Compound Interest

Growth and Depreciation

Sukhiram is the sarpanch of a village and is very good at mathematics. The current population of his village is 18550. From the record of past years, he observed that the population of the village is increasing at the rate of 4% per year. On the basis of the current growth rate, he wants to know the population of the village after 2 years. How can he do so?

Actually, it is a problem of growth (or appreciation). If he knows the formula, then he can easily find out what he wants to know.

Let us understand the formula that should be used here.

The growth per unit time is called the **rate of growth**.

Currently, the population of the village is increasing at the rate of 4% every year.

This means that in a year the population increases by 4% of the previous population. Hence, the rate of growth is 4% and is denoted by r.

$\therefore r = 4\%$

If V_0 is the current measure of quantity and V is the measure of quantity after n years, then

$$\boxed{V = V_0\left(1+\frac{r}{100}\right)^n} \quad ...(i)$$

This is the relation between the current measure of a quantity and the measure of a quantity after n years.

On using this formula, we can find the population of the village after two years. Let us see how.

In the given example,

$r = 4\%$

$V_0 = 18550$

$n = 2$

On substituting these values in equation (i), we obtain

$$V = 18550\left(1+\frac{4}{100}\right)^2$$

$$V = 18550\left(\frac{26}{25}\right)^2$$

$$V = 18550\times\frac{26}{25}\times\frac{26}{25}$$

$$V = 20063.68$$

The population of the village cannot be in decimals. Hence,

V = 20064 (approximately)

In our daily lives, we come across several things that grow. For example: Population of a city or village, prices of goods, height or weight of children etc.

On the other hand, the value of machines or vehicles decreases with time.

The decrease in value per unit time is called **rate of depreciation**.

If the rate of depreciation is constant, then we have the following formula:

$$V = V_0\left(1-\frac{r}{100}\right)^n$$

Here, r is the rate of depreciation per year; V_0 is the current value and V is the value after n years.

When a quantity increases in the first year at the rate of $r_1\%$, then decreases at the rate of $r_2\%$ in the second year and then increases at the rate of $r_3\%$ in the third year, then the formula is

$$V = V_0\left(1+\frac{r_1}{100}\right)\left(1-\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)$$

Here, V_0 is the initial measure and V is the quantity after 3 years.

Let us now solve some problems to understand these formulae better.

Example 1

Priya bought a diamond necklace worth Rs 300000. The value of the necklace appreciates by 6% every year. What will be the value of the necklace after 3 years?

Solution:

The value of the necklace after three years can be calculated by using the compound interest formula.

P = Rs 300000

Rate of appreciation, R = 6% p.a.

Therefore, the value of the necklace after 3 years

$$= \text{Rs } 300000\left(1+\frac{6}{100}\right)^3$$
$$= \text{Rs } 300000\times\left(\frac{53}{50}\right)^3$$
$$= \text{Rs } 300000\times\frac{53\times53\times53}{50\times50\times50}$$
$$= \text{Rs } 357304.80$$

Thus, the value of the necklace after 3 years will be Rs 357304.80.

Example 2

There were 5000 students in a school in the year 2004. In the year 2005, the number of students increased by 5% of the number of students in the previous year. In the year 2006, the number of students decreased by 12% of the number of students in the previous year. How many students did the school have in the year 2006?

Solution:

V_0 = 5000

Rate of appreciation in the year 2005, r_1 = 5%

Rate of depreciation in the year 2006, r_2 = 12%

Number of students in the year 2006 is given by

$$V = V_0\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)$$

Therefore, the number of students in the year 2006

$$= 5000\left(1+\frac{5}{100}\right)\left(1-\frac{12}{100}\right)$$
$$= 5000\left(\frac{21}{20}\right)\left(\frac{22}{25}\right)$$
$$= 4620$$

Thus, the number of students in the school in the year 2006 is 4620.

Compound Interest Using the Concept of Simple Interest

When we deposit some money in a bank, the extra money paid by the bank is called interest.

We know how to calculate simple interest. However, the interest paid by the banks or post offices and charged by the money lenders is not of this kind. The interest paid or charged in such cases is compound interest.

Let us know the method of finding compound interest by taking an example.

Deepak borrowed Rs 16000 from Manish at a rate of 5% per annum for two years. Can you calculate the amount paid by Deepak as compound interest?

Look at the given video to find out the answer.

Now, let us solve one more example to understand the concept better.

Example:

Javed invested Rs 15000 for three years in a bank at the rate of 10% per annum compounded annually. On the other hand, Sujata invested the same sum in some other bank at the same rate for the same duration but on simple interest. Who will earn more interest?

Solution:

It is given that,

Principal = Rs 15000

Rate of interest = 10% per annum

Time = 3 years

Let us first calculate compound interest.

