# Circles

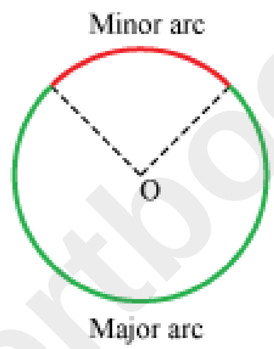
## Circle and Its Attributes

A circle exhibits various interesting properties which make it a special geometric figure.

Let us discuss the same.

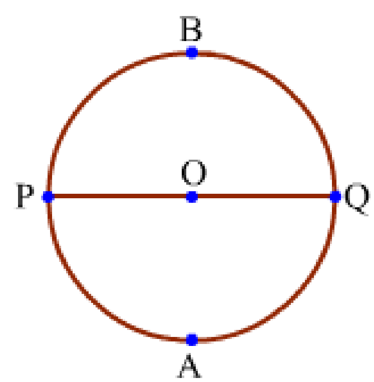
**Minor and major arc:**

An arc less than one-half of the entire arc of a circle is called the minor arc of the circle, while an arc greater than one-half of the entire arc of a circle is called the major arc of the circle.




**Semicircular arc:**

Diameter of a circle divides it into two congruent arcs. Each of these arcs is known as semicircular arc.




In the above figure, PQ is diameter which formed semicircular arcs PBQ and PAQ.

**Finding radius of a circle when its diameter is given:**
We know that the radius of a circle is half of its diameter.

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Let $r$ be the radius and $d$ be the diameter of a circle, then we have $r = \frac{d}{2}$.
Using this formula, we can find the radius of the circle if its diameter is given.

Let us take a look at some examples.

We have to find the radius of the circle when diameter is given.

(i) $d$ = 12 cm

$r = \frac{d}{2}$

$\Rightarrow r = \frac{12}{2}$

$\Rightarrow r = 6$ cm

(ii) $d$ = 25 cm

$r = \frac{d}{2}$

$\Rightarrow r = \frac{25}{2}$

$\Rightarrow r = 12.5$ cm

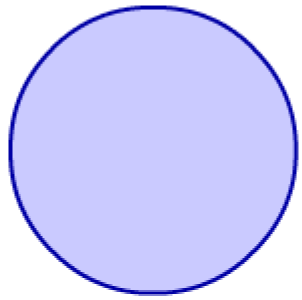
**Finding diameter of a circle when its radius is given:**
We know that the diameter of a circle is twice its radius.
$\therefore d = 2r$
Using this formula, we can find the diameter of the circle when its radius is given.

Let us take a look at some examples.
We have to find the radius of the circle when diameter is given.
(i) $r$ = 15.5 cm
$d = 2r$
$d = 2 \times 15.5$
$d$ = 31 cm

(ii) $r$ = 13 cm
$d = 2r$
$d = 2 \times 13$
$d$ = 26 cm

Let us discuss some more concepts related to circles.

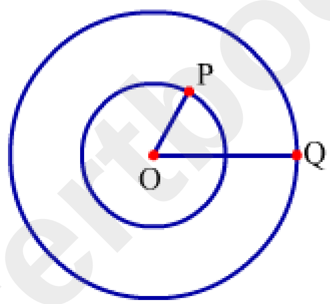
**Circular region:** Look at the following circle.



The whole shaded part is the region of this circle.

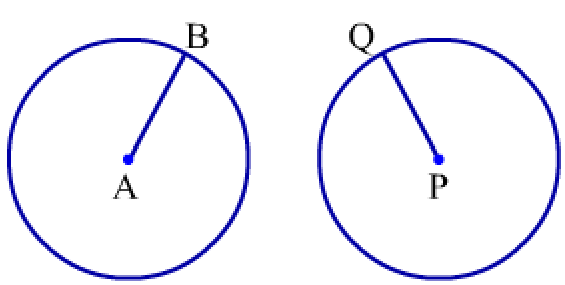
Thus, the interior and boundary together make the region of the circle.

**Concentric circles:** Circles of different radii but having the same centre are known as concentric circles.




In the above figure, two circles have the same centre O but the different radii OP and OQ such that OQ > OP. These circles are concentric circles.

**Congruent circles:** If the radii of two or more circles are equal, then the circles are said to be congruent to each other.




In the above figure, AB and PQ are the radii of the circles such that AB = PQ. Thus, these circles are congruent to each other.

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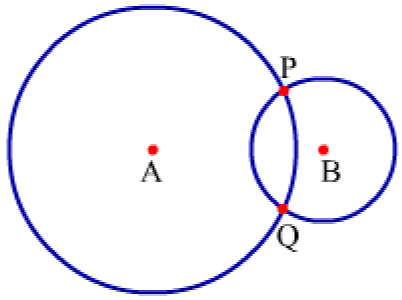
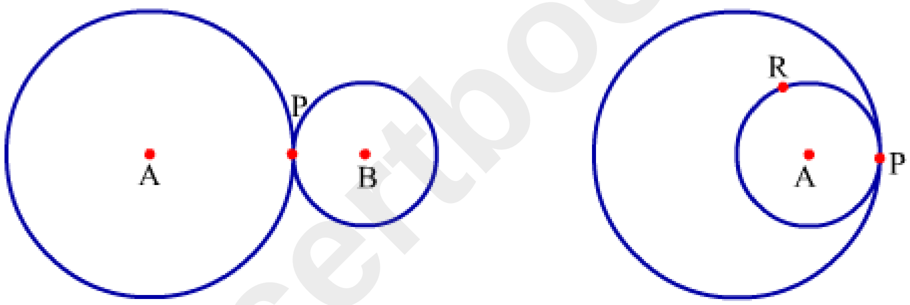
**Intersecting circles:** Two coplanar circles (circles in the same plane) which intersect each other at two distinct points are known as intersecting circles.



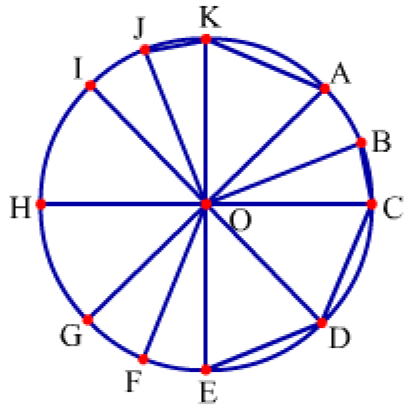

In the above figure, circles with centres A and B intersect each other at two distinct points P and Q. Thus, these are intersecting circles.

If two coplanar circles intersect each other at only one point, then the circles are known as touching circles.




In each of both the above figure, circles touch each other at only one point P. Thus, circles in each figure are touching circles.

Now, observe the following figure.

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Here, OA, OB, OC, ..., OK are all radii of the circle. Similarly, we can draw many more radii of this circle.

