Construction of Polygons

Construction of Quadrilateral When One Angle and Four Sides are Given

We know that if we want to construct a quadrilateral, then we need five elements (measure of sides or angles). How will we proceed if four sides and one angle of a quadrilateral are given?

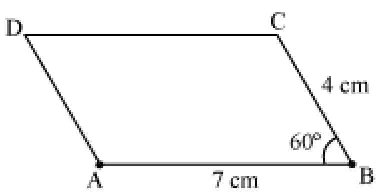
To understand this concept better, let us solve some problems.

Example 1

Construct a parallelogram whose adjacent sides are 4 cm and 7 cm, and the angle between them is 60°.

Solution:

We know that the opposite sides of a parallelogram are equal. Let us draw a rough sketch of parallelogram ABCD, in which AB = 7 cm, BC = 4 cm and ∠B = 60°.

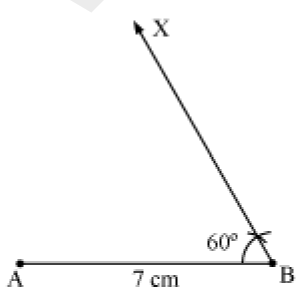


Clearly, CD = AB = 7 cm and AD = BC = 4 cm

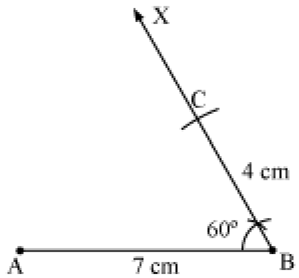
The steps of construction are as follows.

(1) Draw AB = 7 cm

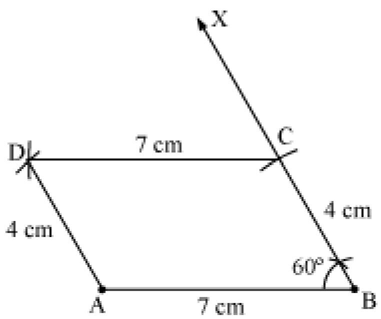
(2) At B, draw ∠ABX = 60°



(3) Taking B as centre, draw an arc of radius BC = 4 cm to cut BX at C.



(4) Taking C as centre, draw an arc of radius AB = 7 cm and taking A as centre, draw an arc of radius BC = 4 cm to intersect the previous arc at D. Join CD and AD.



ABCD is the required parallelogram.

Construction Of Quadrilaterals When Three Sides And Two Included Angles Are Given

We know that to construct a quadrilateral, we require five elements. Suppose three sides and two angles included between them are given. Then how will we construct the quadrilateral?

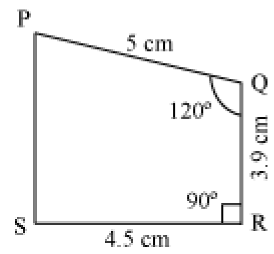
Let us now look at one more example to understand the construction of a quadrilateral better.

Example:

Construct a quadrilateral PQRS with SR = 4.5 cm, QR = 3.9 cm, PQ = 5 cm, ∠R = 90°, and ∠Q = 120°.

Solution:

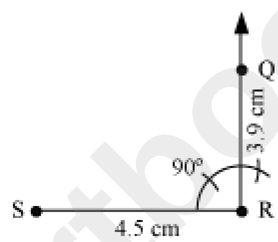
First, we draw a rough sketch of the quadrilateral PQRS as shown in the following figure.



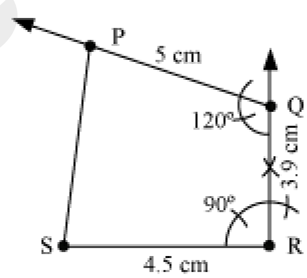
The steps of construction are as follows.

1. We draw a line segment SR of length 4.5 cm.

2. Then we construct an angle of 90° at R and locate the point Q such that RQ = 3.9 cm.



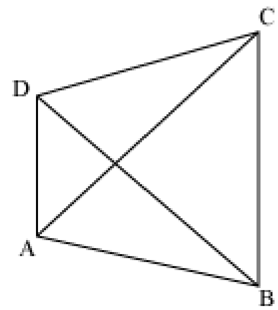
3. Now, we construct an angle of 120° at Q and locate the point P such that QP = 5 cm. Then we join PS.



Thus, PQRS is the required quadrilateral of given measures.

Construction of Quadrilateral when One Diagonal and Four Sides Are Given

Look at the following quadrilateral ABCD.



It has ten elements - four sides (AB, BC, CD, and DA), four angles (∠ABC, ∠BCD, ∠CDA, and ∠DAB), and two diagonals (AC and BD).

If we want to construct a unique quadrilateral, then how many elements are required?

To construct a unique quadrilateral, at least five elements are necessary.

Now, suppose we want to construct a quadrilateral whose one diagonal and four sides are given, then how will we construct it?

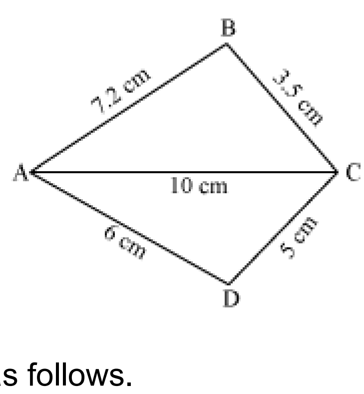
Let us now look at some more examples.

Example 1:

Construct a quadrilateral ABCD with AC = 10 cm, AD = 6 cm, DC = 5 cm, AB = 7.2 cm, and BC = 3.5 cm.

Solution:

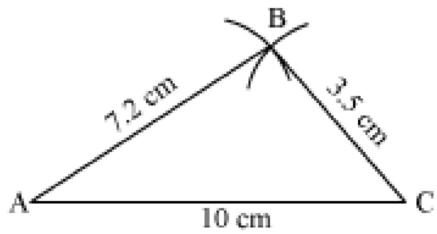
First, we draw a rough sketch of quadrilateral ABCD and indicate the given lengths as shown below.



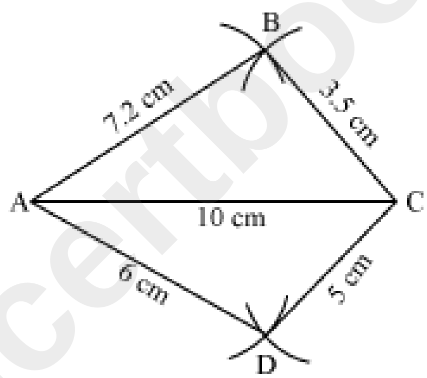
The steps of construction are as follows.

1. Draw AC = 10 cm.

2. Then, draw an arc taking A as centre and 7.2 cm as the radius. Now, draw another arc with C as the centre and 3.5 cm as the radius cutting the previous arc at B. Then, join AB and BC.



3. Draw an arc on the opposite side of B by taking A as centre and 6 cm as the radius. Then, draw another arc with C as the centre and 5 cm as the radius cutting the previous arc at D. Then, join AD and CD.



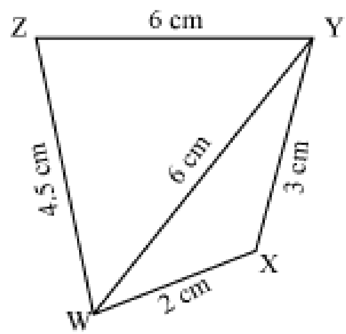
Thus, ABCD is the required quadrilateral.