Interest for the 1st year $= \text{Rs}\ \frac{15000 \times 10}{100}$

= Rs 1500

Amount at the end of 1st year

= Rs 15000 + Rs 1500

= Rs 16500

Interest for 2nd year $= \text{Rs}\ \frac{16500 \times 10}{100}$

= Rs 1650

Amount at the end of 2nd year

= Rs 16500 + Rs 1650

= Rs 18150

Interest for 3rd year $= \text{Rs}\ \frac{18150 \times 10}{100}$

= Rs 1815

∴ Total interest received by Javed after 3 years

= Rs 1500 + Rs 1650 + Rs 1815

= Rs 4965

Simple interest received by Sujata after 3 years

$= \text{Rs}\ \frac{15000 \times 10 \times 3}{100}$

= Rs 4500

Thus, Javed received more interest than Sujata.

Formula for Calculating Compound Interest

Supriya invested Rs 75000 in a bank at the rate of 10% per annum compounded annually. What is the amount received by her after 2 years?

We know the method of calculating compound interest using the concept of simple interest. However, this method is very lengthy as we have to calculate the interest and then amount for each year one by one.

We can also find the amount directly using a formula. Before solving the given problem, let us know about the formula.

When the interest is compounded annually, the amount after *n* years is given by

$$A = P\left(1+\frac{R}{100}\right)^n$$

where P is the principal and R is the rate of interest per annum.

Let's now look at the following video to understand the proof of this formula.

Now, using this formula, let us calculate the amount received by Supriya after 2 years.

In the given problem,

Principal, P = Rs 75000

Rate of interest, R = 10% per annum

Number of years, *n* = 2 years

By the formula, we obtain

$$A = 75000\left(1+\frac{10}{100}\right)^2$$

$$A = 75000\left(\frac{11}{10}\right)^2$$

$$A = 75000\times\frac{11}{10}\times\frac{11}{10}$$

A = 750 × 121

A = Rs 90750

Thus, Supriya received Rs 90750 after 2 years.

Using this formula, we can find any one of the values - amount, time, rate of interest or principal, if the other three values are known to us.

For example, the compound interest on a certain sum invested for two years at the rate of 8% p.a. compounded annually is Rs 3328. The sum can be calculated as:

Let *x* be the sum.

Amount = Principal + Interest

Amount = Rs (*x* + 3328)

Now using the formula, we obtain

$$A = P\left(1+\frac{R}{100}\right)^n$$

$\Rightarrow (x + 3328) = x\left(1+\frac{8}{100}\right)^2$

$\Rightarrow (x + 3328) = x\left(\frac{27}{25}\right)^2$

$\Rightarrow (x + 3328) = x\times\frac{27}{25}\times\frac{27}{25}$

$\Rightarrow x + 3328 = \frac{729x}{625}$

$\Rightarrow 625x + 2080000 = 729x$

$\Rightarrow 729x - 625x = 2080000$

$\Rightarrow 104\ x = 2080000$

$\Rightarrow x = \frac{2080000}{104}$

$\Rightarrow x = 20000$

Thus, the sum is Rs 20000.

Now, let us solve some more examples.

Example 1:

Anurag borrowed Rs 50000 from his friend at the rate of 4% p.a. compounded annually. After some time, he repaid Rs 56243.20 to his friend. For how many years did Anurag borrow the money?

Solution:

Principal, P = Rs 50000

Rate, R = 4% p.a.

Amount, A = Rs 56243.2

Let *n* be the number of years for which Anurag borrowed the money.

Using the formula, $A = P\left(1 + \frac{R}{100}\right)^n$,

$\Rightarrow$ Rs 56243.2 = Rs 50000$\left(1 + \frac{4}{100}\right)^n$

$$\Rightarrow \frac{56243.2}{50000} = \left(1+\frac{4}{100}\right)^n$$

$$\Rightarrow \frac{562432}{500000} = \left(1+\frac{1}{25}\right)^n$$

$$\Rightarrow \frac{17576}{15625} = \left(\frac{26}{25}\right)^n$$

$$\Rightarrow \left(\frac{26}{25}\right)^3 = \left(\frac{26}{25}\right)^n$$

$$\Rightarrow n = 3$$

Thus, Anurag borrowed the money for three years.

Example 2:

Find the rate at which Rs 700 becomes Rs 847 in two years, when the interest is compounded annually.

Solution:

Let R be the rate of interest.