So, it can be said that **a circle has innumerable radii.**

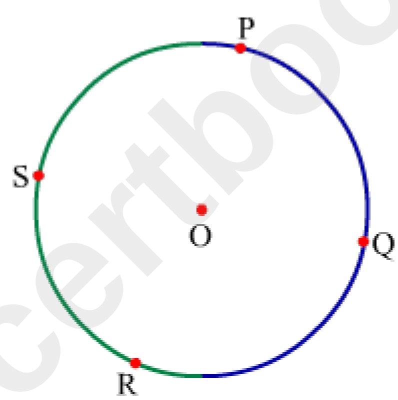
It can be seen that AG, CH, DI and EK all are diameters of the circle. Similarly, we can draw many more diameters of this circle.

So, it can be said that **a circle has innumerable diameters.**

Also, BC, CD, DE, JK and KA are the chords of the circle. Similarly, many more chords of this circle can be drawn.

Thus, it can be said that **a circle has innumerable chords.**

Now, observe the following circle.




It can be seen that points P and R divide this circle into two parts or arcs which are coloured differently. The name "arc PR" does not explain that which of two arcs we are talking about. So, we marked a point on each arc to clarify this. It can be seen that point S is marked on the green arc and point Q is marked on the blue arc. Now, we can give a three letters name to each arc. Thus, green arc can be named as arc PSR or arc RSP whereas blue arc can be named as arc PQR or arc RQP.

Similarly, we can denote any arc by three letters.

Let us discuss some examples to understand this concept better.

**Example 1:**

**With respect to the figure drawn below, name**

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**(a) the centre**

**(b) the diameter**

**(c) any two radii**

**(d) a chord**

**(e) a point lying in the interior of the circle**

**(f) a point lying in the exterior of the circle**

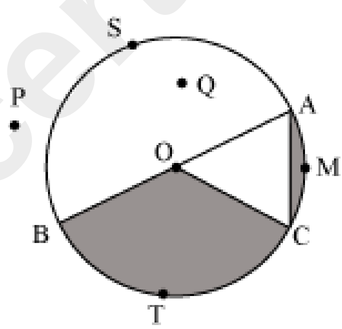
**(g) a sector**

**(h) a segment**

**(i) a point lying on the circle**

**(j) two semi-circles**

**(k) any two arcs**




**Solution:**

**(a)** O is the centre of the circle.

**(b)** $\overline{AB}$ is the diameter of the circle.

**(c)** Two radii of the circle are $\overline{OB}$ and $\overline{OC}$.

**(d)** $\overline{AC}$ is a chord of the circle.

**(e)** Q is a point that lies in the interior of the circle.

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**(f)** P is a point that lies in the exterior of the circle.

**(g)** BOC is a sector of the circle.

**(h)** AMC is a segment of the circle.

**(i)** S is a point that lies on the boundary of the circle (or simply, on the circle).

**(j)** The semi-circles in the given figure are ASB and ATB.

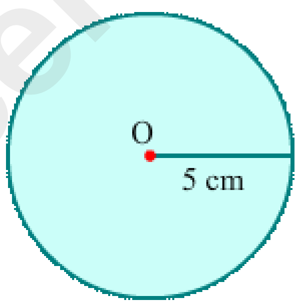
**(k)** BTC and AMC are two arcs of the circle

**Example 2:**

**Using ruler and compass, draw circle of radius 5 cm. Mark its centre and draw the radius.**

**Solution:**

On using a ruler, first we draw the radius 5 cm of the circle and then assuming O as a centre we draw a circle of radius 5 cm by using a compass. Thus, we get a circle of radius 5 cm as shown below.

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Concyclic Points

**Concyclic Points**

You must have seen circular running tracks like the one shown below. You can see concentric circles that divide the track into different paths. Four runners can be seen at different positions on the track.

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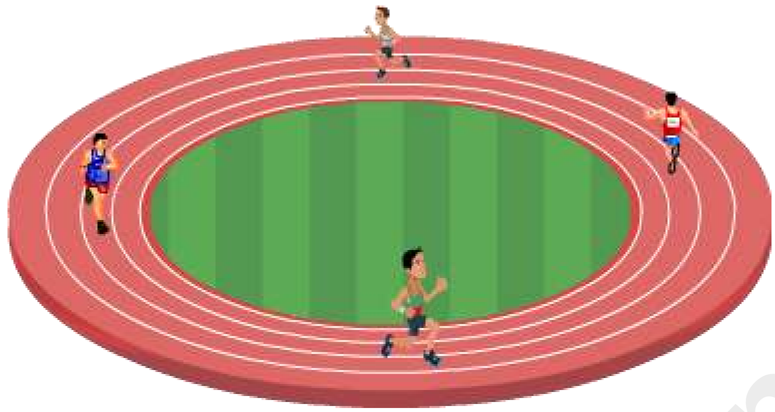

Note how the runners are on the circumference of the same circle on the track. Things which lie on the same circle are referred to as concyclic; so, these runners are also concyclic. Let us go through this lesson to know more about concyclic points and the theorem related to them.

**Theorem Based on Concyclic Points**

**Know More**

A set of more than four points is concyclic if and only if every four-point subset is concyclic. This property is the analogue of concyclicity.

**Whiz Kid**

**General condition for concyclicity**

If *n* distinct points lie on a circle and we join any two points, then $\frac{n(n-1)}{2}$ perpendicular bisectors of the line segments should meet at a single point that is the centre of the circle.

**Solved Examples**

**Easy**

**Example 1**:

**For each figure, state whether or not the points A, B, C and D are concyclic.**

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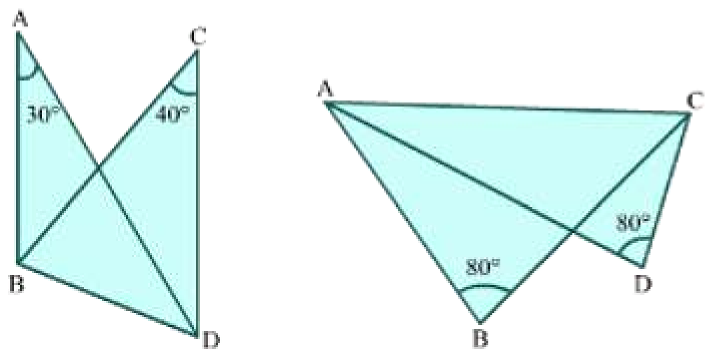


(a) (b)

**Solution**:

**In figure (a)**:

Points A and C are present on the same side of the line joining points B and D.

We have ∠BAD = 30° and ∠BCD = 40°.

Clearly, line segment BD does not subtend equal angles at points A and C that lie on the same side of BD. Therefore, points A, B, C and D do not lie on a circle, i.e., they are not concyclic.