Example 2:

Can you construct a quadrilateral WXYZ with WY = 6 cm, WX = 2cm, XY = 3 cm, YZ = 6 cm, and WZ = 4.5 cm? Justify your answer.

Solution:

First, we draw a rough sketch of the quadrilateral and indicate the given lengths as shown below.



Here, in ΔWXY,

WX + XY = 2 + 3 = 5 **<** WY = 6 cm

Therefore, WX + XY < WY

⇒ Sum of two sides < Third side

Therefore, ΔWXY is not possible. Thus, we cannot construct a quadrilateral with the given lengths.

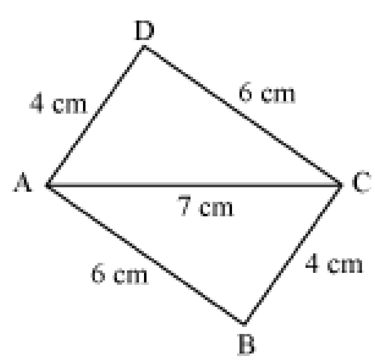
Example 3:

Construct a parallelogram whose adjacent sides are 6 cm and 4 cm and one of its diagonals is 7 cm.

Solution:

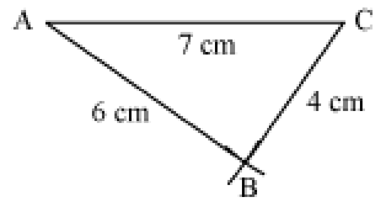
We know that opposite sides of a parallelogram are equal. If ABCD is a parallelogram where the adjacent sides AB and BC are of lengths 6 cm and 4 cm respectively and the diagonal AC is of length 7 cm, then CD = 6 cm and AD = 4 cm

Now, we have the parallelogram ABCD, where AB = CD = 6 cm, BC = AD = 4 cm, and AC = 7 cm. For this, we have to follow the below given steps.

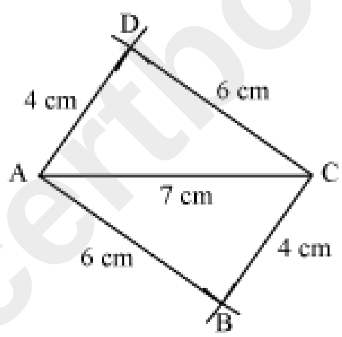


1. Draw AC = 7 cm

2. Draw an arc taking A as centre and 6 cm as the radius. Now, draw another arc with C as the centre and 4 cm as the radius cutting the previous arc at C. Then, join AB and BC.



3. Now, draw an arc taking A as centre and 4 cm as the radius. Then, draw another arc with C as the centre and 6 cm as the radius cutting the previous arc at D. Then, join AD and CD.



Thus, ABCD is the required parallelogram.

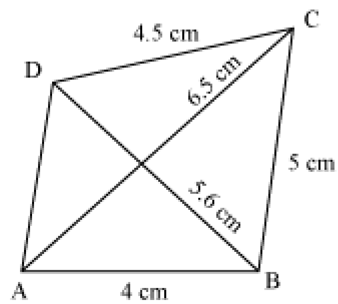
Construction of a Quadrilateral When Three Sides and Both Diagonals Are Given

We know that to construct a quadrilateral, we require five elements. Now suppose three sides and the two diagonals of a quadrilateral are given.

Then, **how will we construct the quadrilateral with these five elements?**

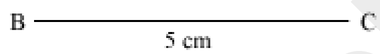
Let us construct a quadrilateral ABCD in which AB = 4 cm, BC = 5 cm, CD = 4.5 cm, AC = 6.5 cm, and BD = 5.6 cm.

First we draw a rough sketch of the quadrilateral ABCD and indicate the given lengths in the figure as shown below.

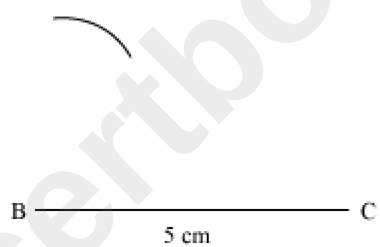


To construct the quadrilateral, we follow the below given steps.

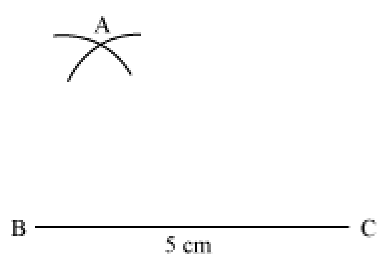
1. Draw BC = 5 cm (the side opposite to the unknown side).



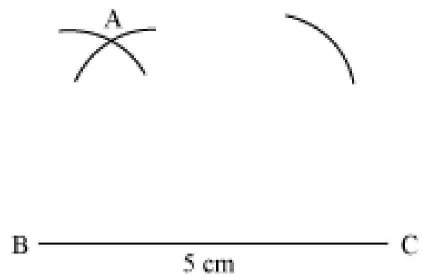
2. Then, draw an arc taking B as centre and AB (4 cm) as radius.



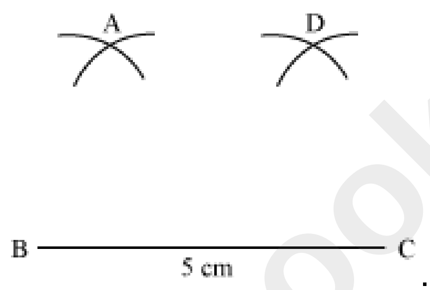
3. Then, draw another arc taking C as centre and AC (6.5 cm) as radius to intersect the previously drawn arc at a point A.



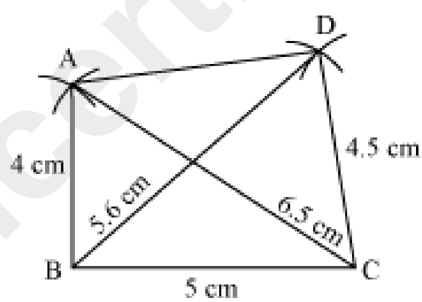
4. Then, draw an arc taking B as centre and BD (5.6 cm) as radius on the same side of BC in which point A lies.



5. Then, draw another arc taking C as centre and CD (4.5 cm) as radius to intersect the arc drawn in step IV at point D.



6. Then, join AB, AD, CD, AC, and BD.



This is the required quadrilateral ABCD.

In this way, we can construct a quadrilateral if we are given three of its sides and the two diagonals.

Let us now solve another example.

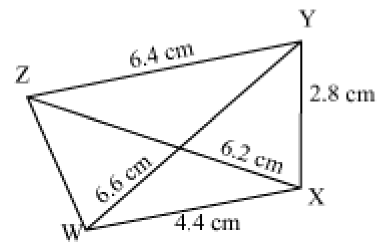
Example 1:

Construct a quadrilateral WXYZ in which WX = 4.4 cm, XY = 2.8 cm,

YZ = 6.4 cm, XZ = 6.2 cm, and WY = 6.6 cm.

Solution:

First, draw a rough sketch of quadrilateral WXYZ and indicate the given lengths as shown below.



The steps of construction are as follows.

(i) Draw XY = 2.8 cm.

