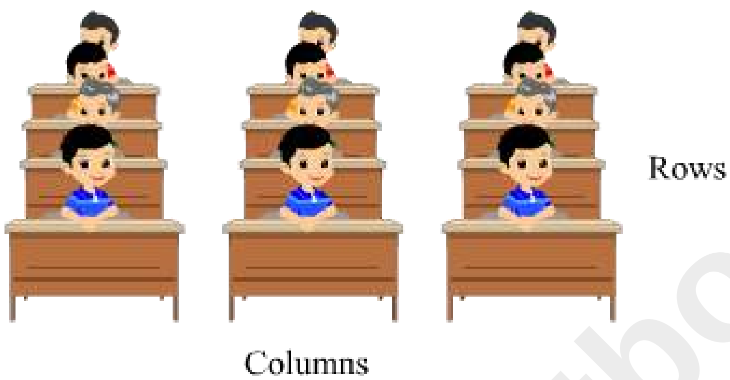
Coordinate Geometry

Cartesian Plane and Reading Location of Points Plotted on It

Need for Coordinate Geometry

Consider a situation wherein we have the students of a class sitting in different rows and we need to locate the position of one particular student. Let us suppose that this student is sitting in the second row. **Using this information, can we ascertain the student's exact location?**



No, we cannot. The given information is insufficient. For us to be able to locate this student in the class, we also need to know the column in which he/she is sitting. So, we require two **variables** to define the student's position—one for the row and the other for the column.

Such situations—wherein we require two attributes to locate points, objects, etc.—can be dealt with by using coordinate **geometry**. It is the geometric system in which the positions of points on a plane are described using ordered pairs of numbers.

Topics to be covered in this lesson:

•Terminology related to coordinate geometry

•Convention of signs in the different quadrants of the Cartesian plane

•Reading the positions of points plotted on the coordinate plane

Did You Know?

The Cartesian plane is named after the famous mathematician and philosopher **René Descartes** to honour his contributions to the field of coordinate geometry.

Know Your Scientist

René Descartes

Born:31 March 1596 **Died**:11 February 1650

In 1637, Descartes' *Geometry* was published. His work on algebra and geometry detailed in this book gave birth to analytical geometry (or Cartesian geometry). For this reason, he is called the 'Father of Analytical Geometry'. René Descartes is also known as the 'Father of Modern Philosophy'.

Ordered Pair

An ordered pair is a pair of two objects taken in particular order. In co-ordinate geometry, an ordered pair means, a pair of two numbers in which order is important. To form an ordered pair, the numbers are written in specific order, separated by a comma, and enclosed in small brackets.

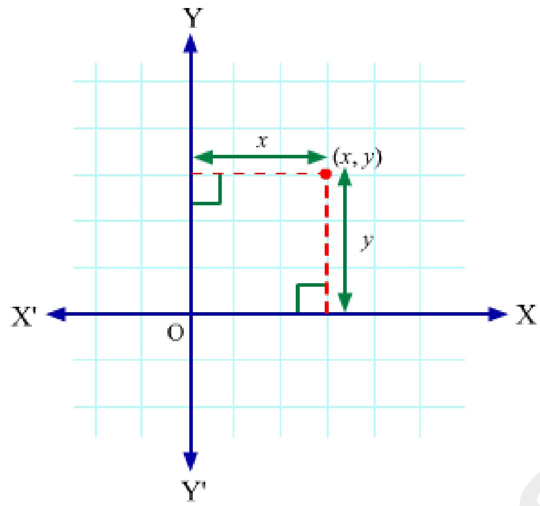
For example:
(3, 5), (−2, 8), (14, 67)

All of the above represents an ordered pair.

Coordinates of a Point

A point on the Cartesian (or coordinate) plane is defined by an ordered pair. The first value in this pair **–** also known as the *x*-coordinate or the **abscissa –** represents the perpendicular distance of the point from the *y*-axis. The second value **–** also known as the *y*-coordinate or the **ordinate –** indicates the perpendicular distance of the point from the *x*-axis.

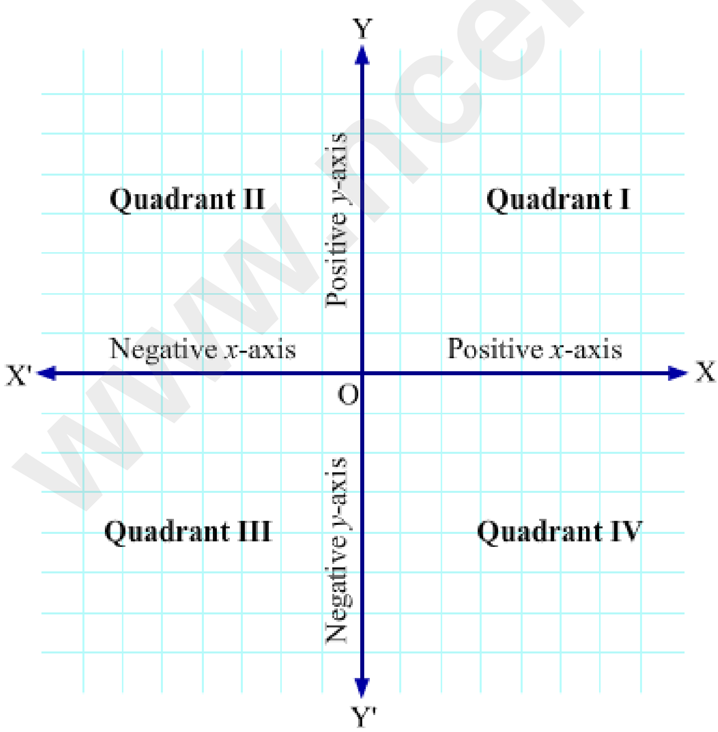
These coordinates are written in the form (*x*-coordinate, *y*-coordinate), i.e., (*x*, *y*).



The line X'OX is called the *x*-axis and Y'OY is called the *y*-axis. The point O is called the origin of the coordinate system. The equation of the *x*-axis is $y = 0$ whereas the equation of the *y*-axis is $x = 0$.

Sign conventions of coordinates

Suppose the coordinates of a point are (x, y). Then, depending upon the quadrant of the plane in which the point lies, the signs of (x, y) will be as follows:



(1) If the point lies in quadrant I, then $x > 0$ and $y > 0$. So, both *x* and *y* will be positive.

(2) If the point lies in quadrant II, then $x < 0$ and $y > 0$. So, x will be negative and y will be positive.

(3)If the point lies in quadrant III, then $x < 0$ and $y < 0$. So, both x and y will be negative.

(4)If the point lies in quadrant IV, then $x > 0$ and $y < 0$. So, x will be positive and y will be negative.

Note: If a point lies on the x-axis, then $y = 0$; if it lies on the y-axis, then $x = 0$.

Solved Examples

Easy

Example 1:

A point lies on the *x*-axis. Find the ordinate of this point.

Solution:

We know that the ordinate of a point is the perpendicular distance of the point from the x-axis. Since the given point lies on the x-axis, its perpendicular distance from the x-axis is zero. Thus, the ordinate of this point is 0.

Medium

Example 1:

In which quadrants will the points (−1, −1), (0, −7), (3, 4), (−2, 0) and (0, 3) lie? Also identify the abscissa and ordinate of each point.

Solution:

We know that the coordinates of a point are written as (x, y), where x is the abscissa and y is the ordinate.

In case of point (−1, −1), $x = -1$ and $y = -1$.

Since $x < 0$ and $y < 0$, this point lies in quadrant III.

In case of point (0, −7), $x = 0$ and $y = -7$.

Since $x = 0$ and $y < 0$, this point lies on the negative y-axis (not in any specific quadrant).

In case of point (3, 4), $x = 3$ and $y = 4$.

Since $x > 0$ and $y > 0$, this point lies in quadrant I.

In case of point (−2, 0), $x = -2$ and $y = 0$.

Since $x < 0$ and $y = 0$, this point lies on the negative x-axis (not in any specific quadrant).

In case of point (0, 3), $x = 0$ and $y = 3$.

Since $x = 0$ and $y > 0$, this point lies on the positive y-axis (not in any specific quadrant).

Reading Locations of Points Plotted on the Coordinate Plane

Whiz Kid

The distance between any two points on the Cartesian plane, with coordinates (x_1, y_1) and (x_2, y_2), is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

For example, the distance between the point A (7, −1) and the origin O (0, 0) is given as follows:

$$\begin{aligned} AO &= \sqrt{(0-7)^2 + [0-(-1)]^2} \\ &= \sqrt{(-7)^2 + (1)^2} \\ &= \sqrt{49+1} \\ &= \sqrt{50} \\ &= 5\sqrt{2} \end{aligned}$$

Activity

Do this activity to practise locating points on the coordinate plane.

•Take some marbles (or buttons) and arrange them in a grid, i.e., in rows and columns. The number of marbles should be the square of an odd number, say n.

•Let $\left(\frac{n+1}{2}\right)^{\text{th}}$ row be the x-axis and $\left(\frac{n+1}{2}\right)^{\text{th}}$ column be the y-axis.

•Let the marble lying at the point of intersection of the x and y axes be the origin O (0, 0).

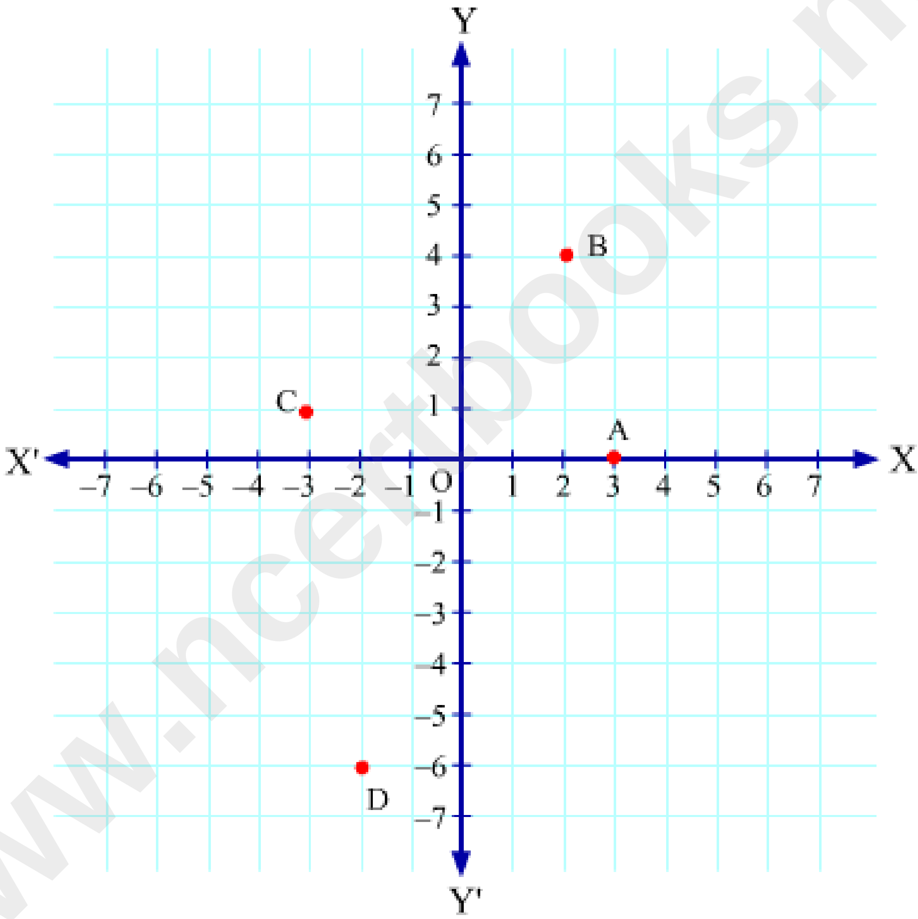
•Now, find the coordinates of all the marbles with respect to the *x*-axis, *y*-axis and origin.