**In figure (b)**:

Points B and D are present on the same side of the line joining points C and A.

We have ∠ABC = ∠ADC = 80°.

Clearly, line segment CA subtends equal angles at points B and D that lie on the same side of CA. Therefore, points A, B, C and D lie on a circle, i.e., they are concyclic.

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Perpendicular from The Centre of a Circle to a Chord Bisects The Chord

**Perpendicular from the Centre to the Chord**

Observe the clock shown below.

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Note how the progression of the second hand from ‘5’ to ‘7’ has been marked on the clock face. We know that the second hand takes five seconds to cover the distance between ‘5’ and ‘6’. Similarly, it takes five seconds to cover the distance between ‘6’ and ‘7’. Clearly, ‘6’ lies in the middle of ‘5’ and ‘7’. Observe how the line marking the position of the second hand at ‘6’ is perpendicular to the straight line marking the distance between ‘5’ and ‘7’. What we have here is a system similar to that obtained on drawing a perpendicular from the centre of a circle to one of its chords.

In this lesson, we will learn about the property of a circle relating to perpendiculars drawn to chords from the centre of the circle. We will also solve some problems based on this property.

**Proof of the Property**

**Know More**

The circle that is centred at the origin with radius 1 is called the **unit circle**.

**Whiz Kid**

- For a given length of perimeter, the circle is the shape with the largest area.
- The circle is a highly symmetric shape as every line passing through its centre forms a line of reflection symmetry and every angle around the centre has rotational symmetry.

- In a circle, parallel chords always cut congruent arcs.

Here, chord AB is parallel to chord CD and arc AC and arc BD are congruent arcs.

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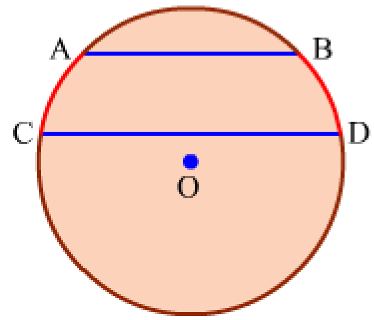



**Converse of the Property**

**Whiz Kid**

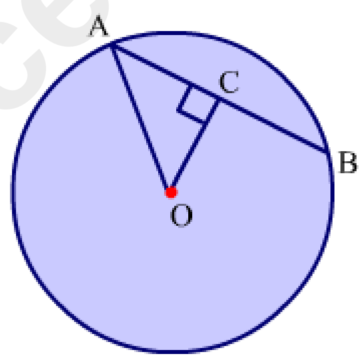
If the intersection of any two chords divides one chord into lengths *a* and *b* and the other into lengths *c* and *d*, then *ab* = *cd*.

**Solved Examples**

**Easy**

**Example 1**:

In the given circle with centre O, if AB = 6 cm and OC = 4 cm, then find the perimeter of ΔOCA.




**Solution**:

We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

O is the centre of the given circle with chord AB and OC ⊥ AB; therefore, OC bisects AB.

⇒ **AC = CB =** $\frac{AB}{2} = \left(\frac{6}{2}\right)\text{cm} = 3\text{ cm}$

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On using the Pythagoras theorem in the right-angled ΔOCA, we get:

$OA^2 = OC^2 + AC^2$

$\Rightarrow OA^2 = (4^2 + 3^2)\ cm^2$

$\Rightarrow OA^2 = (16 + 9)\ cm^2$

$\Rightarrow OA^2 = 25\ cm^2$

$\Rightarrow OA = 5$ cm

So,

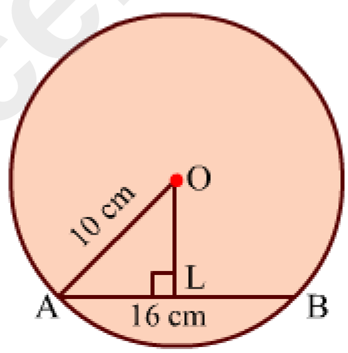
Perimeter of ΔOCA = OA + AC + OC

⇒ Perimeter of ΔOCA = (5 + 3 + 4) cm

⇒ Perimeter of ΔOCA = 12 cm

**Example 2**:

In the given circle centred at O, find the distance of chord AB from the centre.




**Solution**:

We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

O is the centre of the given circle with chord AB and OL ⊥ AB; therefore, OL bisects AB.

$\Rightarrow AL = LB = \frac{AB}{2} = \left(\frac{16}{2}\right)cm = 8$ cm

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On using the Pythagoras theorem in the right-angled ΔOLA, we get:

$OA^2 = OL^2 + AL^2$

$\Rightarrow OL^2 = OA^2 - AL^2$

$\Rightarrow OL^2 = (10^2 - 8^2)\ cm^2$

$\Rightarrow OL^2 = (100 - 64)\ cm^2$

$\Rightarrow OL^2 = 36\ cm^2$

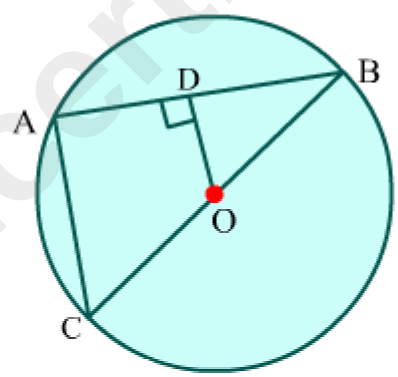
$\Rightarrow OL = 6$ cm

Thus, chord AB is at a distance of 6 cm from the centre of the circle.

**Medium**

**Example 1:**

In the given circle with centre O, prove that AC = 2OD.




**Solution**:

We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

O is the centre of the given circle with chord AB and OD ⊥ AB; therefore, OD bisects AB.

Thus, D is the midpoint of AB.

Since O is the centre, it is the midpoint of diameter BC.

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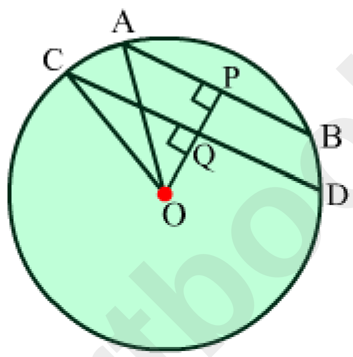
So, in ΔABC, D and O are the midpoints of sides AB and BC respectively.

∴ OD = $\frac{AC}{2}$ and OD||AC (By the midpoint theorem)

⇒ AC = 2OD

**Example 2**:

The given circle centred at O has a radius of 5 cm and two parallel chords AB and CD. If AB = 6 cm and CD = 8 cm, then find the length of PQ.




**Solution**:

We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

O is the centre of the given circle with chord AB and OP ⊥ AB; therefore, OP bisects AB.

Thus, P is the midpoint of AB.