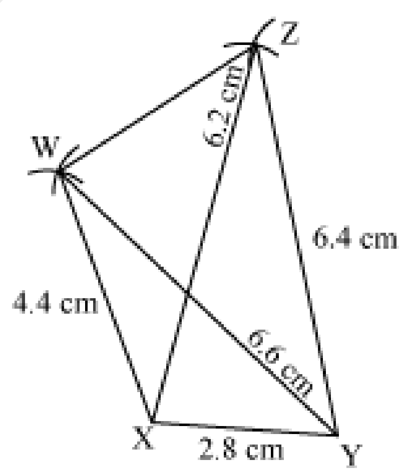
(ii) Draw an arc taking X as centre and WX (4.4 cm) as radius.

(iii) Draw another arc taking Y as centre and WY (6.6 cm) as radius to intersect the previous arc at W.

(iv) Again, draw an arc taking X as centre and XZ (6.2 cm) as radius on the same side of XY in which point W lies.

(v) Draw another arc taking Y as centre and YZ (6.4 cm) as radius to intersect the arc drawn in previous step at Z.

(vi) Join XW, WZ, YZ, XZ, and WY.



Thus, WXYZ is the required quadrilateral.

Construction of Parallelograms

Construction of Parallelograms

We have already studied that "A quadrilateral in which opposite sides are parallel is known as a parallelogram". Also, diagonals of a parallelogram bisect each other.

Owing to these properties, it is possible to construct a parallelogram even if we are provided with only three parameters of the quadrilateral.

Let us learn, how to construct a parallelogram with the help of examples.
There are five different type of construction.

Type : 1 To construct a parallelogram, whose two consecutive sides and the included angle are given.

Let a parallelogram REAL has RE = 4 cm, EA = 9.5 cm, and m ∠REA = 45°.

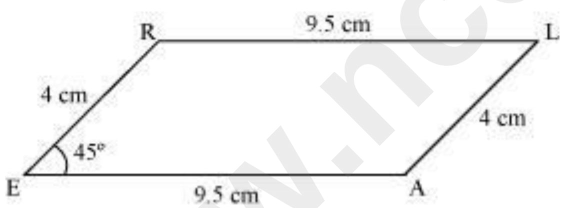
Construction:

We know that opposite sides of a parallelogram are of equal lengths.

Thus, in parallelogram REAL,

RE = AL = 4 cm and EA = LR = 9.5 cm

Firstly, a rough sketch of this quadrilateral is as follows.

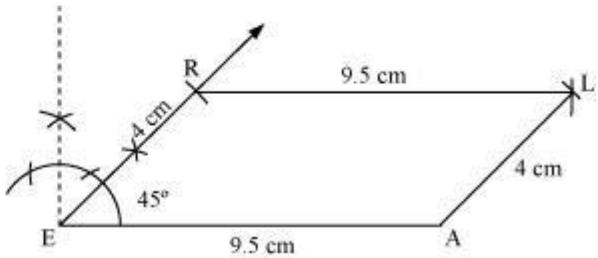


The **steps** of construction for this figure are as follows:

Step 1 Draw a line segment EA of 9.5 cm and an angle of 45º at point E. As vertex R is 4 cm away from vertex E, cut a line segment EA of 4.5 cm on this ray.

Step 2 Vertex L is 9.5 cm and 4 cm away from vertices R and A respectively. By taking radius as 9.5 cm and 4 cm, draw arcs from point R and A respectively. These will be intersecting each other at point L.

Step 3 Join L to R and A.



REAL is the required parallelogram.

Type : 2 To construct a parallelogram, whose one side and both the diagonals are given.

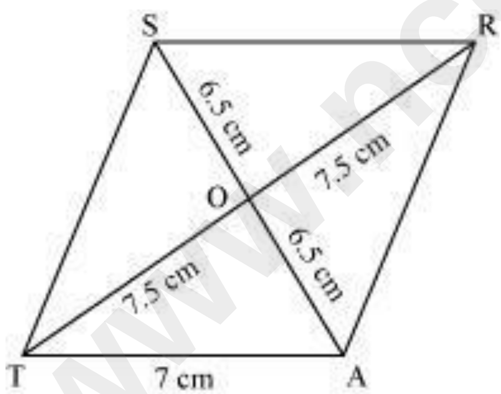
Let a parallelogram STAR has TA = 7 cm, AS = 13 cm, and TR = 15 cm.

Construction:

We know that diagonals of a parallelogram bisect each other. Let the diagonals (AS and TR) of the parallelogram intersect each other at point O.

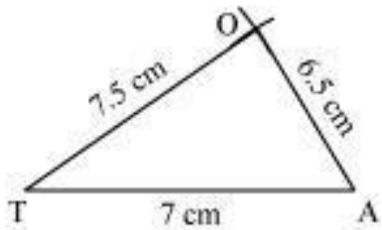
Hence, OT = OR = $\frac{1}{2}$ TR = 7.5 cm and OA = OS = $\frac{1}{2}$ AS = 6.5 cm

A rough sketch of this rhombus can be drawn as follows:



The steps of construction are as follows:

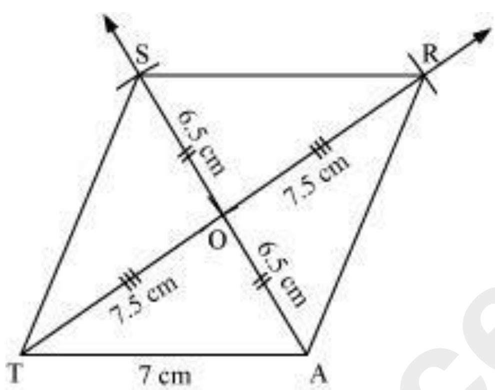
Step 1 ΔOTA can be constructed by using the given measurements as:



Step 2. Vertex TR is on the line joining TO and it is 7.5 cm from point O. Therefore, extend line segment TO and draw an arc of radius OT = 7.5 cm, taking centre as O that cuts ray TO at point R.

Step 3 Extend line segment AO and draw an arc of radius OA = 6.5 cm, taking centre as O that cuts ray AO at point S.

Step 4 Join AR, RS, and ST.



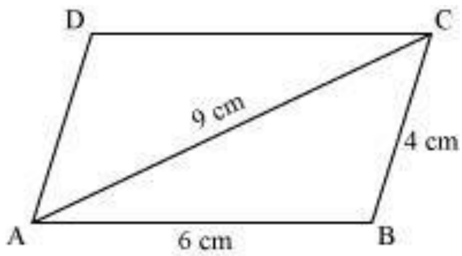
STAR is the required parallelogram.

Type : 3 To construct a parallelogram, whose two consecutive sides and one diagonal is given.

Let a parallelogram has adjacent sides of measures 6 cm and 4 cm and one diagonal of measure 9 cm.

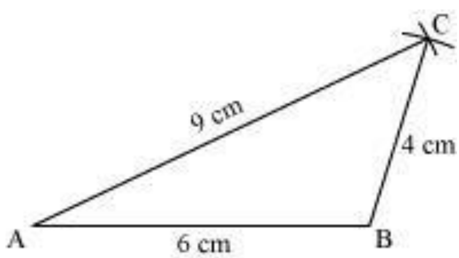
Construction:

A rough sketch of the parallelogram can be drawn as:

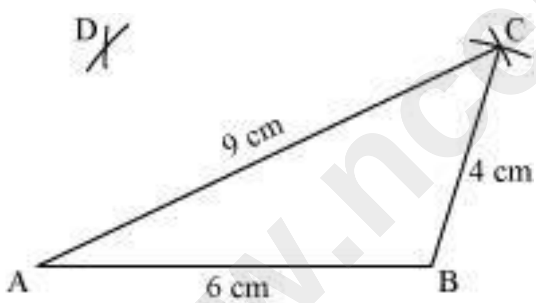


Steps of construction:

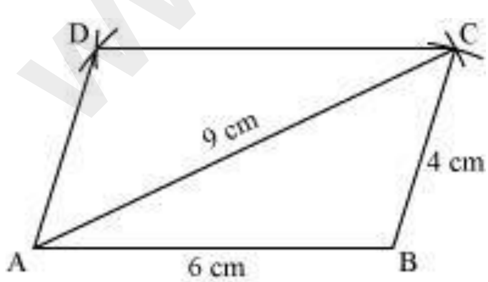
Step 1: Draw line segment AB of length 6 cm. With A and B as centres, and radii 9 cm and 4 cm respectively, draw arcs intersecting each other at C.



