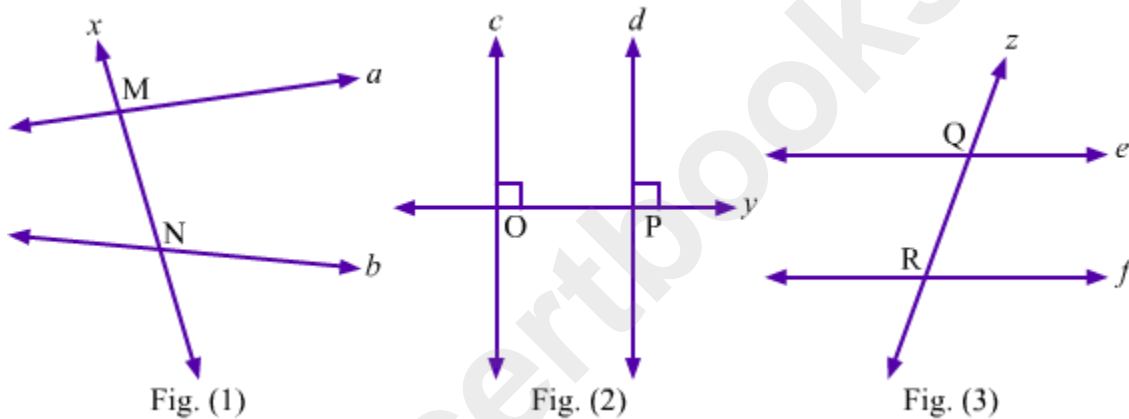
$$PQ = RS$$

In quadrilateral PQRS, we have one pair of parallel and equal opposite sides, i.e.,  $PQ \parallel RS$  and  $PQ = RS$ . Thus, PQRS is a parallelogram.

### Properties of Parallel Lines and Their Transversals with Respect to Intercepts

A line that intersects two (or more) distinct lines at different points is known as a transversal. The portion of the transversal lying between these two distinct lines is known as **intercept**.

To understand intercept better, let us draw two lines and their transversal in a few different ways.



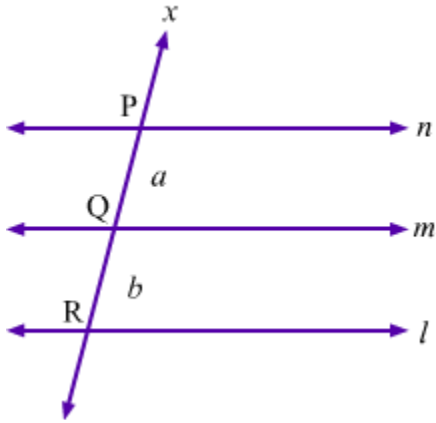
In fig. (1),  $x$  is the transversal which intersects lines  $a$  and  $b$  at points  $M$  and  $N$  respectively. So, line segment  $MN$  is the intercept made by lines  $a$  and  $b$  on transversal  $x$ .

Similarly, in fig. (2),  $OP$  is the intercept made by lines  $c$  and  $d$  on transversal  $y$ .

In fig. (3),  $QR$  is the intercept made by lines  $e$  and  $f$  on transversal  $z$ .

If two lines are parallel, then the transversal exhibit a few interesting properties related to the intercepts formed on it.

Let us consider the following figure.

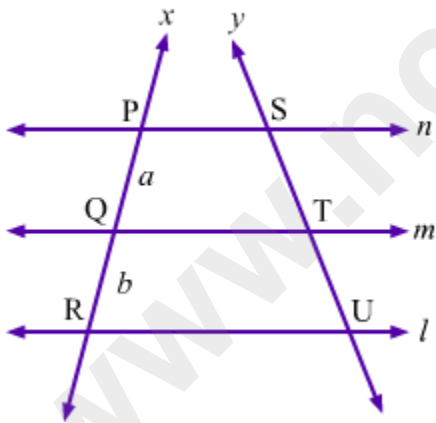


Here, lines  $l$ ,  $m$  and  $n$  are parallel to each other, and line  $x$  is the transversal making intercepts  $PQ$  and  $QR$ .