⇒ AP = PB = $\frac{AB}{2}$ = $\left(\frac{6}{2}\right)$cm = 3 cm

Similarly, Q is the midpoint of CD.

⇒ CQ = QD = $\frac{CD}{2}$ = $\left(\frac{8}{2}\right)$cm = 4 cm

On using the Pythagoras theorem in right-angled ΔOPA, we get:

$OA^2 = OP^2 + AP^2$

$\Rightarrow OP^2 = OA^2 - AP^2$

$\Rightarrow OP^2 = (5^2 - 3^2)\ cm^2$

$\Rightarrow OP^2 = (25 - 9)\ cm^2$

$\Rightarrow OP^2 = 16\ cm^2$

$\Rightarrow OP = 4$ cm

Similarly, in right-angled ΔOQC, we get:

$OC^2 = OQ^2 + CQ^2$

$\Rightarrow OQ^2 = OC^2 - CQ^2$

$\Rightarrow OQ^2 = (5^2 - 4^2)\ cm^2$

$\Rightarrow OQ^2 = (25 - 16)\ cm^2$

$\Rightarrow OQ^2 = 9\ cm^2$

$\Rightarrow OQ = 3$ cm

Now, PQ = OP − OQ

$\Rightarrow PQ = (4 - 3)$ cm

$\Rightarrow PQ = 1$ cm

**Hard**

**Example 1:**

In the given circle with centre O, find the distance between the parallel and equal chords AB and CD.

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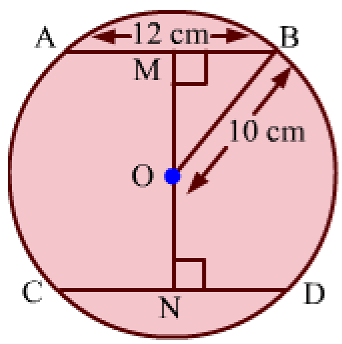


**Solution:**

We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

O is the centre of the given circle with chord AB and OM ⊥ AB; therefore, OM bisects AB.

$\Rightarrow AM = MB = \frac{AB}{2} = \left(\frac{12}{2}\right) cm = 6$ cm

Similarly, ON bisects CD. Since AB = CD, we get:

CN = ND = 6 cm

On using the Pythagoras theorem in right-angled ΔOMB, we get:

$OB^2 = OM^2 + BM^2$

$\Rightarrow OM^2 = OB^2 - BM^2$

$\Rightarrow OM^2 = (10^2 - 6^2)\ cm^2$

$\Rightarrow OM^2 = (100 - 36)\ cm^2$

$\Rightarrow OM^2 = 64\ cm^2$

$\Rightarrow OM = 8$ cm

On joining point O to point D and calculating as above, we get ON = 8 cm.

Now, MN is the perpendicular distance between the given chords.

MN = OM + ON

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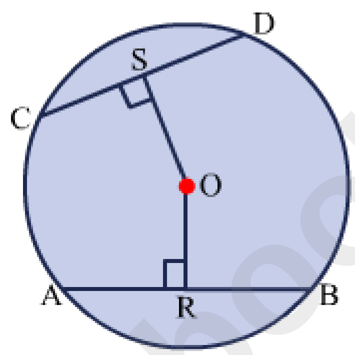
⇒ MN = (8 + 8) cm

⇒ MN = 16 cm

Thus, the distance between the parallel and equal chords AB and CD is 16 cm.

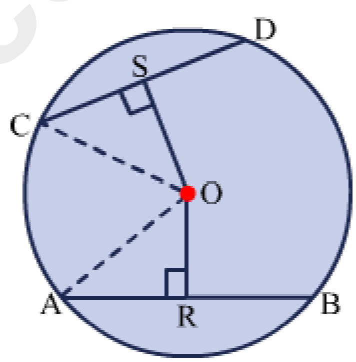
**Example 2**:

In the given circle centred at O, AB = 8 cm, OR = 3 cm and OS = 4 cm. Find the length of CD.




**Solution:**

***Construction***: Join point O to points A and C.




We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

O is the centre of the circle with chord AB and OR ⊥ AB; therefore, OR bisects AB.

⇒ AR = RB = $\frac{AB}{2} = \left(\frac{8}{2}\right)\text{cm}$ = 4 cm

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On using the Pythagoras theorem in right-angled ΔORA, we get:

$OA^2 = OR^2 + AR^2$

$\Rightarrow OA^2 = (3^2 + 4^2)\ cm^2$

$\Rightarrow OA^2 = (9 + 16)\ cm^2$

$\Rightarrow OA^2 = 25\ cm^2$

$\Rightarrow OA = 5$ cm

Clearly, OA and OC are radii of the same circle.

$\therefore OC = OA = 5$ cm

On using the Pythagoras theorem in right-angled ΔOSC, we get:

$OC^2 = OS^2 + CS^2$

$\Rightarrow CS^2 = OC^2 - OS^2$

$\Rightarrow CS^2 = (5^2 - 4^2)\ cm^2$

$\Rightarrow CS^2 = (25 - 16)\ cm^2$

$\Rightarrow CS^2 = 9\ cm^2$

$\Rightarrow CS = 3$ cm

Since $OS \perp CD$, we have:

CS = SD = 3 cm

Now, CD = CS + SD

$\Rightarrow CD = (3 + 3)$ cm

$\Rightarrow CD = 6$ cm

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Distance of Chords from The Centre of The Circle

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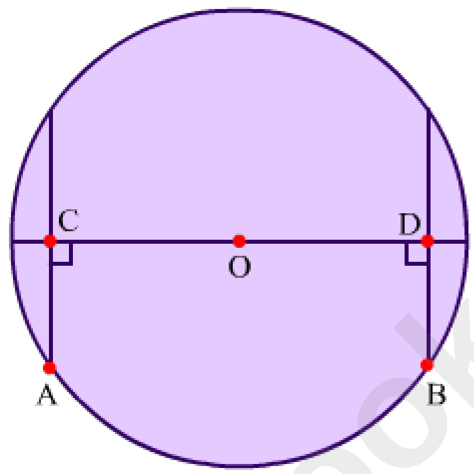
**Equal Chords and Their Distance from the Centre**

Consider a big circular ground in which three ropes are tied to the circumference. Two ropes are of the same length and the third is perpendicular to them, as is shown in the following figure.

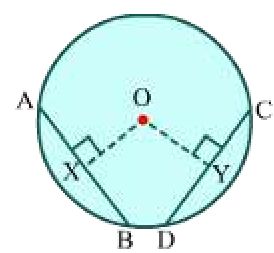



Anita, Bindu and Omkar are standing at points A, B and O respectively. The girls begin walking along the lengths of the shorter ropes to reach points C and D at the same time. Thereafter, they continue in a similar manner toward point O with the same speed as before. Who will reach Omkar first?