Step 2: Taking C and A as centres and radii 6 cm and 4 cm respectively, draw arcs intersecting each other at D.



Step 3: Join AD and CD.



ABCD is the required parallelogram.

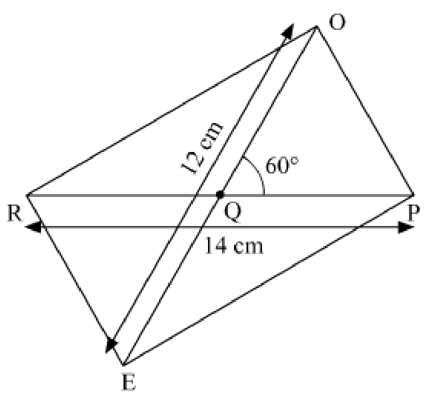
Type : 4 To construct a parallelogram, whose two diagonals and included angle

are given.

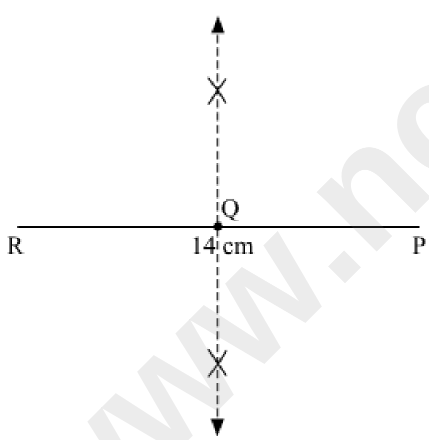
Let parallelogram ROPE has diagonals RP = 12 cm and EO = 14 cm amd the acute angle between the diagonals is 60º.

Construction:

A rough sketch of the parallelogram can be drawn as:



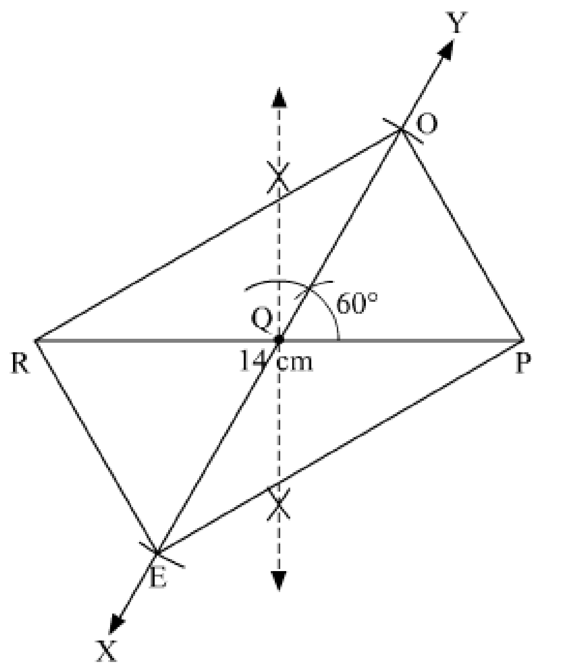
Step 1: Draw RP = 14 cm. Then draw perpendicular bisector of RP to get the midpoint Q.



Step 2: Draw line XY, such that angle YQP = 60º.

Step : 3 From line XY, cut QO = $\frac{EO}{2} = \frac{12}{2} = 6\ cm$ and QE = $\frac{EO}{2} = \frac{12}{2} = 6\ cm$.

Step : 4 Join RO, OP, PE and ER.

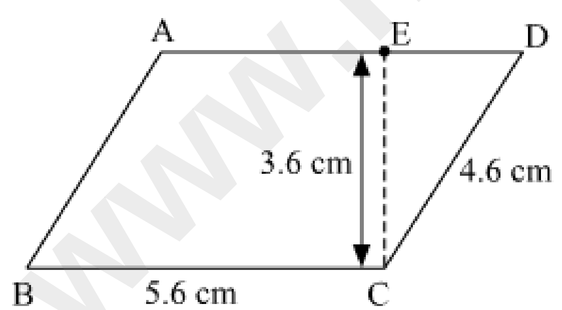


Type : 5 To construct a parallelogram, whose two adjacent sides and height are given.

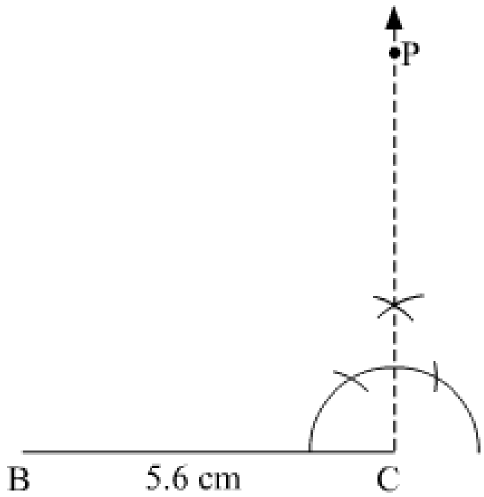
Let parallelogram ROPE has diagonals RP = 12 cm and EO = 14 cm and the acute angle between the diagonals is 60º.

Construction:

A rough sketch of the parallelogram can be drawn as:



Step : 1 Draw BC = 5.6 cm. Then draw perpendicular at C. CP is perpendicular to BC.

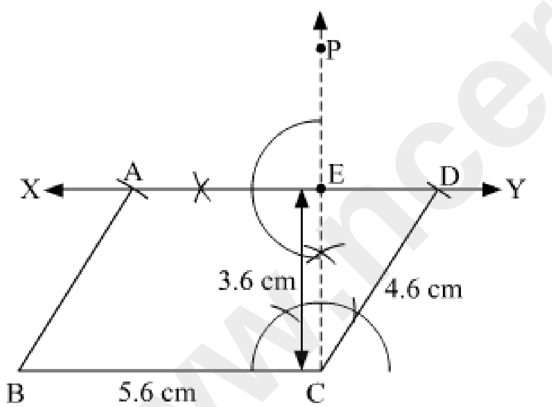


Step : 2 From CP, cut 3.6 cm, i.e height of the parallelogram.

Step : 3 Through E, draw perpendicular to CP to get XY parallel to BC.

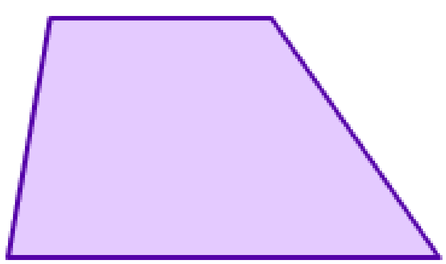
Step : 4 With B as a centre and radius 4.6 cm, draw an arc which cuts XY at A.