Solved Examples

Medium

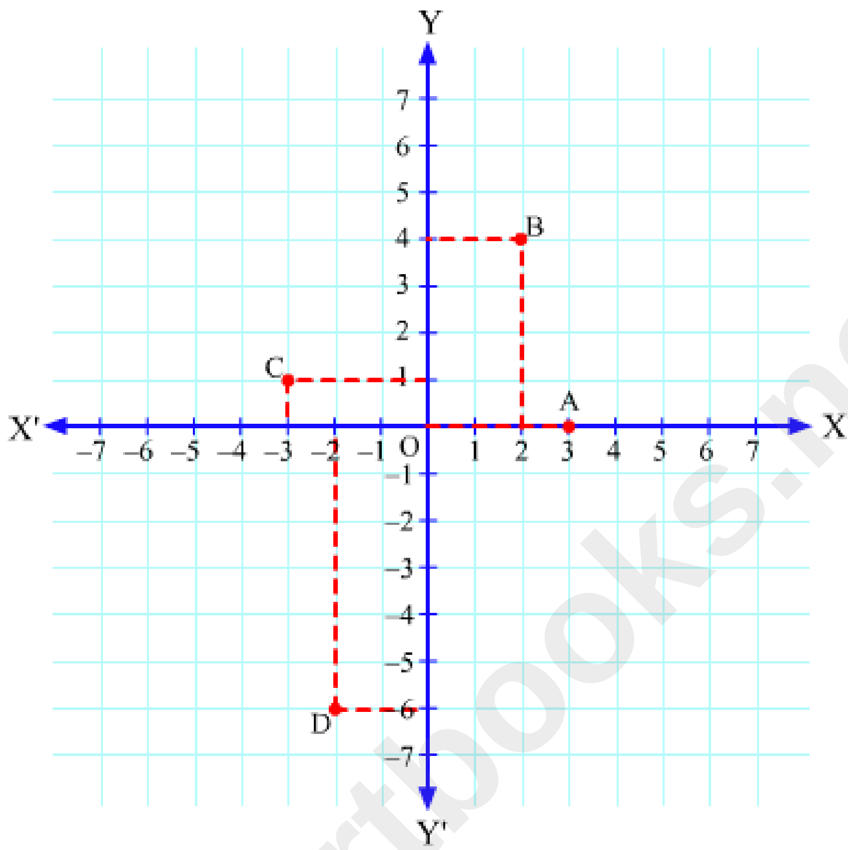
Example 1:

Write the coordinates of the points A, B, C and D plotted in the figure.



Solution:

Draw perpendiculars from A, B, C and D to the coordinate axes as shown in the figure.



We know that the *x*-coordinate (or the abscissa) of a point is the perpendicular distance of the point from the *y*-axis. Let the abscissa of each of the given points be *a*.

Also, the *y*-coordinate (or the ordinate) of a point is the perpendicular distance of the point from the *x*-axis. Let the ordinate of each of the given points be *b*.

The coordinates of each of the given points will then be written as (*a*, *b*).

In case of point A, $a = 3$ and $b = 0$.

So, the coordinates of A are written as (3, 0).

In case of point B, $a = 2$ and $b = 4$.

So, the coordinates of B are written as (2, 4).

In case of point C, $a = -3$ and $b = 1$.

So, the coordinates of C are written as (−3, 1).

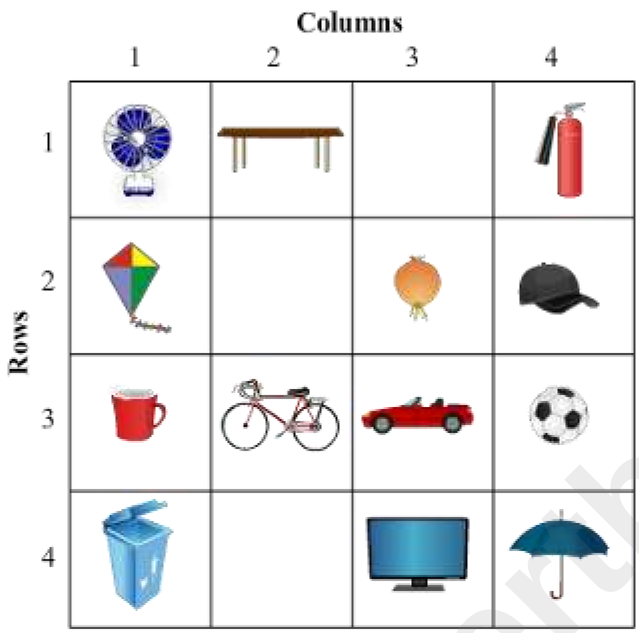
In case of point D, $a = -2$ and $b = -6$.

So, the coordinates of D are written as (−2, −6).

Plotting of Points on a Coordinate Plane

Plotting of Points

Take a look at the following grid of squares containing different items.



Certain squares in the grid are empty. Suppose one of these squares contained an orange before it was taken out, and we need to find that particular square. Let us say the orange was in Column 2. Using this information, can we locate the correct square? No, we cannot. The given information is insufficient as there are two empty squares in Column 2.

To correctly locate the required square, we also need to know the row number. Let us say that the orange was in Row 4. This information helps us to single out the fourth square in the second column. We were able to identify the correct square with the help of two attributes—the column number and the row number. In the same way, to plot a point on a **graph**, we require both its attributes—the abscissa and the ordinate.

In this lesson, we will learn how to plot points with given coordinates on the coordinate plane.

Did You Know?

Global Positioning System (GPS): GPS is a navigation system developed and maintained by the United States government. It is made up of a network of twenty-four satellites placed in Earth's orbit. Anyone with a GPS receiver can freely access GPS.

GPS satellites transmit precise microwave signals. A GPS receiver uses these signals and 'triangulates' to determine the user's exact location and time. The result is accurate to within ten to hundred metres.

GPS receivers determine 2D position (longitude and latitude) by using signals from at least three satellites in view. They ascertain 3D position (longitude, latitude and altitude) from four or more satellites.

GPS is used in clock synchronization, vehicle and aircraft tracking, map making, robotics, etc.

Solved Examples

Easy

Example 1:

Plot the points A (5, −3) and B (−2, 5) on the Cartesian plane.

Solution:

To plot A (5, −3):

(1) Move 5 units along OX and mark the endpoint as M.

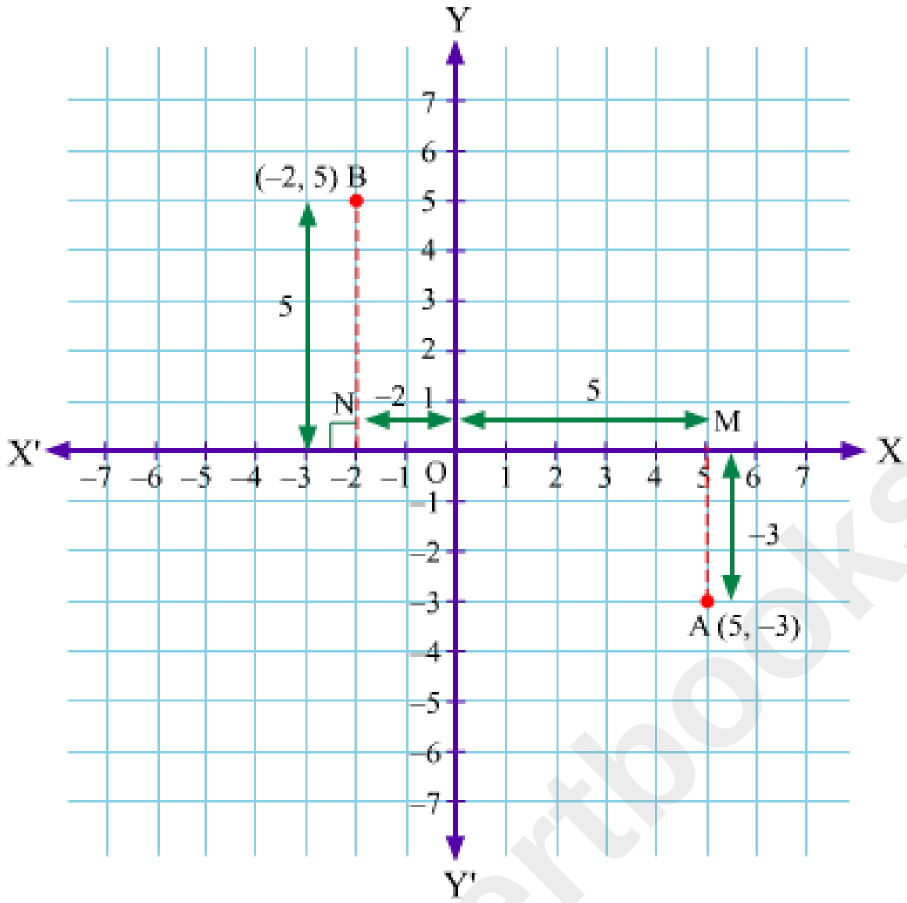
(2) From M and perpendicular to the *x*-axis, move 3 units along OY'. Mark the endpoint as A; this is the location of the point (5, −3) on the Cartesian plane.

To plot B (−2, 5):

(1) Move 2 units along OX' and mark the endpoint as N.

(2) From N and perpendicular to the *x*-axis, move 5 units along OY. Mark the endpoint as B; this is the location of the point (−2, 5) on the Cartesian plane.

Points A and B are plotted in the following graph.



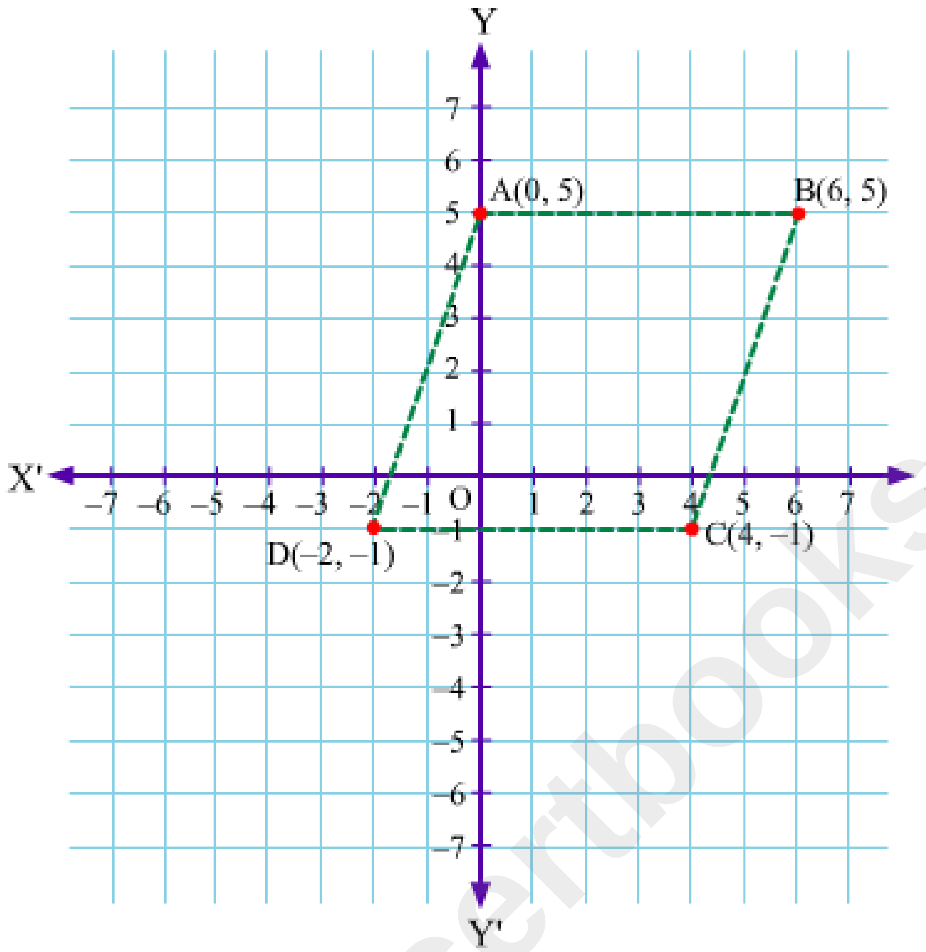
Medium

Example 1:

Draw a quadrilateral ABCD having vertices as A(0, 5), B(6, 5), C(4, –1) and D(–2, –1). Also, name the type of the quadrilateral ABCD.

Solution:

According to the given coordinates, the points A, B, C and D can be plotted as in the given graph.



It can be observed that the distance of points B and C from points A and D respectively is 6 units.

Thus, AB = CD.

From the figure, it can be seen that CD intersects *y*-axis at F(0, –1). Also, perpendicular drawn from B meets CD extended at E.

Now, the distance of points A and B from points F and E respectively is 6 units.