The answer to the above question is based on an important property of chords which we will study in this lesson. This property shows the relation between equal chords in terms of their distances from the centre of a circle.

**Equal Chords Are Equidistant from the Centre of a Circle**

Consider the given circle centred at O with two equal chords AB and CD.




OX and OY are the perpendicular distances of chords AB and CD respectively from centre O. Now, the property that relates these perpendicular distances of equal chords is stated as follows:

***Equal chords are equidistant from the centre of a circle.***

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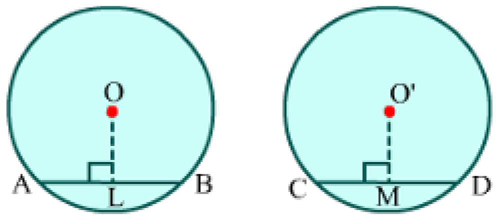
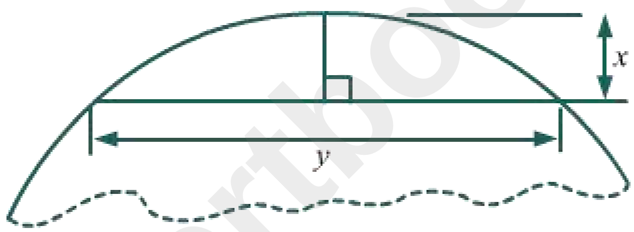
So, according to this property, since AB is equal to CD, their distances from the centre are also equal, i.e., OX = OY. This property is also true in case of congruent circles. Consider, for example, the following congruent circles with centres O and O′, and chords AB and CD. Now, if AB and CD are equal, then AB and CD are equidistant from O and O′, i.e., OL = O′M.




**Know More**

**Sagitta**: It is the perpendicular drawn from an arc of a circle to a chord such that it bisects the chord.




Here, $x = r - \sqrt{r^2 - \left(\frac{y}{2}\right)^2}$, where *r* is the radius of the circle

**Whiz Kid**

**Alternate segment theorem**

If a line touches a circle and a chord is drawn from the point of contact, then the angle between the tangent and the chord is equal to the angles in the corresponding alternate segments.

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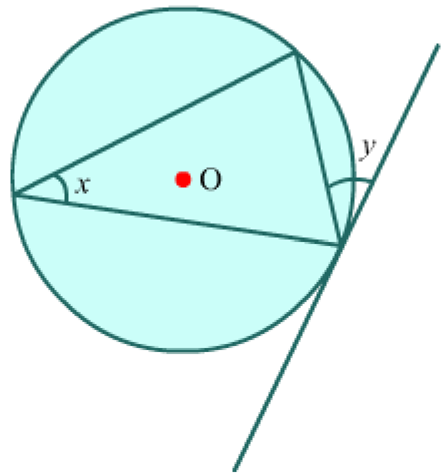
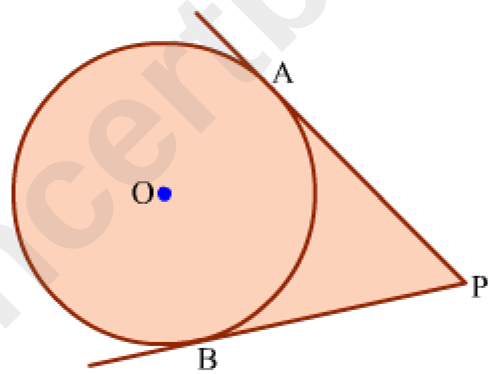



Here, $x = y$

**Proof of the Property**

**Whiz Kid**

The lengths of tangents drawn from a point outside a circle are equal.




Here, PA = PB

**Converse of the Property**

The converse of the property can be stated as follows:

***Chords that are equidistant from the centre of a circle are equal in length.***

Consider, for example, the given circle with centre O and chords AB and CD that are equidistant from O, i.e., OL = OM.

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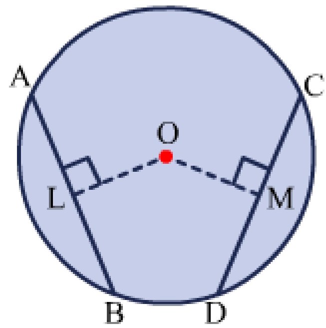
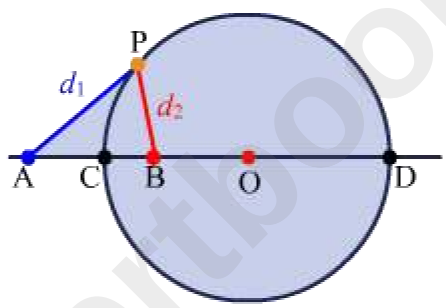



Using the converse of the property, we can say that AB and CD are equal in length.

**Did You Know?**

**Circle of Apollonius**




Apollonius of Perga, an ancient Greek geometer, showed that a circle can be defined as a set of points in a plane having a constant ratio of distances to two fixed foci.

In the given figure, points A and Bare two fixed foci.

Therefore, by the above definition, we have $\frac{AP}{BP} = \frac{AC}{BC}$

Note that the constant ratio cannot be equal to 1.

**Know Your Scientist**

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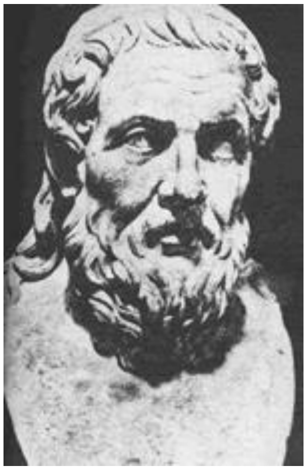


Apollonius (262 BC−190 BC) was an ancient Greek geometer and astronomer. He is known as 'the Great Geometer'. His book *Conics* is one of the greatest scientific works from the ancient world. Terms such as 'parabola', 'ellipse' and 'hyperbola' were introduced in this book. Famous scholars like René Descartes and Isaac Newton were influenced by his innovative methods and the terms he used in the field of conics.