Let the length of the intercept  $PQ$  be  $a$  and that of intercept  $QR$  be  $b$ .

$$\therefore \frac{PQ}{QR} = \frac{a}{b}$$

Now, let us draw another transversal  $y$  with respect to lines  $l$ ,  $m$  and  $n$  making intercept  $ST$  and  $TU$ .



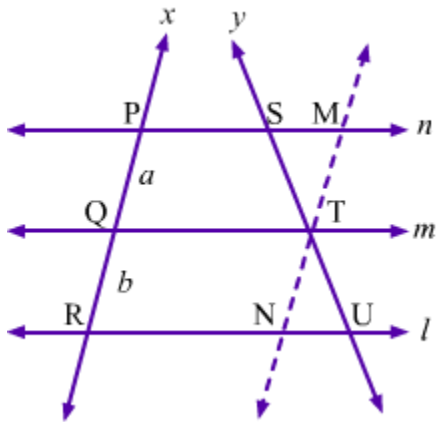
There is a property about intercepts made by three parallel lines on two different transversals. It is stated as follows:

**The intercepts made by three parallel lines on one transversal are in the same ratio as the corresponding intercepts made by the same lines on any other transversal.**

$$\Rightarrow \frac{PQ}{QR} = \frac{ST}{TU} = \frac{a}{b}$$

Let us prove this property.

For this, let us consider the same figure and draw a line parallel to  $x$  through  $T$ , which intersects  $n$  and  $l$  at points  $M$  and  $N$  respectively.



We have,  $PM \parallel QT$  (As  $n \parallel m$ )

$PQ \parallel MT$  (By construction)

Thus,  $PQTM$  is a parallelogram.

$\Rightarrow PQ = MT = a \dots (1)$  (Opposite sides of a parallelogram are equal.)

Similarly,  $QRNT$  is a parallelogram.

$\Rightarrow QR = TN = b \dots (2)$  (Opposite sides of a parallelogram are equal.)

Now, in  $\Delta TSM$  and  $\Delta TUN$ :

$\angle STM = \angle UTN$  (Vertically opposite angles)

$\angle TSM = \angle TUN$  (Alternate angles)

$\angle TMS = \angle TNU$  (Alternate angles)

$\therefore \Delta TSM \sim \Delta TUN$  (By AAA criterion of similarity)

$$\Rightarrow \frac{TS}{TU} = \frac{MT}{NT} \quad (\text{Property of similar triangles})$$

$$\Rightarrow \frac{TS}{TU} = \frac{a}{b} \quad [\text{Using (1) and (2)}]$$

$$\Rightarrow \frac{TS}{TU} = \frac{PQ}{QR}$$

Hence proved.

There may be a case when the intercepts made by three parallel lines on first transversal, i.e.  $x$ , are congruent.

$$\therefore PQ = QR$$

$$\Rightarrow \frac{PQ}{QR} = 1$$

$$\text{Since, } \frac{PQ}{QR} = \frac{TS}{TU},$$

$$\frac{TS}{TU} = 1$$

$$\Rightarrow ST = TU$$

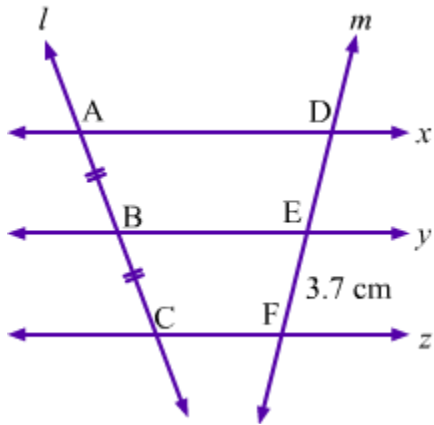
This property can be stated as follows:

**If three parallel lines form congruent intercepts on one transversal, then the intercepts formed by them on other transversals are also congruent.**

Let us go through a few examples to understand the concept better.