Thus FA = EB

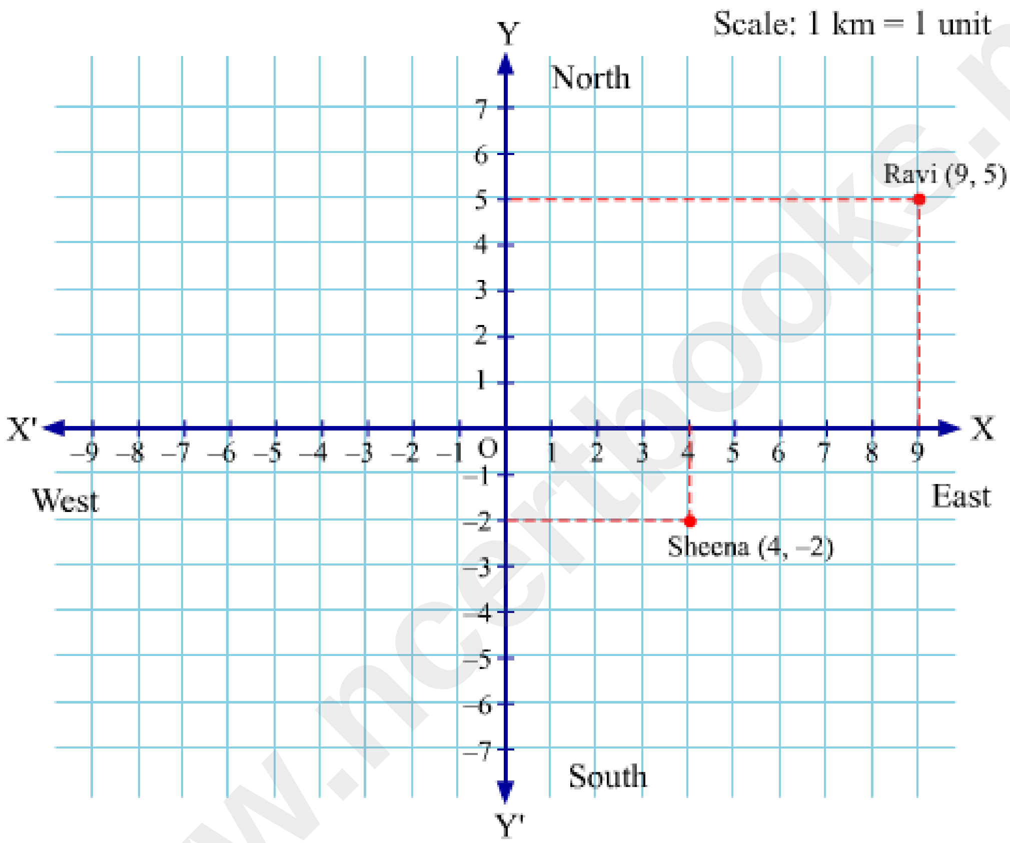
This shows that distance between the sides AB and CD is equal at different points and hence, these sides are parallel.

Since, one pair of opposite sides (AB and CD) is equal and parallel in quadrilateral ABCD, it is a parallelogram.

Example 2:

Ravi travels 9 km east and then 5 km north. Sheena travels 4 km east and then 2 km south. They both start from the same point of origin. Write the coordinates of each person's destination with reference to the magnetic compass. Take 1 km as 1 unit.

Solution:



Let the coordinates of Ravi's destination be (x, y).

Ravi travels 9 km east, i.e., along OX; so, x = 9.

He then travels 5 km north, i.e., along OY; so, y = 5.

Hence, (x, y) = (9, 5)

Let the coordinates of Sheena's destination be (a, b).

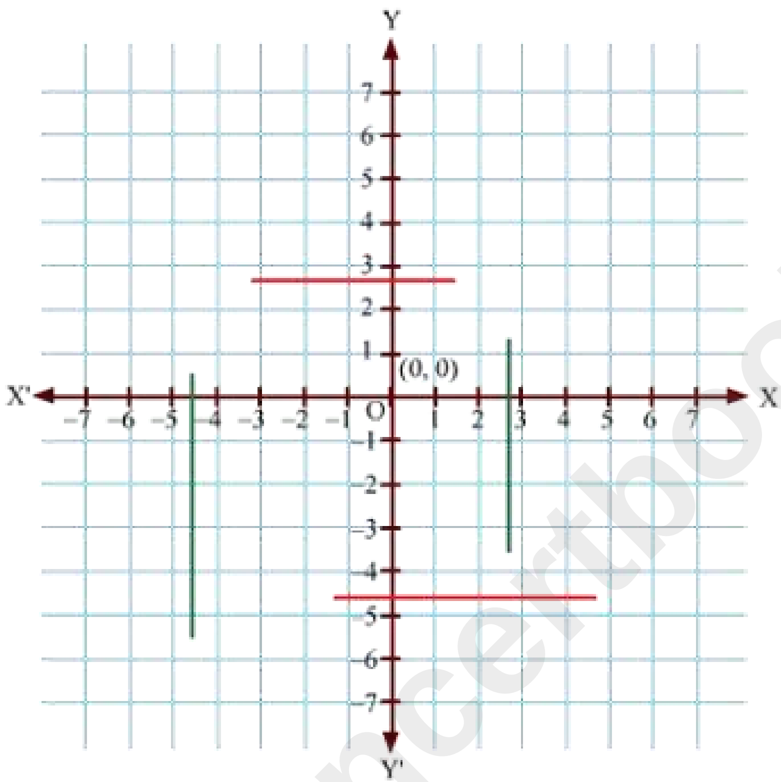
Sheena travels 4 km east, i.e., along OX; so, a = 4.

She then travels 2 km south, i.e., along OY'; so, $b = -2$.

Hence, $(a, b) = (4, -2)$

Graphs of Linear Equations Parallel to Coordinate Axes

Lines Parallel to the Coordinate Axes



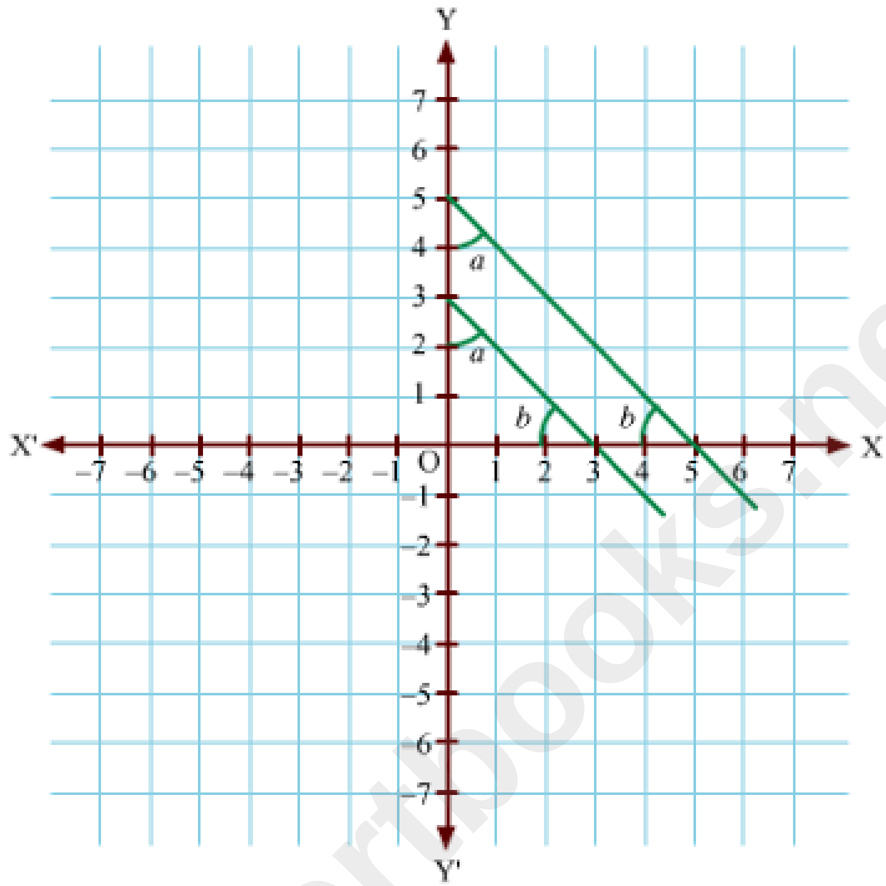
We have studied that the graphs of linear equations are always straight lines that can be easily drawn on the coordinate plane. Sometimes these lines are **parallel** to the coordinate axes. Look at the following figure which shows a few such parallel line graphs.

In the figure, the green graphs are parallel to the *y*-axis and the red graphs are parallel to the *x*-axis. Each of these graphs represents a linear equation.

In this lesson, we will study about linear equations whose graphs are lines parallel to the coordinate axes.

Whiz Kid

Parallel lines have the same slopes. This means that parallel lines make the same angle with a coordinate axis or a common line. For example, take a look at the parallel green lines in the following figure.



In the figure, the parallel green lines make the same angle *a* with the *y*-axis and the same angle *b* with the *x*-axis. So, both the lines have the same slopes with the coordinate axes.

Drawing the Graphs of Linear Equations of the Form '*x* = *a*'

Concept Builder

Consider the linear equation in one variable $4x - 16 = 0$. We know that linear equations in one variable have unique solutions. In this case, the unique solution of the equation is $x = 4$.

Now, we can represent $x = 4$ on a number line as follows:

We can represent other linear equations in one variable in the same way.

Did You Know?

• Since graphs of equations of the form '$x = a$' are always parallel to the y-axis, they are always perpendicular to the x-axis.

• Two lines '$x = a$'and'$x = b$' are always parallel to each other if 'a' and 'b' are non-zero real numbers and'$a \neq b$'.

Solved Examples

Easy

Example 1:

Find the distance between the lines $x = 3.5$ and $x = -1.5$.

Solution:

Distance between the y-axis and the line $x = 3.5$ is 3.5 units. The positive value of x shows that the line is on the right hand side of the y-axis.

Distance between the y-axis and the line $x = -1.5$ is 1.5 units. The negative value of x shows that the line is on the left hand side of the y-axis.

The lines are on different sides of the y-axis, so the distance between the lines will be the sum of their distances from the y-axis.

∴ Distance between the given lines = (3.5 + 1.5) units = 5 units

Medium

Example 1:

Solve the equation $2x - 1 = \frac{x}{3}$ **and represent the solution on:**

i) The number line

ii) The Cartesian plane

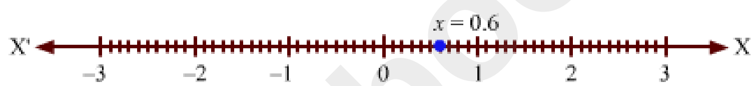
Solution:

The given equation can be reduced as follows:

$$2x-1=\frac{x}{3}$$
$$\Rightarrow 2x-\frac{x}{3}=1$$
$$\Rightarrow \frac{6x-x}{3}=1$$
$$\Rightarrow 5x=3$$
$$\Rightarrow x=\frac{3}{5}$$
$$\Rightarrow x=0.6$$

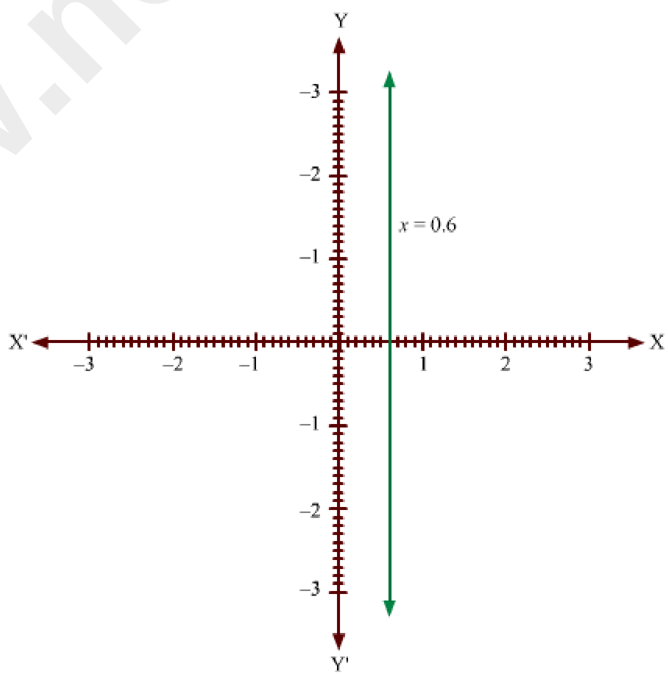
Thus, the equation $2x-1=\frac{x}{3}$ can be rewritten as *x* = 0.6, which is of the form '*x* = *a*'.

i) *x* = 0.6 can be represented on the number line as follows:



ii) We know that the graph of the equation '*x* = *a*' is a line parallel to the *y*-axis, situated at a distanceof '*a*' unitsfrom the *y*-axis. So, the graph of the equation *x* = 0.6 will be a line parallel to the *y*-axis, situated at a distanceof 0.6 unitfrom the *y*-axis.

The graph of the equation *x* = 0.6 is drawn on the Cartesian plane as is shown.



Drawing the Graphs of Linear Equations of the Form '*y* = *b*'

Did You Know?

• Since graphs of equations of the form '$y = b$' are always parallel to the x-axis, they are always perpendicular to the y-axis.

• The line '$y = b$' is always perpendicular to the line '$x = a$' if drawn on the same Cartesian plane.

Solved Examples

Easy

Example 1:

Represent the equation $2y + 3 = 0$ graphically on the Cartesian plane.