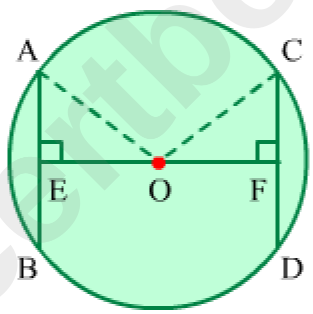
**Proof of the Converse**

**Statement:** Chords that are equidistant from the centre of a circle are equal in length.

**Given**: A circle centred at O with chords AB and CD and OE = OF, where OE ⊥AB and OF ⊥ CD.

**To Prove:** AB = CD

***Construction***: Join point O to points A and C.




**Proof:** It is given that OE ⊥ AB and OF ⊥ CD

We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord. So, we have

$AE = \frac{AB}{2}$ **… (1)**

$CF = \frac{CD}{2}$ **… (2)**

In ΔAEO and ΔCFO, we have:

OA = OC (Radii of the circle)

∠AEO = ∠CFO = 90° (∵ OE and OF are perpendiculars)

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OE = OF (Given)

∴ ΔAEO ≅ ΔCFO (By the RHS congruence rule)

⇒ AE = CF (By CPCT)

⇒ $\frac{1}{2}$ AB = $\frac{1}{2}$ CD (Using equations (1) and (2))

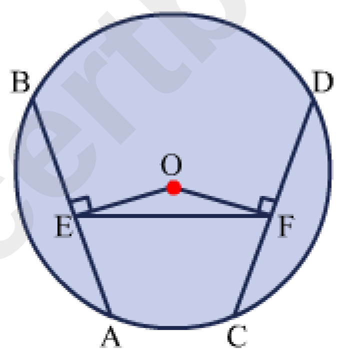
⇒ AB = CD

**Solved Examples**

**Easy**

**Example 1**:

In the given circle centred at O, AB and CD are two equal chords. Prove that ∠AEF = ∠CFE.




**Solution**:

It is given that AB = CD. We know that equal chords are equidistant from the centre of a circle.

∴ OE = OF

In ΔOEF, we have:

OE = OF

∴ ∠OFE = ∠OEF (∵ Angles opposite equal sides are equal)

⇒ 90° − ∠AEF = 90° − ∠CFE (∵ OE and OF are perpendiculars)

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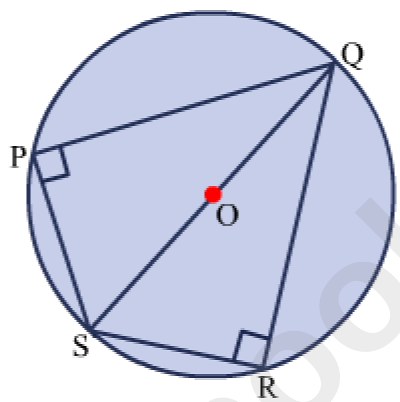
⇒ ∠AEF = ∠CFE

**Medium**

**Example 1**:

In the given circle with centre O, chords PQ and RQ are equidistant from the centre. Prove that diameter SQ bisects ∠PQR and ∠PSR.




**Solution**:

We know that chords which are equidistant from the centre of a circle are equal in length.

∴ PQ = RQ

In ΔQPS and ΔQRS, we have:

PQ = RQ (Proved above)

∠QPS = ∠QRS = 90° (∵ SP and SR are perpendiculars)

QS = QS (Common side)

∴ ΔQPS ≅ ΔQRS (By the RHS congruence rule)

⇒ ∠PQS = ∠RQS and ∠PSQ = ∠RSQ (By CPCT)

Now, ∠PQR = ∠PQS + ∠RQS and ∠PSR = ∠PSQ + ∠RSQ

Thus, diameter QS bisects ∠PQR and ∠PSR.

**Example 2**:

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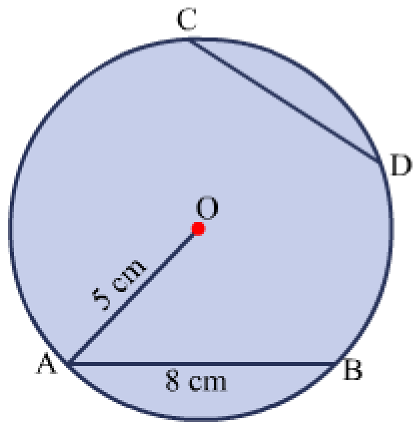
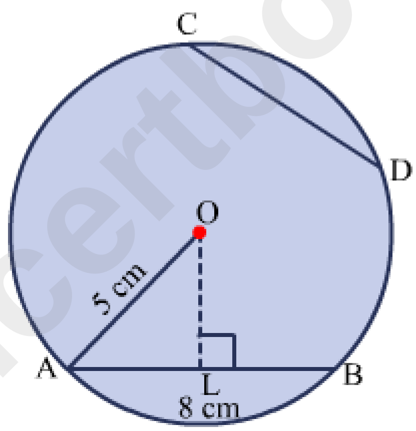
In the given circle, find the length of chord CD which is 3 cm away from centre O.




**Solution**:

***Construction***: Draw a perpendicular OL from centre O to chord AB.




We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

$\therefore$ AL = LB = $\frac{AB}{2}$ = $\left(\frac{8}{2}\right)$cm = 4 cm

On using the Pythagoras theorem in right-angled ΔOLA, we obtain:

$OA^2 = OL^2 + AL^2$

$\Rightarrow OL^2 = OA^2 - AL^2$

$\Rightarrow OL^2 = (5^2 - 4^2)\ cm^2$

⇒ $OL^2 = (25 - 16)\ cm^2$

⇒ $OL^2 = 9\ cm^2$

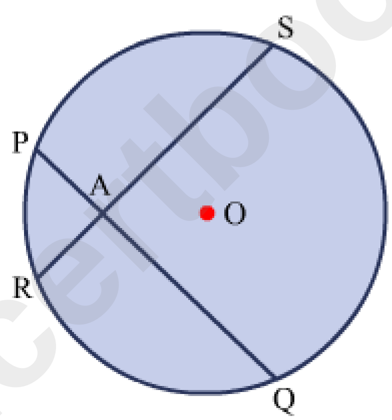
⇒ OL = 3 cm, which is the distance of chord AB from O

It is given that the distance of chord CD from the centre is also 3 cm. We know that chords which are equidistant from the centre are equal in length. Therefore, the length of chord CD is also 8 cm.

**Hard**

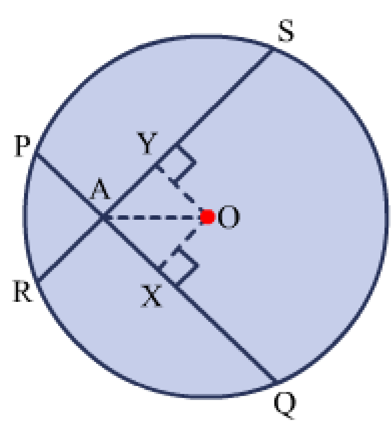
**Example 1**:

In the given circle, PQ and RS are two chords equidistant from centre O and A is the point of intersection of the chords. Prove that AR = AP.




**Solution**:

***Construction***: Draw perpendiculars OX and OY to chords PQ and RS respectively. Join O to A.

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In ΔOXA and ΔOYA, we have:

OX = OY (∵ PQ and RS are equidistant from the centre)

∠OXA = ∠OYA = 90° (∵ OX and OY are perpendiculars)

OA = OA (Common side)

∴ ΔOXA ≅ ΔOYA (By the RHS congruence rule)

⇒ AX = AY … (1) [By CPCT]

We know that chords which are equidistant from the centre are equal in length.