**Example1:**

In the given figure,  $x \parallel y \parallel z$  and  $AB = BC$ . Find  $DF$ , if  $EF = 3.7$  cm.



**Solution:**

It is given that  $x \parallel y \parallel z$ ,  $AB = BC$  and  $EF = 3.7$  cm.

In the given figure,  $l$  and  $m$  are two transversals with respect to parallel lines  $x$ ,  $y$  and  $z$ . Also,  $AB$  and  $BC$  are intercepts formed by three parallel lines  $x$ ,  $y$  and  $z$  on transversal  $l$ , such that  $AB = BC$ .

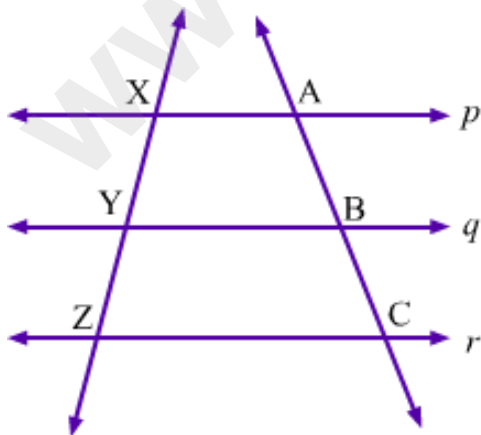
We know that if three parallel lines form congruent intercepts on one transversal, then intercepts formed by them on other transversals are also congruent.

$$\therefore DE = EF = 3.7 \text{ cm (Since } AB = BC)$$

$$\text{Thus, } DF = DE + EF = 3.7 \text{ cm} + 3.7 \text{ cm} = 7.4 \text{ cm.}$$

**Example 2:**

In the given figure,  $p \parallel q \parallel r$ . If  $XY = 2.2$  cm and  $BC = 5$  cm, then what is the numerical value of  $AB \times YZ$ ?



**Solution:**

It is given that  $p \parallel q \parallel r$ ,  $XY = 2.2$  cm and  $BC = 5$  cm.

Now, we know that the intercepts made by three parallel lines on one transversal are in the same ratio as the corresponding intercepts made by the same lines on any other transversal.

$$\therefore \frac{XY}{YZ} = \frac{AB}{BC}$$

$$\Rightarrow AB \times YZ = XY \times BC$$

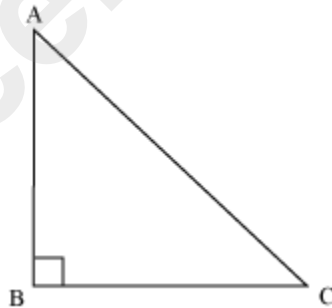
$$\Rightarrow AB \times YZ = 2.2 \times 5$$

$$\Rightarrow AB \times YZ = 11$$

Hence, the required value is 11.

**Pythagoras Theorem and Its Applications**

Look at the following right triangle ABC.



We have the following relationship between the sides of a right-angled triangle ABC.

$$(AC)^2 = (AB)^2 + (BC)^2$$

This relation between the sides of a right-angled triangle is known as **Pythagoras Theorem**.

Let us understand the proof of the theorem with the help of the given video.

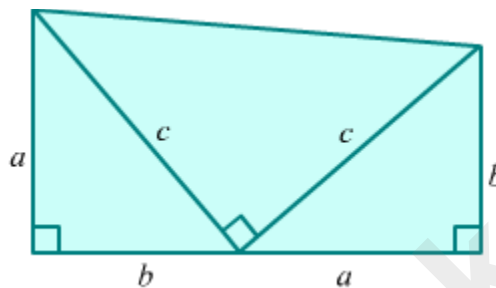
This theorem has several other proofs also. Let us discuss two of them here.