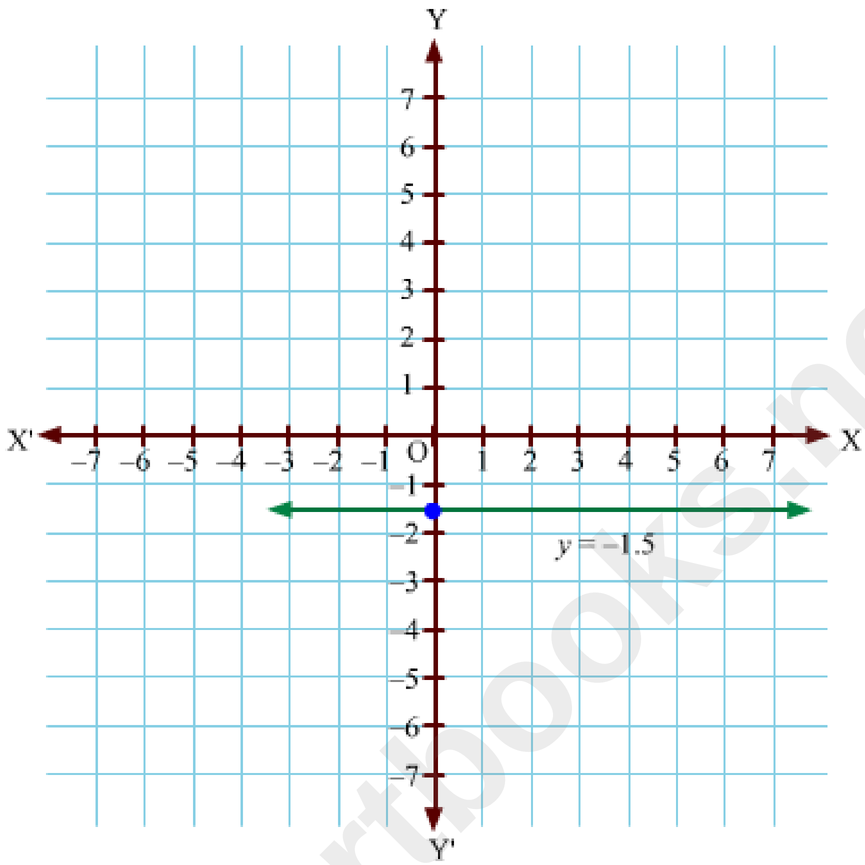
Solution:

The equation $2y + 3 = 0$ can be rewritten as $y = -1.5$, which is of the form '$y = b$'.

We know that the graph of the equation'$y = b$'is a line parallel to the x-axis, situated at a distance of 'b' unitsfrom the x-axis.

So, the graph of $y = -1.5$ will be a line parallel to the x-axis, situated at a distance of 1.5 unitsfrom the x-axis.

The graph of the equation $y = -1.5$ is drawn on the Cartesian plane as is shown.



Medium

Example 1:

Consider the equation $\frac{2y-2}{3}=\frac{3y-3}{2}$ **.**

Represent this equation on:

i) The number line

ii) The Cartesian plane

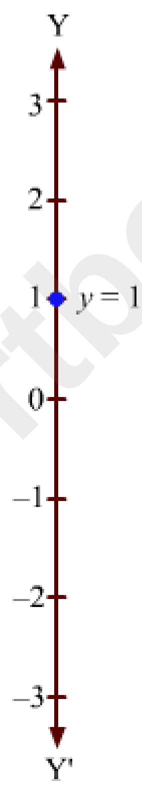
Solution:

The given equation can be reduced as follows:

$$\frac{2y-2}{3}=\frac{3y-3}{2}$$
$$\Rightarrow 4y-4=9y-9$$
$$\Rightarrow -5y=-5$$
$$\Rightarrow y=1$$

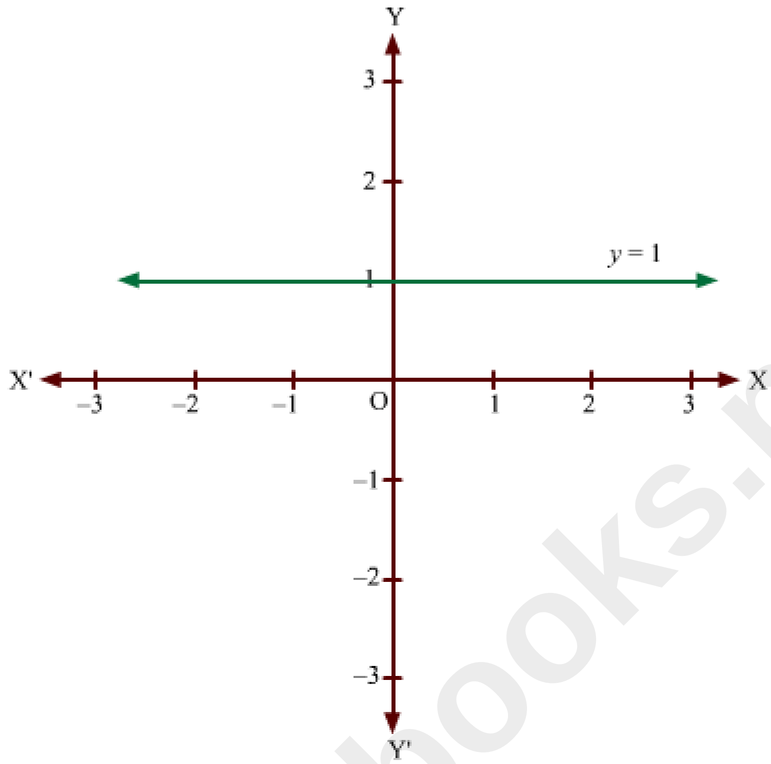
Thus, the equation $\frac{2y-2}{3}=\frac{3y-3}{2}$ can be rewritten as *y* = 1, which is of the form '*y* = *b*'.

i) The equation *y* = 1 can be plotted on the number line as is shown.



ii) Since the equation *y* = 1 is of the form '*y* = *b*', its graph will be a line parallel to the *x*-axis, situated at a distance of 1 unit from the *x*-axis.

The graph of the equation *y* = 1 is drawn on the Cartesian plane as is shown.



Jointly Representing the Lines '*x* = *a*' and '*y* = *b*' on the Cartesian Plane

Solved Examples

Medium

Example 1:

Find the area of the triangle formed by the lines *x* = 3, *y* = 4 and *y* = *x*.

Solution:

The given equations are $x = 3$, $y = 4$ and $y = x$.

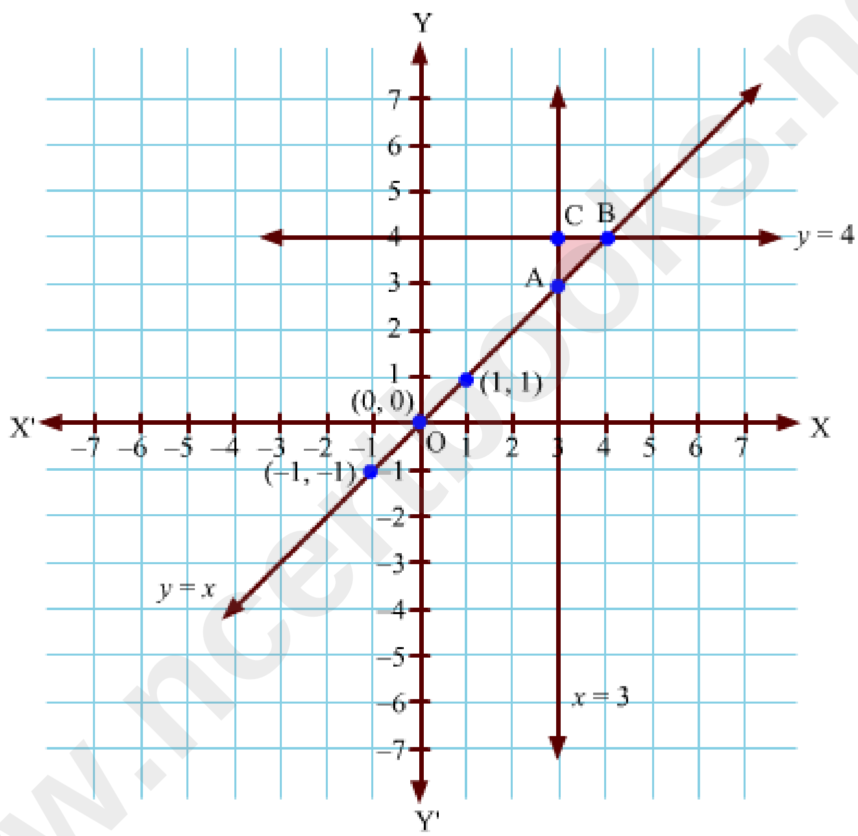
The graph of the equation $x = 3$ will be a line parallel to the *y*-axis, situated at a distance of
3 units from the *y*-axis.

The graph of the equation $y = 4$ will be a line parallel to the *x*-axis, situated at a distance of
4 units from the *x*-axis.

Now, consider the equation $y = x$. Three solutions of this equation are shown in the following table.

x	0	1	−1
y	0	1	−1

The graphs of the given equations are drawn on the Cartesian plane as is shown.



$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times BC \times CA$$

$$= \frac{1}{2} \times 1 \times 1$$

$$= \frac{1}{2}$$

Thus, the area of the triangle formed by the given lines is 0.5 sq. units.

Slope of a Line

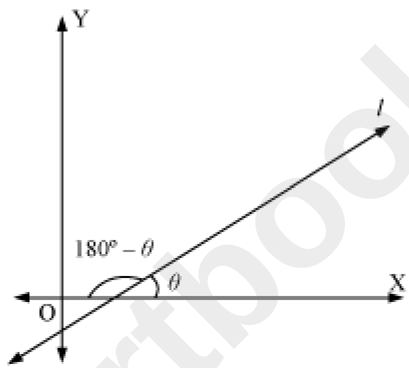
Have you ever wondered why it is difficult to climb a mountain while it is easy to walk down a straight road?

In such cases, we generally use the term ‘slope’ and say that the slope of the mountain is steep.

But do we actually know what slope is and how it is calculated?

Here, we will study about the slopes of straight lines. To understand what we mean by slope, let us first understand what we mean by inclination of a line.

Consider a straight line *l*, as shown in the figure.



Observe that the line *l* makes an angle θ with the positive direction of *x*-axis when measured in the anticlockwise direction. We say that this angle θ is the **inclination** of the line *l*.

The angle which a straight line makes with the positive direction of *x*-axis measured in the anticlockwise direction is called the inclination (or angle of inclination) of the line.

Now, from this definition, we can observe the following points:

1. Inclination of a line parallel to *y*-axis or the *y*-axis itself is 90°.
2. Inclination of a line parallel to *x*-axis or the *x*-axis itself is 0°.

Now that we have understood what we mean by inclination, let us now understand the meaning of the slope of a line.

In the above figure, we have seen that the inclination of line *l* is θ. In this case, we say that tan θ is the slope of line *l*.

If θ is the inclination of a line *l* with the positive direction of *x*-axis, then tan θ is called the slope or gradient of line *l*. The slope of a line is denoted by *m*.

For example, the slope of the line which makes an inclination of 45° with the positive direction of x-axis is given by $m = \tan 45° = 1$

Note:

1. Since $\tan \theta$ is not defined for $\theta = 90°$, we say that the slope of a vertical line is not defined. We also conclude that the slope of y-axis is not defined.
2. The slope of x-axis is 0.

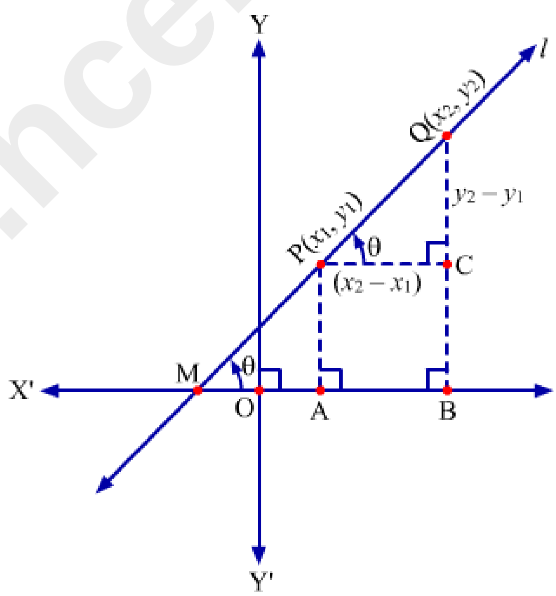
Now, if we have a line which passes through two given points, then can we find the slope of that line?

Yes, we can find the slope of that line using the formula given below.