Step : 5 With C as a centre and radius 4.6 cm, draw one more arc which cuts XY at D.



Construction of Trapeziums

Look at the given figure.



The given figure represents a quadrilateral in which one pair of the opposite sides is parallel. Such a quadrilateral is known as a **trapezium**.

We know that to construct a quadrilateral, we need five elements, i.e. the measures of its sides or angles.

But have you ever wondered how is a trapezium made?

To construct a trapezium, we only need four elements, i.e. the measures of its sides or angles.

For this, any one of the following information is required:

• **Measures of all the sides**

• **Measures of the three sides and one included angle**

• **Measures of the parallel sides and the angles on the longer sides**

Let us discuss each case one by one.

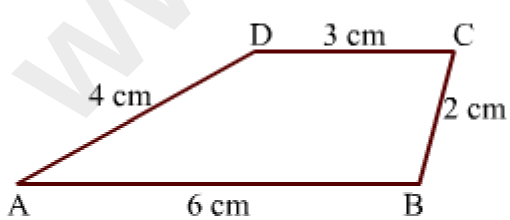
Construction of a trapezium when measures of all the sides are given:

In this case, we divide the trapezium into a parallelogram and a triangle, and then draw each of them separately.

Let us understand this method of construction with the help of an example.

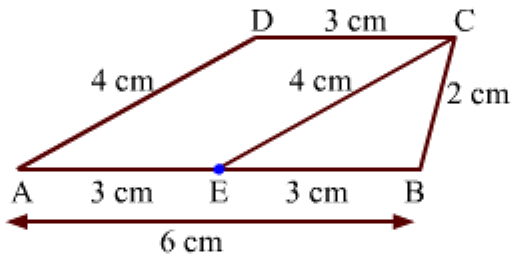
Let us construct a trapezium ABCD, in which AB = 6 cm, BC = 2 cm, CD = 3 cm, DA = 4 cm.

For this, let us first draw a rough sketch of the trapezium ABCD and indicate the given lengths as shown in the following figure.



A suitable point, E, is marked on AB (EB = AB – CD), so as to draw a line segment, EC, parallel to AD.

Here, AB is parallel to DC, so now AECD becomes a parallelogram.



We know that opposite sides of a parallelogram are equal.

∴ EC = AD = 4 cm

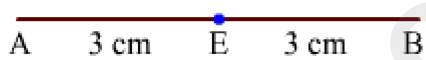
Also, EB = AB – CD = 6 cm – 3 cm = 3 cm

∴ AE = 3 cm

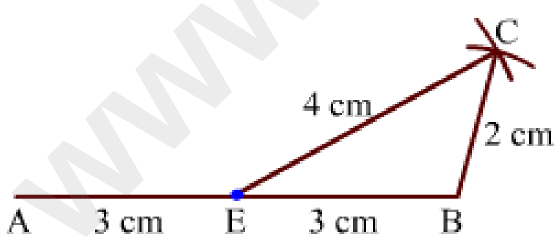
Thus, we now have the measures of all the sides of the parallelogram, AECD, and the triangle, ECB.

Using this information, a trapezium, ABCD, can be constructed by following the given steps of construction:

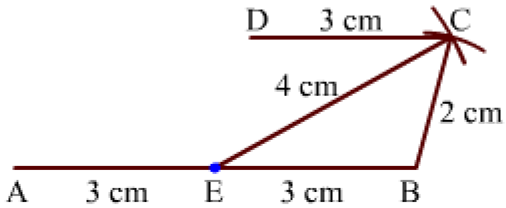
1) Draw a line segment, AB, of length 6 cm as the bottom side of the trapezium. Then, mark a point, E, at a distance of 3 cm from its right end such that EB is of length 3 cm.



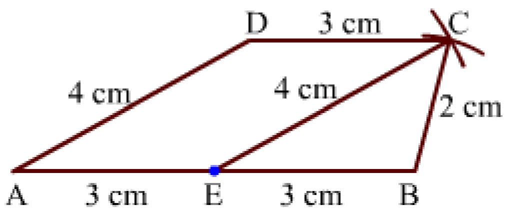
2) Taking E and B as the centres and radii as 4 cm and 2 cm respectively, draw arcs intersecting each other at point C. Join EC and BC.



3) Draw a line segment, DC, of length 3 cm, parallel to the bottom line, AB, through the top vertex of ΔECB.



4) Join AD.



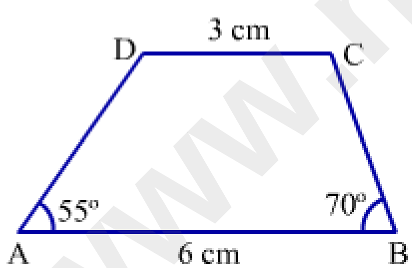
ABCD is the required trapezium.

Construction of a trapezium when the measures of parallel sides and the angles on the longer side are given:

To understand the method of construction, consider the following example.

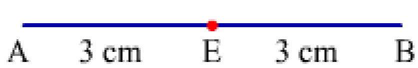
Construct a trapezium, in which AB = 6 cm, ∠A = 55°, ∠B = 70°, CD = 3 cm.

For this, let us firstly draw a rough sketch of the trapezium, ABCD, and indicate the given lengths and angles as shown in the following figure.

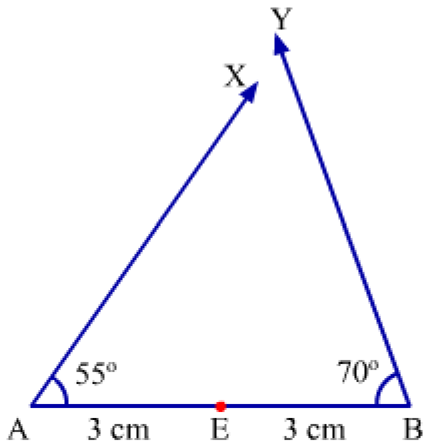


Follow the given steps of construction to obtain the required trapezium.

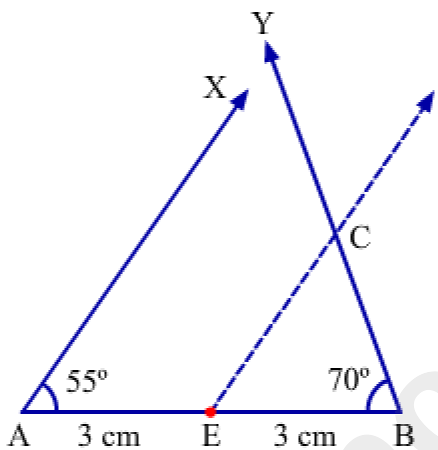
1) Draw a line segment, AB, of length 6 cm as the bottom side of the trapezium. Then, mark a point, E, at a distance of 3 cm from its right end such that EB is of length 3 cm.



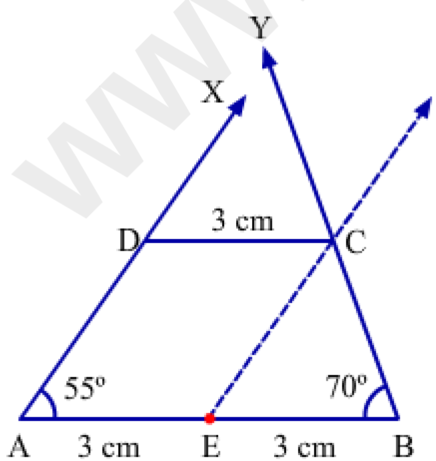
2) Draw ∠BAX = 55° and ∠ABY = 70° at points A and B respectively.