**Proof by American President:**

The 20<sup>th</sup> president of United States, James Garfield proved this theorem taking two right angled triangles having sides as  $a$ ,  $b$  and  $c$  and an other right angled triangle with side  $c$ .

The proof given by him is as follows:

All three triangles are combined as shown in the following figure.



Thus, we got a trapezium.

$$\text{Area of trapezium} = \frac{1}{2} \times (\text{Sum of lengths of parallel sides}) \times \text{Height}$$

$$\Rightarrow \text{Area of trapezium} = \frac{1}{2} \times (a + b)(a + b)$$

$$\Rightarrow \text{Area of trapezium} = \frac{a^2 + 2ab + b^2}{2} \dots(1)$$

And,

$$\text{Sum of areas of 3 triangles} = \frac{ab}{2} + \frac{ab}{2} + \frac{c^2}{2} \dots(2)$$

$$\frac{ab}{2} + \frac{ab}{2} + \frac{c^2}{2} = \frac{a^2 + 2ab + b^2}{2} \quad [\text{Using (1) and (2)}]$$

$$\Rightarrow \frac{2ab + c^2}{2} = \frac{a^2 + 2ab + b^2}{2}$$

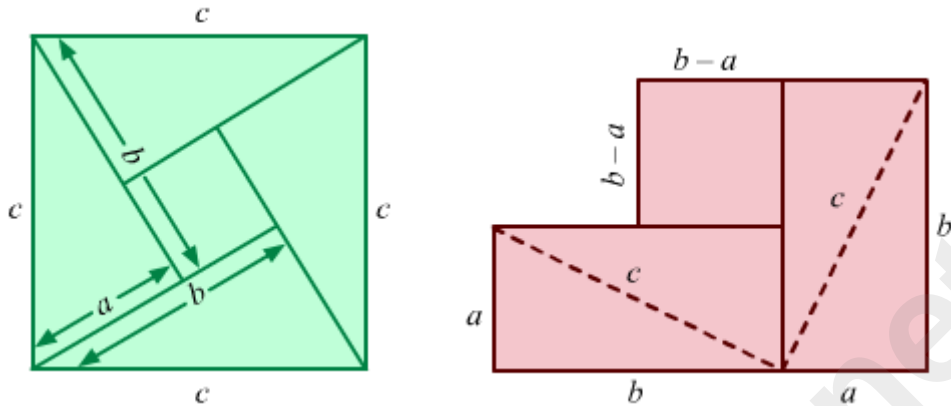
$$\Rightarrow 2ab + c^2 = a^2 + 2ab + b^2$$

$$\Rightarrow c^2 = a^2 + b^2$$

This theorem of right angled triangles was also known to Indian, Chinese, Greek and Babylonian mathematician long before Pythagoras lived. Thus, it was proved by the mathematicians of that time differently.

### Proof by Indian Mathematician:

Bhaskaracharya, the great Indian mathematician of 2<sup>nd</sup> century AD, used the below given diagrams to prove this theorem.



$$\text{Area of 4 triangles} = 4\left(\frac{1}{2}ab\right) = 2ab$$

$$\text{Area of small square} = (b - a)^2 = b^2 - 2ab + a^2$$

Area of big square = Area of 4 triangles + Area of small square

$$\Rightarrow c^2 = 2ab + (b^2 - 2ab + a^2)$$

$$\Rightarrow c^2 = 2ab + b^2 - 2ab + a^2$$

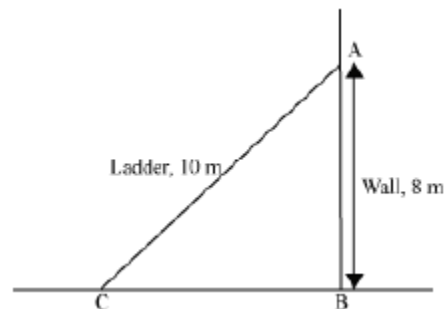
$$\Rightarrow c^2 = b^2 + a^2$$

Bhaskaracharya proved this theorem by using property of similarity also, but we will not discuss it here.