If P(x_1, y_1) and Q(x_2, y_2) are two points on a non-vertical line *l* whose inclination is θ, then the slope of line *l* is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Let us prove this formula.

We have two points P(x_1, y_1) and Q(x_2, y_2) on a line *l* whose inclination is θ as shown in the following figure.



Let us draw perpendiculars from P and Q to X-axis which meet X-axis at A and B respectively.

Also, let us draw PC ⊥ QB.

∴ PC || AB

It can be seen that PQ is transversal with respect to X-axis and PC such that PC || X-axis.

Now,

∠QMB = θ (Given)

∠QPC = ∠QMB (Corresponding angles)

∴ ∠QPC = θ

Also, we have

OA = x_1 and OB = x_2 ∴ AB = $x_2 - x_1$

PA = y_1 and OB = y_2 ∴ QC = $y_2 - y_1$

Since AB = PC

∴ PC = $x_2 - x_1$

In right-angled triangle ΔPQC, we have

∠QPC = θ

$$\tan\theta = \frac{\text{Side opposite to angle } \theta}{\text{Side adjacent to angle } \theta}$$
$$\Rightarrow \tan\theta = \frac{QC}{PC}$$
$$\Rightarrow \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope of line PQ = Slope of line l = tan θ

$$\text{Slope of line PQ} = \text{Slope of line } l = \frac{y_2 - y_1}{x_2 - x_1}$$

Hence proved.

Using this formula, we can find the slope of any line passing through two distinct points.

For example, the slope of the line passing through the points (3, −7) and (5, 1) is $\frac{1-(-7)}{5-3}=\frac{8}{2}=4$

Now, we know that if there are two lines in a coordinate plane, then they will be either parallel or perpendicular. In either of the two cases, a relation between the slopes of the two lines is exhibited. The relation is explained as follows:

- Two non-vertical lines l_1 and l_2 are parallel, if and only if their slopes are equal. In other words, if m_1 and m_2 are the slopes of lines l_1 and l_2 respectively, then the lines l_1 and l_2 are parallel to each other, if $m_1 = m_2$.

- Two non-vertical lines l_1 and l_2 are perpendicular to each other, if and only if their slopes are negative reciprocals of each other. In other words, if m_1 and m_2 are the slopes of lines l_1 and l_2 respectively, then the lines l_1 and l_2 are perpendicular to each other, if $m_1m_2 = -1$.

Let us go through the following video to understand the proof of the above mentioned conditions for parallel and perpendicular lines in terms of their slopes.

Now, if we have three points A, B, and C, then we can conclude the following statement:

Three points A, B, and C will lie on a line i.e., they will be collinear, if and only if the slope of AB is the same as the slope of BC.

Let us now look at some examples to understand the concept of slope better.

Example 1:

A line l_1 passes through points (5, −3) and (4, −6). Another line, l_2, passes through points (8, 1) and (2, 3). Are lines l_1 and l_2 perpendicular, parallel or neither of the two?

Solution:

We will first find the slopes of the two lines.

We know that if a line passes through points (x_1, y_1) and (x_2, y_2), then the slope of that line is given by $m=\frac{y_2-y_1}{x_2-x_1}$.

Thus,

Slope of line l_1 is given by

$$m_1 = \frac{(-6)-(-3)}{4-5} = \frac{-3}{-1} = 3$$

Slope of line l_2 is given by

$$m_2 = \frac{3-1}{2-8} = \frac{2}{-6} = -\frac{1}{3}$$

Here, we can observe that $m_1 m_2 = -1$. Hence, lines l_1 and l_2 are perpendicular to each other.

Example 2:

The line passing through points (0, 2) and (8, 4) is parallel to the line passing through points $\left(4, \frac{8}{5}\right)$ and (2, *p*). Find the value of *p*.

Solution:

We know that two lines are parallel if and only if their slopes are equal. The slope of a line passing through points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Therefore,

Slope of the line passing through points (0, 2) and (8, 4) is given by

$$m_1 = \frac{4-2}{8-0} = \frac{2}{8} = \frac{1}{4}$$

Slope of the line passing through points $\left(4, \frac{8}{5}\right)$ and (2, *p*) is given by

$$m_2 = \frac{p - \frac{8}{5}}{2-4} = \frac{5p-8}{-2\times 5} = \frac{-5p+8}{10}$$

Since the two lines are parallel,

$m_1 = m_2$

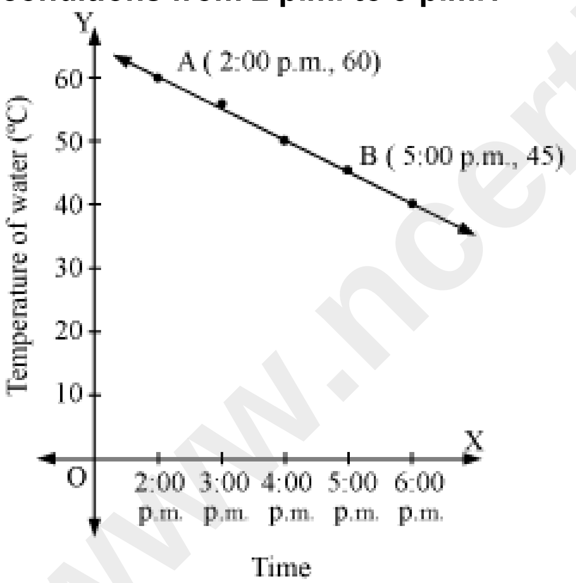
$$\Rightarrow \frac{1}{4} = \frac{-5p+8}{10}$$
$$\Rightarrow 10 = -20p + 32$$
$$\Rightarrow 5 = -10p + 16$$
$$\Rightarrow 10p = 16 - 5 = 11$$
$$\Rightarrow p = \frac{11}{10}$$

Thus, the value of *p* is $\frac{11}{10}$.

Example 3:

The given graph shows the temperature of water, which was kept on fire for some time, at different intervals of time.

What will be the temperature of water at 8 p.m. if it was kept in the same conditions from 2 p.m. to 9 p.m.?



Solution:

Since line AB passes through points A (2:00 p.m., 60°C) and B (5:00 p.m., 45°C), its slope is $\frac{45-60}{5-2} = \frac{-15}{3} = -5$.

Let *y* be the temperature of water at 8:00 p.m. Accordingly, on the basis of the given graph, line AB must pass through point C (8:00 p.m., *y*).

∴ Slope of AB = Slope of BC

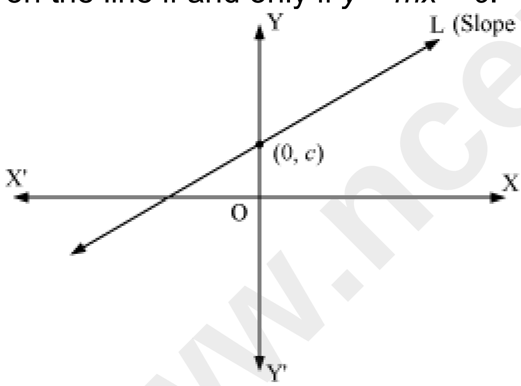
$$\Rightarrow -5 = \frac{y-45}{8-5}$$
$$\Rightarrow -5 = \frac{y-45}{3}$$
$$\Rightarrow -15 = y - 45$$
$$\Rightarrow y = -15 + 45$$
$$\Rightarrow y = 30$$

Thus, the temperature of water will be 30°C at 8:00 p.m.

Slope-Intercept Form of Straight Lines

Slope-intercept Form

- If a line with slope m makes y-intercept as c, then the equation of the line is given by $y = mx + c$.
- In other words, we can say that point (x, y) on the line with slope m and y-intercept c lies on the line if and only if $y = mx + c$.



- If a line with slope m makes x-intercept as d, then the equation of the line is given by $y = m(x - d)$.
- Let us go through the following video to understand how we arrived at the above formulae of slope-intercept form of equation of a line.
- A general equation $Ax + By + C = 0$ can be written in slope-intercept form as follows:
- $y = -\frac{A}{B}x - \frac{C}{B}$, if B ≠ 0, where $m = -\frac{A}{B}$ and $c = -\frac{C}{B}$.

- $x=-\frac{C}{A}$, if B = 0, which is a vertical line whose slope is undefined and whose *x*-intercept is $-\frac{C}{A}$.

Solved Examples

Example 1:

The equation of a line is given by 12*x* + 8*y* − 9 = 0. Find the angle made by this line with the positive direction of the *x*-axis.

Solution:

The equation of the line is given by

12*x* + 8*y* − 9 = 0

⇒ 8*y* = 9 − 12*x*

$$\Rightarrow y=\frac{9}{8}-\frac{12}{8}x$$
$$\Rightarrow y=\frac{9}{8}-\frac{3}{2}x$$

Comparing this equation with the general form *y* = *mx* + *c*, we obtain the slope of the line as

m = $-\frac{3}{2}=\tan\theta$

Thus, the angle made by the line with the positive direction of the *x*-axis is $\tan^{-1}\left(-\frac{3}{2}\right)$.

Example 2:

Find the equation of the line that makes *x*-intercept as 5 and is perpendicular to the line 16*x* + 4*y* = 5.

Solution:

It is given that the line is perpendicular to the line 16*x* + 4*y* = 5.

The slope of this line can be calculated as

$4y = -16x + 5$

$$\Rightarrow y = -4x + \frac{5}{4}$$

Thus, the slope of this line is −4. Therefore, the slope of the required line is $\frac{1}{4}$.

Also, it is given that the line makes *x*-intercept as 5.

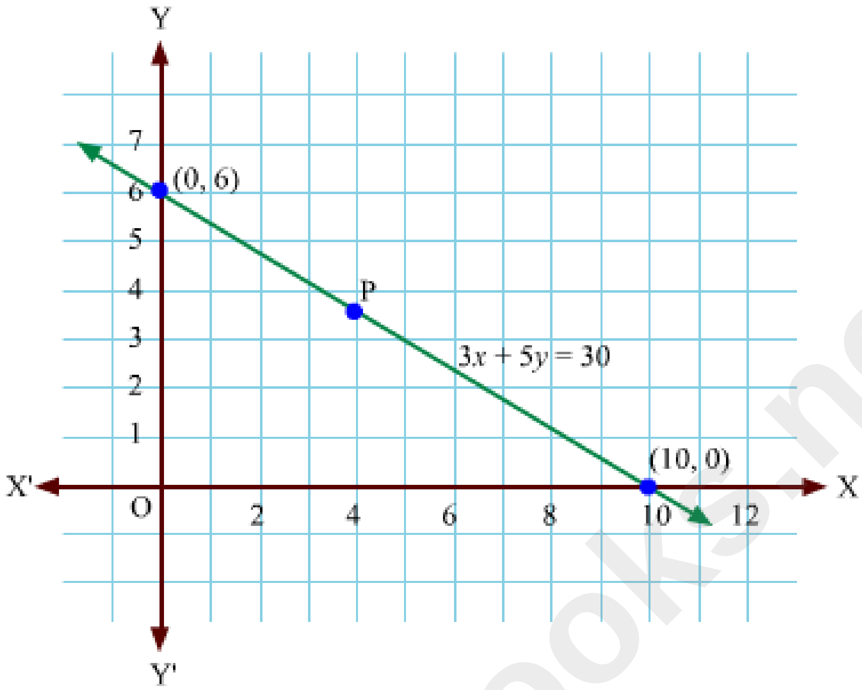
By using the slope-intercept form, we get the required equation of the line as

$$y = \frac{1}{4}(x-5)$$
$$\Rightarrow 4y = x-5$$
$$\Rightarrow x-4y-5=0$$

Graphical Solution of a Linear Equation In Two Variables

Finding the Solution of a Linear Equation by Using Its Graph

We have learned to draw graphs for linear equations in two variables. Take, for example, the linear equation $3x + 5y = 30$. Its graph is drawn as is shown.



The drawn line represents the equation $3x + 5y = 30$. Every point lying on this line is a solution of the equation. Point P is one such solution of the equation.

In this lesson, we will learn how to find solutions of linear equations using the graphs of those equations.

Finding the Solution of a Linear Equation by Using Its Graph

Whiz Kid

Graphical method is also used to find the solutions of a system of linear equations. In this case, we draw the lines for the linear equations and mark the points where the lines intersect. The points of intersection are solutions of the system of linear equations.