3) Draw a ray parallel to ray AX from point E such that it intersects ray BY at point C.



4) Draw a line segment from point C, parallel to the bottom line, AB, which intersects the ray AX at point D.



ABCD is the required trapezium

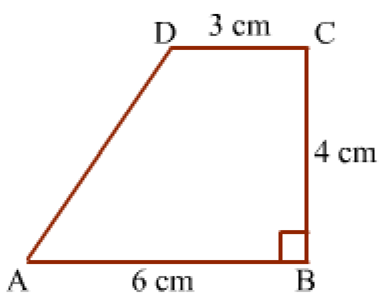
Construction of a trapezium when measures of three sides and one included angle are given:

Suppose we have to construct a trapezium, ABCD, and the following elements of the trapezium are given.

AB = 6 cm, CD = 3 cm, BC = 4 cm and ∠ABC = 90°

How will we construct the trapezium with this information?

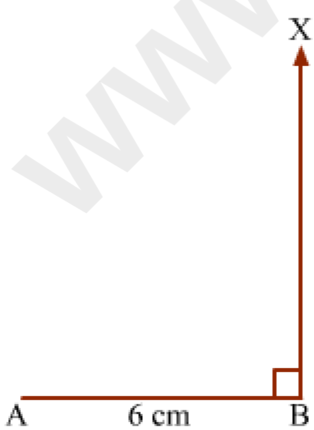
For this, let us firstly draw a rough sketch of the trapezium, ABCD, and indicate the given lengths and angle as shown in the following figure.



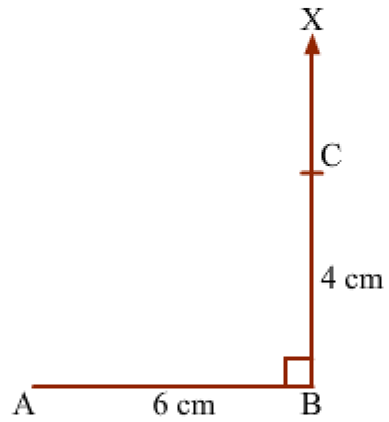
To understand the method of construction, go through the following steps of construction:

1) Draw a line segment, AB, of length 6 cm as the bottom side of the trapezium.

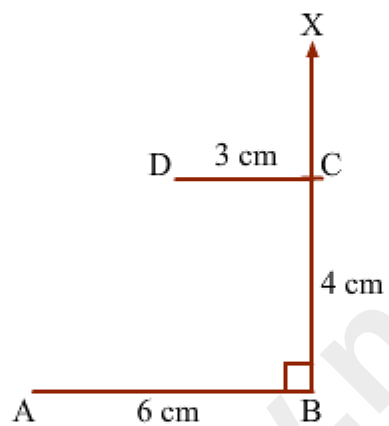
2) Draw ∠ABX = 90° at point B.



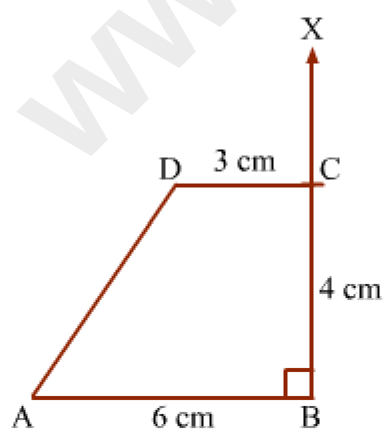
3) Taking B as the centre and radius 4 cm, draw an arc on ray BX and name the point of intersection as C.



4) From point C, draw a line segment CD parallel to the bottom line segment, AB, of length 3 cm.



5) Join AD.



ABCD is the required trapezium.

Construction of Special Quadrilaterals

Suppose the diagonals of the rhombus PQRS are PR = 5.8 cm and SQ = 6 cm.

Can we construct the rhombus PQRS?

Now, let us consider the case of a square.

Suppose a side of the square ABCD is 6.2 cm. **Can we construct the square?**

Let us now consider the case of a rectangle.

Suppose we have to construct a **rectangle** STUV with ST = 5 cm and TU = 4 cm. How will we construct it? The given video will guide you through the various steps of construction.

We can construct square and rectangle if the measurements of their sides are not given. Let us learn the same.

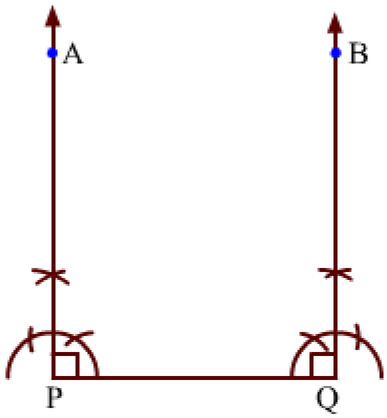
Constructing a square when the length of the side is not known:

We know the properties of square that all of its sides are of equal length and each angle measures 90°. So, if the length of the side of square is not given then we can construct a square having side of any length.

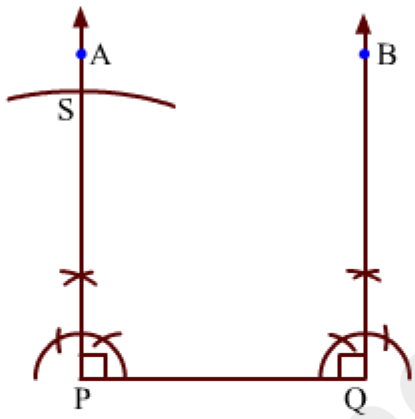
The steps of construction are as follows:

(1) Draw a line segment PQ of any length.

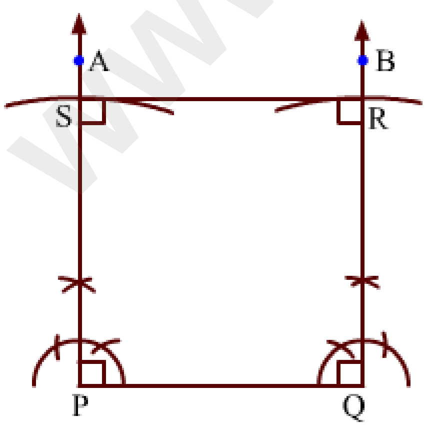
(2) Draw a ray PA perpendicular to PQ at point P. Also, draw another ray QB perpendicular to PQ at point Q.



(3) Place the point of compass at point P and take the radius equal to the length of PQ. Taking this radius and placing the point of compass at point P, draw an arc intersecting the ray PA at S.



(4) Without changing the radius and placing the point of compass at point Q, draw another arc intersecting the ray QB at R. Join R to S.



Thus, we obtain a square PQRS in which the length of side is not known.

Similarly, we can construct a rectangle of unknown length and breadth.

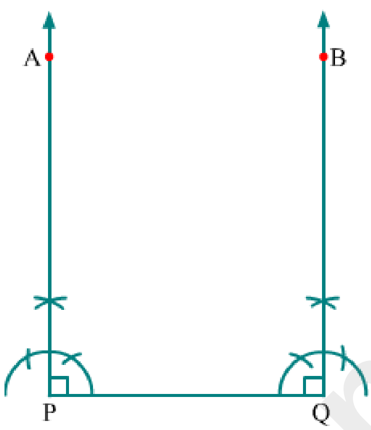
Constructing a rectangle when length and breadth are not known:

In a rectangle, opposite sides are of equal length and each angle measures 90°. These properties can be used to construct a rectangle whose length and breadth are not given.