**Note:** Conventionally in  $\Delta ABC$ , we consider the lengths of sides opposite to vertices A, B and C as  $a$ ,  $b$  and  $c$  respectively.

In real life, we come across many situations where a right angle is formed. Let us consider such a situation.

A 10m long ladder is placed on a wall such that the ladder touches the wall at 8m above the ground. This situation can be shown geometrically as follows.



In the above figure, AB is the wall of height 8 m and AC is the ladder of length 10 m. We know that a wall is perpendicular to the floor, i.e. AB is perpendicular to BC. Thus,  $\angle ABC$  is a right angle.

**Now, can we calculate the distance of the foot of the ladder from the base of the wall?**

We can calculate the distance of the foot of the ladder from the base of the wall by using Pythagoras theorem. To understand how Pythagoras theorem is helpful in this case, look at the following video.

In this way, we can use Pythagoras theorem in many situations where right-angled triangle is formed.

**Is the converse of Pythagoras theorem also true?**

Yes, the converse of Pythagoras theorem is also true.

Its converse can be stated as follows:

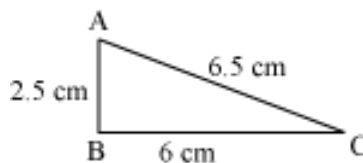
**“In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle”.**

But how will we prove it?

Thus, in a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

Using the converse, we can check whether the given triangle is a right triangle or not.

Let ABC be a triangle with sides AB = 2.5 cm, BC = 6 cm and CA = 6.5 cm. Can we say that the triangle ABC is a right-angled triangle?



$$\text{Here, } (AB)^2 = (2.5)^2 = 6.25$$

$$(BC)^2 = 6^2 = 36$$

$$\text{And, } (CA)^2 = (6.5)^2 = 42.25$$

Therefore, we obtain

$$(AB)^2 + (BC)^2 = 6.25 + 36 = 42.25 = (CA)^2$$

Thus, using the converse of Pythagoras theorem, we can say that the angle opposite to the side CA, i.e.  $\angle B$ , is a right angle.

**Pythagorean triplet:** Any three natural numbers  $a, b, c$  form a Pythagorean triplet if it satisfies  $a^2 = b^2 + c^2$  irrespective of order.

**General form to find the Pythagorean Triplets:**

For natural number:  $2n, (n^2 - 1), (n^2 + 1)$  where  $n$  may be even or odd.

For odd natural numbers:  $n, 1/2(n^2 - 1), 1/2(n^2 + 1)$  where  $n$  is odd,  $n \in \mathbb{N}$ .

Any number of pythagorean triplets can be generated by giving values to  $n$ .

**For example**, the number 39, 80, and 89 forms a Pythagorean triplet.

$$39^2 = 1521$$

$$80^2 = 6400$$

$$89^2 = 7921$$

$$\text{Now, } 1521 + 6400 = 7921$$

$$\therefore 39^2 + 80^2 = 89^2$$

Here, the square of a number is equal to the sum of the squares of the other two numbers. Therefore, we can say that 39, 80, and 89 forms a Pythagorean triplet.

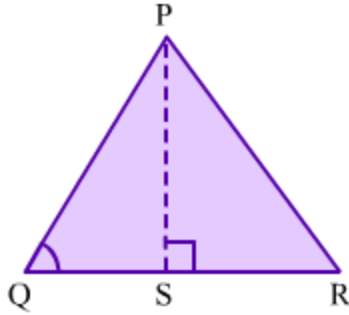
**Applications of Pythagoras theorem:**

Though Pythagoras theorem holds true for only right angled triangle, it can be applied to obtuse and acute angled triangles as well. The obtained results are discussed below:

**(1) In acute angled  $\Delta PQR$ , if  $PS \perp QR$ ,  $QSR$  is a line and  $\angle Q < 90^\circ$  then**

$$PR^2 = PQ^2 + QR^2 - 2QR \cdot QS$$

**Proof:** Observe the acute angled triangle  $\Delta PQR$ .



Here,  $PS \perp QR$ , QSR is a line and  $\angle Q < 90^\circ$ .