Solved Examples

Easy

Example 1:

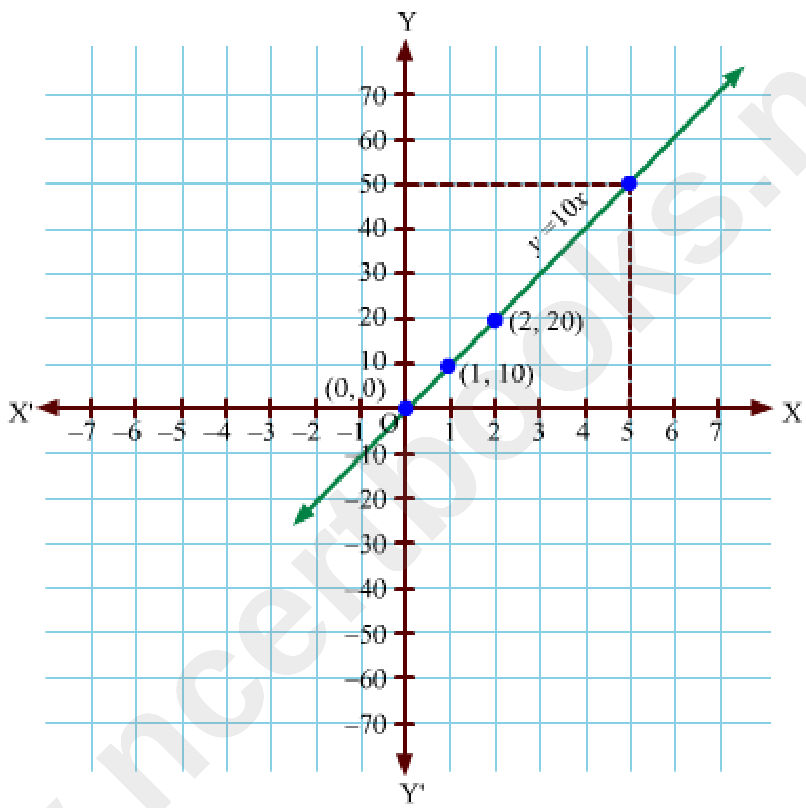
Using the graph of $y = 10x$, find the solution of the equation when $y = 50$.

Solution:

The given equation is $y = 10x$. Three solutions of this equation are shown in the following table.

x	0	1	2
y	0	10	20

By plotting and joining the points (0, 0), (1, 10) and (2, 20), we obtain the following graph.



The drawn line represents the graph of the equation $y = 10x$. On this line, the value of x corresponding to $y = 50$ is 5.

Example 2:

Draw the graph of $4x - 3y + 12 = 0$. Using the graph, find the solution of the equation when:

i) $x = 3$

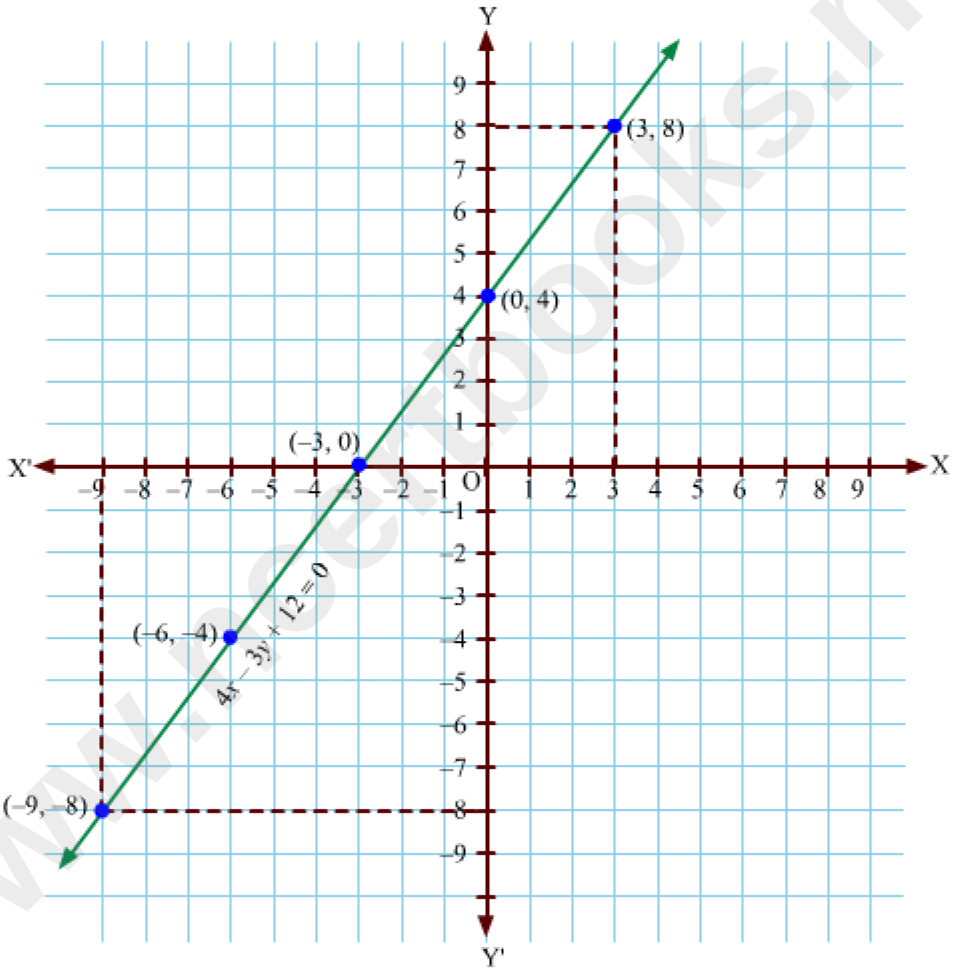
ii) $y = -8$

Solution:

The given equation is $4x - 3y + 12 = 0$. Three solutions of this equation are shown in the following table.

x	0	−3	−6
y	4	0	−4

By plotting and joining the points (0, 4), (−3, 0) and (−6, −4), we obtain the following graph.



The drawn line represents the graph of the equation $4x - 3y + 12 = 0$. On this line:

i) The value of y corresponding to $x = 3$ is 8.

ii) The value of x corresponding to $y = -8$ is −9.

Medium

Example 1:

A boy has some twenty-five paise coins and some fifty paise coins which add up to Rs 2.25. Form a linear equation in two variables for this information. Using the graph of the equation, find the number of fifty paise coins if there are 3 coins of twenty-five paise.

Solution:

Let x be the number of twenty-five paise coins and y be the number fifty paise coins.

We know that Re 1 = 100 paise

∴ Rs 2.25 = 225 paise

Amount obtained from twenty-five paise coins = $25x$

Amount obtained from fifty paise coins = $50y$

According to the given information, we have:

$25x + 50y = 225$

$x + 2y = 9$

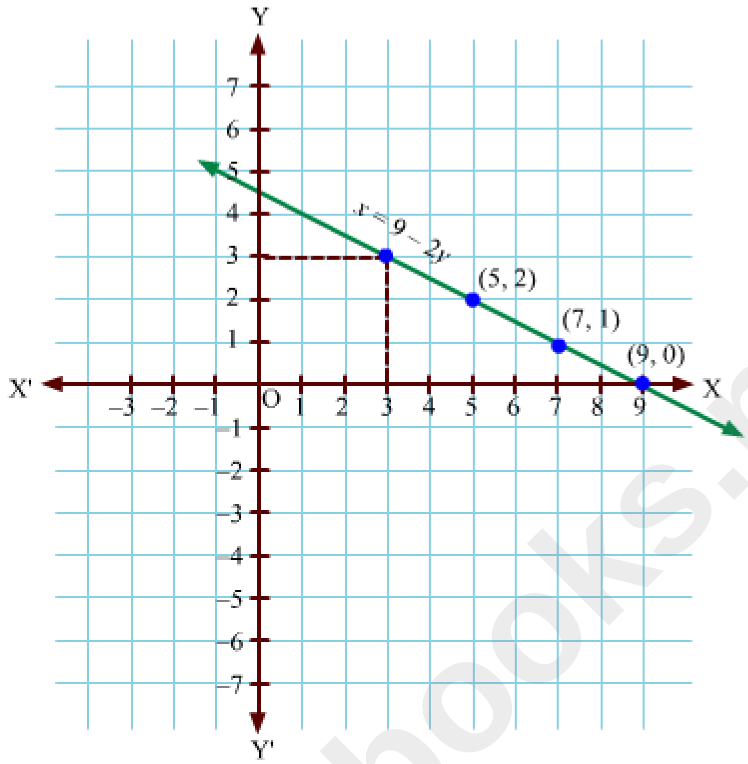
$x = 9 - 2y$

This is the required linear equation in two variables for the given information.

Three solutions of this equation are shown in the following table.

x	9	7	5
y	0	1	2

By plotting and joining the points (9, 0), (7, 1) and (5, 2), we obtain the following graph.



The drawn line represents the graph of the equation $x = 9 - 2y$. On this line, the value of y corresponding to $x = 3$ is 3. Therefore, if there are 3 coins of twenty-five paise, then there will be 3 coins of fifty paise.

Example 2:

Aarushi is driving a car with a uniform speed of 60 km/hour. Draw the distance−time graph. Using the graph, find the distance travelled by Aarushi in:

i) Two and half hours

ii) Half an hour

Solution:

Uniform speed of the car = 60 km/hr(Given)

Let D be the distance travelled by Aarushi in t hours.

We know that:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\Rightarrow 60 = \frac{D}{t}$$

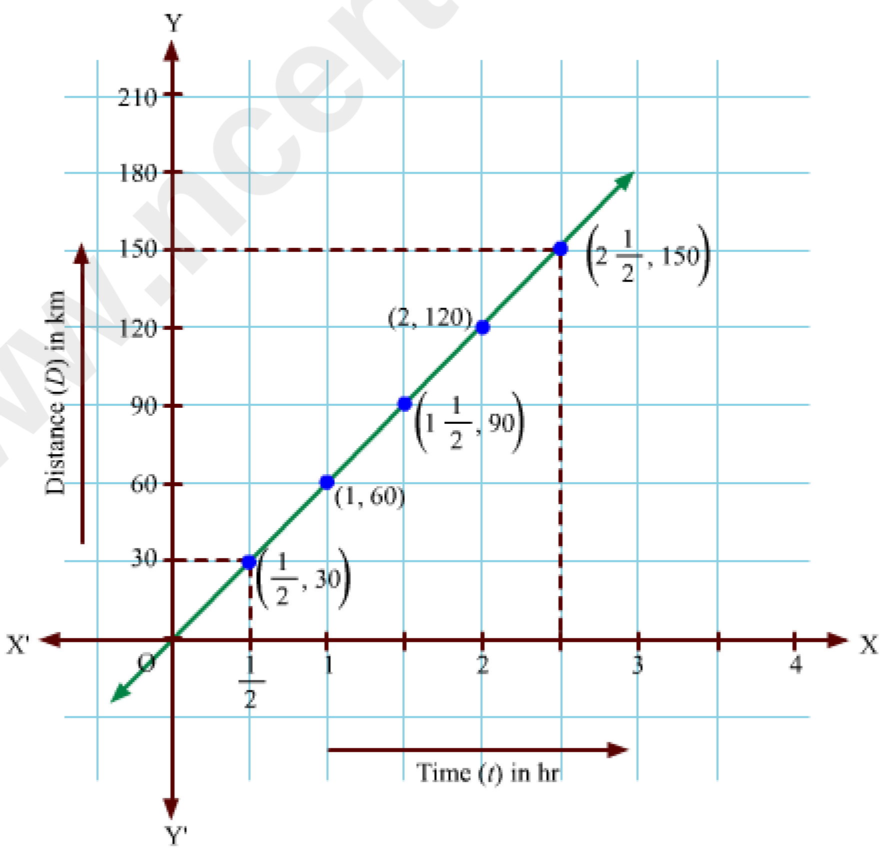
$$\Rightarrow D = 60t$$

Three solutions of this equation are shown in the following table.

t	1	$1\frac{1}{2}$	2
D	60	90	120

Let us keep time (*t*) along the *x*-axis and distance (*D*) along the *y*-axis on the Cartesian plane.

By plotting and joining the points (1, 60), $\left(1\frac{1}{2}, 90\right)$ and (2, 120), we obtain the following graph.



The drawn line represents the distance−time equation $D = 60t$. On this line:

i) The value of D corresponding to $t = 2\frac{1}{2}$ is 150. So, the distance travelled by Aarushi in two and a half hours is 150 km.

ii) The value of D corresponding to $t = \frac{1}{2}$ is 30. So, the distance travelled by Aarushi in half an hour is 30 km.

Hard

Example 1:

The linear equation $C = \frac{5F - 160}{9}$ is used to compare the temperatures on the Celsius (*C*) and Fahrenheit (*F*) scales. Draw the graph for the equation and find the following information from the graph.

i) If the temperature is 0°F, then what is the approximate temperature in degree Celsius?

ii) Find and locate the point where the numerical value of temperature is the same on both the scales?