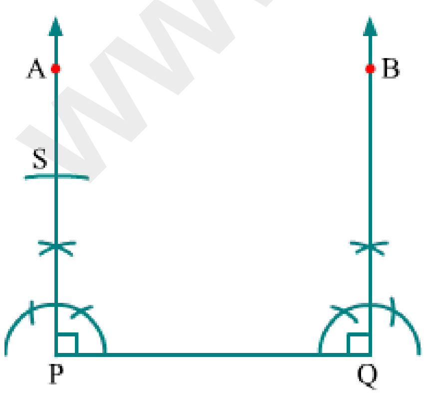
The steps of construction are as follows:

(1) Draw a line segment PQ of any length.

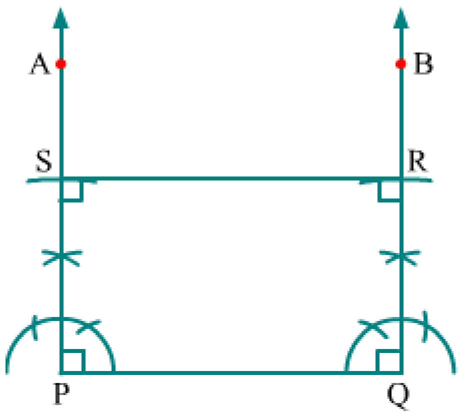
(2) Draw a ray PA perpendicular to PQ at point P. Also, draw another ray QB perpendicular to PQ at point Q.



(3) Taking any radius which is not equal to the length of PQ and placing the point of compass at point P, draw an arc intersecting the ray PA at S.



(4) Without changing the radius and placing the point of compass at point Q, draw another arc intersecting the ray QB at R. Join R to S.



Thus, we obtain a rectangle PQRS in which the length and breadth are not known.

Construction of a Regular Hexagon

We know that a regular hexagon is a regular polygon of six sides.

Suppose we have to construct a regular hexagon ABCDEF of side 7 cm.

How will we proceed?

Let us see.

One can construct a regular hexagon by making use of any of the following rules relating to a regular hexagon.

(1) Each interior angle of a regular hexagon = 120°

(2) Length of a side of a regular hexagon = Radius of its circumcircle

First of all, let us construct the regular hexagon ABCDEF of side 7 cm using the first rule, i.e., each interior angle of a regular hexagon is equal to 120°. The following video will show you how to construct it.

Now, let us construct the same regular hexagon ABCDEF of side 7 cm using the second rule, i.e., the length of a side of a regular hexagon is equal to the radius of its circumcircle. The following video will show you how to do so.

Let us see one more example in order to gain a better understanding of the above two methods of construction.

Example:

Draw a regular hexagon ABCDE of side 4.5 cm.

Solution:

One can construct a regular hexagon by making use of any of the following rules relating to a regular hexagon.

(1) Each interior angle of a regular hexagon = 120°

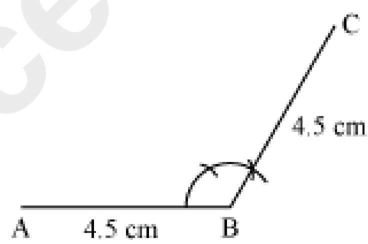
(2) Length of a side of a regular hexagon = Radius of its circumcircle

First of all, let us construct the regular hexagon ABCDEF of side 4.5 cm using the first rule, i.e., each interior angle of a regular hexagon is equal to 120°.

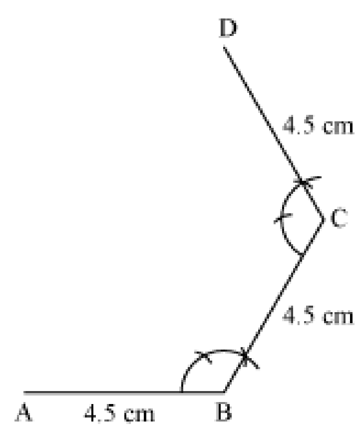
The steps of construction are as follows:

(1) Draw AB = 4.5 cm.

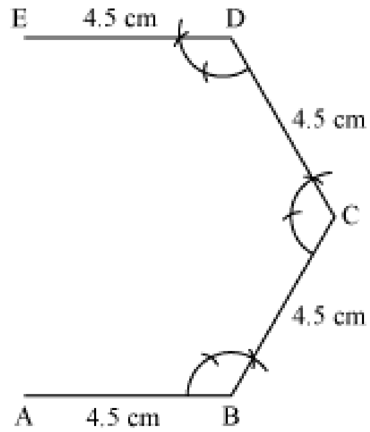
(2) At B, draw ∠ABC = 120°, where BC = 4.5 cm.



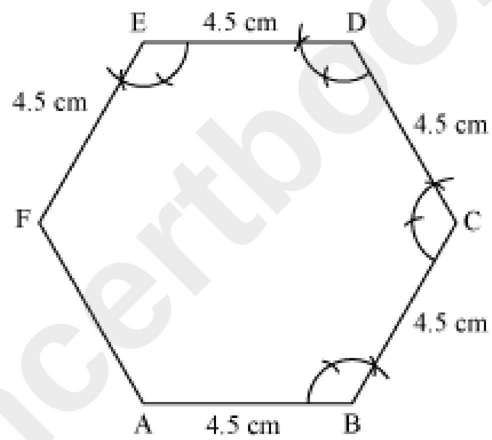
(3) At C, draw ∠BCD = 120°, where CD = 4.5 cm.



(4) At D, draw ∠CDE = 120°, where DE = 4.5 cm.



(5) At E, draw ∠DEF = 120°, where EF = 4.5 cm. Join AF.

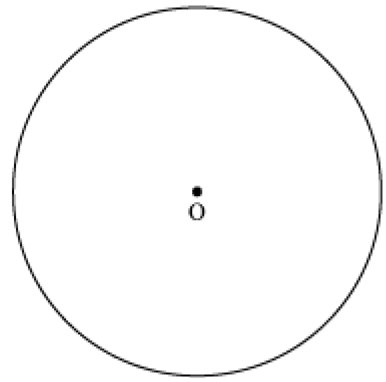


ABCDEF is the required regular hexagon of side 4.5 cm.

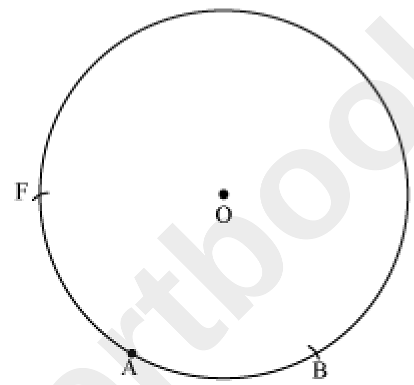
Now, let us construct this regular hexagon using the second rule, i.e., the length of a side of a regular hexagon is equal to the radius of its circumcircle.

The steps of construction are as follows:

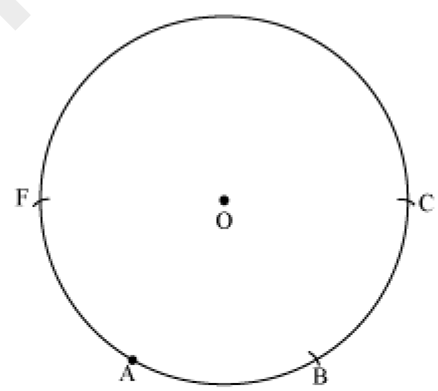
(1) Draw a circle of radius 4.5 cm.