∴ PQ = RS … (2)

We also know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

$$\therefore PX = \frac{PQ}{2} \text{ and } RY = \frac{RS}{2}$$

Using equation 2, we obtain:

PX = RY … (3)

On subtracting equation 1 from equation 3, we obtain:

PX − AX = RY − AY

⇒ AP = AR

**Example 2**:

When two equal chords AB and CD of a circle with centre O are extended, they meet at a point E as is shown in the figure. Prove that BE = DE and AE = CE.

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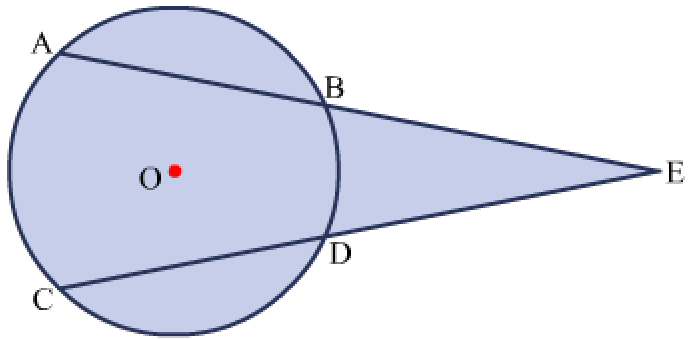
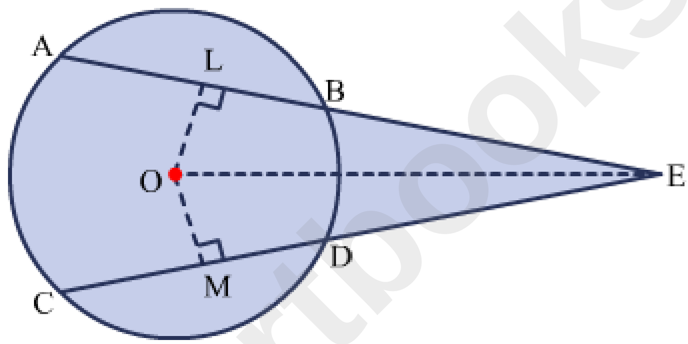



**Solution**:

***Construction***: Join O to E and construct OL ⊥ AB and OM ⊥ CD.




It is given that AB = CD. We know that equal chords are equidistant from the centre.

∴ OL = OM

In ΔOLE and ΔOME, we have:

OL = OM (Proved above)

∠OLE = ∠OME = 90° (∵ OL and OM are perpendiculars)

OE = OE(Common side)

∴ ΔOLE ≅ ΔOME  (By the RHS congruence rule)

⇒ LE = ME … (1)  [By CPCT]

We know that the perpendicular drawn from the centre of a circle to a chord bisects the chord. Thus L and M are the midpoints of AB and CD respectively.

∴ BL = DM… (2)  [∵ AB = CD]

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On subtracting equation 2 from equation 1, we get:

LE − BL = ME − DM

⇒ BE = DE

Now, AB = CD and BE = DE

∴ AB + BE = CD + DE

⇒ AE = CE

**Strengthen this topic**TAKE A TOPIC TEST

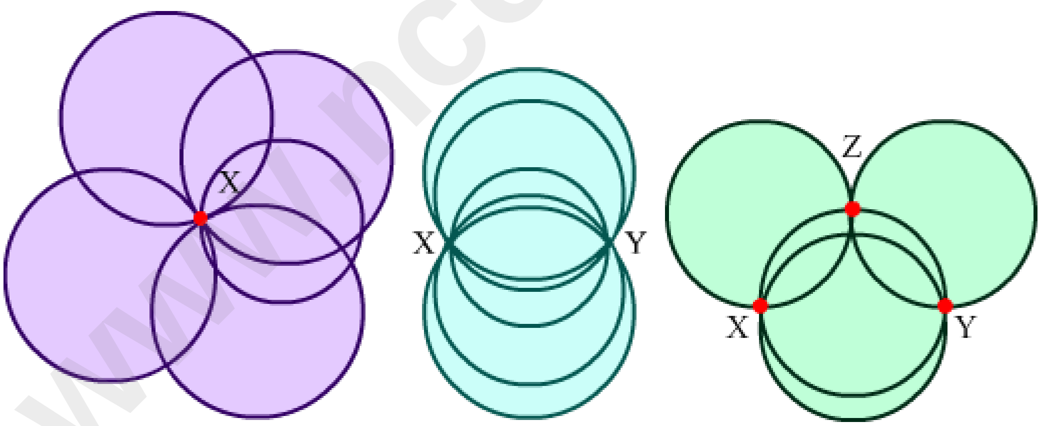
SCROLL DOWN FOR THE NEXT TOPIC

Only One Circle can Pass Through Three Non-Collinear Points

**Limitation of Points Shared by Circles**

We know that the most important point required to draw a circle is its centre which is equidistant from all other points lying on the boundary of the circle. We can also draw infinitely many circles of different radii with the same centre.

Now, let us observe some points shared by circles on their boundaries. A few circles passing through common points X, Y and Z are shown below.




It can be observed that when point X is taken alone, we can draw infinitely many circles passing through it. Similarly, when X and Y are taken together, we can get infinitely many circles passing through them. However, when we take the three points X, Y and Z together, we obtain only one circle passing through them. Thus, we can conclude that to draw a unique circle, we require at least three non-collinear points. In this lesson, we will study more about this conclusion.

**Only One Circle Can Pass through Three Non-Collinear Points**

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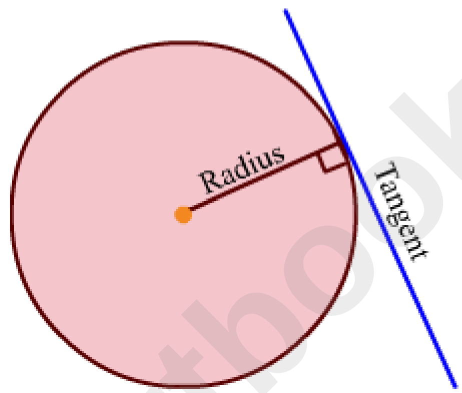
**Did You Know?**

- Circles were worshiped in ancient Rome as they were thought to be divine and holy.
- A circle is a shape that does not exist in nature. It is a mental construct and a symbolic representation that was invented in a manner similar to how the alphabet and language were invented.

**Whiz Kid**

A tangent is a line that touches a circle at only one point. It always forms a right angle with a radius of that circle.