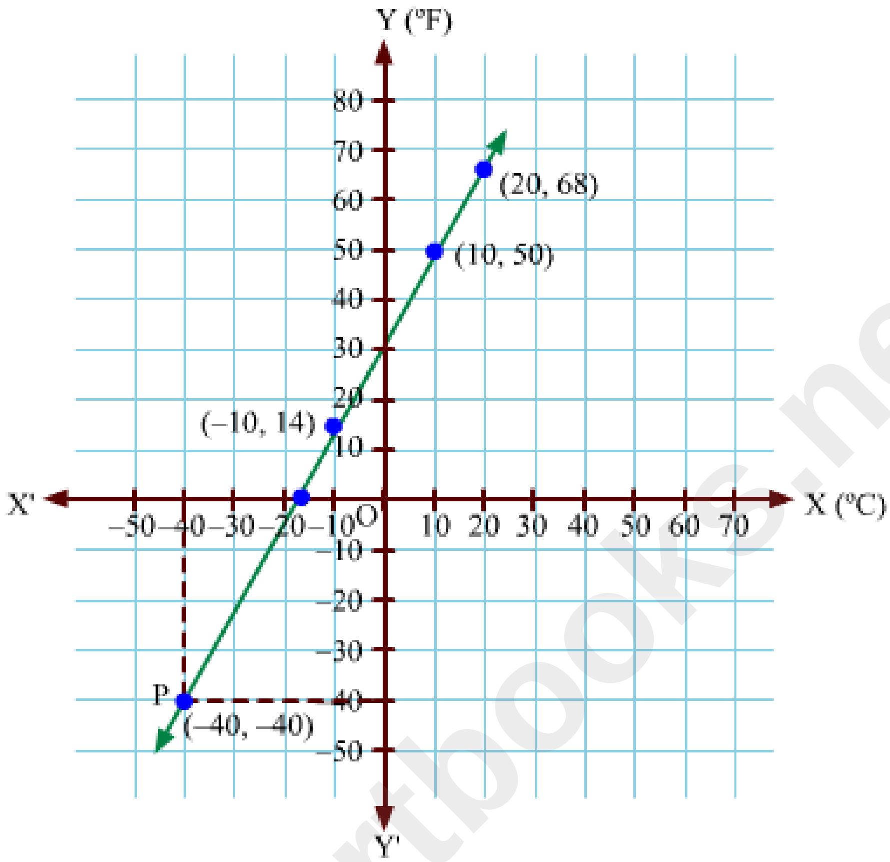
Solution:

The given equation is $C = \frac{5F - 160}{9}$. Three solutions of this equation are shown in the following table.

C	10	−10	20
F	50	14	68

Let us keep temperature in degree Celsius along the *x*-axis and temperature in degree Fahrenheit along the *y*-axis on the Cartesian plane.

By plotting and joining the points (10, 50), (−10, 14) and (20, 68), we obtain the following graph.



The drawn line represents the Celsius−Fahrenheit equation $C = \frac{5F - 160}{9}$. On this line:

i) The approximate value of *C* corresponding to *F* = 0 is −20. So, when the temperature is 0°F, the temperature on the Celsius scale is approximately −20°C.

ii) At point P, both *C* and *F* have the same value, i.e., −40. So, −40 is the point at which the Celsius and Fahrenheit scales coincide, i.e., −40°C = −40°F.

Example 2:

The acceleration *x* of a small object having a mass of 6 kg is directly proportional to the force *y* applied on it. Express this statement as a linear equation in two variables and draw its graph. Using the graph, find the force required to produce an acceleration of:

i) 5 cm/sec^2

ii) 6 cm/sec^2

Solution:

It is given that the force applied is *y* and the acceleration produced is *x*.

$\therefore y \propto x$

$\Rightarrow y = mx$

The constant mass 'm' of the object is given as 6 kg.

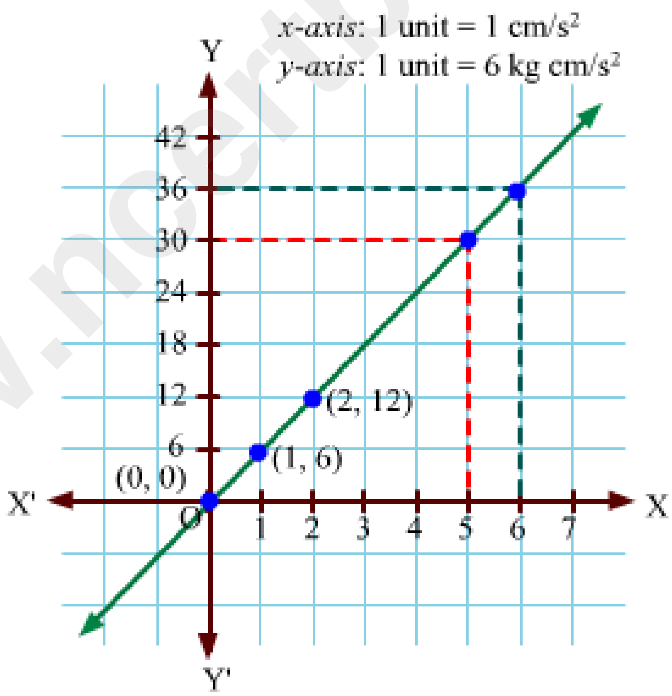
$\therefore y = 6x$

This is the required linear equation in two variables.

Three solutions of this equation are shown in the following table.

x	0	1	2
y	0	6	12

By plotting and joining the points (0, 0), (1, 6) and (2, 12), we obtain the following graph.



The drawn line represents the equation $y = 6x$. On this line:

i) The value of *y* corresponding to $x = 5$ is 30. So, a force of 30 kg cm/s^2 is required to produce an acceleration of 5 cm/s^2.

ii) The value of y corresponding to $x = 6$ is 36. So, a force of 36 kg cm/s^2 is required to produce an acceleration of 6 cm/s^2.

Graphical Solution of a Pair of Linear Equations In Two Variables

Suppose you go to a stationary shop to buy some registers and pens. If you buy two registers and two pens, then you have to pay Rs 30. However, if you buy two registers and four pens, then you have to pay Rs 40.

Now, how can we represent this information algebraically, i.e., in the form of equations?

Let us denote the cost of each register by Rs x and that of each pen by Rs y.

Now, it is given to us that two registers and two pens cost Rs 30.

$\therefore 2x + 2y = 30$

$\Rightarrow x + y - 15 = 0$ … (1)

We also know that two registers and four pens cost Rs 40.

$\therefore 2x + 4y = 40$

$\Rightarrow x + 2y - 20 = 0$ … (2)

Thus, we now have two linear equations in two variables from the given situation.

$x + y - 15 = 0$ … (1)

$x + 2y - 20 = 0$ … (2)

These two equations are known as a pair of linear equations in two variables. If we try to individually solve each equation, we will be unable to do so. We can arrive at an answer only if we try and solve both these equations together.

The **general form** of a pair of linear equations in two variables is written as

$a_1x + b_1y + c_1 = 0$, where a_1, b_1, c_1 are real numbers, and $a_1^2 + b_1^2 \neq 0$ (i.e., a_1, $b_1 \neq 0$)

$a_2x + b_2y + c_2 = 0$, where a_2, b_2, c_2 are real numbers, and $a_2^2 + b_2^2 \neq 0$ (i.e., a_2, $b_2 \neq 0$)

A pair of linear equations represents two straight lines. When their graphs are drawn, there are three possibilities.

(i) The two lines may intersect at one point.

(ii) The two lines may be parallel.

(iii) They may represent the same line, i.e. they may be coincident.

The **solution of a pair of linear equations** is the point of intersection of their graphs.

Let us understand the above stated statement with the help of the video.

Now, according to this, a pair of dependent equations is represented by coincident lines. Let us verify this graphically with the help of the given video.

Similarly, you can also graphically prove two lines to be inconsistent. Try it out yourself.

Limitations of graphical method:

- If the large values of variables are involved in the solution of system of linear equations then it is not convenient to use graphical method.
- If the fractional values of variables are involved in the solution of system of linear equations then it is not possible to obtain accuracy through graphical method.
- If the angle between two lines is small then the exact point of intersection cannot be obtained through graphical method.

Now, let us solve some examples to understand this concept better.

Example 1:

Anshuman and Vikram have 10 marbles with them. Twice the number of marbles with Anshuman is equal to three times the number of marbles with Vikram. Solve this question graphically.

Solution:

Let us suppose that Anshuman has x marbles and Vikram has y marbles.

From the given information, we can form two equations as

$x + y = 10$

$\Rightarrow x + y - 10 = 0$ … (i)

$2x = 3y$

$\Rightarrow 2x - 3y = 0$ … (ii)

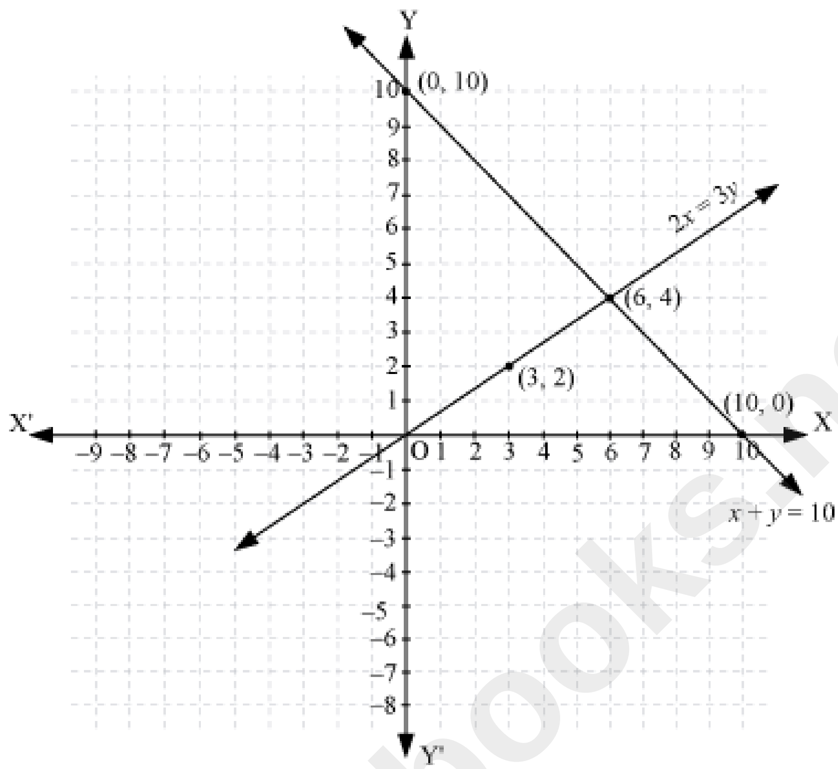
A table can be drawn for the corresponding values of *x* and *y* for equation (i) as

x	***y* = 10 − *x***
0	10 − 0 = 10
10	10 − 10 = 0

Similarly, a table can be drawn for the corresponding values of *x* and *y* for equation (ii) as

x	$y = \frac{2x}{3}$
3	$\frac{2\times3}{3} = 2$
6	$\frac{2\times6}{3} = 4$

The two equations can be plotted on a graph as



As seen in the graph, the two lines intersect at the point (6, 4).

This implies that the solution for the pair of linear equations is $x = 6$ and $y = 4$.

Thus, Anshuman has 6 marbles while Vikram has 4 marbles.

Example 2:

Find graphically whether the following pairs of linear equations are consistent or inconsistent? If the equations are consistent, then find their solutions as well.

(a) $2x + 3y = 2$

$4x + 3y = 4$

(b) $x + y = 5$

$2x + 2y = 10$

Solution:

(a) $2x + 3y = 2$

$4x + 3y = 4$

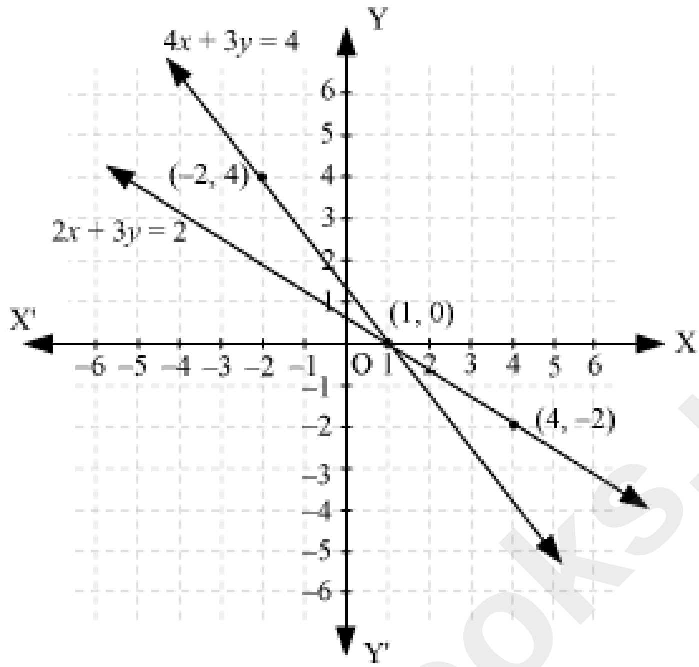
A table can be drawn for the corresponding values of *x* and *y* for the equation 2*x* + 3*y* = 2 as

x	$y = \frac{2-2x}{3}$
1	$\frac{2-2\times 1}{3} = \frac{2-2}{3} = \frac{0}{3} = 0$
4	$\frac{2-2\times 4}{3} = \frac{2-8}{3} = \frac{-6}{3} = -2$

Similarly, a table can be drawn for the corresponding values of *x* and *y* for the equation 4*x* + 3*y* = 4 as

x	$y = \frac{4-4x}{3}$
1	$\frac{4-4\times 1}{3} = \frac{4-4}{3} = \frac{0}{3} = 0$
−2	$\frac{4-4\times(-2)}{3} = \frac{4+8}{3} = \frac{12}{3} = 4$

These points can now be plotted and joined to obtain the graphs of the equations as



As seen in the graph, the two lines intersect at the point (1, 0). Thus, the given pair of linear equations is consistent with its solution as $x = 1$ and $y = 0$.