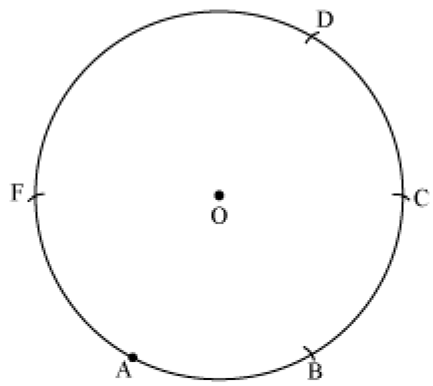
(2) Take any point A on its circumference. With A as the centre and radius as 4.5 cm, draw two arcs to cut the circle at points B and F.



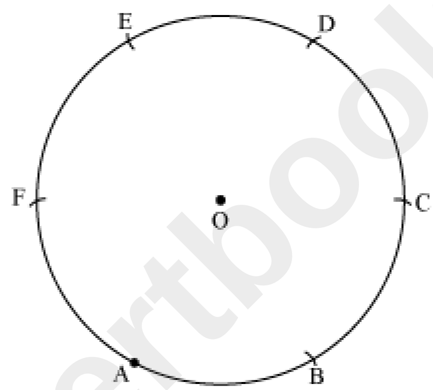
(3) With B as the centre and radius as 4.5 cm, draw an arc that cuts the circle at point C.



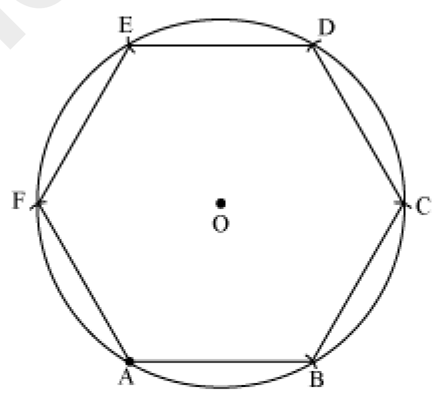
(4) With C as the centre and radius as 4.5 cm, draw an arc that cuts the circle at point D.



(5) With D as the centre and radius as 4.5 cm, draw an arc that cuts the circle at point E.



(6) Join AB, BC, CD, DE, EF and AF.



ABCDEF is the required regular hexagon of side 4.5 cm.

Construction Of Circumcircle And Incircle Of A Given Regular Hexagon

We know that circumcircle of a given polygon touches all of its vertices and its incircle touches all of its sides.

Suppose ABCDEF is a regular hexagon of side 5 cm and we have to construct its circumcircle as well as incircle.

How will we construct them?

Let us see.

The following video will guide you to construct the incircle of a regular hexagon ABCDEF of side 5 cm.

Let us look at one example in order to understand the above concepts better.

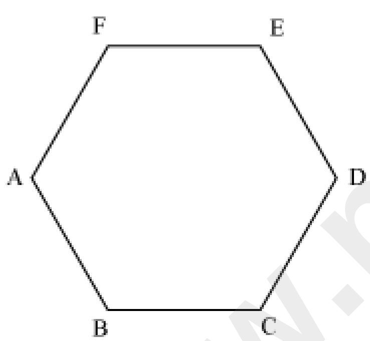
Example 1:

Draw circumcircle and incircle of a regular hexagon of side 8.5 cm.

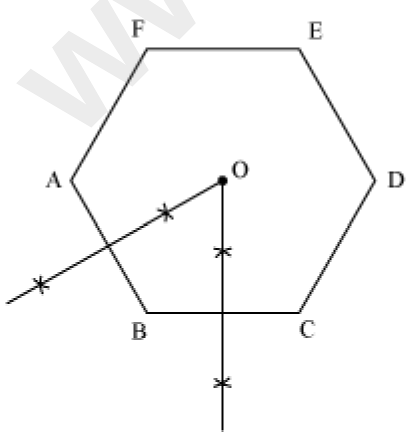
Solution:

The steps of construction to construct the circumcircle are as follows:

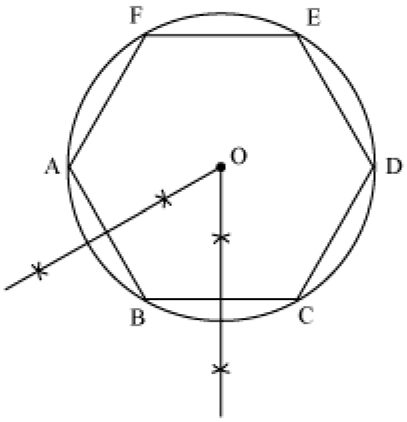
(1) Draw a regular hexagon ABCDEF of side 8.5 cm.



(2) Draw perpendicular bisectors of sides AB and BC. Let them intersect at a point O.



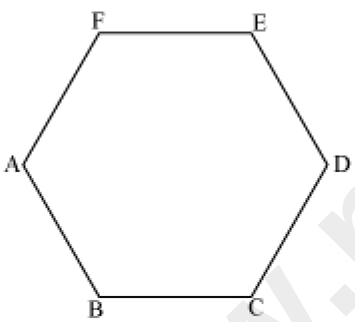
(3) With O as centre and radius equal to OA, draw a circle.



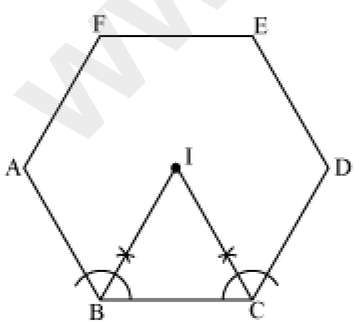
We observe that the circle touches all the vertices A, B, C, D, E, and F of the regular hexagon ABCDEF. Hence, it is the required circumcircle of the regular hexagon.

The steps of construction to construct the incircle of regular hexagon ABCDEF of side 8.5 cm are as follows:

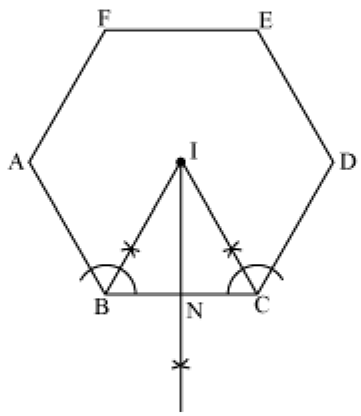
(1) Draw a regular hexagon ABCDEF of side 8.5 cm.



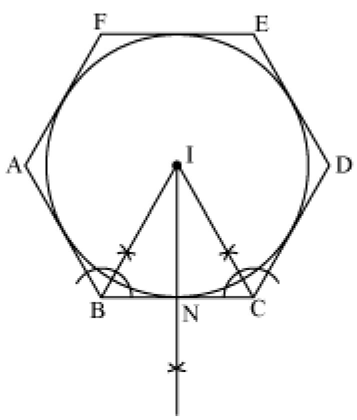
(2) Draw the bisectors of ∠B and ∠C. Let them meet at I.



(3) From I, draw IN perpendicular to BC.



(4) Taking I as centre and radius equal to IN, draw a circle.



We observe that the drawn circle touches each side of regular hexagon ABCDEF. Hence, it is the required incircle.