In  $\Delta PSQ$ , we have

$$PQ^2 = PS^2 + QS^2 \quad \dots(1) \quad (\text{By Pythagoras theorem})$$

In  $\Delta PSR$ , we have

$$PR^2 = PS^2 + SR^2 \quad (\text{By Pythagoras theorem})$$

$$\Rightarrow PR^2 = PS^2 + (QR - QS)^2$$

$$\Rightarrow PR^2 = PS^2 + QR^2 + QS^2 - 2QR \cdot QS$$

$$\Rightarrow PR^2 = (PS^2 + QS^2) + QR^2 - 2QR \cdot QS$$

$$\Rightarrow PR^2 = PQ^2 + QR^2 - 2QR \cdot QS \quad [\text{Using (1)}]$$

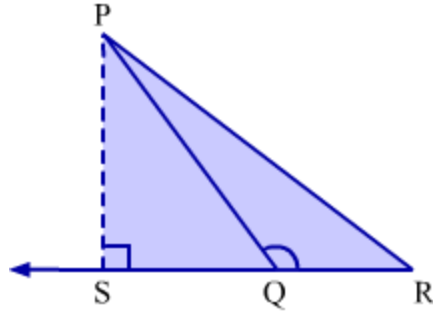
Hence proved.

**(2) In obtuse angled  $\Delta PQR$ , if  $PS \perp QR$ , SQR is a line and  $\angle Q > 90^\circ$  then**

$$\mathbf{PR^2 = PQ^2 + QR^2 + 2QR \cdot QS}$$

**Proof:**

Observe the acute angled triangle  $\Delta PQR$ .



Here,  $PS \perp QR$ ,  $SQR$  is a line and  $\angle Q > 90^\circ$ .

In  $\triangle PSQ$ , we have

$$PQ^2 = PS^2 + QS^2 \quad \dots(1) \quad (\text{By Pythagoras theorem})$$

In  $\triangle PSR$ , we have

$$PR^2 = PS^2 + SR^2 \quad (\text{By Pythagoras theorem})$$

$$\Rightarrow PR^2 = PS^2 + (QR + QS)^2$$

$$\Rightarrow PR^2 = PS^2 + QR^2 + QS^2 + 2QR \cdot QS$$

$$\Rightarrow PR^2 = (PS^2 + QS^2) + QR^2 + 2QR \cdot QS$$

$$\Rightarrow PR^2 = PQ^2 + QR^2 + 2QR \cdot QS \quad [\text{Using (1)}]$$

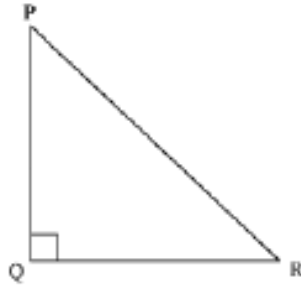
Hence proved.

Now, let us discuss some more examples based on Pythagoras theorem and its converse.

**Example 1:**

$\triangle PQR$  is an isosceles triangle, right angled at  $Q$ . Prove that  $PR^2 = 2PQ^2$ .

**Solution:**



Here, PQR is an isosceles triangle, right angled at Q. Therefore,

$$PQ = QR \dots (1)$$

Now, using Pythagoras theorem, we obtain

$$(PR)^2 = (PQ)^2 + (QR)^2$$

Using equation (1), we obtain

$$(PR)^2 = (PQ)^2 + (PQ)^2$$

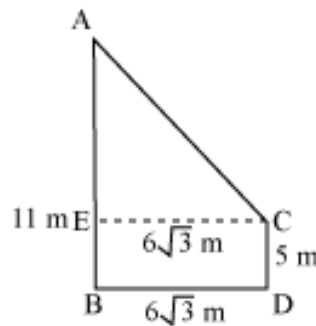
$$(PR)^2 = 2(PQ)^2$$

**Example 2:**

Two poles are of length 5m and 11m. The distance between the feet of the poles is  $6\sqrt{3}$  m. Find the distance between the tops of the poles.