Principal, P = Rs 700

Amount, A = Rs 847

Number of years, *n* = 2

Now, using the formula, $A = P\left(1+\frac{R}{100}\right)^n$, we obtain

$$\Rightarrow 847 = 700\left(1+\frac{R}{100}\right)^2$$

$$\Rightarrow \frac{847}{700} = \left(1+\frac{R}{100}\right)^2$$

$$\Rightarrow \frac{121}{100} = \left(1+\frac{R}{100}\right)^2$$

$$\Rightarrow \left(\frac{11}{10}\right)^2 = \left(1+\frac{R}{100}\right)^2$$
$$\Rightarrow \frac{11}{10} = 1+\frac{R}{100}$$
$$\Rightarrow \frac{11}{10} - 1 = \frac{R}{100}$$
$$\Rightarrow \frac{1}{10} = \frac{R}{100}$$
$$\Rightarrow R = 10\%$$

Thus, the rate of interest is 10%.

Problems Where the Interest is not Compounded Annually

We know how to find the compound interest using the following formula:

Amount, $A = P\left(1+\frac{R}{100}\right)^n$

Here, *P* is the principal, *R* is the rate of interest per annum, and *n* is the number of years.

Let us now try to solve an example.

What is the amount received after $1\frac{1}{2}$ years when Rs 20000 are invested at the rate of 6% p.a. compounded half yearly?

Do you find any difference between this question and the questions you have solved before?

Yes, in this example, the interest is compounded half yearly. In the questions we have solved before, the interest was compounded annually.

The time interval after which the interest is compounded is known as the conversion period.

In the given example, the conversion period is six months as the interest is compounded half yearly.

To solve such questions where the interest is not compounded annually, we have to change the rate and time accordingly.

In the given example, the time period is given as $1\frac{1}{2}$ years.

Here, the conversion period is six months.

$\therefore$ Number of conversion periods = 3

Rate of interest = 6% per annum

= 3% per half yearly

Now, we can solve this question using the same formula.

$$A = P\left(1+\frac{R}{100}\right)^{n}$$

Where,

n is the number of conversion periods and *R* is the rate per conversion period.

$$A = 20000\left(1+\frac{3}{100}\right)^{3}$$

$$A = 20000\left(\frac{103}{100}\right)^{3}$$

$$A = 20000\times\frac{103}{100}\times\frac{103}{100}\times\frac{103}{100}$$

$$A = \text{Rs } 21854.54$$

Hence, the amount received after $1\frac{1}{2}$ years is Rs 21854.54.

Before solving more examples, let us see the following table that will help us find the number of conversion periods and the rate of interest per conversion period.

Let *R* be the rate of interest per annum and *t* be the number of years.

Then,

When the interest is compounded	Number of conversion periods (n)	Rate of interest per conversion period (r)
Half yearly	t × 2	$\frac{R}{2}$

Quarterly	t × 4	$\frac{R}{4}$
Monthly	t × 12	$\frac{R}{12}$

Now, let us solve some more examples.

Example 1:

The compound interest on a certain sum invested for one year at the rate of 10% per annum compounded half yearly is Rs 5125. Find the sum.

Solution:

Let P be the sum.

Compound interest = Rs 5125

Amount, A = P + 5125

Number of conversion periods, n = 1 × 2 = 2

Rate of interest per conversion period $= \frac{10}{2}\% = 5\%$

We know that,

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$\Rightarrow (P+5125) = P\left(1+\frac{5}{100}\right)^2$$

$$\Rightarrow (P+5125) = P\left(\frac{21}{20}\right)^2$$

$$\Rightarrow (P+5125) = \frac{441}{400}P$$

$$\Rightarrow 400P + 2050000 = 441P$$

$$\Rightarrow 441P - 400P = 2050000$$

$$\Rightarrow 41P = 2050000$$

$$\Rightarrow P = \frac{2050000}{41}$$

$$\Rightarrow P = 50000$$

Hence, the sum is Rs 50000.

Example 2:

Lalit borrowed Rs 125000 from his friend at the rate of 8% per annum compounded quarterly. After some time, he re-paid Rs 132651 to his friend. For how long did Lalit borrow the money?

Solution:

Principal, P = Rs 125000

Amount, A = Rs 132651

Let *n* be the number of conversion periods.

Rate of interest per conversion period, $r = \frac{8}{4}\% = 2\%$

We know that,

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$132651 = 125000\left(1+\frac{2}{100}\right)^n$$

$$\frac{132651}{125000} = \left(1+\frac{1}{50}\right)^n$$

$$\left(\frac{51}{50}\right)^3 = \left(\frac{51}{50}\right)^n$$

$$\Rightarrow n = 3$$

∴ Number of conversion periods = 3

Here, the interest is compounded quarterly and we know that, one quarter = 3 months

∴ 1 conversion period = one quarter = 3 months

∴3 conversion periods = 3 × 3 months = 9 months

Hence, Lalit borrowed the money for a period of 9 months.