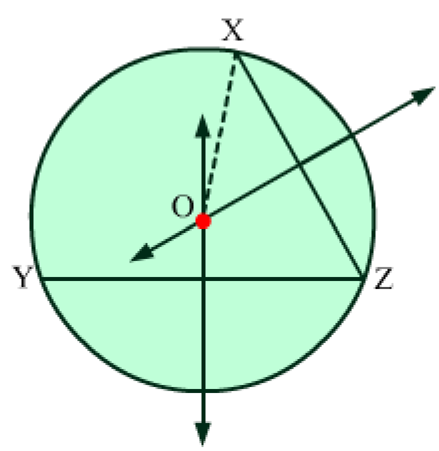

**Know More**

Two lines cannot intersect at more than one point.

**Verification of the Uniqueness of a Circle**

Let us verify the fact that only one circle passes through three non-collinear points.

The given circle with centre O passes through points X, Y and Z.

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Let us assume that another circle with centre O′ and the same radius can pass through X, Y and Z. Then, O′ must lie on the perpendicular bisectors of XZ and YZ.

We know that two lines cannot intersect at more than one point. So, O′ must coincide with O.

Hence, one and only one circle can pass through three non-collinear points.

SCROLL DOWN FOR THE NEXT TOPIC

Angles Subtended by Congruent Arcs

**Angles Subtended by Congruent Arcs**

The given figure shows five children playing ball in a circular park. They are positioned at points A, B, C, D and E. The child standing at point A is at the centre of the circle, while the others are at its circumference.


The lines joining the different points represent the paths followed by the ball during play. Note how paths BC and DE act as chords of the circle. If we assume that the distances BC and DE are equal, then the arcs corresponding to them will also be equal or congruent. Clearly, the angles subtended at centre A by arcs BC and DE are equal, i.e., ∠BAC and ∠DAE are equal. By this, we can conclude that congruent arcs subtend equal angles at the centre of a circle. Let us understand this property and solve some problems based on it.

**Understanding the Property and Its Converse**

**Examples Based on the Property**

**Solved Examples**

**Easy**

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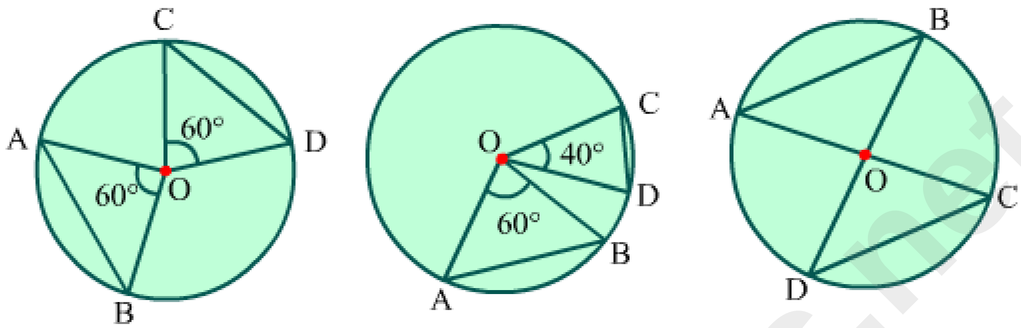
**Example 1**:

For each figure, state whether or not the arcs AB and CD are equal.




(i) (ii) (iii)

**Solution**:

We know that arcs subtending equal angles at the centre of a circle are congruent.

**In figure (i):**

∠AOB = ∠COD = 60°

⇒ Arc AB = Arc CD

**In figure (ii):**

∠AOB = 60° and ∠COD = 40°

⇒ Arc AB ≠ Arc CD

**In figure (iii):**

∠AOB = ∠COD (Vertically opposite angles)

⇒ Arc AB = Arc CD

**Example 2**:

In the given circle with centre O, arcs AB and CD are equal and arcs AB and EF are equal. If ∠AOB = 55°, then find the measure of ∠FOE and ∠COD.

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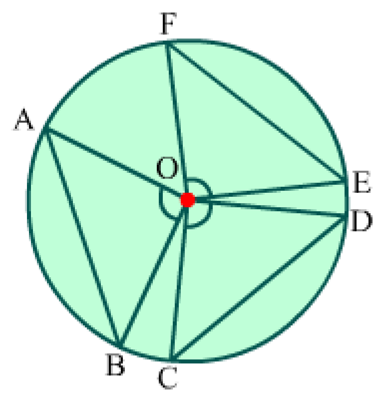



**Solution**:

It is given that:

Arc AB = Arc CD

Arc AB = Arc EF

∴ Arc CD = Arc EF

We know that congruent arcs subtend equal angles at the centre of a circle.

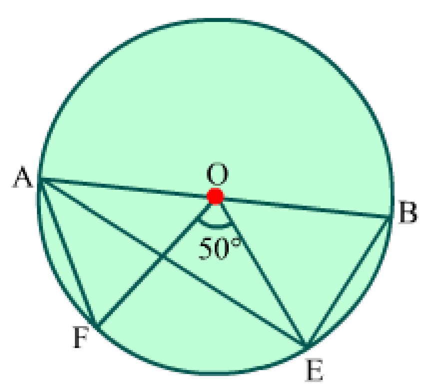
∴ ∠AOB = ∠COD = ∠EOF

Now, ∠AOB = 55°

∴ ∠COD = ∠EOF = 55°

**Example 3**:

In the given circle with centre O, AF = BE and ∠FOE = 50°. Find the measure of ∠AOF.

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**Solution**:

It is given that:

Chord AF = Chord BE

We know that congruent arcs subtend equal angles at the centre of a circle.

∴ ∠AOF = ∠BOE … (1)

AOB is a straight line; therefore, we have:

∠AOF + ∠FOE + ∠BOE = 180°

⇒ 2∠AOF + 50° = 180°  (By equation 1)

⇒ 2∠AOF = 180° − 50°

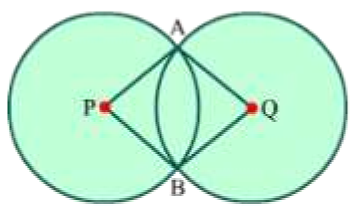
⇒ 2∠AOF = 130°

⇒ ∠AOF = 65°

**Medium**

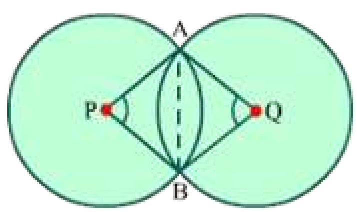
**Example 1**:

The given figure shows two congruent circles with centres P and Q, and intersecting each other at points A and B. Show that ∠APB = ∠AQB.




**Solution**:

***Construction***: Draw a chord AB that is common to the given circles.

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We know that if two chords of congruent circles are equal, then their corresponding arcs are congruent.

We have AB as the common chord of the two given circles. Therefore, the length of arc AB is the same in both circles.

We know that congruent arcs subtend equal angles at the centre of a circle.

$\therefore \angle APB = \angle AQB$

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