**Solution:**

The figure for the given situation can be drawn as follows.



In the above figure, the poles are denoted by AB and CD, where AB = 11 m and CD = 5 m. The distance between the feet of the poles, i.e. BD, is  $6\sqrt{3}$  m.

Let us draw a perpendicular CE from C on AB.

AC is the distance between the top of the poles.

$$\text{Here, } BD = CE = 6\sqrt{3} \text{ m}$$

$$\text{And } AE = AB - BE$$

$$= AB - CD \text{ [since } BE = CD]$$

$$= (11 - 5) \text{ m}$$

$$= 6 \text{ m}$$

Using Pythagoras theorem in  $\triangle ACE$ , we obtain

$$(AC)^2 = (AE)^2 + (EC)^2$$

$$(AC)^2 = (6)^2 \text{ m}^2 + (6\sqrt{3})^2 \text{ m}^2$$

$$= (36 + 108) \text{ m}^2$$

$$= 144 \text{ m}^2$$

$$\Rightarrow AC = \sqrt{144 \text{ m}^2}$$

$$\Rightarrow AC = 12 \text{ m}$$

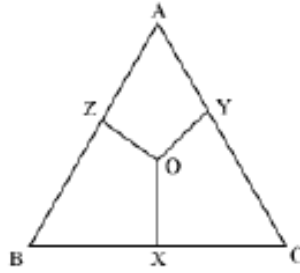
Thus, the distance between the tops of the poles is 12 m.

**Example 3:**

**O** is any point in the interior of  $\triangle ABC$  and **OX**, **OY**, and **OZ** are the perpendiculars drawn from **O** to **BC**, **CA**, and **AB** respectively.

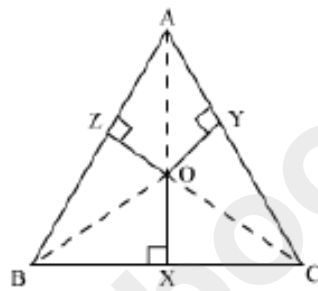
**Prove that**

$$AZ^2 + BX^2 + CY^2 = AY^2 + CX^2 + BZ^2$$



**Solution:**

Let us join OA, OB, and OC in the given figure.



Using Pythagoras theorem in  $\Delta OZA$ , we obtain

$$OA^2 = OZ^2 + AZ^2$$

$$\text{or } AZ^2 = OA^2 - OZ^2 \dots (i)$$

Similarly, in  $\Delta BOX$  and  $\Delta COY$  respectively, we obtain

$$BX^2 = OB^2 - OX^2 \dots (ii)$$

$$\text{and } CY^2 = OC^2 - OY^2 \dots (iii)$$

On adding equations (i), (ii) and (iii), we obtain

$$AZ^2 + BX^2 + CY^2 = OA^2 + OB^2 + OC^2 - OZ^2 - OX^2 - OY^2$$

$$AZ^2 + BX^2 + CY^2 = (OA^2 - OY^2) + (OB^2 - OZ^2) + (OC^2 - OX^2) \dots (iv)$$

Using Pythagoras theorem in  $\Delta OYA$ , we obtain

$$OA^2 = OY^2 + AY^2$$

$$\text{Or } AY^2 = OA^2 - OY^2$$

Similarly,  $BZ^2 = OB^2 - OZ^2$

and  $CX^2 = OC^2 - OX^2$

Using these in equation (iv), we obtain

$$AZ^2 + BX^2 + CY^2 = AY^2 + BZ^2 + CX^2$$

Hence, proved.