(b) $x + y = 5$

$2x + 2y = 10$

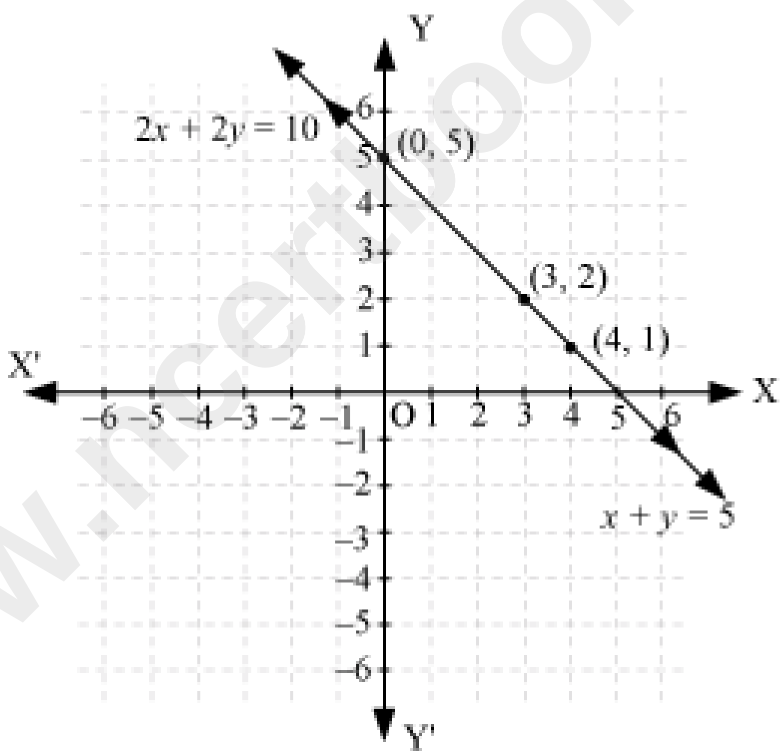
A table can be drawn for the corresponding values of x and y for the equation $x + y = 5$ as

x	$y = 5 - x$
0	$5 - 0 = 5$
4	$5 - 4 = 1$

Similarly, a table can be drawn for the corresponding values of x and y for the equation $2x + 2y = 10$ as

x	$y = \frac{10-2x}{2}$
3	$\frac{10-2\times 3}{2} = \frac{10-6}{2} = \frac{4}{2} = 2$
0	$\frac{10-2\times 0}{2} = \frac{10-0}{2} = \frac{10}{2} = 5$

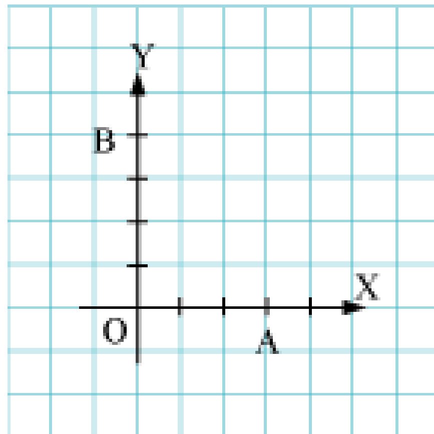
These points can now be plotted and joined to obtain the graphs of the lines as



As seen in the graph, the two equations are represented by the same line. This means that the given pair of linear equations is dependent. Any point lying on this line is a solution to the pair of equations.

Distance Formula

Let us consider the following graph.



In the above figure, O is the origin, A is a point at a distance of 3 units on *x*-axis, and B is a point at a distance of 4 units on *y*-axis.

Can we find the distance between the two points A and B?

We shall find this out in the given video, where first we will see the basic approach and then work out a formula to calculate the distance between any two points.

In coordinate geometry, the distance formula has a lot of applications. Let us look at one of its applications.

What can you say about the three points (–6, 10), (–1, 1), and (3, –8)? Are they collinear?

Let us see.

If the sum of the distances of any point from the other two is equal to the distance between the other two points, then we can say that the given points are collinear.

For example, let (–6, 10), (–1, 1), and (3, –8) be denoted by A, B, and C respectively.

Now, using distance formula, $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$, we obtain

$$\begin{aligned} AB &= \sqrt{(-1+6)^2+(1-10)^2} \\ &= \sqrt{(5)^2+(-9)^2} \\ &= \sqrt{25+81} \\ &= \sqrt{106} \end{aligned}$$

$$BC = \sqrt{\left[3-(-1)\right]^2 + (-8-1)^2}$$
$$= \sqrt{(4)^2 + (-9)^2}$$
$$= \sqrt{16+81}$$
$$= \sqrt{97}$$

$$AC = \sqrt{\left[3-(-6)\right]^2 + (-8-10)^2}$$
$$= \sqrt{(9)^2 + (-18)^2}$$
$$= \sqrt{81+324}$$
$$= \sqrt{405}$$
$$= 9\sqrt{5}$$

Here, AB + BC $= \sqrt{106} + \sqrt{97}$

≠ AC

Therefore, the points A, B, and C are not collinear.

Let us solve some more examples based on the distance formula.

Example 1:

Find the distance between the point (7, –1) and origin.

Solution:

Let the points (7, –1) and origin (0, 0) be denoted by A and O respectively. Then, using distance formula, we obtain

$$AO = \sqrt{(0-7)^2 + \left[0-(-1)\right]^2}$$
$$= \sqrt{(-7)^2 + (1)^2}$$
$$= \sqrt{49+1}$$
$$= \sqrt{50}$$
$$= 5\sqrt{2} \text{ units}$$

Thus, the distance between the points (7, –1) and (0, 0) is $5\sqrt{2}$ units.

Example 2:

Find the value of *a*, if the distance between (*a*, –1) and (8, 3) is $4\sqrt{5}$.

Solution:

Let points (*a*, –1) and (8, 3) be denoted by A and B respectively.

Then, using distance formula, we obtain

$$AB = \sqrt{(8-a)^2 + [3-(-1)]^2}$$

The distance is given as $4\sqrt{5}$. Therefore, we can write

$$4\sqrt{5} = \sqrt{(8-a)^2 + (4)^2}$$

On squaring both sides, we obtain

$16 \times 5 = 64 + a^2 - 16a + 16$

$\Rightarrow 80 - 64 - 16 = a^2 - 16a$

$\Rightarrow 80 - 80 = a^2 - 16a$

$\Rightarrow a^2 - 16a = 0$

$\Rightarrow a\,(a - 16) = 0$

$\therefore a = 0$ and 16

Thus, the values of *a* are 0 and 16.

Example 3:

Find the point on *y*-axis that is equidistant from (–5, 2) and (9, –2).

Solution:

The *x*-coordinate of all points on the *y*-axis is zero. Therefore, let (0, *b*) be the point which is equidistant from (–5, 2) and (9, –2).

∴ Distance between (0, *b*) and (–5, 2) = Distance between (0, *b*) and (9, –2)

Using distance formula, we obtain

$$\sqrt{(-5-0)^2+(2-b)^2}=\sqrt{(9-0)^2+(-2-b)^2}$$
$$\Rightarrow \sqrt{25+4+b^2-4b}=\sqrt{81+4+b^2+4b}$$

On squaring both sides, we obtain

$29 + b^2 - 4b = 85 + b^2 + 4b$

$\Rightarrow 8b = 29 - 85$

$\Rightarrow 8b = -56$

$\Rightarrow b = -7$

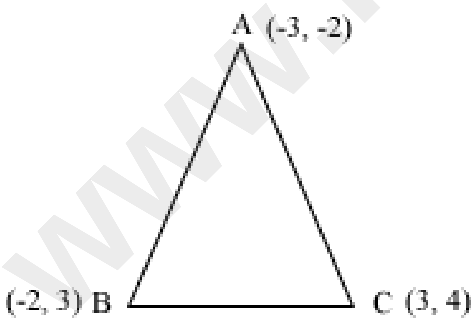
Thus, the point (0, –7) on the *y*-axis is equidistant from (–5, 2) and (9, –2).

Example 4:

Check whether the points (–3, –2), (–2, 3), and (3, 4) are the vertices of an isosceles triangle or not.

Solution:

Let the vertices (–3, –2), (–2, 3), and (3, 4) be denoted as A, B, and C respectively as shown in the figure.



Using distance formula, we obtain

$$AB = \sqrt{[-2-(-3)]^2 + [3-(-2)]^2}$$
$$= \sqrt{(1)^2 + (5)^2}$$
$$= \sqrt{1+25}$$
$$= \sqrt{26} \text{ units}$$

BC $= \sqrt{[3-(-2)]^2 + (4-3)^2}$

$$= \sqrt{(5)^2 + (1)^2}$$
$$= \sqrt{25+1}$$
$$= \sqrt{26} \text{ units}$$

AC $= \sqrt{[3-(-3)]^2 + [4-(-2)]^2}$

$$= \sqrt{(6)^2 + (6)^2}$$
$$= \sqrt{36+36}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2} \text{ units}$$

Now, AB = BC = $\sqrt{26}$ units

Thus, (–3, –2), (–2, 3), and (3, 4) are the vertices of an isosceles triangle.

Dependent And Independent Variables

In our daily life, we must have seen many instances where one quantity depends on another. For example,

(1) If we make more calls, then our phone bill will be high.

(2) If we purchase more quantity of a product, then the cost will be high, etc.

In the given example (1), the phone bill depends on the number of calls made by us. Here, the number of calls of phone is an independent variable and the phone bill is a dependent variable as it depends upon the number of calls we make.

We can define a dependent and an independent variable as follows.

“**The variable, which changes with the change in the other variable, is called dependent variable and the other variable is called independent variable (or control variable).”**

In the second example, the cost of the product depends upon the quantity of product that we purchase. Hence, the quantity of the product is an independent variable and the cost is a dependent variable.

Let us discuss an example to understand the concept of dependent and independent variables.

Example 1:

Trace out the dependent and independent variables from the following situations.

(a) Time period and simple interest, where rate of interest is fixed.

(b) The area of cultivated land and the crop harvested.

(c) Speed of a person and distance covered by him in a fixed time.

(d) Amount of a material and number of particles in it.

Solution:

(a) For a fixed rate of interest, if we deposit the principal for more time period, we will get more simple interest. Thus, the amount of simple interest depends upon time period. Hence, the time period is an independent variable and the simple interest is a dependent variable.

(b) If we increase the area of cultivated land, then we will harvest more amount of crop. Here, the amount of crop harvested depends upon area of land cultivated. Hence, the area of cultivated land is an independent variable and the crop harvested is a dependent variable.

(c) If a person will increase his speed, then he will cover more distance in a fixed time. Here, distance covered by the person depends upon his speed. Hence, speed of the person is an independent variable and distance covered is a dependent variable.

(d) If the amount of material increases, then the number of particles also increases. Here, the number of particles depends upon the amount of material. Hence, the amount of a material is an independent variable and the number of particles present in it is a dependent variable.

Example 2:

The volume of a cuboid is given by the formula $V = l \times b \times h$,
where *l*, *b,* and *h* respectively are the length, breadth and height of the cuboid.
Find the dependent and independent variables among *V, l, b,* and *h.*

Solution:

If we increase the length of a cuboid, then its size will be increased and hence its volume will be increased. Similarly, if we change any side of the cuboid, then its volume will be changed. We can say that the volume of the cuboid depends on the length, breadth and height of the cuboid.

Thus, *V* is a dependent variable and *l*, *b,* and *h* are independent variables.